Dynamic design of footbridges

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Dynamic Design of Footbridges

Analyse dynamique de passerelles lors du passage de piétons

Dynamische Untersuchung von Fussgängerbrücken

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SUMMARY

Generally footbridges for pedestrians are designed so that the pedestrians may be regarded as static load. Some such footbridges are sometimes found to produce uncomfortable vibration by passage of pedestrians. This paper describes the characteristics of dynamic fluctuation of the pedestrian's load, the response of bridges by the experiment carried out on five footbridges in Tokyo and the complemental calculation to analyze dynamic response of bridges. The feastibility of vibration absorbers is also discussed, and its practical installation is reported.

RÉSUMÉ

Les piétons sont en général considérés comme une charge statique lors du calcul de passerelles pour piétons. Mais il arrive que certaines passerelles produisent des vibrations désagréables lors du passage de piétons. Cet article décrit les caractéristiques de la variation dynamique de la charge du piéton, le comportement de ponts lors d'expériences réalisées sur cinq passerelles à Tokyo, et le calcul complémentaire permettant de déterminer le comportement dynamique des ponts. Les possibilités de réalisation d'amortisseurs de vibrations, ainsi que leur installation pratique, sont également discutées.

ZUSAMMENFASSUNG

Beim Entwurf von Fussgängerbrücken werden in der Regel die Fussgänger als statische Belastung angesehen. Bei einigen dieser Brücken treten beim Durchmarsch von Fussgängern unangenehme Schwingungen auf. Die Autoren beschreiben die Kennwerte der dynamischen Fussgängerbelastung, das experimentell bestimmte Verhalten von fünf Fussgängerbrücken in Tokio sowie die dazugehörigen dynamischen Untersuchungen. Schliesslich werden die theoretischen und praktischen Probleme in Zusammenhang mit dem Einbau von Schwingungsdämpfern erörtert.

1. INTRODUCTION

Recent thinking on structural design has centered on whether a structure can be safely used for the prescribed period, taking into consideration service load, strength of materials, precision of construction, serviceability of the structure, etc. Thus, not only the fracture of structures byt also their deformation and vibration within the limits of human tolerance, must be studied.

According to the Japanese Footbridge Design Code, the footbridges shall be designed, so that pedestrians are regarded as static load. Some such bridges are known to vibrate considerably in use, causing discomfort to those who are walking across them. In this study, the fluctuating load imposed on bridges by pedestrians and the dynamic response of bridges according to the loads were studied through a vibration experiment with five simply supported footbridges in Tokyo. As a result of this study, specially designed absorbers were installed to insulate against vibration and the possibilities for their practical use were studied.

2. CHARACTERISTICS OF FLUCTUATING LOAD CAUSED BY PEDESTRIANS

Before the vibration tests of the footbridges was carried out, a preliminary experiment was carried out to study the characteristics of pedestrian load. First, in order to investigate the pace of pedestrians, 505 persons walking naturally along roads were sampled, and their paces were measured. The results are shown in Fig. 1. The broken line in Fig. 1 indicates a normal distribution with the same mean value and standard deviation as the results of the experiment. Accoding to this, their pace was on the average about two steps a second. It can be regarded as a normal distribution and the standard deviation from the mean value is very small.

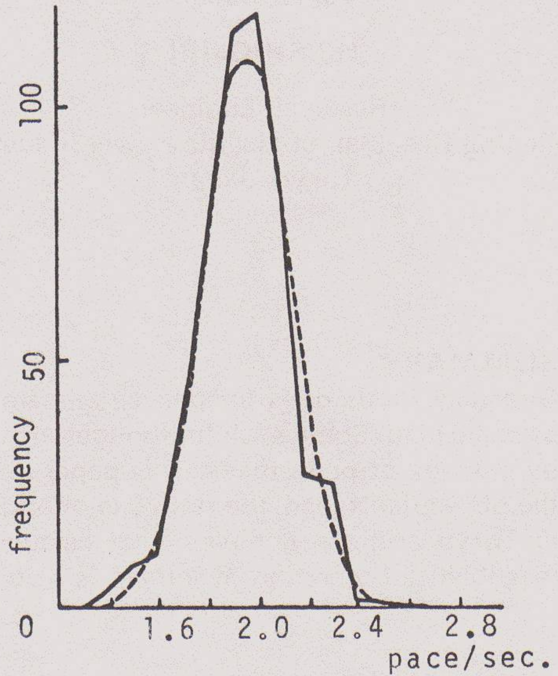


Fig 1. Pace distribution of pedestrians

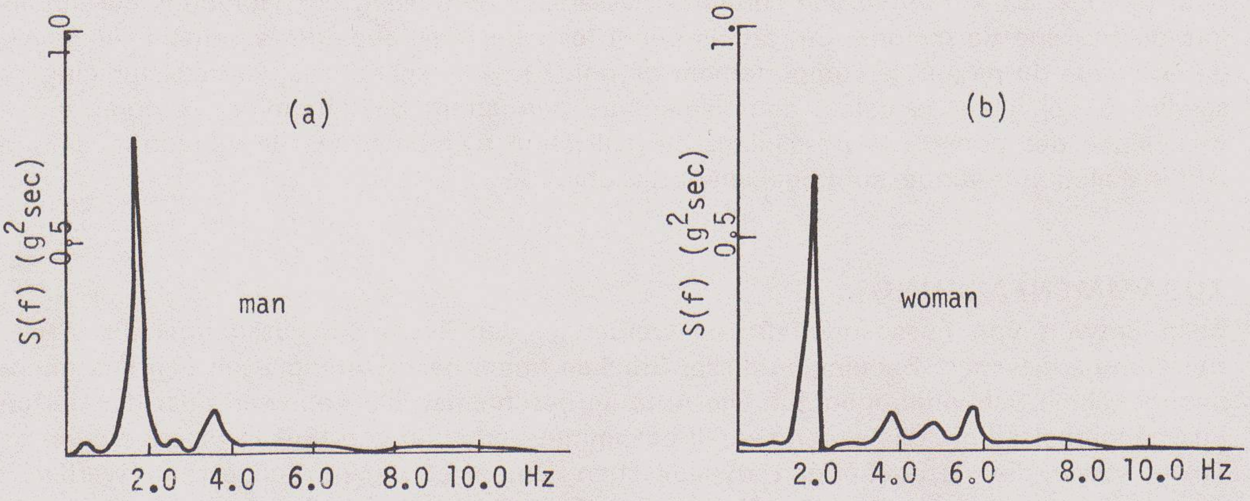


Fig.2 Typical power spectra of pedestrians

The load applied to a footbridge by a pedestrian was mainly considered to be his weight and fluctuating load by his walking. To investigate the nature of their fluctuating load, the acceleration of four male and four female pedestrians between the age of 20 and 40 walking on a rigid concrete floor was measured at the position of their waist. Fig. 2(a) and 2(b) show the typical power spectra of acceleration obtained by treating the observed waveforms with a hybrid computer. The pace frequency stands out conspicuously. It means that the influence of pace will dominate the vibration of a footbridge due to pedestrians. Fig. 3 is the probability density distribution of acceleration amplitude obtained from their acceleration waveforms. The standard deviation, or the mean acceleration amplitude, was 0.342 g in men and 0.210 g in women.

3. OBSERVATION OF FOOTBRIDGES

Five footbridges, which stood at Meidaimae, Hatagaya, Shibuya 1-Chome, Shibuya East Exit, and Shibuya West Exit in Tokyo, were investigated. Some of the characteristics of bridges are shown in Table 1. In these studies, accelerometers were fixed half-way and a quarter of the way across the span. The measurements were taken at night when there was no traffic. When a person crossed these bridges the acceleration was measured in various way of walking. From these, it could be seen that the maximum acceleration appeared at the mid-point of the span. The vibration waveforms at the mid-point for normal walking are indicated in Fig. 4, and the squares of absolute values of the Fourier transform, in Fig. 5. In Fig. 4 fo shows the 1st natural frequency and δ the logarithmic decrement. Fig. 5 and Table 1 make it clear that the 1st natural frequency of each

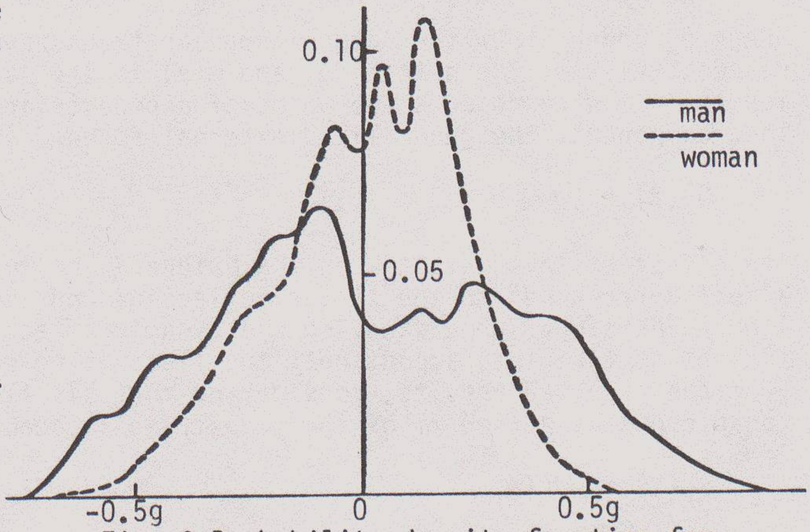


Fig. 3 Probability density function for acceleration of pedestrians

Table 1 Observed Characteristics of Footbridges

Bridges	Materials	Span Length	1st Natural Frequency	Logarithmic Decrement	Maximum Acceleration
Meidaimae	Steel	20.4^{m}	4.00^{Hz}	0.097	5.70×10^{-3}
Hatagaya	pc	21.1	6.02	0.101	3.61×10^{-3}
Shibuya 1-Chome	Steel	30.7	2.83	0.064	1.16×10^{-2}
Shibuya East Exit	Steel	40.3	2.51	0.026	5.70×10^{-3}
Shibuya West Exit	Steel	48.5	2.09	0.049	2.34×10^{-2}

bridge is predominant, and that very strong vibration was caused when a pedestrian's pace was equal to the 1st natural frequency, as in the Shibuya West Exit Bridge. Fig. 6 shows the relation between the spans and the 1st natural frequencies of the observed bridges. It is clear that a bridge whose the 1st natural frequency is approximately equal to the pace of pedestrians have a span near 50 meters.

4. VIBRATION OF FOOTBRIDGE CAUSED BY ONE PEDESTRIAN

To analyze the vibration of a footbridge brought about by one pedestrian, $\eta(x,t)$, the following expression may be used with its vibration modes, $\phi_n(x)$.

$$\eta(x,t) = \sum_{n=1}^{\infty} q_n(t)\cdot\phi_n(x) \qquad (1)$$

If the damping is small, and the effect of damping between modes in disregarded,

$$\ddot{q}_n(t) + 2p_nh_n\dot{q}_n(t) + p_n^2q_n(t) = Q_n(x,t)/\int_0^1\{\Phi_n(x)\}^2m(x)dx \qquad (2)$$

where p_n and h_n show the natural angular frequency and damping constant, respectively, of the n-th mode, and $m(x)$ is the mass of the footbridge per unit length. If W represents the weiht of a pedestrian, and $\Delta W(t)$ his fluctuating load component, the generalized external force $Q_n(t)$ is given as follows:

$$Q_n(t) = \{W + \Delta W(t)\}\Phi_n(vt) \qquad (3)$$

The effect of this weight W on the bridge as he moves at normal walking speed almost corresponds to the static deflection and the dynamic increment is almost 1.00. Therefore, when studying his dynamic effect upon the bridge, it is sufficient to take into accout only the fluctuating component $\Delta W(t)$ of the load. From the observed results, considering that his fluctuating component was conspicuous at period ω_0 of the pedestrian's pace, and assuming that

$$\Delta W(t) = \Delta W\sin\omega_0 t \qquad (4)$$

the vibration due to $\Delta W(t)$ was simulated by an analogue computer. Fig. 7 shows the example of the Hatagaya Bridge. The ordinate shows the ratio of the dynamic deflection to the static deflection of the bridge when ΔW is loaded at the center of the span. Also calculations were made by substituting the observed values for the Shibuya West Exit Bridge into Eq. 2 and using the valued for the same person as was used for the above experiment. It followed from this that the maximum acceleration of the bridge was 2.14×10^{-2} g, very similar to the observed value 2.34×10^{-2} g. Fig. 8 shows the vibration increment for the ratio of the 1st natural frequency of the bridge to pace frequency. The ordinate represents dynamic deflection divided by static

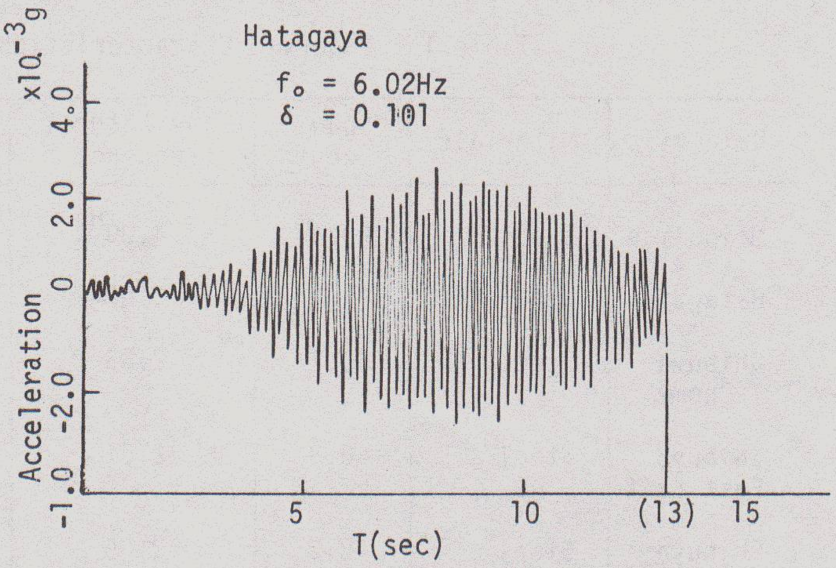


Fig. 4 Actual response at 1/2 of the span

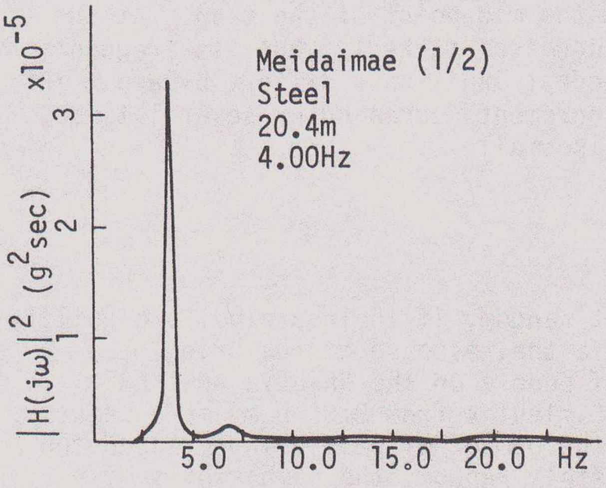


Fig. 5(a) Response of acceleration

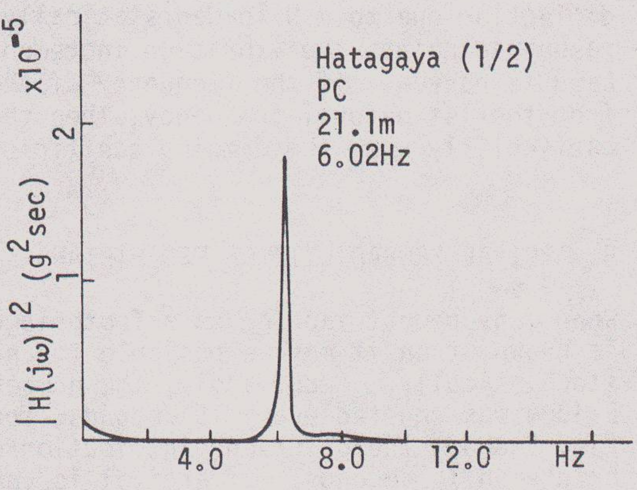


Fig. 5(b)

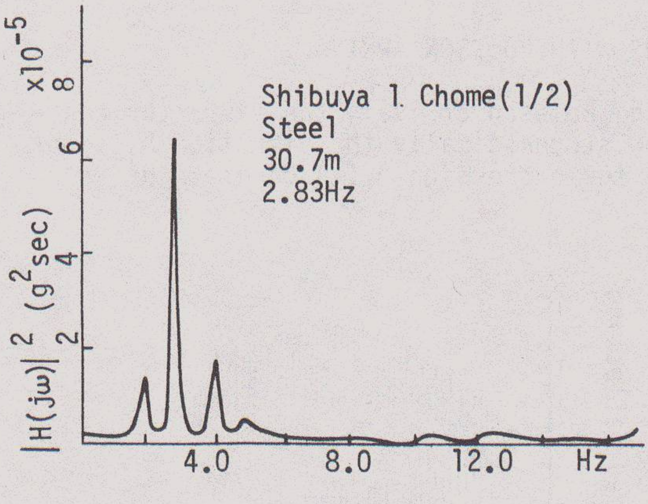


Fig. 5(c)

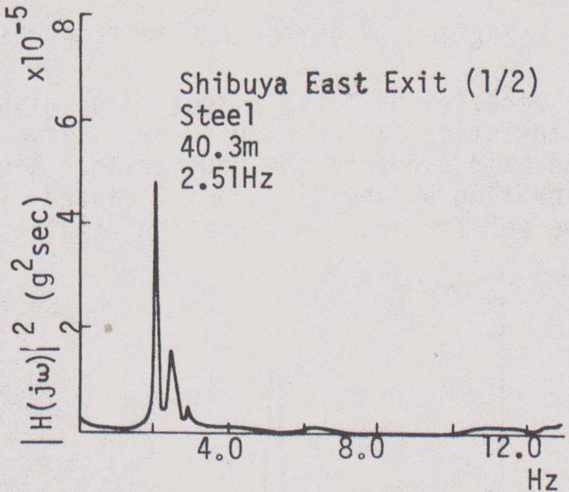


Fig. 5(d)

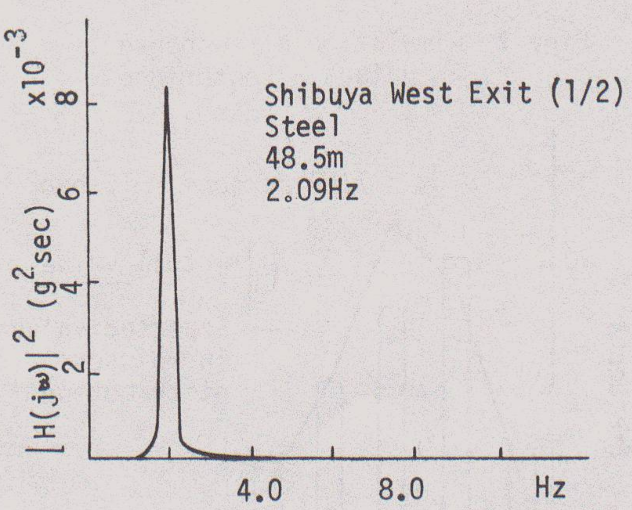


Fig. 5(e)

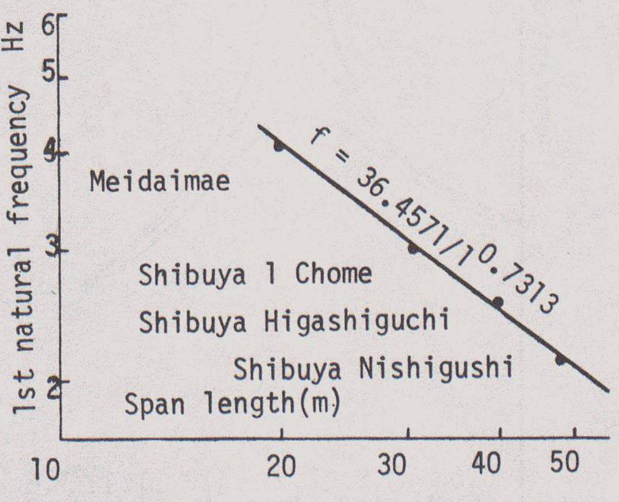


Fig. 6 Span length and 1st natural frequencies of footbridges

deflection due to ΔW loaded statically at the mid-point of the span. At the resonance point, the vibration increment increases markedly, but its frequency band is narrow. If the frequency of the pedestrian's pace differs by avout 10% from the 1st natural frequency, then the increment decreased by several times, particularly when the damping coefficient is small.

5. ARRIVAL PROBABILITY OF PEDESTRIANS

When many pedestrians cross a footbridge at random, if their arrival probability is known, then it may be possible to analyze the response of the bridge stochastically. Accordingly, the number of people on the Shibuya West Exit Bridge was counted every 15 seconds, for 15 minutes from 8:00 p.m. on a weekday. Fig. 9 shows the observed distribution of the number of people who crossed the bridge in 15 seconds. If arrival is completely random, the number of people crossing in a given time will follow the Poisson distribution ("Poisson arrival"). The conformity with the Poisson distribution for the same mean value was warranted by means of the X^2 test at the 5% significant level.

6. VIBRATION OF BRIDGE DUE TO PEDESTRIANS WITH POISSON ARRIVAL

If pedestrians cross a footbridge with the Poisson arrival, the vibration characteristics can be studied by superposing stochastically the vibration $h_i(t)$ of the bridge due to one pedestrian. Since the expression $h_i(t)$ denotes the vibration at any fixed point caused by one pedestrian, the vibration $\eta(t)$

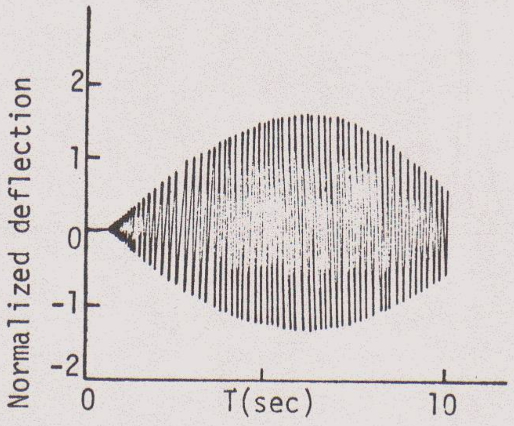


Fig. 7 Simulation of response at Hatagaya Footbridge

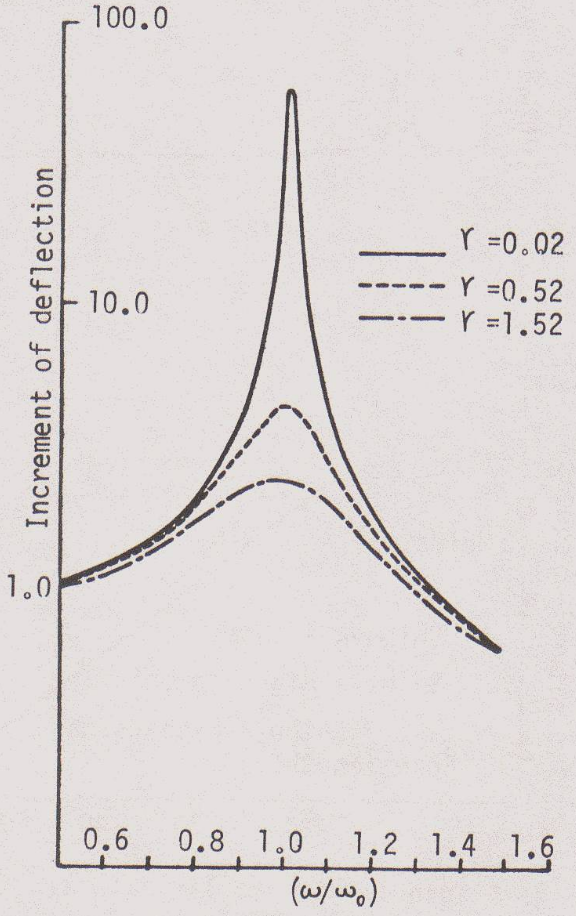


Fig. 8 Vibration increment at 1/2

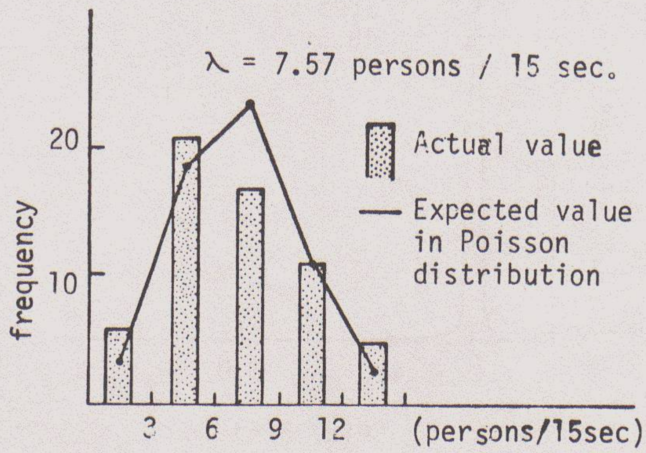


Fig.9 Arrival probability of pedestrians

caused by many pedestrians is as follows:

$$\eta(t) = \sum_{i=1}^{\infty} h_i(t - t_i) \tag{5}$$

where t_i is the time when each pedestrian reaches the bridge, and has random values in the case of the Poisson arrival. If each $h_i(t)$ is the same, independent of the pedestrian, Eq. 5 shows "shot noise", but it actually differs from one to another. Thus, if the vibration $h_i(t)$ probability of those producing each pattern follows the Poisson distribution, then the mean value of $\eta(t)$, m_η, the variance, σ_η^2, and the power spectrum $S_\eta(\omega)$ are shown as follows:

$$\begin{aligned} m_\eta &= \lambda T_0 E[\textstyle\int_0^\infty h_i(t)dt\,] = \lambda T_0 E[\, H_i(0)\,] \\ \sigma_\eta^2 &= \lambda T_0 E[\textstyle\int_0^\infty h_1^2(t)dt\,] = \lambda T_0 E[\,\overline{S_i(\omega)}\,] \\ S\eta(\omega) &= 2\pi\lambda^2 T_0^2 E[\, H_1^2(0)\,] + \lambda T_0 E[\, |H(j_\omega)|^2] \end{aligned} \tag{6}$$

where $H_i(j\omega)$ = the Fourier transform of $h_i(t)$, $\overline{S_i(\omega)}$ = the mean power of $h_i(t)$, λ= the overall mean arrival rate, and T_0 = the time required for crossing the footbridge.

If only the fluctuating part of $h_i(t)$ is considered, $H_i(0) = 0$; therefore, when the arrival of pedestrians is in conformity with the Poisson distribution, the variance and power spectrum are equal to λT_0 times the expected values of those in the vibration $h_i(t)$ of the bridge produced by one pedestrian. In the case of the Poisson arrival, the mean amplitude of the bridge is obviously $\sqrt{\lambda T_0}$ times, because the mean amplitude is the standard deviation σ_η.

When the pedestrian's paces were assumed to have the normal distribution, indicated by the broken line in Fig. 1, and the vibration $h_i(t)$ of the bridge was given as Eq. 1, the expected values for mean amplitude and power spectrum at the mid-point of the span for one pedestrian were calculated. In the calculation, the body weights of all pedestrians were assumed the same, all frequency components except pace frequency were disregarded, and the decrement σ was taken as 0.02 of the minimum value in the observation. Table 2 shows the normalized expected values of the deflection and acceleration amplitude for one pedestrians, and the expected values estimated by assuming a pedestrian's mean body

Table 2 Expected Values of Deflection and Acceleration Amplitude by One Passenger (Poisson Arrival).

Bridges	1st natural frequency	Expected values (normalized)		Mean weight 60 kg ΔW/W = 0.342	
		Def. Amp.*	Acc. Amp.**	Def. Amp.+	Acc. Amp.++
Meidaimae	4.00Hz	1.00	0.253	0.0078 cm	0.0012 g
Hatagaya	6.02	1.00	0.112	0.0013	0.0002
Shibuya 1 Chome	2.83	1.34	0.653	0.0134	0.0023
Shibuya East Exit	2.51	3.81	2.809	0.0677	0.0124
Shibuya West Exit	2.09	5.21	4.920	0.120	0.0196

* $\dfrac{\pi EI}{2\Delta Wl^3}\sqrt{E[S(\omega)]}$ ** $\dfrac{\pi EI}{2\Delta Wl^3\omega_0^2}\sqrt{E[\omega^4 S(\omega)]}$ + $\sqrt{E[S(\omega)]}$ ++ $\sqrt{E[\omega^4 S(\omega)]}$

weight 60 kg and a $\Delta W = 0.342W$. Fig. 10 shows the expected values of its power spectrum of deflection amplitude. It is normalized using the amplitude when ΔW is loaded statically in the mid-point of the span, and the ordinate is a logarithmic scale. Therefore, the normalized power spectrum of the vibration, when pedestrians have the Poisson arrival, is determined by multiplying the values shown in Fig. 10 by λT_0. The actual external force which a pedestrian exerts on a footbridge is not so simple as given by Eq. 4, and it involves some frequency components other than the pace frequency, as shown in Fig. 2(a) and 2(b). However, the observed results indicates the facts that the external force affecting the vibration of the bridge is mainly the pedestrian's pace, and that the primary natural frequency is more conspicuous than any other frequency.

From Table 2 and Fig. 10, it is clear that as the mean value of the frequency of a pedestrian's pace nears the 1st natural frequency, the response of the bridge increases markedly. In the Meidaimae and Hatagaya Bridges, which have a natural frequency of over two times the pace frequency, the mean amplitude ratio is 1.00 and the dynamic influence can be disregarded in a stochastic point of view, while in bridges such as the Shibuya East Exit and West Exit Bridges, in which the difference between the 1st natural frequency and the frequency of a walking pace is less than 25%, the influence of vibration cannot be regarded. For example, for the Shibuya West Exit Bridge, if the mean arrival rate of pedestrians is taken the observed value of 7.57 persons per 15 seconds and a pedestrian's mean velocity is assumed to be 1.05 meters per second, then the mean deflection and acceleration amplitudes due to the fluctuating load are estimated to be about 0.581 cm and about 0.0947 g respectively. According to the reports on human sensitivity to acceleration, 0.1 g is the threshold of a man's insecurity and discomfort.(7) It means that the Shibuya West Exit Bridge may give discomfort.

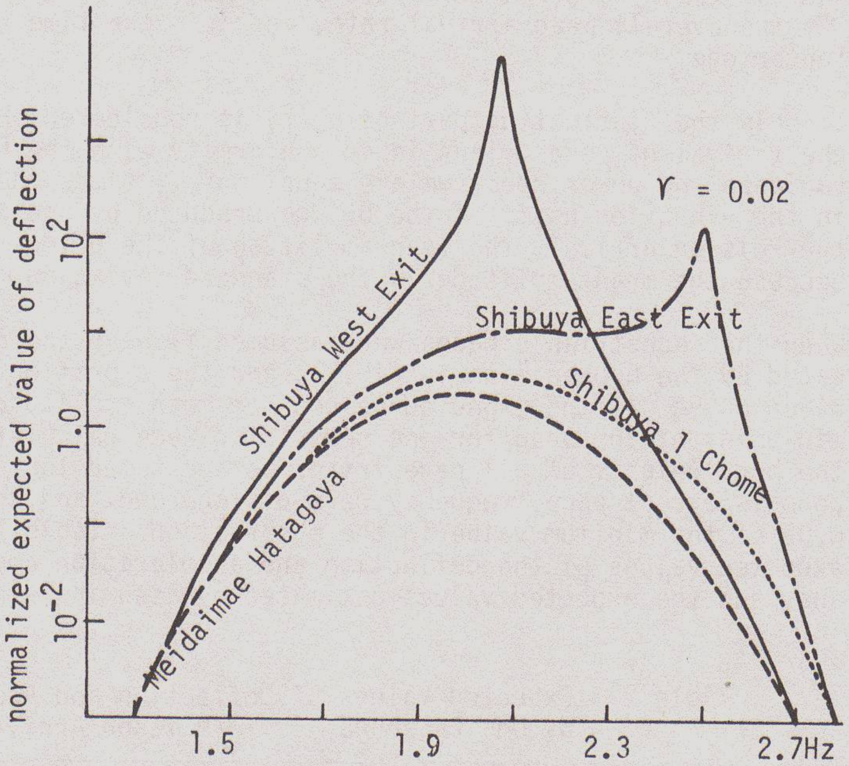


Fig.10 Expected values of power spectrum by one pedestrian

7. EXTREME VALUE AND ITS RETURN PERIOD

In order to find out whether footbridges will at any time become unsuitable for use, it is important to know the extreme value of amplitude and is return period as well as the standard deviation. In this section, the relation between the extreme value and its return period is examined under the assumption that η, the amplitude of deflection produced on a footbridge, could exhibit the normal distribution with a mean of 0. The return period, $T(\eta_0)$, in which the amplitude of deflection of a footbridge recurs at a certain level η_0 is given by

$$T(\eta_0) = \frac{2\pi}{\sqrt{\sigma_4^2 \ \sigma_2^2}\ P(\eta \geqslant \eta_0 \mid T)} \tag{7}$$

where σ_2^2 and σ_4^2 represent respectively the expected values of the second and the fourth moments of power spectrum $S(\omega)$, and $P(\eta > \eta_0 \mid T)$ represents the probability of the amplitude η exceeding η_0 within a time period T. $\sqrt{\sigma_4^2 / \sigma_2^2}$ is the frequency of peaks within a unit time.

The expected values of the power spectra when pedestrians arrives at each footbridge in the Poisson distribution with the pace distribution of Fig. 1, are shown in Fig. 10. The return period for the extreme value can be estimated form Fig. 10, if the maximum arrival rate of pedestrians at a footbridge is assumed to be 1 person/sec. per 1 meter of bridge width for given extreme value of deflection and acceleration amplitudes of bridges. In this study, the extreme value of acceleration amplitude was taken to be 0.1 g which will cause human beings a feeling of discomfort and insecurity.(7) As for deflection amplitude, the extreme value for fluctuating amplitude of deflection was defined as the value obtained by subtracting the mean of static deflection generated by W from the design limit of deflection in the Japanese Footbridge Design Code.

The results of calculation are shown in Table 3. With regard to acceleration amplitude, a value over 0.1 g is not expected to occur during the lives of the Meidaimae or Hatagaya Footbridge. whereas it will occur about once a year on the Shibuya 1-Chome Footbridge. As for the Shibuya East Exit and West Exit Footbridges, even the mean acceleration amplitude given by standard deviation exceeds 0.1 g. With regard to deflection amplitude, no problem exists except on the Shibuya West Exit Footbridge. For this footbridge, the return period for an extreme value over the design limit of deflection is about once per 10 days. After this experiment, some reinforcement was carried out on the Shibuya West Exit and East Exit Footbridges, and now their dynamic properties are considerably improved.

Table 3 Return Period of Extreme Value (λ = 1 person/sec/m)

Bridges	Standard Deviation		Return Period	
	Mean Def. Amp.	Mean Acc. Amp.	Def. Amp. (Extreme Value)	Acc. Amp. Extreme Value=0.1g
Meidaimae	0.052 cm	0.0081 g	Over 100 years (4.4 cm)	Over 100 years
Hatagaya	0.007	0.0012	Over 100 years (5.19 cm)	Over 100 years
Shibuya 1 Chome	0.080	0.0151	Over 100 years (6.86 cm)	Over 1 years
Shibuya East Exit	0.663	0.1216	Over 100 years (6.92 cm)	Always
Shibuya West Exit	1.417	0.2308	Over 10 days (6.93 cm)	Always

8. VIBRATION INSULATION USING ABSORBERS

As described above, it is known that vibration unpleasant for pedestrians occurs when the natural frequency of a footbridge coincides with the frequency of pace of the pedestrians, and the bridge cannot be kept in a state which is agreeable for use. To eliminate such vibration, the following have been considered: increase of damping, decrease of the live load-dead load ratio, and discord between the natural frequency of the bridge and the frequency of pace of pedestrians.

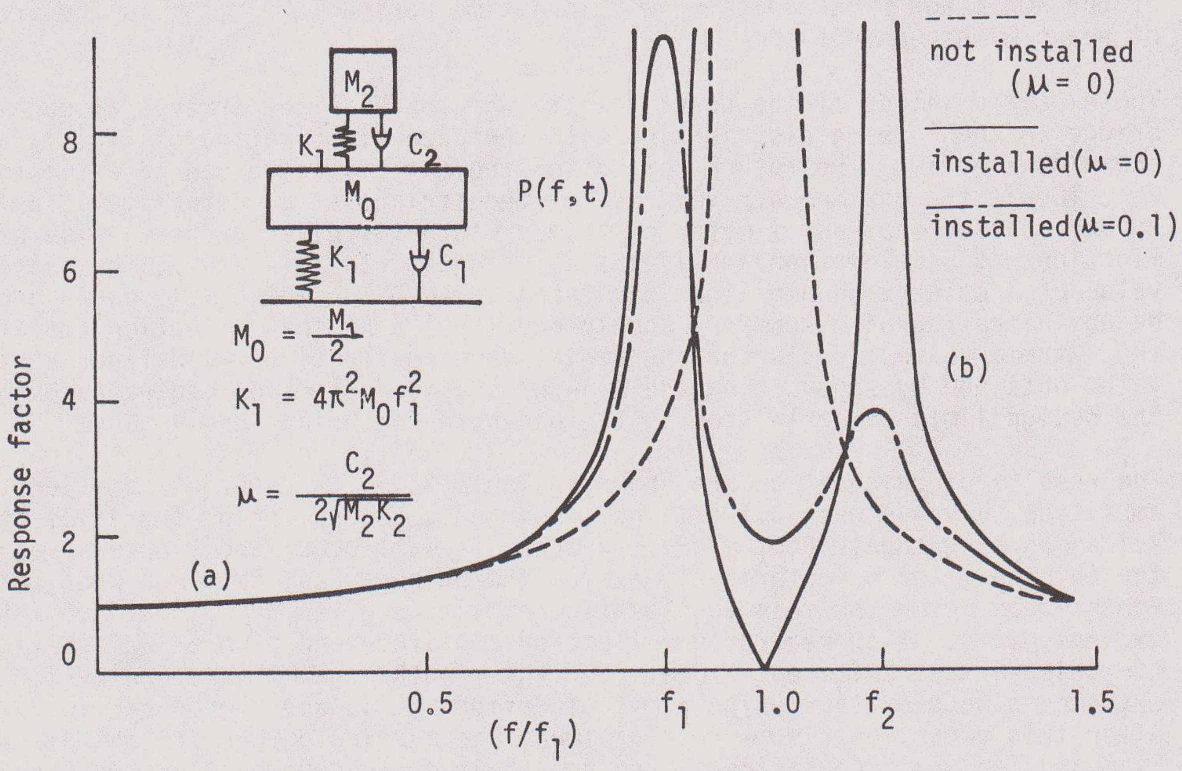


Fig. 11 Resonance curve

As a way of preventing such vibration, the absorber was prepared and its capacity for vibration insulation was observed. Fig. 11(a) shows the system of the absorber, and Fig. 11(b) shows the resonance curves of the bridge. When the calculated natural frequency of the absorber coincides with the pace frequency, most of the energy given by pedestrians to the bridges is absorbed by the absorber and the bridge does not vibrate even if the natural frequency of the bridge coincides with the pace frequency. Since the vibration system including the absorber may be regarded as a system with two degrees of freedom, there are two resonance points above and below the resonance point before installed. If the distance between these resonance points f_1 and f_2 is sufficiently great, and these are out to the limits of pace frequency shown in Fig. 1, the absorber, is effective. When the natural frequency of the bridge near the pace frequency is altered by absorbers, the distance between these resonance points $f_2 - f_1$ varies according to the mass ratio between the bridge and the absorber, M_1/M_2, as shown in Table 4. When the M_1/M_2 ratio is small, the absorber will be effective, but heavy, which will affect the design of the footbridge.

Table 4 Distance of resonance points and ratio of masses

M_1/M_2	20	30	40	50
$f_2 - f_1$	0.33	0.26	0.24	0.20

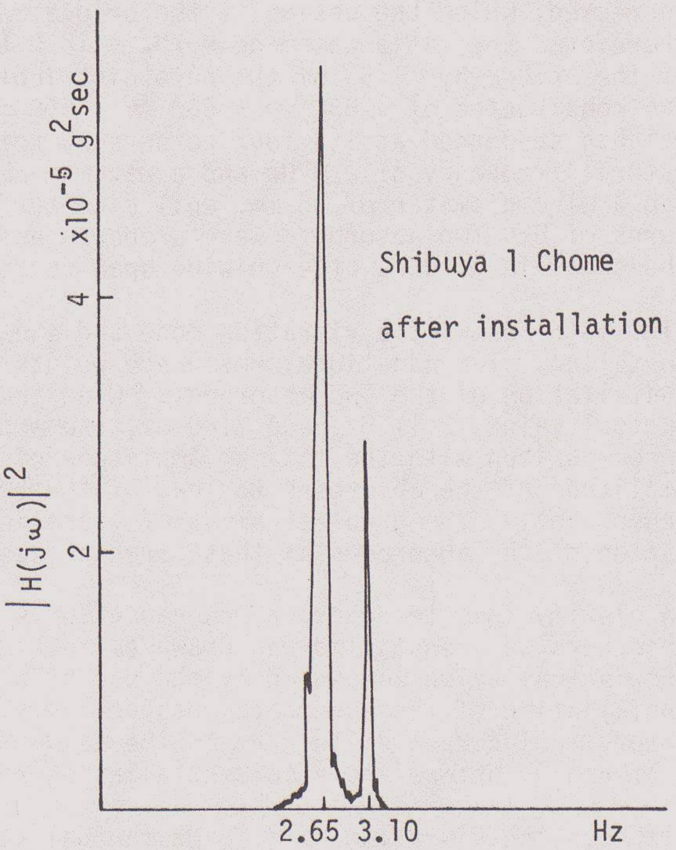


Fig. 12 Response of acceleration

Preliminary experiment, prior to practical installation of absorbers on footbridges, was carried out to make clear the effect of the absorber at the Shibuya 1-Chome Bridge. The ratio, M_1/M_2 was chosen as 50, and spring constant was taken so that the natural frequency of the absorber coincides with that of the bridge, 2.8 Hz. The absorber was installed at the mid-point of the bridge, which was vibrated by a vibrator and random walks of pedestrians. Fig. 12 shows power spectrum of acceleration at the mid-point of the bridge after installation. Compared with Fig. 4(c), which shows the power spectrum before installation, it is clear that two resonance points appear after installation above and below the original 1st natural frequency of the bridge. The effect of installation of absorbers on a bridge is to absorb vibrations in a narrow frequency band, as caused by a pedestrian on the bridge, by means of the vibration of the absorber, and also the natural frequency of the bridge from one near the pace frequency so that it lies in a range in which the bridge cannot resonate easily.

It happened to construct a slender footbridge at Kawashima where Route 50 crossed the Kinu river, and absorbers were installed and their effects were investigated. This bridge, the Kawashima Footbridge, is a deck-type steel girder bridge with an overall length of 447.1 m, consisting of two 3-span continuous girders and one 2-span continuous girder. Absorbers were installed at the centers of both side spans of the 3-span continuous girder (47.850 m + 47.875 m + 47.850 m) near the right bank.

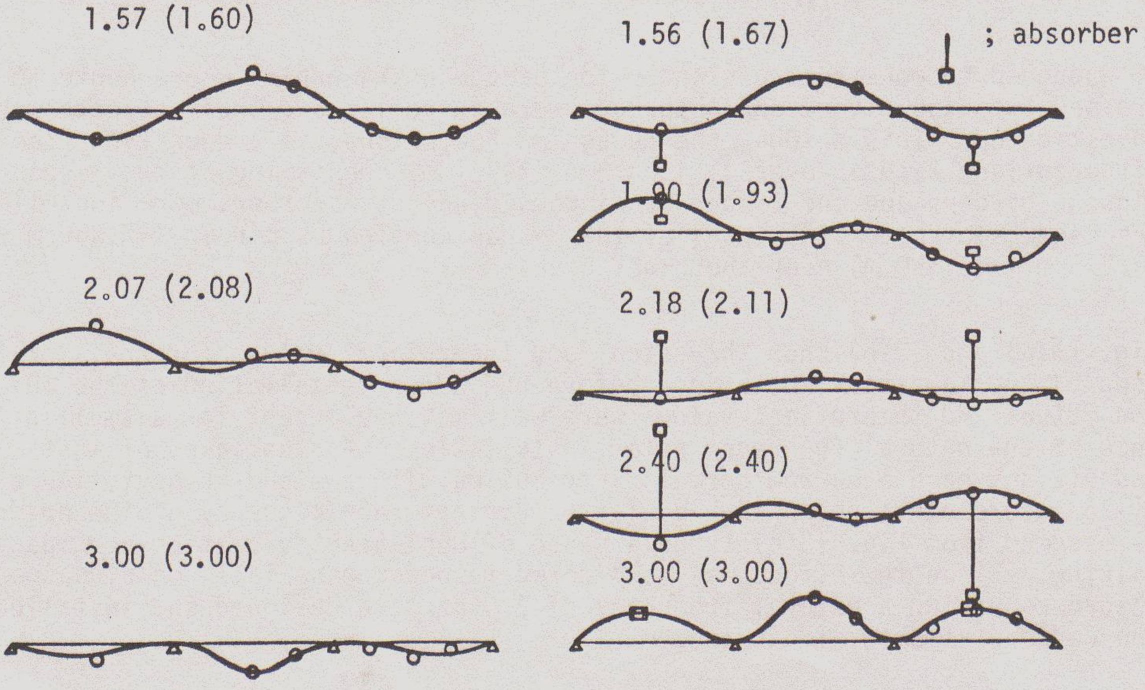
Fig. 13(a) and 13(b) show the actual and theoretical values for the vibration mode of the Kawashima Footbridge before and after installation of the absorbers. The actual and theoretical values were well matched except for a small difference of the natural frequency after installation. As the paces of most pedestrians have a narrow normal distribution with a standard deviation of $\sigma =$ 0.178 pace/sec, as shown in Fig. 1, the 2nd natural frequency of the bridge can be assumed from Fig. 13(a) to be a cause of unpleasant vibration accompanying walking. Since the effect was condidered to occur especially on side spans, absorbers having a natural frequency of 2.0 Hz were designed and installed at the centers of both side spans.

To obtain greater vibration energy in absorbers, and a considerable shift of the resonance point through the installation of the absorbers it is necessary to make the ratio between the mass of the absorbers and that of the bridge, M_1/M_2 as small as possible. As a result, however, the mass of the absorbers is increased, and the dead load and, in turn, the rigidity of the bridge are increased, which the design of the bridge makes undesirable. For this bridge, therefore, the ratio was made $M_1/M_2 = 18.0$ to move the 2.07 Hz resonance point of the bridge by 1.5 of the pace distribution of pedestrians. Each absorber was constructed of a 830 mm x 630 mm x 500 mm steel box containing steel plate weights suspended at its four corners by springs, and was designed to have a natural frequency of 2.0 Hz and a maximum amplitude of 8 cm. The four springs had a wire diameter of 30 mm, coil diameter of 199 mm, and a number of active turns of 8. Two absorbers were prepared and installed between the centers of the two main girders of each side span so as not to affect the appearance.

Fig. 13(b) shows the vibration mode and the frequency after the absorbers were installed. Two additional resonance points were found as a result of the installation of the two absorbers. When the natural frequencies were 2.18 Hz (actual value, 2.11 Hz) and 2.40 Hz, the amplitude of the absorbers was large in comparison with the maximum amplitude of the bridge, and the ratios of the amplitude of the absorbers to that of the bridge were 9.5 and 4.9 respectively. Consequently, the greatest amout of vibration energy was consumed by the vibration of the absorbers at these frequencies.

To clarify this tendency, a resonance curve of the bridge at the place where the absorbers were installed was drawn as in Fig. 14. Small circles and triangles show actual values measured by the use of a vibrator before and after the installation of the absorbers, respectively, and solid and broken lines show theoretical curves calculated on the assumption that the damping constant is 0.1, again, before and after installation respectively.

Although the vibration of 2.18 Hz (actual value, 2.11 Hz) in Fig. 13(b) does not show clearly, a peak is found in the actual value because of damping. In any case, the vibration amplitude of the bridge at the resonance point after the



(a) before installation

(b) after installation

o,( ) ; actual value

Fig. 13 Vibration modes and natural frequencies

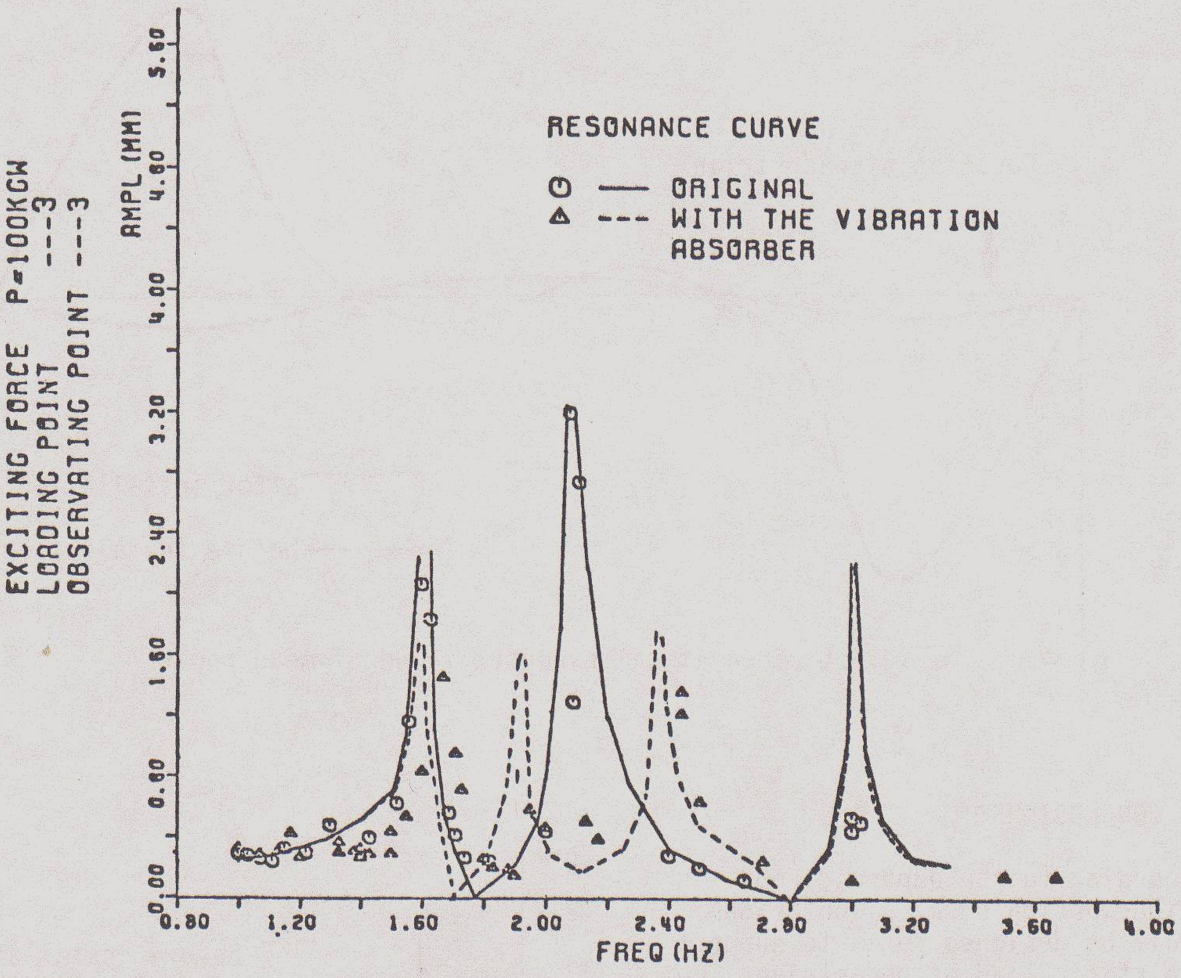


Fig. 14 Resonance curve before and after installation

installation of the absorbers is small in comparison with that before installation, and most of the vibration energy has obviously been absorbed by them.

As described above, the installation of absorbers on a footbridge results in the alteration of the resonance frequency of the bridge so that it lies outside the range of the pace frequencies. The pronounced peak at 2.07 Hz (2.08 Hz) before the installation of the absorbers had decreased after installation, and peaks at 1.90 Hz (1.93 Hz) and 2.40 Hz were substituted.

The expected value of the mean power of deflection when a pedestrian walks on the bridge was obtained on the assumption that the pace distribution of pedestrians conforms with a cormal distribution having the same mean value and standard deviation. Fig. 15 shows the results. These are the expected values of the mean power of deflection measured at any points when the pedestrian walks on the first side span 12 m from the end, and as obvious decrease of mean power in observed after the installation of the absorbers.

In general, there are a number of pedestrians crossing a bridge at the same time. Therefore twelve people were selected and they walked at random at their own normal pace. Fig. 16 shows the power spectrum of amplitude at the place where the absorbers were installed. The peak at the resonance point observed before the installation of the absorbers had decreased considerably. The ratio of mean powers before and after installation is 1:20 or less, and that of mean amplitudes is less than 1:4.

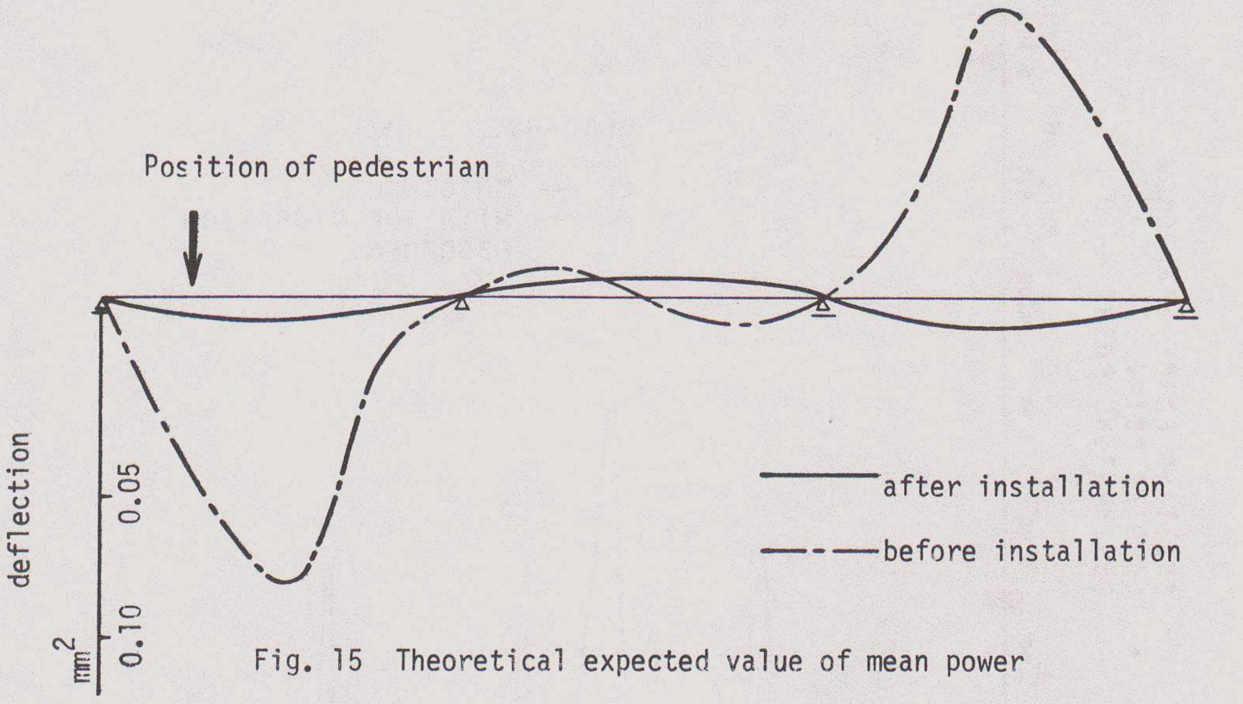


Fig. 15 Theoretical expected value of mean power

9. CONCLUSIONS

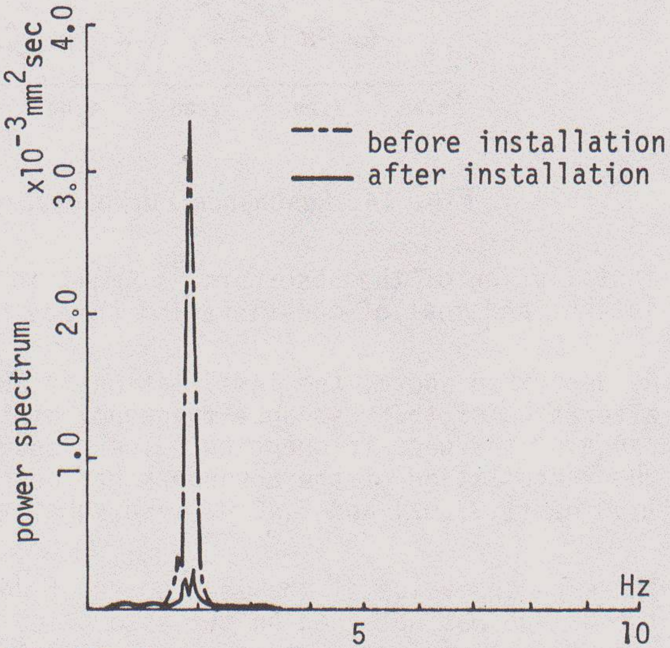


Fig. 16 Power spectrum
(12 persons' random walk)

According to the Japanese Footbridge Design Code, a footbridge shall be designed so as to support the static load of pedestrians, but the effect of the dynamic component on the bridge is not directly considered. However, there are some bridges which vibrate as they are walked on, causing discomfort to pedestrians. In this study, the vibration due to walking was investigated and it was seen that the main cause of unpleasant vibration lies in the primary natural frequency of the bridge. In particular, it was found that it was the resonance of the natural frequency and the pace frequency that led to the discomfort. Statistically, if the arrival of pedestrians at the bridge accords with the Poisson distribution, the vibration of the footbridge due to numbers of pedestrians is similar to that caused by a single pedestrian. It is therefore desirable, in footbridge design, to avoid the condition in which the primary natural frequency of the bridge coincides with the frequency of pedestrian's pace, i.e. approx. 2.0 Hz. Accoding to the relation between the spans and the natural frequency of actual bridges, this phenomenon seemed to occur in the bridges having spans near 50 meters. As a means of improving the characteristics of existing footbridges producing unpleasant vibration, absorbers were designed and installed on an existing bridge. The absorbers have the effects not only to absorb vibration caused by pedestrians, but also to alter the natural frequency of the bridge from the one near the pace frequency. It became clear that absorbers installed on footbridges can be expected to give effective insulation against vibration.

10. ACKNOWLEDGEMENT

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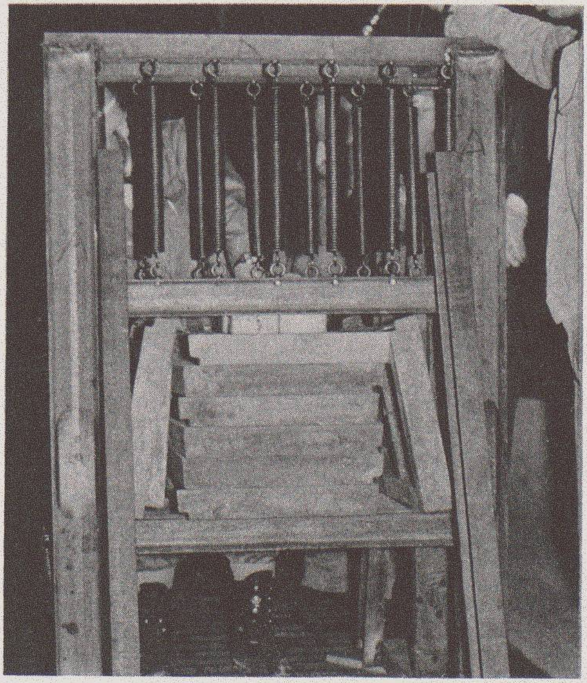
Picture 1 Experimental absorber installed on Shibuya-1-chome footbridge



Picture 2 General view of absorber and vibrator on Shibuya-1-chome footbridge

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