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Load-Bearing Masonry Walls Analyzed as Beam-Columns

Analyse de murs porteurs en briques considéré comme un système de poutres-colonnes

Backsteintragwände als Rahmenstützen untersucht

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SUMMARY

The behavior of unreinforced load-bearing masonry walls made of bricks has been represented by that of beam-columns. Their complete load-deformation relationships are predicted by a computer program that can account for the difference in the compressive and tensile yield stresses and the finite lengths of their stress-strain yield plateaux. For the simple cases in which cracking of the material is permitted, but crushing is not allowed, analytical solutions have been obtained. To further simplify design, the results are plotted in the form of charts.

RÉSUMÉ

Le comportement de murs porteurs en briques non armés est représenté comme celui de poutres-colonnes. La relation charge-déformation est établie à l'aide d'un programme de calcul à l'ordinateur, qui tient compte des différences dans les contraintes de rupture à la traction et à la compression, et des hauteurs respectives des plateaux de fluage. Des solutions analytiques ont été obtenues pour les cas simples où la fissuration est tolérée, mais non l'écrasement du matériau. Les résultats sont représentés en forme de diagrammes.

ZUSAMMENFASSUNG

Das Verhalten von unbewehrten Backsteintragwänden wird durch Rahmenstützen dargestellt. Die vollständigen Spannungs-Dehnungsverhältnisse werden mittels eines Komputerprogramms vorausgesagt, das den Unterschied zwischen Druck- und Zuggrenzspannung und die vorgegebenen Längen der Spannungs-Dehnungs-Grenzplateaux berücksichtigen kann. Für die enfachen Fälle, wo Rissbildung erlaubt, Bruch aber unerlaubt ist, sind analytische Lösungen gefunden worden. Um die Bemessung zu vereinfachen, sind die Ergebnisse als Kurven dargestellt.

1. INTRODUCTION

Unreinforced load-bearing walls made of fired-clay bricks constitute an efficient structural system for supporting gravity loads and resisting wind loads in low-rise building structures. In countries where the cost of concrete, steel, and timber is prohibitive, they are particularly suitable for buildings which are a few stories high, requiring no advanced technology for their construction. Because the safety of the building as a whole depends upon their structural integrity, it is important that sufficient care be given to their design.

Clearly, one of the prerequisites for a safe design is a thorough knowledge of the behavior of the constituent materials. The basic properties such as compressive and tensile strengths, modulus of elasticity, and initial rate of absorption of water are generally dependent upon a host of factors including binding material, porosity, and shape of the brick unit. Standard test procedures are available for determining these properties. [8,9,10,12,13]

However, even with a knowledge of these basic properties, the designer is still faced with two obstacles before he can predict the behavior of a load-bearing wall and the margin of safety against collapse. Firstly, he needs to establish adequate failure criteria which will be representative of the ultimate strength of the wall. Although some failure criteria have been proposed and some degree of success has been obtained for certain loading conditions [10], a suitable formulation covering the more general loading situation of compressive axial force with bending and shear still needs to be derived and tested.

Secondly, the designer must be able to determine the forces in the most highly stressed region of a wall in order to apply the relevant failure criteria. Some analytical methods have been proposed for the case of eccentrically loaded walls [11,14]. In these methods, it is assumed that the solid brick behaves as a linear-elastic material with compressive strength, but with no tensile strength. Algebraic expressions relating the compressive stress to the applied axial load are derived for three cases, namely (a) when the compressive force lies outside the kern, (b) when the compressive force lies within the kern at one end of the wall, but outside the kern in some parts of the wall due to deflections, and (c) when the compressive force is situated everywhere within the kern.

The purpose of this paper is to present a method for analyzing unreinforced load-bearing walls subjected to general and common loading cases in which axial loads, end-moments, and lateral loads are present. Essentially, it considers a unit width of the wall and represents its behavior by that of a beam-column. In obtaining solutions for the beam-column, shear deformations are assumed to be small compared with flexural deformations. A check can be made at the end of the analysis to ensure that this is so. In this method, the tensile strength of the brick may be considered. Another method which accounts for these effects was previously developed by Chen. [2]

Two types of analysis are performed. One uses numerical integration along the column to predict the ultimate strength behavior, and considers crushing of the brick in compression. It is, thus, suitable for assessing the margin of safety against collapse. The second analysis restricts the stress level to those which do not cause crushing of the brick, and is suitable for design, especially when the analytical data are presented in the form of interaction charts.

2. IDEALIZATION OF WALL

The behavior of each vertical segment of a load-bearing wall may be idealized into that of a beam-column subjected to compressive axial forces, end-moments, and lateral loads. The magnitude of the axial forces depends on the intensity of loading on the floors being supported. The end-moments, on the other hand, depend upon the type of connections used at the upper and lower ends of the wall, as well as the floor loading.

Figure 1 shows two building structures with load-bearing walls. In Fig. 1(a), the floor resting on the walls acts as a rigid diaphragm transferring wind

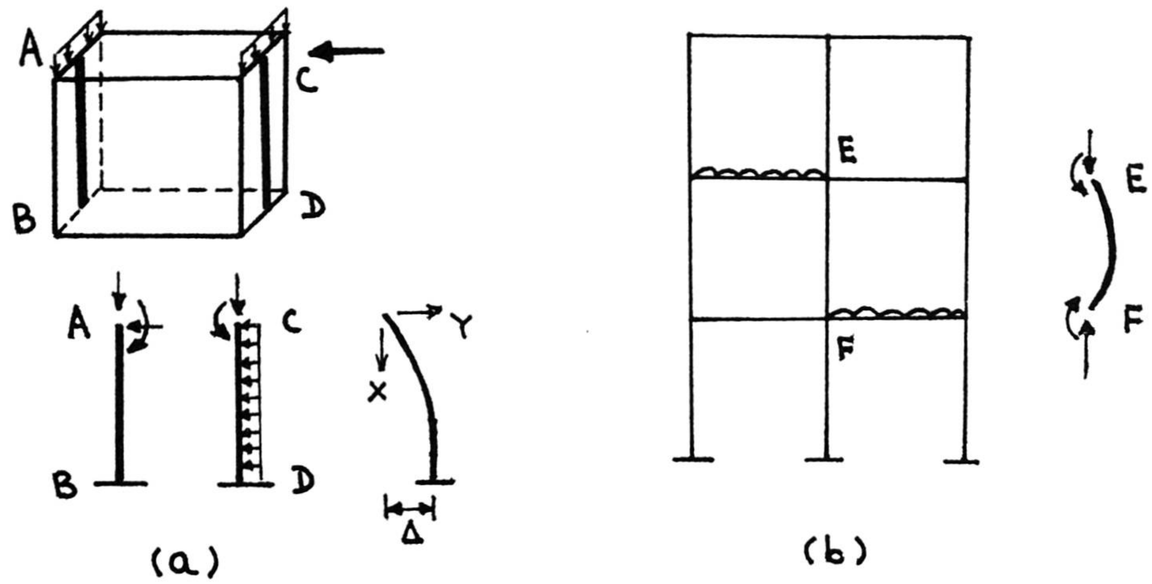

FIG. 1

forces from one wall to the other. Assuming that the bases of the walls are rigidly fixed to the foundation, wall segments AB and CD may be idealized into laterally-loaded beam-columns as shown. In Fig. 1(b), a checkerboard type of loading is imposed on the floors so that wall segment EF bends in single curvature with approximately equal moments applied at the ends. If no lateral forces are present, column EF is subjected to pure bending only with no shear deformations.

3. BASIC RELATIONSHIPS

In this study, the solid wall is made of bricks which have an elastic-perfectly-plastic stress-strain relationship as shown in Fig. 2. The modulus of elasticity, E, is identical in tension and compression, the yield stress level in tension, σ_{ty}, is less than that in compression, σ_{cy}, and the ratio of these two quantities is denoted as μ, where $0 < \mu < 1$. The yield strains in tension and compression are given by $\varepsilon_{ty} = \sigma_{ty}/E$ and $\varepsilon_{cy} = \sigma_{cy}/E$, respectively. The cracking strain is represented by ε_{to} and the crushing strain by ε_{co}.

Generally, the brick and mortar have properties which are different. For example, the modulus of elasticity, E, is not the same for the two materials. Consequently, the stress-strain diagram of Fig. 2 is to be taken as the unit average of the brick and mortar obtained from tests on small assemblages

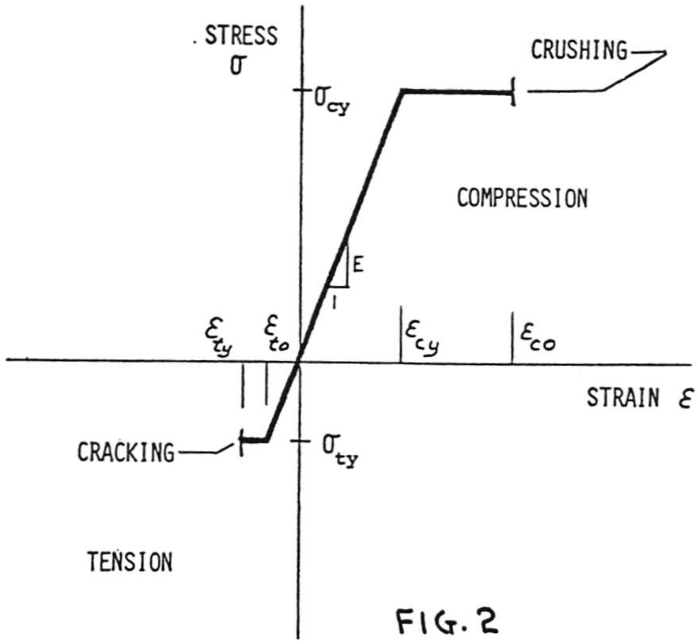

FIG. 2

averaging their heterogeneous actions.

Using the idealized stress-strain relationship of Fig. 2, it is possible to derive the moment-curvature-thrust relationships of a rectangular section of unit width and thickness t , assuming that plane sections remain plane after deformation. Three different moment-curvature-thrust relationships exist.

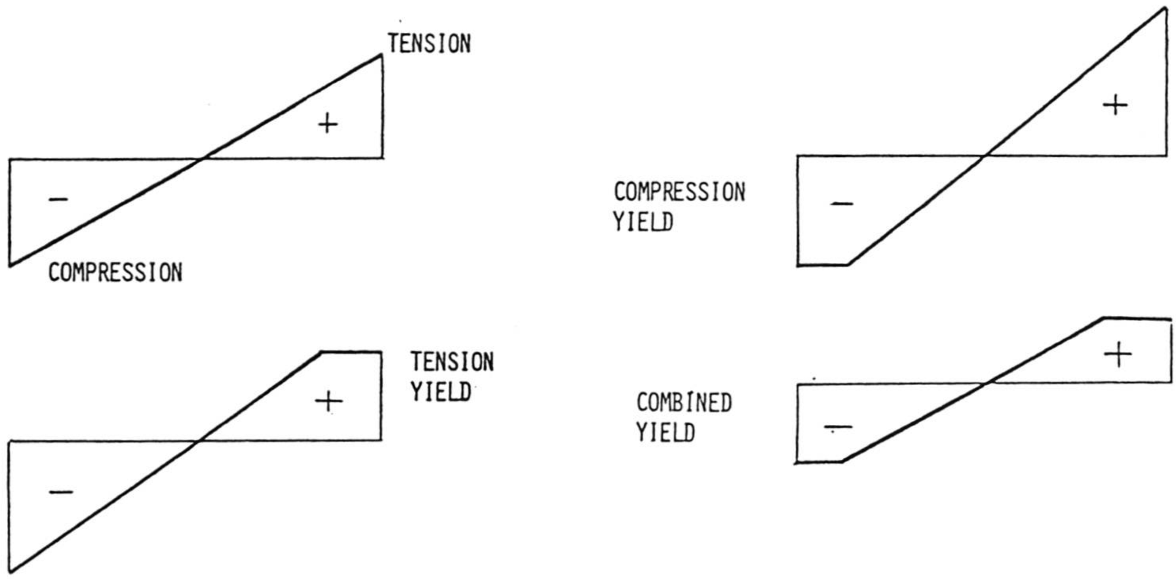

FIG. 3

They represent the cases when the wall is elastic, when cracking or crushing occurs, and when both cracking and crushing are present, Fig. 3. These relationships have been derived by Chen [2] and are reproduced above in dimensionless form.

3.1 Moment-Curvature Relationships

The relevant equations are simplified by using nondimensionalized quantities:

$$\bar{\varepsilon} = \frac{E}{\sigma_{cy}} \;;\; p = \frac{P}{t\sigma_{cy}} \;;\; m = \frac{M}{M_y} \;;\; \phi = \frac{\phi}{\phi_y} \qquad (1a)$$

$$M_y = \sigma_{cy}\frac{t^2}{6} \;;\; \phi_y = \frac{M_y}{EI_c} = \frac{2\sigma_{cy}}{Et} \qquad (1b)$$

where P = axial load per unit width of wall, M = bending moment on unit width of wall, M_y = bending moment to cause σ_{cy} to be reached assuming no tension yielding, ϕ = curvature corresponding to M , ϕ_y = curvature corresponding to M_y , and I_c = moment of inertia of elastic section.

Figure 4 shows a typical moment - curvature - thrust curve [1,2]. The curve

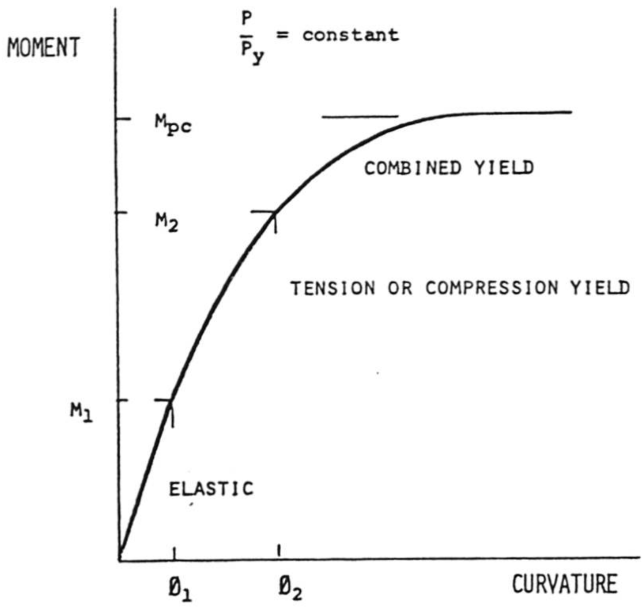

FIG. 4

is separated into 3 regions by the points (m_1 , ϕ_1) and (m_2 , ϕ_2). The curvatures ϕ_1 and ϕ_2 are given by Eq. 2,

$$\phi_1 = \frac{1+\mu}{2} - \left|\frac{1-\mu}{2} - p\right| \qquad (2a)$$

$$\phi_2 = \frac{(1+\mu)^2}{4\phi_1} \qquad (2b)$$

where $\mu = \sigma_{ty}/\sigma_{cy}$ and the notation $|\;|$ denotes absolute values. The three portions of the moment - curvature - thrust curve are defined as follows,

(a) elastic regime

$$m = \phi \qquad \phi \leqslant \phi_1 \qquad (3)$$

(b) tension or compression yield regime

$$m = b - \frac{c}{\phi^{1/2}} \qquad \phi_1 < \phi \leq \phi_2 \tag{4}$$

(c) combined yield regime

$$m = m_{pc} - \frac{f}{\phi^2} \qquad \phi > \phi_2 \tag{5}$$

where $b = 3\,\phi_1$; $c = 2\,\phi_1^{3/2}$; $f = (1 + \mu^3)/16$;
and $m_{pc} = 3(1 - p)(\mu + p)/(1 + \mu)$.

3.2 Strain Limits

A criterion often used to determine the strength of a wall is the attainment of the strain limits ε_{co} and ε_{to} at the extreme fibers in the most highly stressed region. The curvatures ϕ_{to} and ϕ_{co} at which cracking and crushing occurs are given by Eqs. 6, 7, and 8 [2]:

(a) elastic regime , ($\phi_{to} \leq \phi_1$, $\phi_{co} \leq \phi_1$)

$$\phi_{to} = \bar{\varepsilon}_{to} + p$$

$$\phi_{co} = \bar{\varepsilon}_{co} - p \tag{6}$$

(b) tension of compression yield regime,

($\phi_1 < \phi_{to} \leq \phi_2$, $\phi_1 < \phi_{co} \leq \phi_2$)

$$\phi_{to} = \frac{\varepsilon_{to} + p}{2} + \frac{1}{2}\left[(\bar{\varepsilon}_{to} + p)^2 - (\mu - \bar{\varepsilon}_{to})^2 \right]^{1/2}$$

$$\phi_{co} = \frac{\bar{\varepsilon}_{co} - p}{2} + \frac{1}{2}\left[(p - \bar{\varepsilon}_{co})^2 - (1 - \bar{\varepsilon}_{co})^2 \right] \tag{7}$$

(c) combined yield regime, ($\phi_2 < \phi_{to}$, $\phi_2 < \phi_{co}$)

$$\phi_{to} = \frac{1}{1-p}\left(\frac{1+\mu}{2}\varepsilon_{to} + \frac{1-\mu^2}{4}\right)$$

$$\phi_{co} = \frac{1}{\mu+p}\left(\frac{1+\mu}{2}\varepsilon_{co} - \frac{1-\mu^2}{4}\right) \tag{8}$$

In the above equations, $\bar{\varepsilon}_{to} = \varepsilon_{to}/\varepsilon_y$ and $\bar{\varepsilon}_{co} = \varepsilon_{co}/\varepsilon_y$. It can be readily predicted whether tension or compression yielding occurs first. If $p < (1-\mu)/2$, tension yielding occurs first [2]. Note that if the brick material is idealized into one with zero tensile strength, $\mu = 0$.

4. NUMERICAL COLUMN INTEGRATION

Several numerical methods exist for determining the ultimate strength of beam-columns. However, the column integration technique presented in Ref. 4 is used here. The advantages of this method are that it requires very little computer storage and it enables solutions for many loading cases to be obtained without iteration [3]. Such a solution is made possible by the fact that the numerical technique generates two quantities simultaneously, the moment M, which satisfies equilibrium, and the slope of the bending moment diagram, dM/dx, which satisfies compatibility. By substituting these values into the basic equilibrium equations, the forces which are needed for maintaining equilibrium are obtained. Deformations are also computed directly from these quantities.

The main equations used in the column integration technique are [4] :

(a) elastic regime

$$\frac{dm}{dx} = K\,(T - m^2 - Qm)^{1/2} \tag{9}$$

(b) compression or tension yield zone

$$\frac{dm}{dx} = K\,\left(S - \frac{2c^2}{b-m} - Qm\right)^{1/2} \tag{10}$$

(c) combined yield zone

$$\frac{dm}{dx} = K\,\left[R + 4f^{1/2}\,(m_{pc} - m)^{1/2} - Qm\right]^{1/2} \tag{11}$$

in which R, S, T = constants of integration; $Q = 2qK^{-2}M_y^{-1}$; $K = (P/EI_c)^{1/2}$, and q = uniformly distributed load.

The integration of moments along a column is started at a point where the boundary condition is known and ends when the length of the member is reached. As an example, consider column AB in its equilibrium deflected shape, Fig. 1 (a). Taking the origin at the end A, integration can start at A by specifying a shape dM/dx (see Appendix.) Checking whether M_o is in the elastic zone, or in the yielded zones, either T, S, or R, is calculated from Eqs. 9, 10, and 11. The moment M_i at a point Δx below A is then obtained as $M_i = M_o + (\Delta x)\, dM/dx$. The moment M_i is substituted in the appropriate equation, Eqs. 9, 10, or 11, to compute a new dM/dx and the entire process is repeated until the end B is reached where M_B and $(dM/dx)_B$ are now known. For equilibrium to be satisfied the moment at B must be the computed moment M_B. Compatibility is satisfied in the following way. At point B, the slope dy/dx = 0. Since,

$$M_x = Py + Qx + M_o \tag{12}$$

$$\frac{dM_x}{dx} = P \cdot \frac{dy}{dx} + Q \tag{13}$$

$$\left(\frac{dM}{dx}\right)_B = Q \tag{14}$$

Hence the shear force Q is equal to the slope dM/dx at B. Further, since Q is known and $M_B = Qh + P\,\Delta$, the deflection Δ can be readily calculated. By specifying different values of dM/dx at A in order to start the column integration, different values of Q and Δ are calculated. The maximum shear capacity of the column is determined from a plot of Q vs. Δ , such as Fig. 5.

For columns CD and EF, integration would be started at the column center. References 3 and 4 provide a detailed description of the entire column integration process and the ways in which the required end forces and rotations

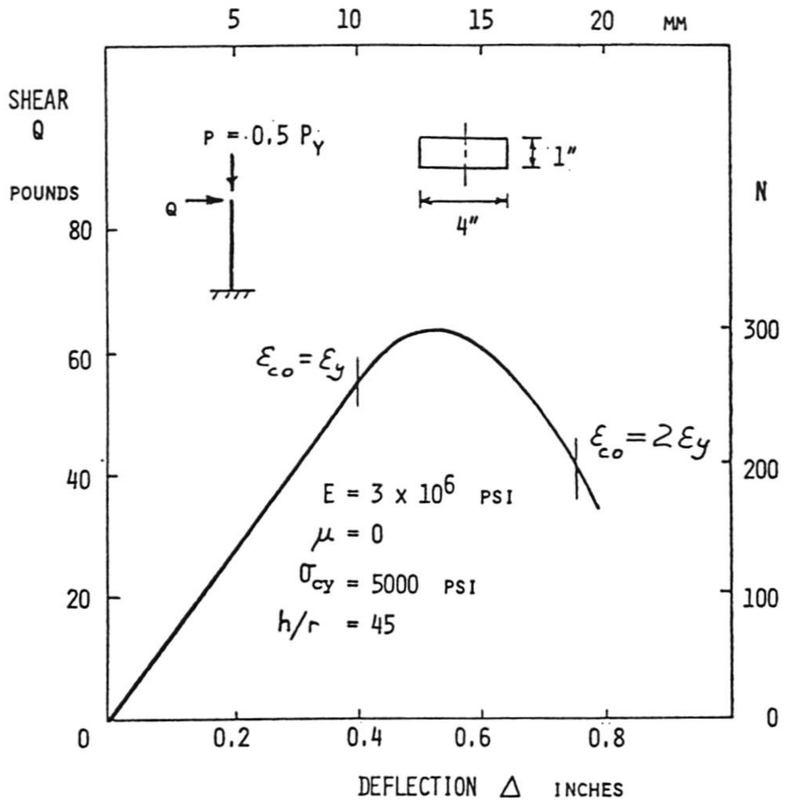

FIG. 5

are obtained by satisfying equilibrium and compatibility.

5. ILLUSTRATIVE EXAMPLE -- ULTIMATE STRENGTH

In order to check the accuracy of the computer program which makes use of the column integration technique described above, the parameter μ was set to 1.0 to represent steel which has identical strengths in tension and compression. A steel beam-column with a rectangular cross section was then analyzed and the results compared with data available in the literature [4]. A close agreement was obtained.

As a first illustration of the applicability of the computer program to masonry walls, the beam-column AB in Fig. 1(a) is analyzed. For a given column with one free end, it is required to determine the maximum shear force Q that can be carried. The cross section of the column is rectangular with dimensions 4 in. x 1 in. (100 mm x 25 mm.) Bending is about the major axis (parallel to the shorter dimension.) The column slenderness ration h/r = 45, σ_{cy} = 5 ksi (22.3 kN), and P = 0.5 P_y . Using the dm/dx relationships, Eqs. 9 to 11, integration is started at the free end by specifying an initial slope dm/dx. Integration is carried out until the full column length is reached. No consideration is given to strain limits during the integration process. Several values of Q and Δ are obtained by specifying as many initial slopes dm/dx. The results are plotted in Fig. 5. After detecting that the maximum value of Q has passed, the computer program calculates the bending moments and deflections, Δ , corresponding to the cracking or crushing strains. For the example chosen, it was assumed that no cracking occurs by using a large value of ε_{to} (= 20 ε_y) and a zero value for μ .

The results for a brick having no compressive yield plateau ($\varepsilon_{co} = \varepsilon_y$) and for one having a short compressive yield plateau ($\varepsilon_{co} = 2\,\varepsilon_y$) are given in Fig. 5. The following information is conveyed. The maximum value of the shear force Q is 65 lb (290 N) if no crushing failure occurs such as when $\varepsilon_{co} = 2\varepsilon_y$. If the material has no yield plateau, $\varepsilon_{co} = \varepsilon_y$, the maximum shear that can be carried is 54 lb (240 N) which is lower than the instability load based on no strain limits. Note that at the maximum load of 65 lb (290 N), the bending deflection is 0.53 in. (14 mm.) Using elastic theory, the shear deformation corresponding to the same shear is about 0.0006 in. when Poisson's Ratio is taken as zero.

The results for a beam-column pinned at one end and acted upon by a moment at the upper end are shown in Fig. 6(a). Again, strain limits are ignored in

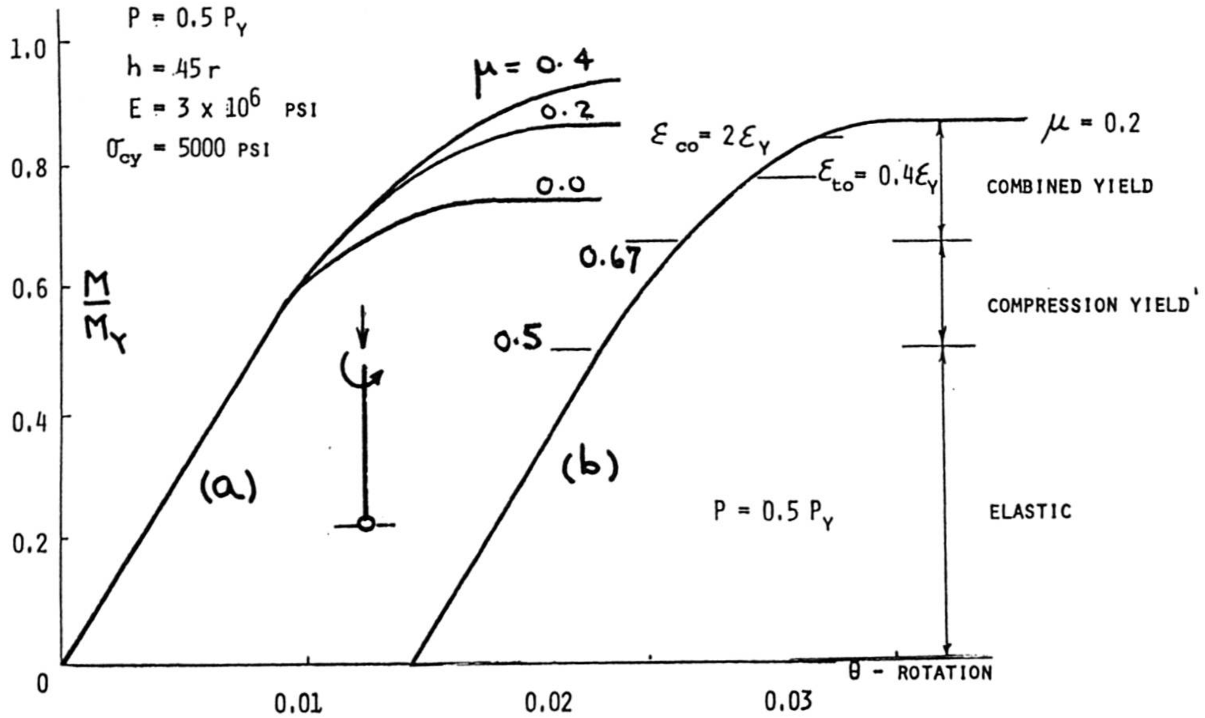

FIG. 6

the generation of the moment-rotation curves. These curves illustrate the effect of tensile strengths upon the ultimate strengths. The curve corresponding to $\mu = 0$ is for a brick material with no tensile strength. It is seen that the effect of tensile strengths is more marked when μ is small and that this effect decreases with increasing μ. An identical observation was made by Chen. [2]

Shown in Fig. 6(b) is the same curve corresponding to $\mu = 0.2$ in Fig. 6(a). The various stages in the behavior of the beam-column are depicted. Compression yielding starts at a moment $M = 0.5\,M_y$ and failure is in the combined yield zone. Note that if the brick material does not have a compressive yield plateau, $\varepsilon_{co} = \varepsilon_{cy}$, and failure occurs at $M = 0.5\,M_y$. If the brick material has a short tensile yield plateau, $\varepsilon_{to} = 0.4\,\varepsilon_y$, and a somewhat longer compressive yield plateau, $\varepsilon_{co} = 2\,\varepsilon_y$, failure occurs in the combined yield zone at $M = 0.77\,M_y$. Thus, the ultimate strength is dependent upon the strain limits in compression and tension.

6. ANALYTICAL COLUMN INTEGRATION

For the preliminary design, it is usual to assume that crushing of the brick does not take place at working loads. Under these circumstances, the masonry wall can behave in one of the three modes depicted in Fig. 7, depending upon

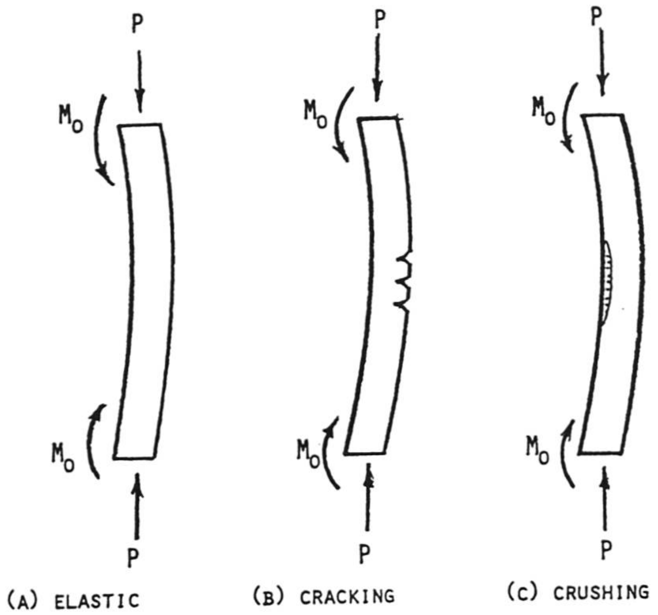

FIG. 7

the axial stresses and height of the wall. Case (a) represents the elastic solution in which all the column cross sections are under compression and crushing of the material does not occur. In case (b), cracking occurs first in the tension fibers, but crushing is not permitted in the compression fibers. In case (c), crushing is just about to take place in the extreme fibers with all sections under compression.

The solution to case (a), Fig. 7, is straight-forward and need not be derived here. Cases (b) and (c) are two limiting conditions which are of interest to designers. The solutions to cases (b) and (c) have been given in detail in Ref. 7. Consequently, only the results are presented below.

As seen above, the moment-curvature relationship applicable to a cracked section is different from that applicable to an uncracked section. Therefore, for case (b) in Fig. 7, different equations will govern the behavior of the cracked and uncracked portions of the masonry walls. However, compatibility and equilibrium will still be preserved at the junction of the two portions. For the uncracked portion, the solution is

$$m = A \sin(Kx) + B \cos(Kx) \tag{15}$$

For the cracked segment,

$$\frac{K\,S}{2c^2}(D - x) = \cosh^{-1}\sqrt{\gamma} + \sqrt{\gamma}\sqrt{\gamma - 1} \tag{16}$$

$$\gamma = \frac{S\,(b - m)}{2c^2} \tag{17}$$

where A, B, D, and S are constants of integration which are calculated by substituting the appropriate boundary conditions. In particular, if a column of length L (with $\mu = 0$) is loaded axially by force $p < \frac{1}{2}$ and by equal end-moments m_o, Eqs. (15) and (16) may be solved to give

$$m_o = p \cos(KL_1) - [2p - 4p^2]^{1/2} \sin(KL_1) \tag{18}$$

$$KL_2 = KD - (2p)^{3/2} \cosh^{-1}(1) \tag{19}$$

where

$$KD = (2p)^{3/2} \left[\cosh^{-1}\sqrt{\frac{1}{2p}} + \sqrt{\frac{1}{2p}}\sqrt{\frac{1}{2p} - 1}\,\right] \tag{20}$$

and

$$L = 2(L_1 + L_2) \tag{21}$$

In a similar manner, the solution to case (c) of Fig. 7 has been obtained as [7]

$$m_o = (1 - p) \cos\left(\frac{KL}{2}\right) \tag{22}$$

To simplify design procedures, Eqs. 15, 16, and 22 have been plotted in the form of charts, Fig. 8, 9, and 10. The application of these charts to loading cases in which the end-moments are unequal has been explained in Ref. 7.

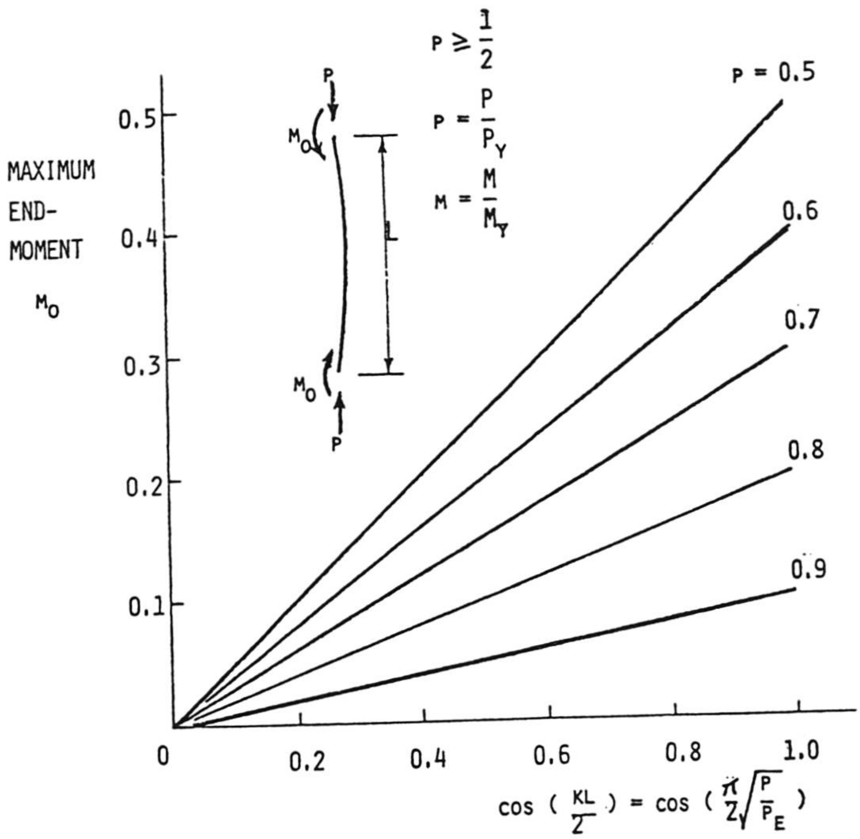

FIG. 8: MAXIMUM END-MOMENT TO JUST CAUSE CRUSHING

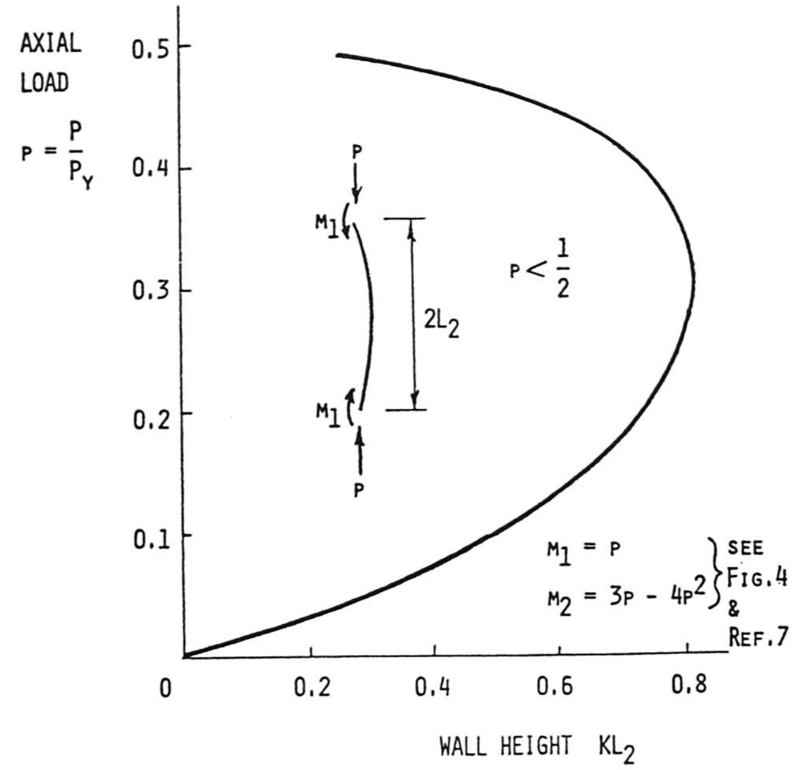

FIG. 9: CHARACTERISTICS OF WALL WITH CRACKED SECTION

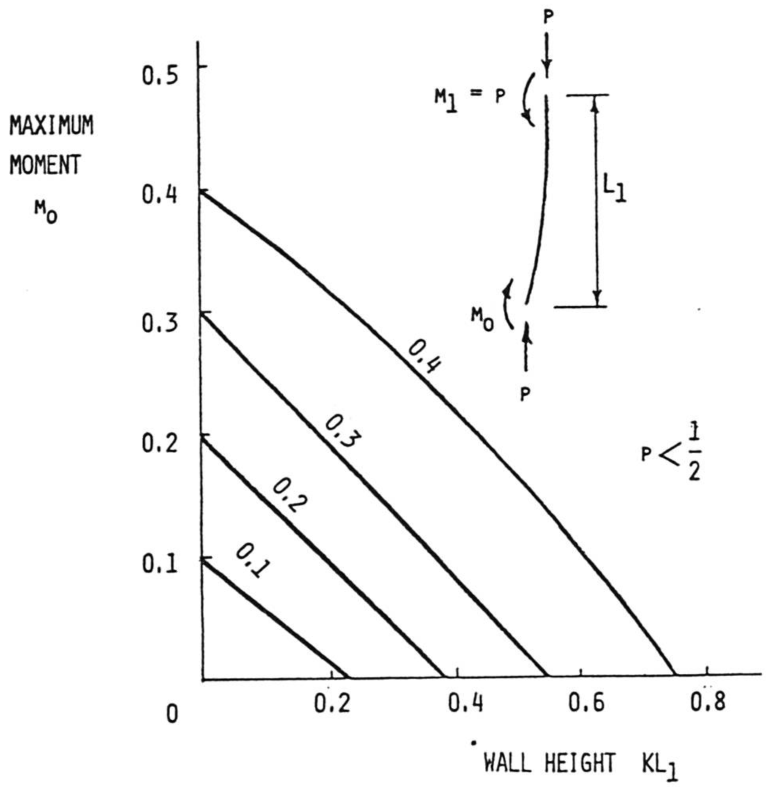

FIG. 10: CHARACTERISTICS OF WALL WITH UNCRACKED SECTION

6. COMMENTS

In the ultimate strength analysis, it is assumed that the criteria for failure are known. If no strain limits are imposed on the beam-column (no cracking or crushing occurs,) the ultimate strength is represented by the peak in its load-deformation response curve. This situation corresponds to the case when the compressive and tensile yield plateaus of the stress-strain curve are infinitely long. The other criterion used is that failure occurs when either the cracking or crushing strain is reached. In this case the compressive and tensile yield plateaus are of finite lengths. The mode of failure depends upon the relative lengths of these plateaus.

These two failure criteria have been used in the examples illustrating the application of the computer program, not only because they are reasonable, but also because a more accurate failure criterion is not available at this stage. As the behavior of walls made of bricks and mortar is further explored under all possible load combinations, suitable failure criteria will be developed and these can then be directly incorporated in the computer program.

To check the applicability of the ultimate strength design method presented, two brick walls were fabricated and tested to failure. The results, which are reported in Ref. 6, are in good agreement with the theory.

7. SUMMARY

The behavior of an unreinforced load-bearing masonry wall made of bricks can be represented by that of a beam-column with the same axial load, end-moments, shears, and boundary conditions. Computer programs have been prepared for analyzing these masonry beam-columns, taking into account the tensile strength and finite length of the stress-strain yield plateaus. In the solution process, the complete load-deformation behavior is first predicted, assuming no restriction on the strain limits. Appropriate failure criteria may then be applied to determine the true maximum strengths. In the examples provided, the failure criterion used is the attainment of the maximum compressive or tensile strain at the extreme fibers. Comparison with two test results have shown good agreement with theory.

In addition, as an aid to design, analytical expressions have been derived which give the maximum bending moments that can be applied to the wall to avoid crushing of the material. To simplify the design further, these expressions have been translated into charts.

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APPENDIX: SELECTION OF INITIAL dM/dx

When applying the column integration technique described above, it is necessary to choose an initial slope dM/dx to start the numerical integration process. In cases where symmetry exists such as in column EF of Fig. 1(b), the slope dM/dx can be readily selected since it is known at the column's mid-height [dM/dx = 0]. Where dM/dx is theoretically zero, a small initial slope (usually 10^{-6}) must be specified for the generation of the moment curve to proceed.

In non-symmetrical cases such as in column AB of Fig. 1(a), the initial dM/dx at A can be chosen with care by calculating the elastic response of the column, as follows. In the usual notation, the moment M at point x in the column is given by

$$M = Py + Qx \pm M_o \tag{23}$$

$$M = A \sin\mu x + B \cos\mu x \tag{24}$$

where A and B are constants of integration, and $\mu^2 = P/EI_c$. By satisfying the boundary conditions at A and B, it can be shown that the slope $(dM/dx)_o$ at point A [x = 0] is

$$\left(\frac{dM}{dx}\right)_o = \mu \frac{(M_B \pm M_o \cos\mu L)}{\sin\mu L} \tag{25}$$

where M_B is the moment at B. Noting that $M_B \leqslant M_{pc}$, $(dM/dx)_o \leqslant \mu(M_{pc} \pm M_o \cos L)/\sin L$. Hence an initial slope of $(dM/dx)_x = 1.1\ (M_{pc} + M_o \cos L)/\sin L$ can be slected, and subsequently increased by 5% until the complete load-deformation curve is obtained.

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NOMENCLATURE

b, c, f, m_{pc} =	constants defining $m - \emptyset - p$ curves
E	modulus of elasticity
I_c	elastic column moment of inertia
K	$\sqrt{P/EI_c}$
M	bending moment
M_y	first yield moment in compression assuming no tension yield
m	M/M_y
m_1, m_2	moments defining boundaries of yielded regions
P	axial load
P_y	axial yield load
p	P/P_y
Q	lateral load
$R, S, T,$	constants of integration

Δ	=	deflection
ε		strain
ε_{co}		strain when first crushing occurs
ε_{cy}		strain at compression yield
ε_{to}		strain at first cracking
ε_{ty}		strain at tension yield
$\bar{\varepsilon}$		nondimensional value of
μ		σ_{ty}/σ_{cy}
σ_{cy}		compressive yield stress
σ_{ty}		tensile yield stress
ϕ		curvature
ϕ_y		curvature at first compressive yield
ϕ_1, ϕ_2		curvatures defining boundaries of yielded regions
ϕ_{co}		curvature at first crushing
ϕ_{to}		curvature at first cracking
ϕ		ϕ/ϕ_y