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Ultimate Load of Trusses buckling in their Plane

Résistance ultime des poutres en treillis dans leur plan

Traglast von ebenen Fachwerkträgern

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SUMMARY

About twenty trusses have been simulated numerically on computer by using a finite element program, which takes into account both geometrical and material nonlinearities. For the trusses, whose compressed bars have a slenderness ratio larger than 50, the results show that the computation of the secondary bending moments is useless; the bars may be designed, after the computation of the effective length, by centric buckling under the normal force; this effective length may be found by an elastic method applied to a simplified structure.

RÉSUMÉ

Une vingtaine de treillis ont été simulés numériquement sur ordinateur au moyen d'un programme électronique qui prend en compte les non-linéarités géométriques et matérielles. Pour des treillis dont les barres comprimées ont un élancement supérieur à 50, les résultats montrent que le calcul des moments secondaires est inutile; les barres peuvent être dimensionnées, après avoir calculé une longueur de flambement, en les considérant comme soumises à l'effort normal seul; la longueur de flambement peut être trouvée par une méthode élastique appliquée à une structure simplifiée.

ZUSAMMENFASSUNG

Etwa zwanzig ebene Fachwerkträger wurden mit Hilfe eines Computerprogramms, welches sowohl geometrische als auch materialtechnologische Nichtlinearitäten berücksichtigt, numerisch berechnet. Bei Druckstäben, deren Schlankheitsgrad 50 übersteigt, zeigen die Ergebnisse, dass die Berechnung der Zwängungsmomente unnötig ist. Diese Stäbe können als zentrisch belastete Knickstäbe behandelt werden, deren Knicklänge mittels einer elastischen Methode an einem vereinfachten System bestimmbar ist.

1. INTRODUCTION

The essential problem, in the design of rigidly connected trusses, resides in its compressed bars. The following questions immediately arise:

- Is the concept of effective length valid ?
- Which effective length should be adopted for in-plane buckling ?
- Should the secondary moments be considered and introduced (in an interaction formula) to compute the ultimate strength ?

Some engineers could believe that these questions have been solved for many years in the various national codes. However, successive and frequent changes in the design rules of these codes indicate clearly that a definitive solution is not available. This situation derives from the parallel - but not simultaneous - evolution of several basic points:

- the semi-probabilistic theory and the ultimate strength control of structures;
- the development of the so-called "European buckling curves";
- the computation method of the ultimate load of trusses.

Most of the time, the compressed bars of a truss are designed:

- at collapse by using the European curves;
- by using effective lengths which date from a time when the computations were made with the allowable stress method and the buckling curves of EULER, TETMAYER (in Europe), JOHNSON (in the States), etc.

In these conditions, several questions arise:

- Is such a mixed method always on the safe side ?
- Is a more elaborate design, incorporating the secondary moments, really necessary ?

This boils down to the question of how a truss must be computed. The present study is limited to the behaviour of the truss in its plane, assuming bracing against lateral buckling. Moreover, only in-plane buckling of bars is considered.

2. PRESENT STATE OF THE QUESTION

2.1. The codes

The codes may be usefully scrutinized to the extent that, being a compendium of practical design rules, they reflect present knowledge. This study shows that, for about half a century, the buckling (or effective) lengths ℓ_b of truss bars were taken as equal to 0,8 ℓ. The first changes of attitude in the code committees appeared about ten years ago. The general tendency is to increase the buckling length. With $\ell_b = k\ell$, the past and present values of k of some codes are given in Table 1.

It can be seen that, if k has effectively increased, not all countries have adopted k=1 like the United States. In the American code, the choice is justified by remarking that, in truss design for equal strength, no bar can help the adjacent one because, by definition, all bars have the same collapse load.

This attitude appears somehow too pessimistic, to the extent that the chords often have, for technological reasons, the same cross section along their whole length, which shows that uniform strength design is rather unrealistic.

TABLE 1

			chord member	web member		chord member	web member
U.S.A.	A.I.S.C.				1969	1,0	1,0
Germany	DIN 4114	1952	1,0	1,0	1978	1,0	0,9
Netherlands	NEN 3851	old specifications			1974	1,0	0,7<k<1
Tchecoslovakia	CSN 731401	(no data available)			1976	1,0	0,5<k<1
Canada	C.I.S.C.				1976	0,9	0,9
Belgium	NBN B51-001	1977	0,8	0,8	1980	0,9	0,9
France	C.M.	1956	0,9	0,8	1966	0,9	0,8
Switzerland	SIA 161	1956	0,8	0,8	1979	0,9	0,8
Great Britain	BS 5400	1956	0,7	0,7	1980	0,85	0,7

2.2. The computation methods

The rigorous study of structures composed of bars is very complex and the determination of the exact collapse load yields intricate computations. The methods are essentially numerical, are based on second order theory and may be classified in two categories, according to the behaviour of the material:

- elastic;
- elastoplastic (with or without strain hardening).

When the material obeys HOOKE's law ($\sigma = E\varepsilon$), buckling lengths are generally obtained by using the displacement method where the unknowns are the displacements of the rigid nodes (straightforward generalization of the classical slope deflection method). TIMOSHENKO [T1], MERCHANT [M1], KUANG HAN CHU [K1], etc.... have developed corresponding tables and charts. If the chosen unknowns are the moments at the bar ends, the structure can be analysed iteratively by generalizing the CROSS method: HOFF [H1], WINTER [W2], LUNDQUIST [L1], etc... have developed this approach. At present, the computations are made by a computer program solving the linearized stability eigenvalue problem [F3].

As soon as the constitutive law of the material is nonlinear, analytical developments become practically impossible. For this reason, all authors use electronic computer programs based on large displacement theory ([F2], [L2], etc...). This approach seems the most advanced and the most realistic at the present time.

On the basis of a study of this type, DUBAS [D2] states about trusses that:
"The collapse load, computed from the buckling lengths furnished by the bifurcation theory, gives results in agreement with those derived from a detailed elastoplastic computation. However, one may fall on the unsafe side, especially in the case of welded bars".

It appears from present study that the foregoing conclusion does not apply as a general rule (see section 4.2.3 "Bifurcation approach").

3. WORKING ASSUMPTIONS

The present study [D1] uses the nonlinear finite element program developed by FREY [F1], [F2].

3.1. The basic assumptions are large displacements, small strains and elastic or elastoplastic constitutive laws, so that both geometric and material nonlinearities are taken into account.

A truss bar is discretized in a certain number n of "engineering beam elements" (Fig.1) formulated in approximate updated Lagrangian description [F1]. The optimum number of elements which would together represent most faithfully the bar subjected to buckling varies as a function of the slenderness ratio $\lambda = \ell/i$ of the bar. Numerical experimentation has yielded the empirical relationship:

$$n \simeq 4 + \lambda/20$$

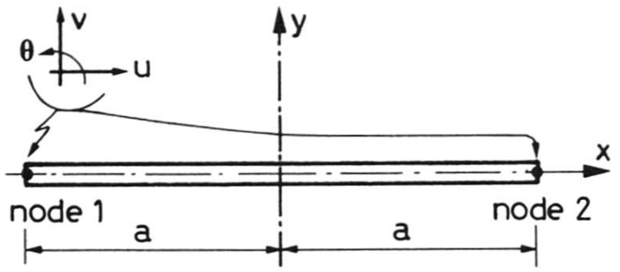


Fig.1.
Engineering beam element.

Numerical simulation of the simple truss presented by DUBAS [D2] has been used to test the agreement between his results and those obtained by FINEL G [F2]. The collapse loads agree within 0,7%, a negligible discrepancy (instability of the compressed chord).

A comparison between a test undertaken in the laboratory of the senior author and its numerical computation was also made [D1]. The tested truss was composed of tubes and some of the bars were assembled with definite eccentricities. The collapse modes observed and computed were characterized by the yielding of the corresponding welded connections. The experimental collapse load was found to exceed the computed value by 3,2%.

These two verifications have emphasized the reliability of the program for the study of such problems (see also [F1] for further verifications).

3.2. In structural engineering, we can distinguish two main types of trusses:

- those loaded by moving loads (road and rail bridges);
- those loaded by fixed loads (industrial halls of any kind).

In the second case, the design may be such that all bars will be fully used for the same loading (equal strength design). Buckling should therefore be more dangerous for this type of structure.

3.3. The study is limited to truss geometries which are the most popular in bridges and roofs:

- the N (PRATT) truss;
- the V (WARREN) truss.

The trusses retained are shown on Fig.2; it was decided to study the buckling of:

- diagonals in the V truss (Fig.2.a);
- chords in the N truss (Fig.2.b);
- chords in the third truss (Fig.2.c).

The interest of the last truss lies in the unequal lengths of the bars of the compressed chord, where the normal force is constant. Thus we could study the influence of this particularity on the effective length of the chord.

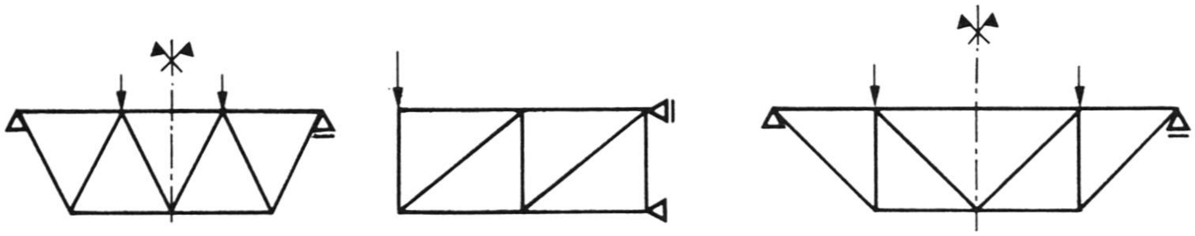


Fig.2. Studied trusses.

3.4. There are different kinds of cross sections currently used in practice (H, Π, ⊥, O, □,...).

We have retained the H section (flanges in the truss plane) and the hollow rectangular section. If the former is quite usual, the latter is beginning to invade the market for reasons of aesthetics and maintenance.

3.5. In the V truss, the sections adopted are H sections (weak axis buckling). The studied parameters [D1] were:

- the type of loading: symmetrical or unsymmetrical;
- the slenderness of the chord or diagonal compressed bars.

On the other hand, the N truss is composed of tubes with rectangular cross section. We have especially studied the influence of:

- the initial deflections of the bars;
- the eccentricities due to a non-concordance of the axis (as a consequence of the practical advantage of making a single cut in the tube and of displacing the bar axis by some centimeters; see Fig.3);
- the welding residual stresses at the joints.

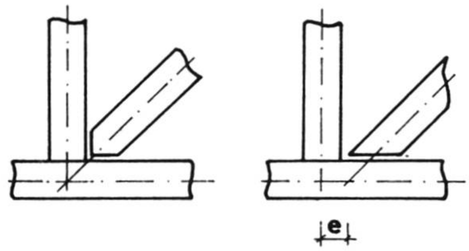


Fig.3. Connections without or with eccentricities.

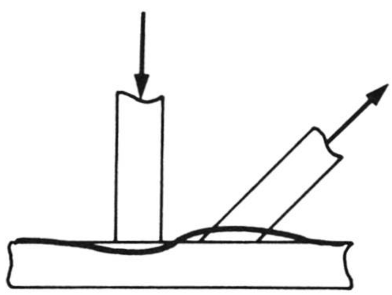


Fig.4. Local deformations in tubular joints.

3.6. However, due to the flexibility of the nodes, the numerical computation of a truss composed of rectangular tubes raises the eternal problem: to which extent does the theory represent the reality ?

Indeed, for the computer, the connections are assumed to be perfectly rigid, and the finite element used does not take into account either the local buckling or the local deformation of the walls of the tubes (Fig.4). As such phenomena may have an influence on the general buckling of the structure, we have tried to find in which cases the numerical calculation is acceptable.

Many authors have already undertaken experimental researches (CIDECT researchers [C1], [C2], SFINTESCO [S1], DAVIES and ROPER [D3], JANIN and GIRARD [J1],...) or theoretical studies (CZECHOWSKI and BRODKA [C3], MOUTY [M3], VENANZI [V1],...) on these connections. The latter compute the collapse load of the connections by applying the yield line theory (Fig.5), assuming the chord tube to be a frame on which the diagonal is attached.

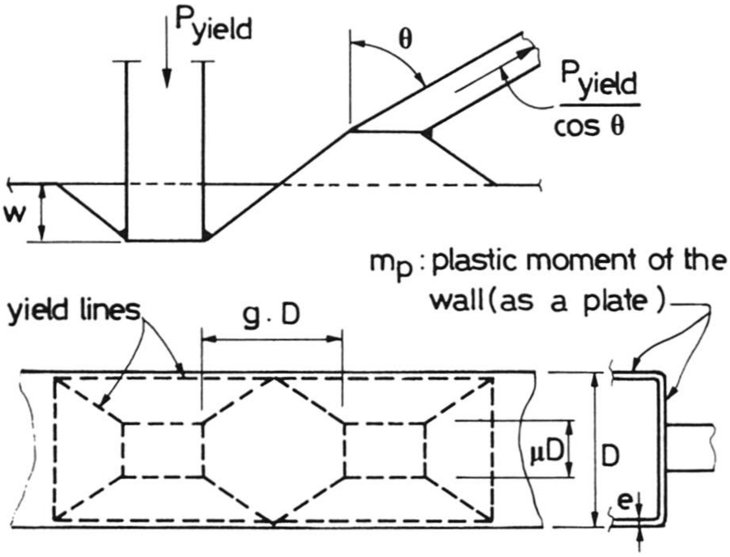


Fig.5. Computation of a joint by the yield line theory.

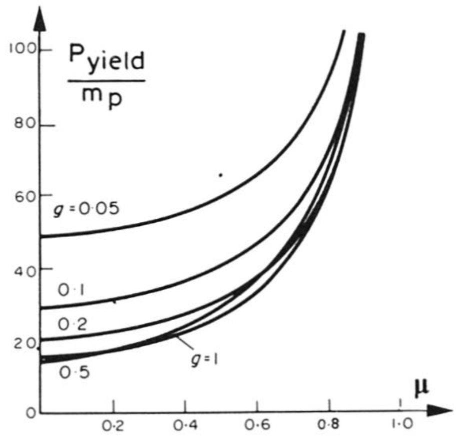


Fig.6. Influence of parameters μ and g [D3].

In general, these methods give good results, if it is considered [D1] that collapse of the connection occurs when the relative displacement w/D (Fig.5) is 1%. These various researches enable one to state that:

- the connections remain rigid up to collapse provided that μ exceeds 0,8 and g is maintained relatively small (Fig.5 and 6);
- if D/e is greater than 20, the cross section area of the web members should be reduced to 30 to 40% in the connection itself;
- nearly up to collapse, the behaviour of the connection is linear elastic and the various relative deflections w/D remain smaller than D/100.

Finally, as already mentioned in section 3.1, the comparison between the truss test and its electronic computation has also demonstrated the reliability of the above assumptions.

3.7. The design of the studied trusses (see [D1] for details) is based on the principle of equal strength. The compressed bars are computed by the European buckling curves with the following buckling lengths ℓ_b :

$$\ell_b = 0{,}8\,\ell \text{ for the diagonals} \; ; \; \ell_b = \ell \text{ for the chords.}$$

The quality of steel is Fe 360 (AE24); residual stress patterns are those used by the ECCS; initial deflections of bars are sine curves (L/1000 for the weakest compressed bar).

3.8. For the N trusses (Fig.2), the following cases are examined:

- bars with or without initial deflection;
- effect of welding residual stresses at the joints;
- existence or absence of eccentricities at the joints (Fig.3).

The collapse of these trusses occurs by the buckling of the compressed chord.

From now on, it can be remarked that [D1] :

- the role of the initial eccentricities is not negligible ($\simeq 10\,\%$);
- the eccentricities may induce substantial local yielding at the connections;
- taking into account the welding stresses does not modify at all the overall behaviour of the truss.

3.9. For the V trusses, we have studied the influence of:

- the slenderness of the compressed bars;
- the type of loading (see appendix).

3.10. For the trusses with unequal meshes, we have looked for the influence of the normal force distribution. Here, it was constant in the whole compressed chord. The parameter was the slenderness of the compressed bars (see appendix). It appears that the above geometry is not more dangerous for the buckling of the chord than a regular one.

4. INTERPRETATION OF THE RESULTS

4.1. Collapse mode

Owing to the development of numerical methods adapted to the computer, the present tendency is to forget the concept of effective length in favor of a direct computation of the collapse load. Indeed, this load depends on several factors:

- the type of loading;
- the truss geometry;
- the shape of the cross sections of the bars;
- the type of design used (for example: equal strength).

According to this approach, each truss would be a separate entity that should be designed globally.

Now, the use of the concept of effective length obliges us to isolate the bar under review from its environment. This way of handling the problem is obviously open to the additional criticism that, for an ultimate strength design, no account can be taken of the internal resultant redistribution, when some sections are yielding.

These remarks show clearly that the concept of effective length has no theoretical basis for rigidly jointed trusses.

However, the concept is highly useful in practice, because its knowledge facilitates very much the designer's work, and as we shall see further, numerical experimentation makes it worth accepting it in the frame of the present study (statically determinate plane trusses).

To clarify, we have distinguished two concepts:

- the collapse length (ℓ_c), which is an "effective length" of the bar considered in the actual truss, as it is given by the computer or the experiment;
- the equivalent buckling length ($\ell_{eq,b}$), which is the "effective length" of the bar computed for a given truss by means of an approximate method.

Indeed, a careful survey of the results has shown that the collapse of a truss was induced by the buckling of one bar, in spite of the equal strength design.

As shown by the deflections of two trusses (Fig.7,8) and the yielding pattern of the second truss (Fig.9; see [D1] for more details), the collapse mode is peculiar to "statically determinate" trusses, because:

- collapse always occurs by buckling of one bar;
- as soon as this bar buckles, the entire truss collapses.

Once the collapse load has been exceeded (post-critical range), we observe the following phenomena:

- the deflection of ONE bar continues to increase very quickly;
- the yielding pattern of this bar shows that it is strongly bent;
- the OTHER bars unload elastically.

Thus, although all bars interact, a truss does not collapse as a whole, and failure is concentrated on a localized zone; at the limit, if we were in a laboratory, we could test the truss again by replacing only the failed bar. But, if it can be said that the truss perishes through the buckling of one of its bars, it must carefully be kept in mind that this bar is restrained by the adjacent ones. The yielding pattern after collapse shows that clearly.

One must note that the above mentioned phenomena develop only after the collapse load is reached. A computation program able to study the post-critical range is thus necessary to make the collapse mode of the structure appear correctly (for instance [F1] enables post-critical analysis).

These remarks emphasize the following fact: the collapse of a truss is essentially different from that of a rigid frame, although both are structures with rigid nodes:

- In a sway frame, the stability of any bar depends on the degree of lateral and angular restraint of its end sections. This degree depends itself on the displacements of all nodes and, therefore, on the rigidity of the whole system. Consequently, the whole set of bars composing the frame is involved in the collapse.
- In a truss, on the contrary, the nodes are practically fixed, because the truss is formed of a series of triangular "rigid" cells. Consequently, the degree of restraint of a single bar depends essentially on the adjacent bars. Moreover, the type of loading is very different in a truss. The loads act only at the nodes and do not subject directly the bars to bending. The normal force plays therefore a paramount role (see 5. CONCLUSIONS). And, even if the nodes are rigid, the rotational degree of freedom (Fig.1) plays only a minor role. It could be said that a truss with rigid or hinged nodes has the same degree of redundancy.

Such a fact has been observed experimentally during the test (mentioned in section 3.1) up to collapse of a truss. Four nodes became completely plastified. To continue the test, one had to stiffen these joints be means of additional gussets. It was observed that the overall rigidity of the structure was the same before and after this reinforcement. This shows that node rigidity has a very small influence on the behaviour of the truss as a whole (not on an isolated member, naturally).

Thus, the truss which is "statically determinate" for the normal force will perish when one of its bars buckles.

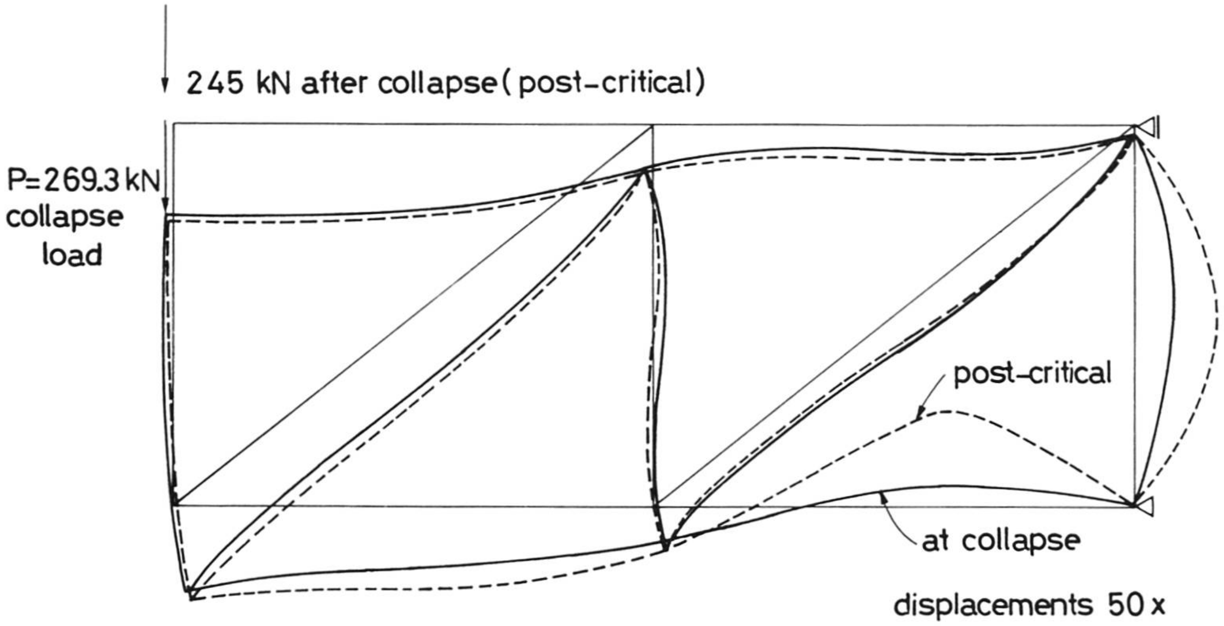


Fig.7. Deflection of the truss nr 4 b) at and after collapse (see appendix).

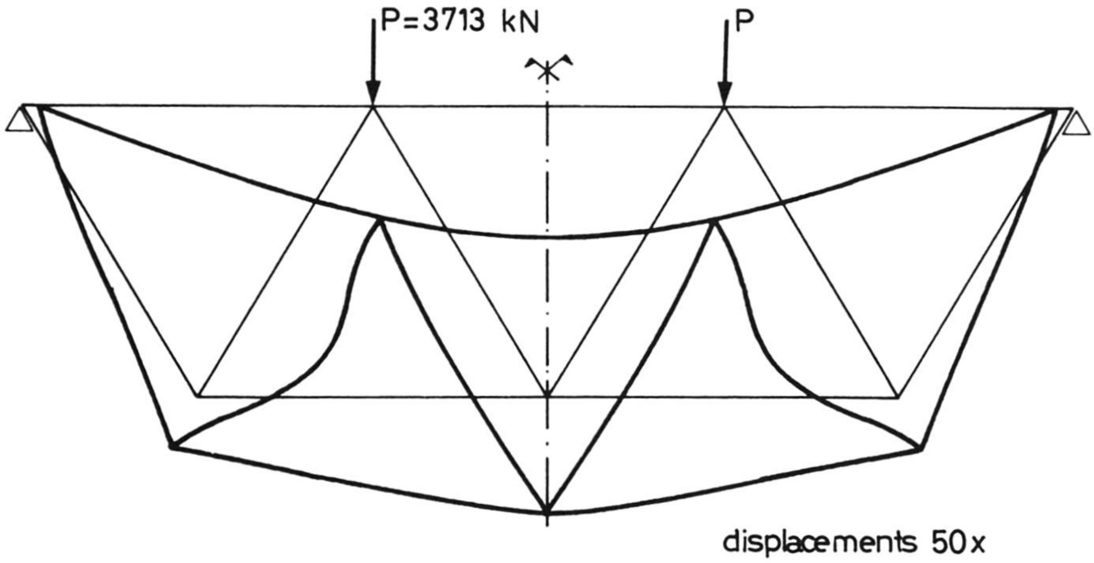


Fig.8. Deflection of the truss nr 7 a) at collapse (see appendix).

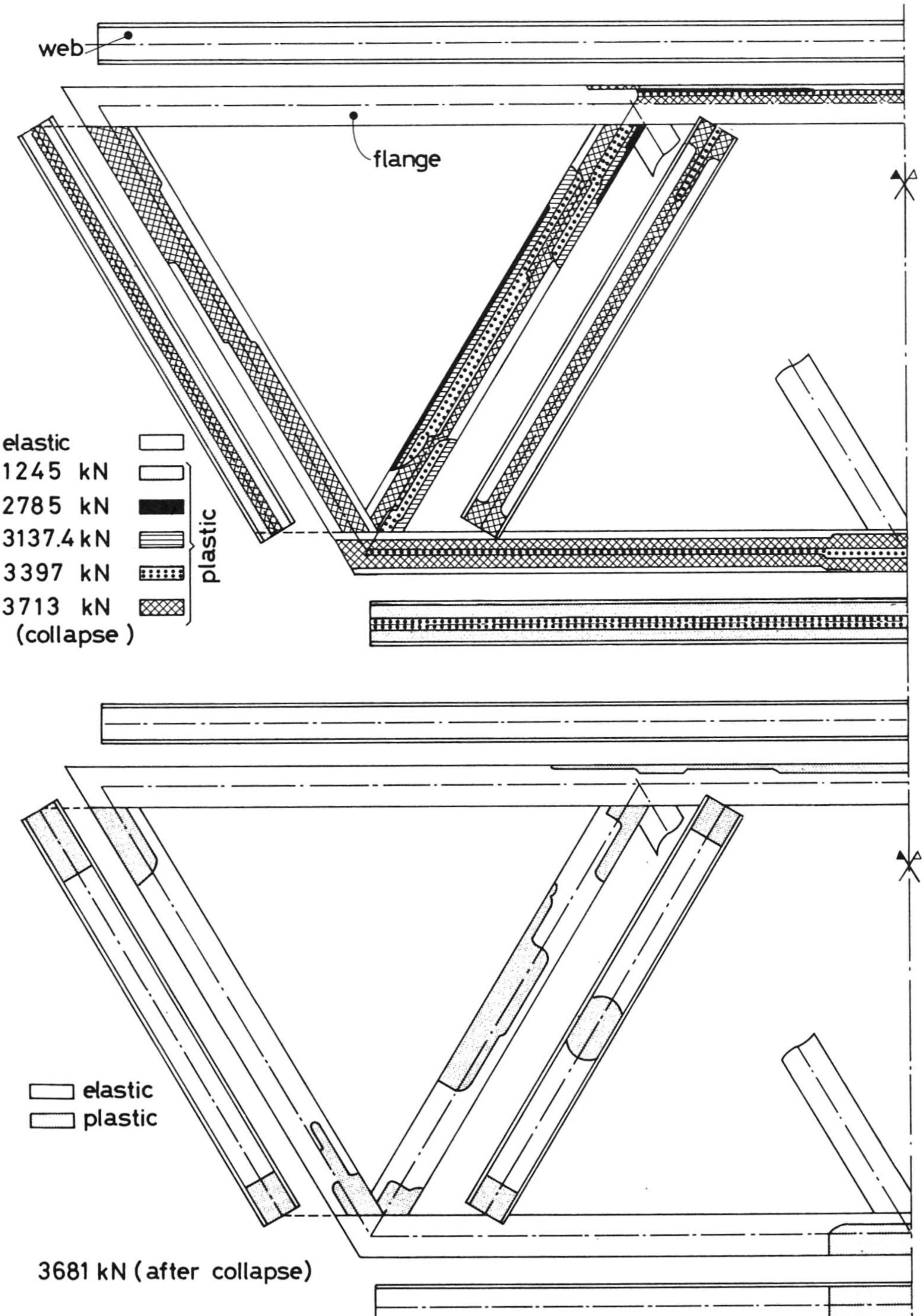


Fig. 9. Yielding pattern of the truss nr 7 a).

These two very different behaviours give us hints about the method to use for design:

- For a frame, a global design is imperative to find the collapse load. Moreover, this design should be second order and elastoplastic, as recommended by the ECCS rules; for instance the RANKINE formula may help to find an order of magnitude.
- For a truss, the "local" design, bar by bar - taking into account the geometry of the problem and the type of loading - seems indicated. One should naturally use the "European buckling curves", which apply to isolated hinged columns loaded centrically. By computing in the reverse sense the slenderness ratio from the computed collapse load of the truss, one finds the so-called "collapse length", and the problem is reduced to the prediction of this "collapse length".

This approach to the problem is corroborated by the numerical step by step calculations. It was observed, in all cases studied, that the behaviour of the truss is quasi linear up to collapse, although plasticity spreads and bars deflect. In particular, the normal forces in the bars of the computed truss are quasi identical (within 2% max) to that of a perfectly hinged truss, even at collapse.

This shows that no normal force redistribution does occur before collapse and that, practically, a truss bar behaves like an isolated bar with elastic end restraints.

Based on the above justifications, the design of all truss bars could be done by means of the European buckling curves, each bar behaving as a pin-jointed one and having as effective length its collapse length. If this approach is sound, the problem to solve is the accurate prediction of the collapse length. This prediction should be based on a linear elastic stability approach, and, in the next section, various attempts are presented.

4.2. Numerical results

We have used the following methods:

- computation of the collapse length by the ECCS curves;
- estimation of the collapse load by:
  - elastoplastic formulae,
  - elastic formulae,

as will be discussed in detail hereafter.

4.2.1. Computation of the collapse length

For each bar investigated, we have adopted the geometric and structural imperfections and the yield stress as recommended by the ECCS:

$f_{max} = (1/1000)\ell$; $\sigma_{res} = \sigma_{res}$ recommended by ECCS ; $\sigma_r = \sigma_r$ (ECCS).

It is therefore logical to obtain the collapse length ℓ_c as follows:

$\overline{N} = N_{max}/A\sigma_r$; $\overline{N} \to \overline{\lambda}$ via the adequate ECCS curve ; $\overline{\lambda} \to \lambda \to \ell_c$.

For the N truss without initial deflection, ℓ_c is computed using both EULER formula and a_0 curve of ECCS; the true value of ℓ_c lies in between.

If we write the collapse length under the form $\ell_c = k_c \cdot \ell$, Table 2 summarizes the values of k_c.

TABLE 2

Truss (*)	ECCS curve used		k_c	
1	a_0	Code 1	0,807	Chords
2 a)	a_0	Code 1	0,847	
b)	a_0	Code 1	0,755	
3 a)	a_0	Code 1 and EULER	0,506 < k < 0,862	
b) (**)	a_0	Code 1 and EULER	0,838 < k < 0,970	
c) (**)	a_0	Code 1 and EULER	0,802 < k < 0,948	
4 a)	a_0	Code 1	0,80	
b) (**)	a_0	Code 1	0,912	
c) (**)	a_0	Code 1	0,876	
5 a)	c	Code 1	0,665	
b)	c	Code 1	0,782	
6	c	Code 1	0,532	Diagonals
7 a)	b	Code 3	0,683	
b)	b	Code 3	0,686	
c)	b	Code 3	0,720	
8	c	Code 1	0,512	
9	b	Code 2	0,704	

(*) see Appendix.
(**) truss with eccentric connections.

4.2.2. Elastoplastic methods

MERCHANT-RANKINE formula

This formula reads

$$\frac{1}{P_c} = \frac{1}{P_p} + \frac{1}{P_E}$$

where P_c = the actual collapse load;

P_p = the limit load given by simple plastic theory, neglecting the change of geometry (first order theory);

P_E = EULER critical buckling load.

MERCHANT, as well as HORNE, have shown that the value given by the generalized RANKINE formula was a good approximation of the actual load for portal frames, including multi-storey frames [M1],[M4].

Figure 10 shows the curve given by RANKINE formula written as follows:

$$P_c/P_p = \frac{1}{1+P_p/P_E}$$

and, for each truss, the points of coordinates $(P_p/P_E, P_c/P_p)$ with P_c = computed

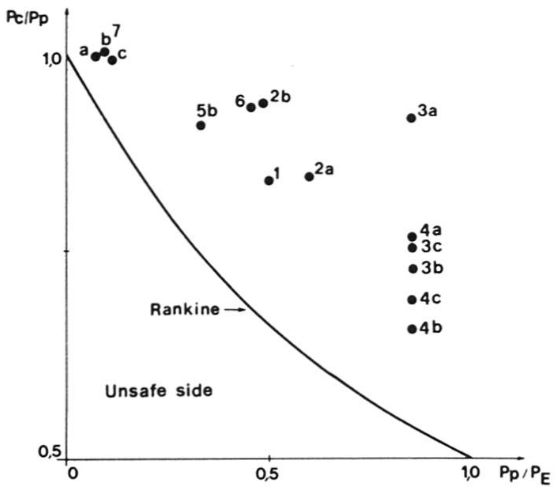


Fig.10. RANKINE formula.

collapse load (program FINEL G).

Thus, for a truss, P_C always exceeds $P_{RANKINE}$; the mean percentage difference is about 20%. Therefore, the use of RANKINE formula, initially developed for beam columns (large values of M), is not advisable for the design of trusses.

Belgian-American interaction formula [M4]

The plastic version of this formula reads

$$\frac{S_r}{\bar{N}\, A\sigma_r} + \frac{M_{eq}}{M_p(1-P/P_E)} = 1 \qquad (4.2.1)$$

where S_r : normal force in the bar;

$\bar{N}$: reduction factor given by the adequate ECCS curve for the collapse length ℓ_c ;

$A\sigma_r$, M_p : squash load and plastic moment of the bar;

$P_E = \pi^2 EI/\ell_c^2$: EULER load of the bar;

$M_{eq} = C\, M_{max}$: equivalent bending moment where:

M_{max} is the maximum first order moment in the bar;

C is a minoration factor depending on the shape of the moment diagram.

As $M_{max}/(1-P/P_E)$ is an approximate value of the second order bending moment in the bar at collapse, it may be replaced by the actual value obtained from the computer program. Thus, ℓ_c remains as the only unknown in (4.2.1) and may be computed.

Table 3 gives the value of $k_I = \ell_c/\ell$ obtained in this way (see below).

A look at the values of Table 3 yields the following comments:

- for the chord, $k_I \simeq k_c$; this shows that the effect of the secondary moments is small;
- for the diagonal, it is not possible to find a value of ℓ_c , because to satisfy equation (4.2.1), we should adopt values of $\bar{N}$ larger than 1. This is due to the influence of the normal force term which is much larger than that of the bending moment term.

However, now that certain values ℓ_c are obtained, it is interesting to apply formula (4.2.1) with the first order secondary moments (the only ones at disposal in an engineer's bureau) to see whether the formula is fulfilled. This computation shows that, if

$$a = S_r/(\bar{N}\, A\sigma_r) \ , \ b = M_{eq}/M_p(1-P/P_E)$$

the following results are obtained:

truss	$P_{collapse}$(kN)	k_I	a	b	a + b
1	815	0,3	0,640	0,049	0,689
4 a)	779,5	0,51	0,829	0,025	0,854
b)	671,6	0,715	0,792	0,124	0,916
c)	707,0	0,25	0,709	0,062	0,771

It is seen that:

- k_I given by (4.2.1) is on the unsafe side (because $a + b < 1$);
- the very small values of b indicate that bending is far from playing a dominant role in a truss.

These results confirm those found in Table 3: the interaction formula, developed to predict the collapse of a beam-column, is not convenient to predict the behaviour of a truss bar which is essentially compressed.

TABLE 3

Truss	k_I	k_H	k_E	k_c	
1	0,29	(0,96)	0,756	0,807	CHORDS
2 a)	- (*)	(0,98)	0,846	0,847	
b)	- (*)	(0,98)	0,760	0,755	
3 a)	0,845	(0,962)	0,763	$0{,}506 < k < 0{,}862$	
b)	0,89	(0,962)	0,761	$0{,}838 < k < 0{,}970$	
c)	0,85	(0,962)	0,763	$0{,}802 < k < 0{,}948$	
4 a)	0,51	(0,962)	0,763	0,80	
b)	0,715	(0,962)	0,761	0,876	
c)	0,25	(0,962)	0,763	0,912	
5 a)	0,45	(0,990)	-	0,665	
b)	0,70	(0,904)	0,752	0,782	
6	(0,3)	0,899	0,639	0,532	DIAGONALS
7 a)	(<0,4)	0,888	0,661	0,683	
b)	(<0,4)	0,876	0,696	0,686	
c)	(<0,4)	0,910	0,638	0,720	
8	(<0,36)	0,862	-	0,512	
9	(<0,2)	0,858	-	0,704	

(*) the bending moment diagrams for these trusses are not given in [D2].

4.2.3. Elastic methods

The elastic methods have a main drawback: they neglect plasticity (decrease of end restraints, increase of bearing capacity of the whole member). However, as explained at the end of 4.1., they should enable to compute an equivalent buckling length to be used as collapse length in connection with the adequate ECCS curve.

Dutch formula

We call Dutch formula the formula given in the Dutch specifications [N1],[N2], which enables to compute $\ell_{eq,b}$ for the web members of the truss:

$$\ell_{eq,b} = k_H \ell$$

$$k_H = 0,7 + 0,3\,\psi$$

$$\psi = \frac{\Sigma\, EI_i/\ell_i}{\Sigma\, EI_k/\ell_k}$$

index i applies to the studied bar + adjacent compressed bars;

index k applies to the studied bar + whole set of adjacent bars.

The values of k_H are summarized in Table 3 (above). It may be noted that:

- the values for the chords are only given as references;
- the formula takes into account the type of loading by treating differently the compressed bars and those subjected to tension;
- for the diagonals, the formula seems to give values always exceeding 0,85; we are thus on the safe side;
- in addition, k_H does not vary in the same sense as k_c .

The general conclusion is that, if the Dutch formula is on the safe side, it will, nevertheless, not help us in increasing the accuracy of the design of truss bars.

Bifurcation approach

When stability is studied assuming linear elastic material and small displacements, a "bifurcation load" is obtained (linearized stability theory). Using a computer code (STABIF [F3]) to analyse the global stability of the whole truss structure, we get:

- the critical load corresponding to the lowest buckling mode (first eigenvalue);
- the associated deflected shape (first eigenvector).

If the weakest bar of the truss is known, one may compute

$$\ell_{eq,b} = \pi\sqrt{EI/S_{cr}} \quad \text{and} \quad k_E = \ell_{eq,b}/\ell .$$

Table 3 (above) gives the values of k_E obtained in this way.

Trusses 5a), 8 and 9 have no k_E values. Indeed, the results given by STABIF do not make sense in these cases. Let us recall that, in these trusses, unlike the others, the compressed chords and diagonals have very different slendernesses. Now, the bifurcation method overestimates very much (like Euler's formula) the buckling strength of short members (Fig.11). Therefore, the weakest bar, in this approach, will be the most slender bar, even if actually it is not the case.

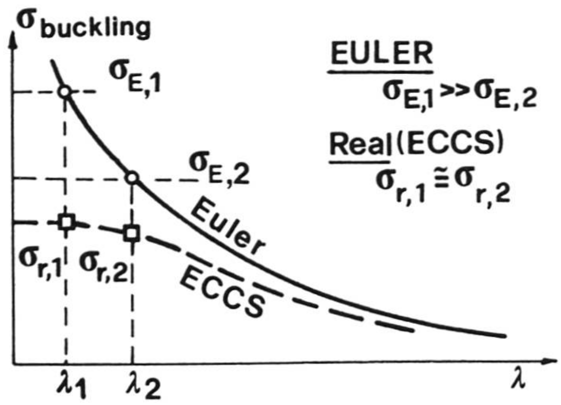


Fig.11. EULER and ECCS buckling curves.

Moreover, Table 3 shows that k_E is by far not always on the safe side, even if the difference is sometimes small. Therefore, this method cannot be retained.

Approximate bifurcation formula [D1]

The bifurcation method takes into account the real elastic restraint conditions of the bar under study, that is, the real geometric shape of the structure. But it often gives too small values of k. On the contrary, the Dutch method protects us against this danger by imposing k values larger than 0,7 (which corresponds to a bar pinned at one end and built-in at the other).

These two remarks were the starting point for the development by one of the authors of an approximate bifurcation formula [D1]. Looking at Figure 12, the

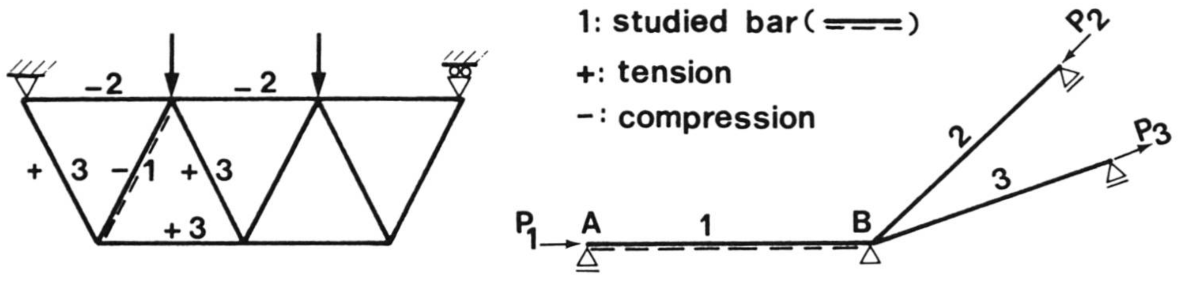


Fig.12. Actual structure (left) and simplified subassemblage (right).

study of the compressed member was based on the following scheme:

- Type 1 bar : is the bar under study, with characteristics E, I_1, ℓ_1, compressed by a force P_1.
- Type 2 bar : all the compressed bars adjacent to bar 1 are replaced by one bar with characteristics E, I_2^*, ℓ_2^*, P_2^*.
- Type 3 bar : all the tensile bars adjacent to bar 1 are replaced by one bar with characteristics E, I_3^*, ℓ_3^*, P_3^*.

When a set of bars with characteristics P_i, I_i, ℓ_i is replaced by a single bar, its characteristics P^*, I^*, ℓ^* are given by the relations

$$I^* = \sum_i (I_i \ell^* / \ell_i) \quad \text{and} \quad P^* = \sum_i (P_i \ell_i / \ell^*)$$

This structural scheme fulfills the various requirements emphasized above, namely:

- like the actual truss, the subassemblage is a fixed node structure;
- a compressed truss bar is never pinjointed at either end; the subassemblage should therefore lead to a safe result;
- k can only become equal to 0,7 if I_2 and I_3 tend to infinity.

In line with TIMOSHENKO [T1], the critical multiplier λ_{cr} is the solution of the following transcendental equation:

$$\left[I_1K_1\psi(U_2) + I_2K_2\psi(U_1)\right]\varphi(U_3) - K_3I_3\psi(U_1)\psi(U_2) = 0 \qquad (4.2.2)$$

with

$$\psi(U_i) = \cot K_i\ell_i - 1/K_i\ell_i$$

$$\varphi(U_3) = \coth K_3\ell_3 - 1/K_3\ell_3$$

$$K_i = \sqrt{\lambda_{cr}P_i/EI_i}$$

This equation may be solved by iteration. As it possesses many roots, we must make sure that the approximations converge towards the multiplier λ_{cr} of the first buckling mode with $\ell_{eq,b} > 0,7\,\ell$. Its solution has been programmed for a pocket computer.

Table 4 gives three types of effective length computed by this formula:

$$\ell_{eq,b} = k_{Ei}\ell$$

k_{E1} : takes into account all bars;

k_{E2} : takes into account the tensile bars only;

k_{E3} : is like k_{E2}, but with all tensile forces made equal to zero.

The disadvantages and advantages of this approximation may be summarized as follows:

Disadvantages:
- the material is purely elastic;
- the use of the method is less convenient than WOOD's or similar charts (see hereafter).

Advantages :
- except for trusses with eccentricities, k_{E1} is always on the safe side, while remaining reasonably accurate;
- k_{E1} changes always in the same sense as k_C ;
- the method is based on the concept of one bar restrained at one end by the adjacent ones. The concept of global buckling is therefore left aside and the problem created by trusses with bars having very different slenderness ratios does not exist anymore;
- the bar restraints depend not only on the geometric characteristics of the adjacent bars but also on the values of the normal forces in these bars and therefore on the external loading;
- the comparison of the values k_{E1} and k_{E2} indicates whether or not the bar under study is supported by the adjacent compressed bars.

DONNELL's formula - JOHNSTON's and WOOD's charts

We started our investigation by the global bifurcation method where the structure was studied as a whole. Because of its disappointing results, we have simplified to obtain formula (4.2.2), from which k_{E1}, k_{E2} and k_{E3} were computed. Still simplifying, we assume now the bar under study to be elastically restrained in rotation at both ends. Therefore, it may be asked whether the use of a formula or chart giving the effective length of such a restrained bar cannot give better results.

Before answering this question, it is necessary to underline two statements:

- The use of such a formula (or chart) should not be a better approach than the solution provided by the global bifurcation method, which, as we have seen, does not give satisfactory results.
- The various approaches for restrained bars (DONNELL's formula, JOHNSTON's or WOOD's charts, etc...) utilize elastic restraint coefficients which do not take into account either the sign or the value of the adjacent normal force. Now, we have seen (cf. Table 4) that, sometimes, the compressed adjacent bars, instead of supporting the bar under study, increase its instability. This is the reason why, to test the above approach, only the bars subjected to tension will be taken into consideration.

As in preceding developments, the adjacent bars are assumed to be hinged at their far ends.

Using DONNELL's approximate formula [M2], the critical load of the restrained bar (Fig.13) is given by

$$P_{cr} = n\,\pi^2 EI/\ell^2$$

$$\text{with} \quad n = \frac{1+2,9(f_1+f_2)+7,2f_1f_2}{1+1,4(f_1+f_2)+1,8f_1f_2} \quad \text{and} \quad f_i = \frac{\ell}{6,5\,EI} R_i$$

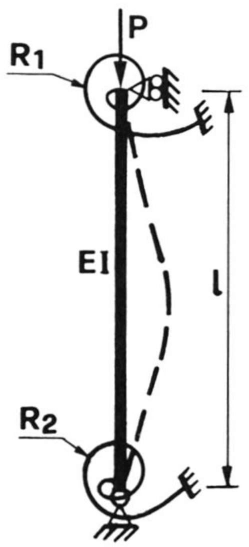


Fig.13. Subassemblage for DONNELL's formula.

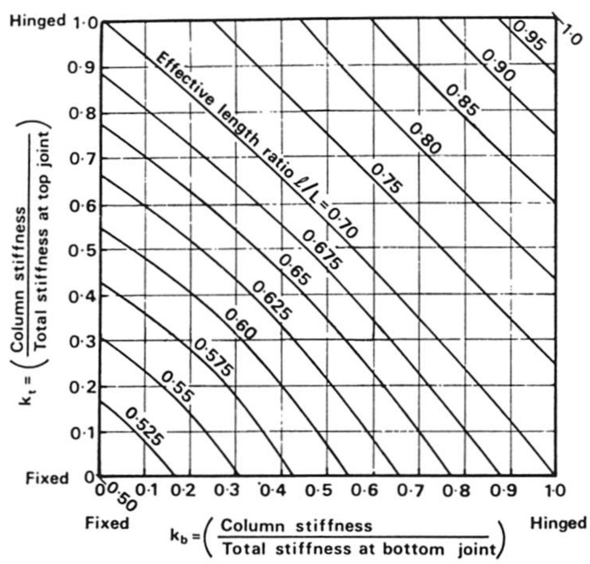


Fig.14. WOOD's chart [W1].

The coefficient $R_i = \sum_j 3 \frac{EI_j}{\ell_j}$ characterizes the restraint of the j adjacent bars.

We deduce $k_D = \sqrt{n}$ (see Table 4).

We recall here briefly only the chart developed by WOOD [W1] (Fig.14). JOHNSTON's approach differs only in its presentation [J2]. Results are evidently nearly identical to those found by DONNELL, because all three treat the same problem (see k_W in Table 4).

Finally, we could evaluate as follows the merits of this approach:

Advantages : - k_D is slightly less than k_{Ei} for the diagonals;
- k_D is always on the safe side;
- DONNELL's formula is easy to use.

Disadvantages: - k_D is larger than k_{Ei} for the chords;
- the elastic restraint coefficients do not depend on the load.

TABLE 4

	Approximate bifurcation formula [D1]			DONNELL	WOOD and JOHNSTON		
	k_{E1} ($S \gtrless 0$)	k_{E2} ($S>0$)	k_{E3} ($S=0$)	k_D	k_W	k_c	
1	0,860	0,889	0,889	0,879	0,89	0,807	CHORDS
2a)	0,953	0,933	0,965	0,961	0,965	0,847	
b)	0,841	0,933	0,965	0,961	0,965	0,755	
3a)	0,817	0,849	0,905	0,899	0,90	0,506 < k < 0,862	
b)	0,817	0,849	0,905	0,899	0,90	0,838 < k < 0,970	
c)	0,817	0,849	0,905	0,899	0,90	0,802 < k < 0,948	
4a)	0,817	0,849	0,905	0,899	0,90	0,80	
b)	0,817	0,849	0,905	0,899	0,90	0,912	
c)	0,817	0,849	0,905	0,899	0,90	0,876	
5a)	0,772	0,933	0,933	0,920	0,920	0,665	
b)	0,901	0,908	0,908	0,89	0,90	0,782	
6	0,739	0,750	0,779	0,702	0,705	0,532	DIAGONALS
7a)	0,744	0,749	0,778	0,717	0,72	0,683	
b)	0,754	0,749	0,778	0,717	0,72	0,686	
c)	0,749	0,759	0,791	0,791	0,785	0,720	
8	0,718	0,735	0,767	0,698	0,690	0,512	
9	0,739	0,751	0,771	0,722	0,73	0,704	

5. CONCLUSIONS

5.1. For or against elastoplastic computation

The rigidly jointed frames are essentially bent structures, while trusses are essentially axially loaded. It is therefore understandable that refined computations reveal a different behaviour for each of the two kinds of structures. A frame and a truss obey respectively to interaction curves (N/N_p , M/M_p) of the types 1 and 2 (Fig.15).

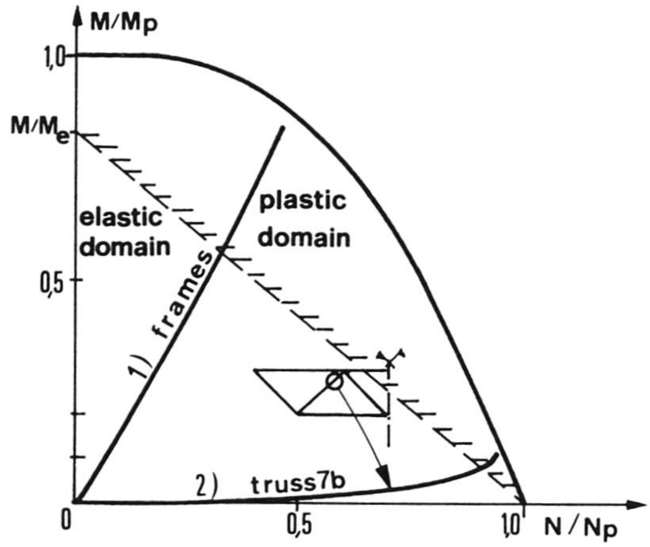


Fig.15. Interaction curve (N/N_p , M/M_p).

In opposition to the truss, the frame reaches very early the yielding zone, which justifies fully the redistribution of internal forces due to plastification. Then, the preceding considerations explain that the interaction formula (which is the basis for the design of beam-columns) is not adequate for truss bars.

In these conditions, the use of methods based on a purely elastic behaviour of the material seems adequate to determine the effective length of buckling of truss bars.

Indeed, we think (recall Fig.7 to 9) that the type of collapse observed shows that the design "bar by bar" and the concept of effective length are correct.

5.2. Computation of the collapse load

It has been shown that trusses may be designed with sufficient accuracy one bar at a time, by using the concept of effective length and by distinguishing trusses with centric bars from trusses with eccentric bars. Once computed, the effective length is considered as equal to the collapse length and the bar is designed by using the European buckling curves ECCS.

5.2.1. Trusses with centric bars

Above all, it must be emphasized that the present study [D1] applies only to truss members with slenderness larger than 50.

With this restriction, the results discussed show that

- the buckling effective lengths are , for the chords, comprised between 0,75 and 0,95 ℓ and, for the diagonals, less than 0,8 ℓ;
- these effective lengths should be determined by an approximate method based on the bifurcation theory. The global bifurcation approach should, however, be abandoned.

Two methods are recommended. Their choice will depend on the aim of the designer.

a) Approximate bifurcation formula

As this formula takes into account the value and the sign of the forces in the adjacent bars, we may consider the stiffness of the compressed bars without fear of overestimating the elastic restraint. Moreover, the comparison of k_{E1} and k_{E2} enables to have an idea of the equal strength design:

- if $k_{E1} < k_{E2}$, bar 2 induces buckling of bar 1;
- if $k_{E1} > k_{E2}$, bar 2 restraints bar 1;
- if $k_{E1} = k_{E2}$, "simultaneous" buckling of bars 1 and 2 may occur.

b) DONNELL-JOHNSTON-WOOD approaches

These three methods are equivalent; they are based on the concept of the bar axially loaded and elastically restrained against rotation at both ends. To obtain safe results, it is recommended to neglect the stiffness of the adjacent compressed bars and to consider that the stretched bars are hinged at their farther ends. The advantage of this second approach is its simplicity.

5.2.2. Trusses with eccentric bars

For these trusses, it is recommended:

1.- to avoid eccentricities (first source of yield) if the slenderness of the compressed bar is small (second source of yield);

2.- for the design against buckling, to choose one of the following two possibilities:
  a.- use $\ell_c = \ell$;
  b.- use $\ell_c = k\ell$ (where k is computed as recommended in section 5.2.1) and utilize the interaction formula (4.2.1) after having computed the secondary moments
    - either exactly;
    - or as follows:

      the moment due to eccentricity is $M \simeq Te/2$ (Fig.16); the part of this moment taken by the bar under study is $M_1 = k_1 M$ with $k_1 = (I_1/\ell_1)/(I_1/\ell_1 + I_2/\ell_2)$.

As the bar 1 is supposed to be hinged at the far end, the distribution of moments is triangular and C = 0,548. Refined numerical calculations have shown that, in this way, satisfactory results are obtained.

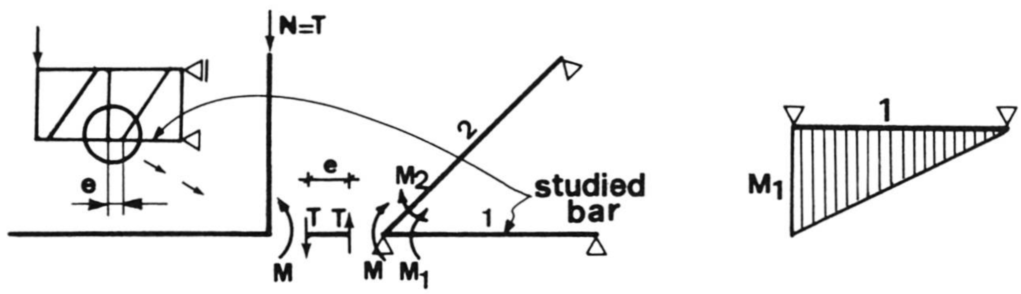
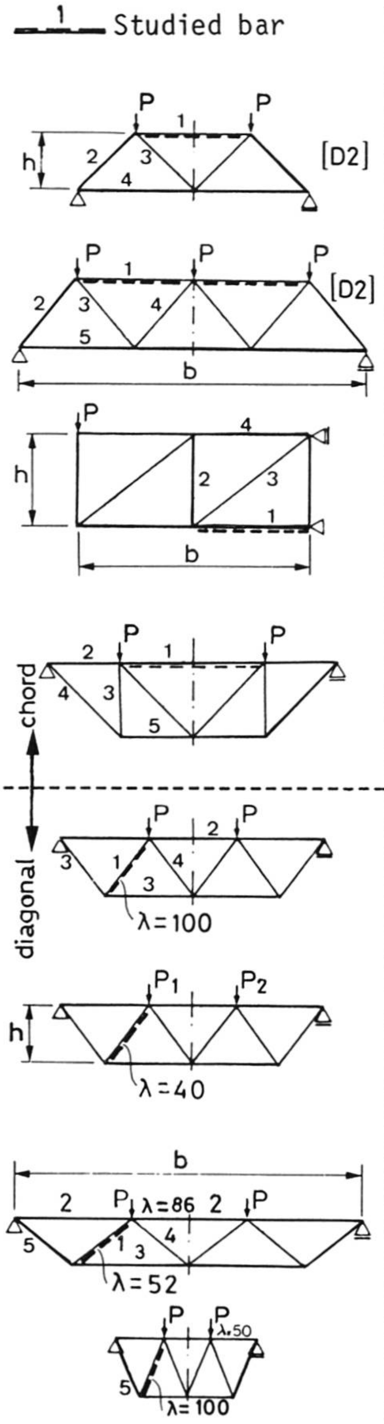


Fig.16. Simplified method for secondary moment M_1.

5.3. Future developments

The present study is uniquely devoted to the problem of flexural buckling in the plane of the truss. The study should be completed by a closer examination of the behaviour of trusses containing very stocky bars ($\lambda = 25 - 30$). Indeed, for such bars, plastic effects should be more important and the behaviour of these bars would therefore be closer to that of the columns of a rigid frame. Then, one should tackle the spatial instability of trusses.

APPENDIX - Trusses simulated on computer



Truss nr	b (cm)	h (cm)	Type of cross-section for bar nr: 1	2	3	4	5
1	1000	250	HEA200	HEA200	HEA200	HEA120	
2 a)	1500	250	HEA200	HEA200	HEA100	HEA120	HEA180
b)	1500	250	HEA200	HEA200	HEA100	HEA180	HEA180
3 & 4 a)	1000	400	MSH 100 ×10	MSH 100 ×6,3	MSH 100 ×10	MSH 120 ×60 ×6,3	
b)	1000	400					
c)	1000	400					
	3: bars without initial deflections 4: bars with initial deflections b), c) : joints with eccentricities						
5 a)	1000	250	HEB180	HEB160	HEB160	HEB160	HEA180
b)	1000	250	HEA260	HEM140	HEM140	DIL240	DIL200
6	900	250	HEB100	HEB120	HEA100	HEA100	
7 a)	900	250	HEM280	HEM280	HEM240	HEB240	
b)	900	250	HEM280	HEM260	HEM240	HEB240	
c)	900	250	HEM280	HEM280	HEM240	HEB240	
	a) and b): $P_1 = P_2 = P$; c): $P_1 = 1{,}5\ P$; $P_2 = 0$						
8	1500	150	HEA220	HEM220	HEM220	HEB160	HEB220
9	450	250	HEA120	HEM100	HEB120	HEA100	HEM100

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MAIN NOTATIONS

ℓ : length of the bar between centers of connections.

λ : slenderness ratio of the bar computed as ratio of its length ℓ by its radius of gyration i : $\lambda = \ell/i$.

ℓ_b : the buckling length is the length of a perfect fictitious member, hinged at both ends, which would have the same critical load as the given member, supposed equally perfect (this definition is only valid if the bar is isolated, loaded only at its ends, of constant cross section, and perfectly straight).

ℓ_c : the collapse length is the length of the fictitious member hinged at both ends that has:

- the same geometric and mechanical properties as the actual bar;
- the same collapse load as the bar placed in a structure in a definite environment (loading, type of truss, etc...).

$\ell_{eq,b}$: same definition as ℓ_c , but by adopting a purely elastic bar.

k : ratio of the buckling or collapse length of the bar by its length (ℓ_b/ℓ , ℓ_c/ℓ , ...).