Design of Slender Reinforced Concrete Columns

Dimensionnement de poteaux élancés en béton armé

Bemessung von schlanken Stahlbeton-Druckgliedern

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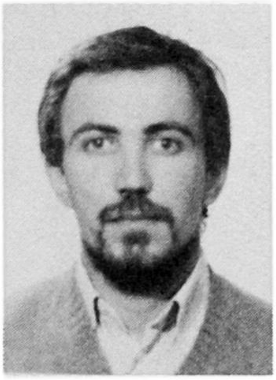

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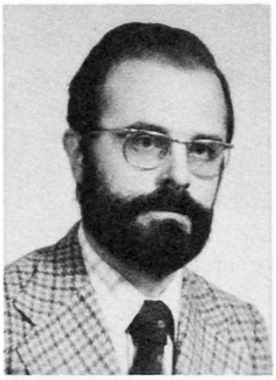


Francisco Morán, born 1940, got his civil engineering degree at the Polytechnical University of Madrid. He does research work on Structural Concrete and has been an active member of CEB since 1970.

SUMMARY

The so-called Reference Curvatures Method is presented. This is a general method for the direct design of slender hinged reinforced concrete columns of constant cross-section (concrete and reinforcement). Design charts and approximate formulae for practical application can be easily derived from the method, as shown, giving a direct design of slender columns. Some numerical examples are also presented.

RÉSUMÉ

L'article traite de la méthode des courbures de référence. Cette méthode permet le dimensionnement direct de poteaux élancés, biarticulés, en béton armé à section constante (béton et armatures). Abaques et formules approchées sont aisément déduits de la méthode et permettent son application pratique. Quelques exemples numériques sont présentés.

ZUSAMMENFASSUNG

Die sogenannte Referenzkrümmungs-Methode wird vorgestellt. Diese ist eine generelle Methode für die direkte Bemessung von schlanken, zweigelenkigen Stahlbeton-Druckgliedern mit gleichbleibendem Querschnitt (Beton und Bewehrung). Bemessungsdiagramme und Näherungsformeln für die praktische Anwendung können aus dieser Methode abgeleitet werden, deren Gebrauch durch ein numerisches Beispiel erklärt wird.

1. INTRODUCTION

The analysis of slender reinforced concrete columns is made difficult by the facts of the mechanical non-linear behaviour of reinforced concrete as a material and of the geometrical non-linear behaviour of the column as an one member structure. To compute the true deflections of the column under these circumstances, the reinforcement must be known. Therefore, the design is an iterative process, and still more time consuming than the analysis. Existing methods, like the Model Column Method [2] and the Sinusoidal Deflection Method [3], [4], allow direct design, but only if unpractically big collections of design charts have been previously prepared.

The Reference Curvatures Method [1] aims at simplifying the design for the standard and most usual case of the slender hinged reinforced concrete column with constant cross-section (concrete and reinforcement). The design problem is that of finding the strict (minimum) reinforcement quantity for the cross-section of the column (see $4 below).

The mechanical non-linearity of reinforced concrete is taken into account through the moment-curvature diagrams of the column cross-section, which are used in form of internal eccentricity-curvature diagrams, called mechanical behaviour curves (fig. 1).

The geometrical non-linearity of the column is taken into account adopting a known deflection shape (or curvature distribution) for the column. This hypothesis defines an external eccentricity-curvature diagram, called geometrical behaviour curve (fig. 1).

A slender column can reach two different ultimate limit states: strength or instability failure. The method makes a clear distinction between them.

The basic idea of the method is to use for the design not the whole set of mechanical behaviour curves for different reinforcement ratios, but only two characteristic points of each curve, namely one for strength and another for instability failure. These points form the so-called reference curvature curves, which can be easily defined and facilitate the design.

2. HYPOTHESIS ABOUT THE BEHAVIOUR OF THE CROSS-SECTION

The following hypothesis are used to define the behaviour of the column cross-section, according to CEB [2] :

- A defined stress-strain diagram for the concrete (for practical reasons the parabola-rectangle diagram was chosen, but any other diagram could have been selected).
- A defined stress-strain diagram for the reinforcement (a bilinear diagram was chosen, but other diagram could have been selected).
- A definition of the possible strain diagrams at the ultimate limit state or strain criterium for strength failure (this was chosen according to the CEB Model Code).

The moment-curvature diagrams, or the mechanical behaviour curves (fig. 1) can be easily derived from these hypothesis.

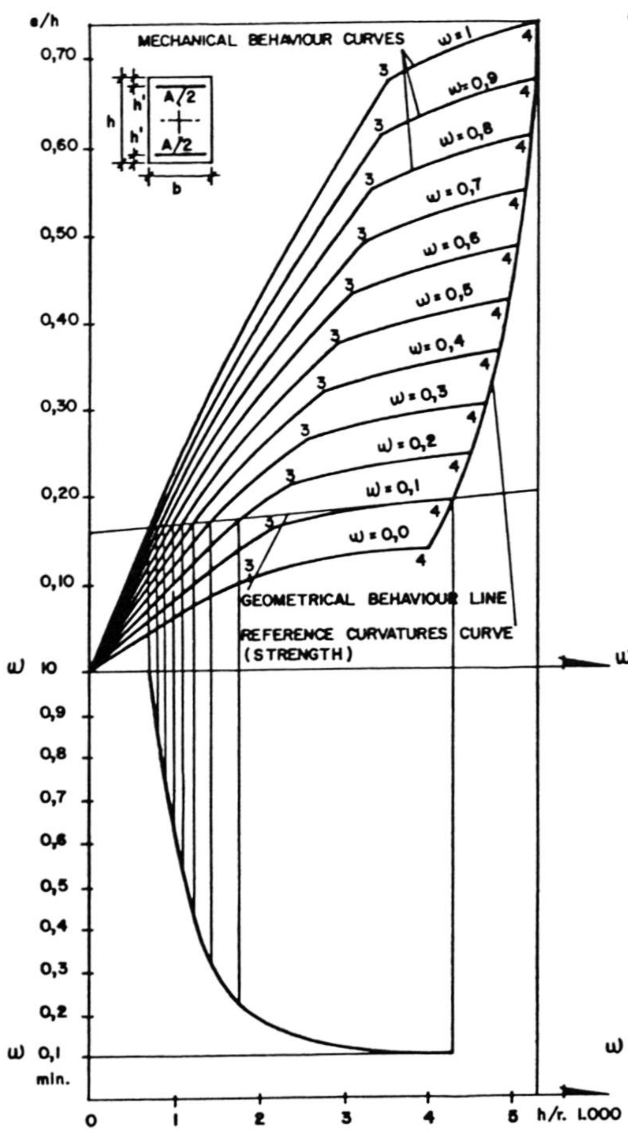


Fig. 1 Ultimate limit state of strength ν = -0, 6

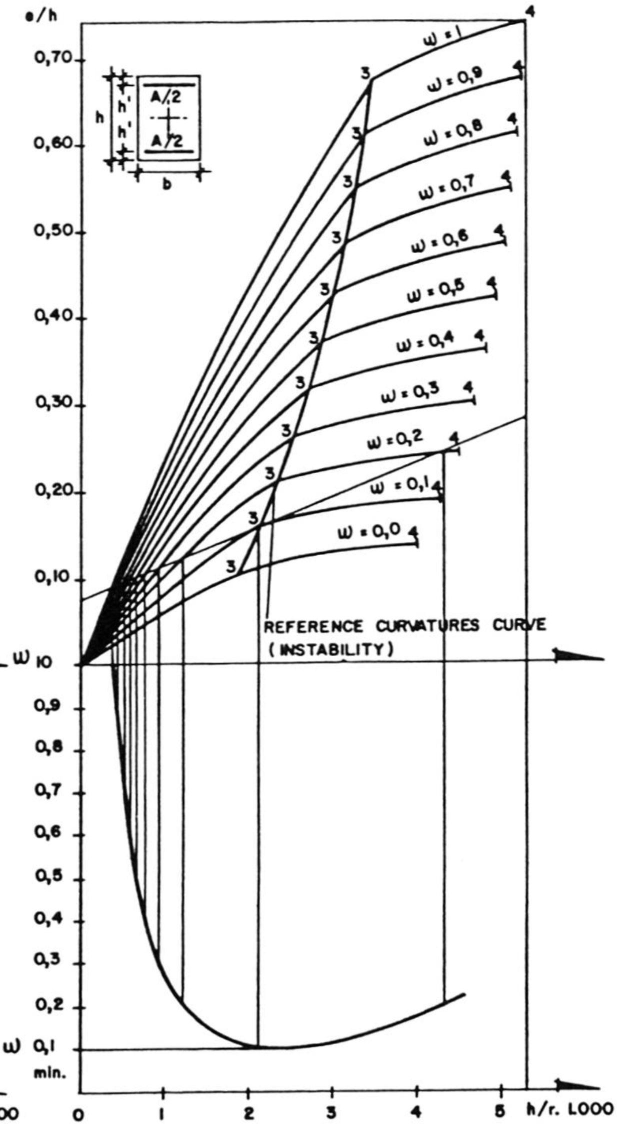


Fig. 2 Ultimate limit state of instability ν = -0, 6

3. HYPOTHESIS ABOUT THE BEHAVIOUR OF THE COLUMN

The Reference Curvatures Method can admit any predetermined deflection shape or curvature distribution for the column, resulting in a relationship between the total external eccentricity at the critical cross-section and the curvature at this section, i.e., a geometrical behaviour curve (fig. 1).

For example, the hypothesis of sinusoidal curvature distribution can be adopted, as in the Model Column Method [2] , or the hypothesis of sinusoidal deflection, as in the Sinusoidal Deflection Method [3] , [4] . This latter hypothesis is believed to be more correct for a column with different end eccentricities.

4. THE REFERENCE CURVATURES METHOD

The data for the design of a reinforced concrete slender column are:

- l_o = effective buckling length, i.e., length of equivalent hinged column;
- e^I_1, e^I_2 = first order eccentricities at the ends;
- N = axial force.

The designer can select, applying practical and constructional criteria:

- form of the cross-section and dimensions of it;
- distribution (arrangement) of the reinforcement;
- f_{cd}, f_{yd} = design strengths of concrete and reinforcing steel.

The design problem is to find the strict (minimum) reinforcement quantity for the cross-section of the column.

If the hypothesis of column behaviour of the Model Column Method is adopted, a linear relationship between the total external eccentricity at the critical cross-section and the curvature at this same section, or geometrical behaviour line, can be defined for the column (fig. 1):

$$\frac{e}{h} = \frac{e^I}{h} + \frac{h}{r}\,\frac{\lambda^2}{10} ,$$

where

$$\frac{e^I}{h} = \frac{e^I_1}{h} = \frac{e^I_2}{h} \qquad \text{for} \quad \frac{e^I_1}{e^I_2} = 1 ;$$

$$\frac{e^I}{h} = \left(0,6 + 0,4 \cdot \frac{e^I_1}{e^I_2}\right) \frac{e^I_2}{h} \qquad \text{for} \quad \frac{e^I_1}{e^I_2} \neq 1 .$$

On the other hand, with the (reduced) axial force, the form of the cross-section, the distribution (arrangement) of reinforcement and the design strengths of the materials, the moment-curvature diagrams for different reinforcement ratios can be drawn. These diagrams are conveniently represented in terms of internal eccentricities-curvatures, or mechanical behaviour curves of the cross-section, and are shown also in fig. 1.

a) Let it be a column in which the strength failure of the critical cross-section is reached, as is the case in fig. 1. The strict (minimum reinforcement) design is the one corresponding to the mechanical behaviour curve which intersects at its rightmost (strength failure) point the geometrical behaviour line.

For a design with a reinforcement ratio ω higher than this minimum, the column is in a stable equilibrium state at the point in which the mechanical behaviour curve intersects the geometrical behaviour line. In this case, the failure is not reached.

If these designs are represented in an ω - h/r diagram, the result is a descending curve, whose rightmost point corresponds to the strict design (fig. 1).

Therefore, the set of mechanical behaviour curves, of cumbersome determination, can be omitted and substituted by the so-called reference curvatures curve of strength (fig. 1). This curve is formed by the last (rightmost) points of the mechanical behaviour curves, and represents the relationship between eccentricity and ultimate curvature (e/h versus h/r) for a reduced axial force (ν) and different reinforcement ratios (ω). The intersection of this curve with the geometrical behaviour line gives directly the reinforcement ratio ω corresponding to the strict design of the column. In a short column ($\lambda = 0$) this line is horizontal and the resulting ω is the same as using an interaction diagram.

b) Let it be now a column in which the instability failure is reached, as is the case in fig. 2. The strict design is the one corresponding to the mechanical behaviour curve which is tangent to the geometrical behaviour line in a point whose curvature is lower than the strength failure curvature. For a design with a higher ratio ω, the column can find a stable equilibrium state and the failure is not reached.

If these designs are represented in an ω - h/r diagram, the result is a curve with a left descending zone and a right ascending zone. The minimum of the curve gives the mechanical ratio corresponding to the strict design. The left zone corresponds to stable equilibrium states and the right zone corresponds to unstable equilibrium states.

The point of the mechanical behaviour curve at which the instability occurs can be any of the points of this curve, depending on the geometrical behaviour line. Nevertheless, a detailed study of the mechanical behaviour curves has resulted in the following conclusions:

1. For mechanical behaviour curves corresponding to low axial forces ($\nu \geq -0,4$)the point corresponding to the yield of the lower layer steel is a critical point at which the slope of the curve makes a sudden change (point 2 in the upper curve of fig. 3).

2. For mechanical behaviour curves corresponding to medium and high axial forces ($\nu \leq -0,4$) the point corresponding to the yield of the upper layer steel is a critical point at which the curve makes a sudden change of slope (point 3 in the lower curves of fig. 3).

3. In most cases the ultimate limit state of instability corresponds to one of these points.

4. The ω - h/r curve has a relative minimum for the curvature corresponding to the strict design. This curve is rather plane about this point. Hence, any error in the curvature will result in a small error in the design value of ω and always be on the safe side.

Consequently, a reference curvatures curve of instability can be defined (fig. 2). This curve is formed by the critical points (i.e., those points which have sudden changes of slope) of the mechanical behaviour curves, and represents the relationship between eccentricity and probable instability curvature (e/h versus h/r) for a reduced axial force (ν) and different reinforcement

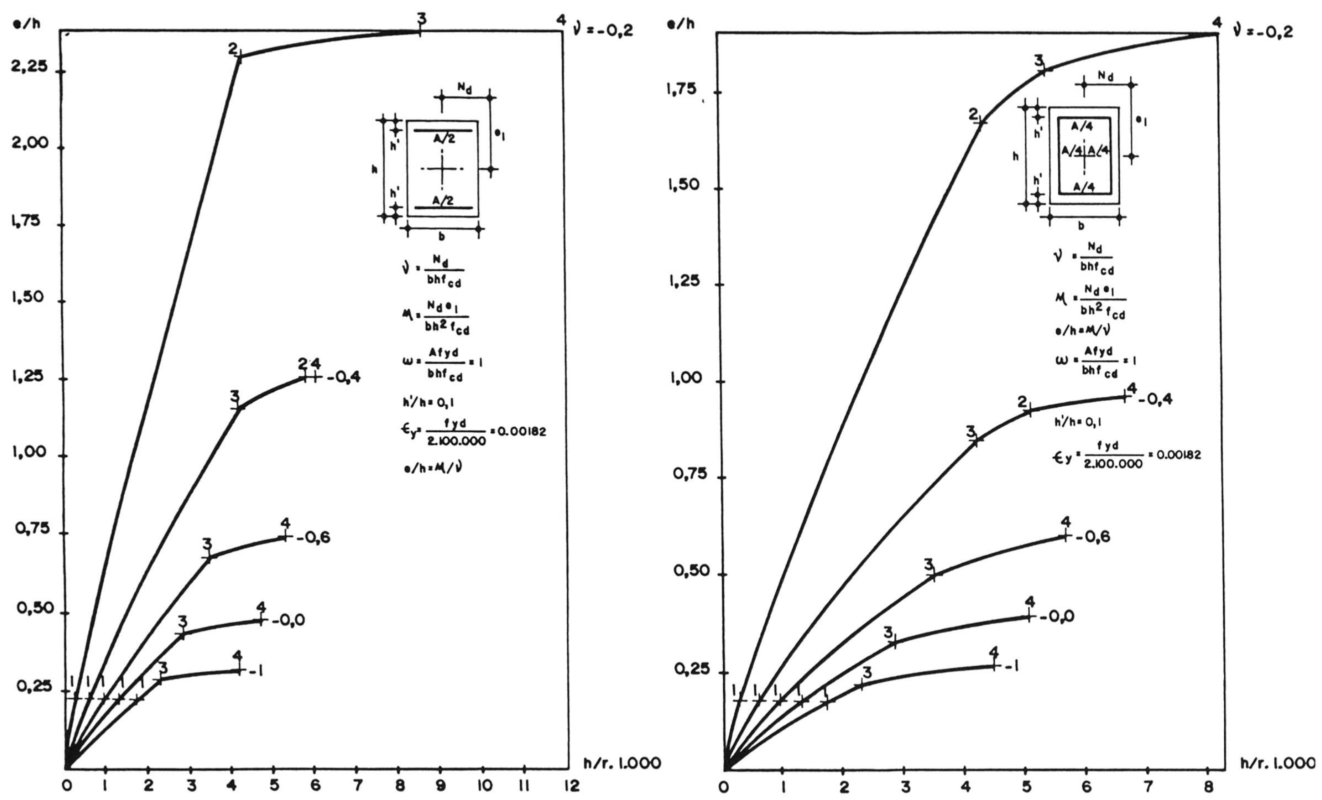


Fig. 3 Characteristic points of the mechanical behaviour curves

ratios (ω). The intersection of this curve with the geometrical behaviour line gives directly the reinforcement ratio ω corresponding to the strict design in case of instability failure, or at least a good estimate of it which will always lie on the safe side.

As we usually do not know which type of failure (strength or instability) will be reached by a given column, we must try the intersection points of the geometrical behaviour line with both reference curvature curves. We shall obtain two different values of the reinforcement ratio ω . The lower of them will correspond to the strict design of the column.

5. DESIGN CHARTS

Design charts for different types of cross-sections have been prepared for the graphical application of the Reference Curvatures Method. Some of them are presented on figs. 4 and 5. Two charts must be prepared for each section type, one of them for axial forces $\nu \geq -0,4$ (fig. 4) and another for axial forces $\nu \leq -0,4$ (fig. 5). The reference curvatures curves of strength and instability for the different values of ν are presented on the right hand side of the charts. Each point of these curves corresponds to a value of the reinforcement ratio ω. To obtain these values two sets of lines ω - e/h are presented on the left hand side of the graphs. These two sets correspond to the strength and instability failures, and to the same values of ν as the curves on the right hand side.

The procedure for the use of the charts is as follows:

1. The chart corresponding to the cross-section type and to the value of ν (i.e. $\nu \geq -0,4$ or $\nu \leq -0,4$) must be selected.

2. The geometrical behaviour line of the column must be drawn on the coordinate system h/r - e/h of the right hand side of the chart (fig. 5). This line will intersect the reference curvatures curves of strength and instability in two points.

3. Two horizontal straight lines must be drawn from these points leftwards until they intersect the corresponding lines on the left hand side of the charts on two points (fig. 5).

4. The reinforcement ratios ω for strength and instability failure corresponding to these two points can be directly read on the abscissae axis ω (fig. 5).

5. The lower of these two values of ω will give the strict design of the column and indicate the type of failure (strength or instability) of it.

The creep effect due to permanents loads can be taken into account using the linear approach method ([2] , paragraph 3.2.1), i.e., introducing an additional eccentricity e_c due to creep deformation.

To check the accuracy of the RCM based Design Charts, an analytical comparison has been made with the Model Column Method [2] and with the Sinusoidal Deflection Method [3], [4]. In this comparison 450 interaction diagrams have been considered, namely for:

- rectangular cross-section with reinforcement distributed along two opposite sides; with 8 equal bars; with r. distributed along the four sides.
- $e^I_1/e^I_2 = 1;\ 0;\ -1$.

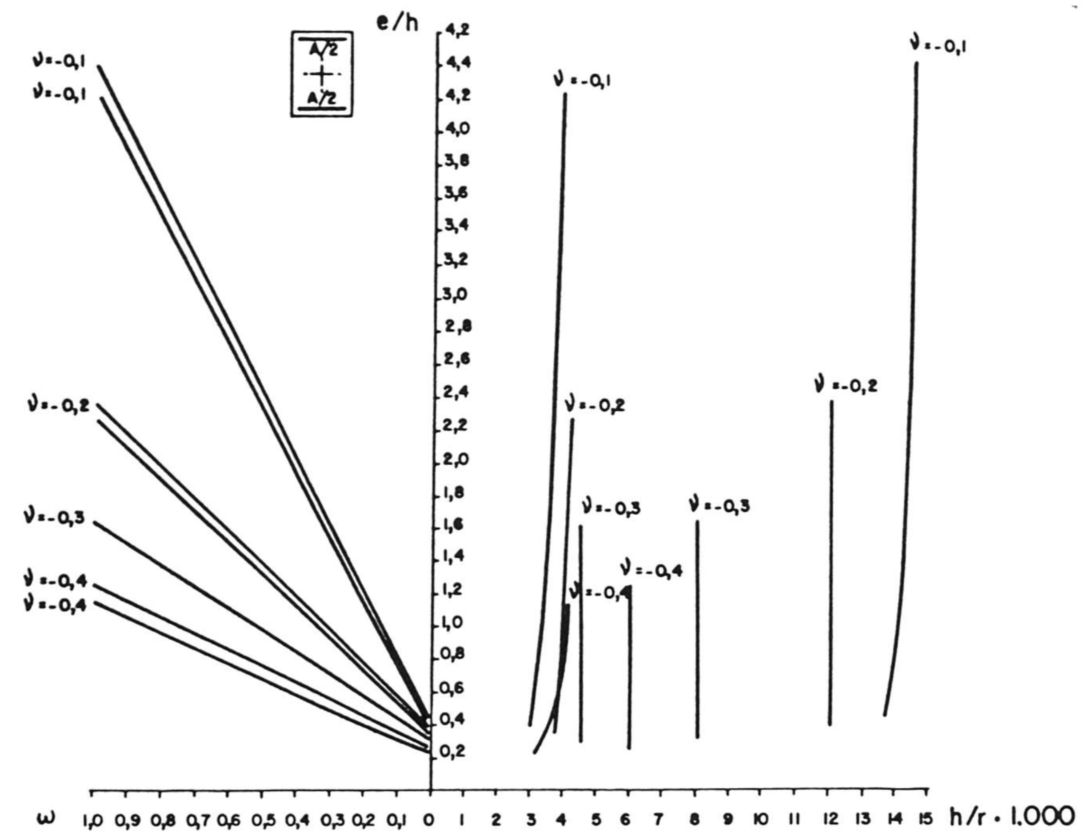


Fig. 4 Design chart for the Reference Curvatures Method $\nu \geq -0,4$

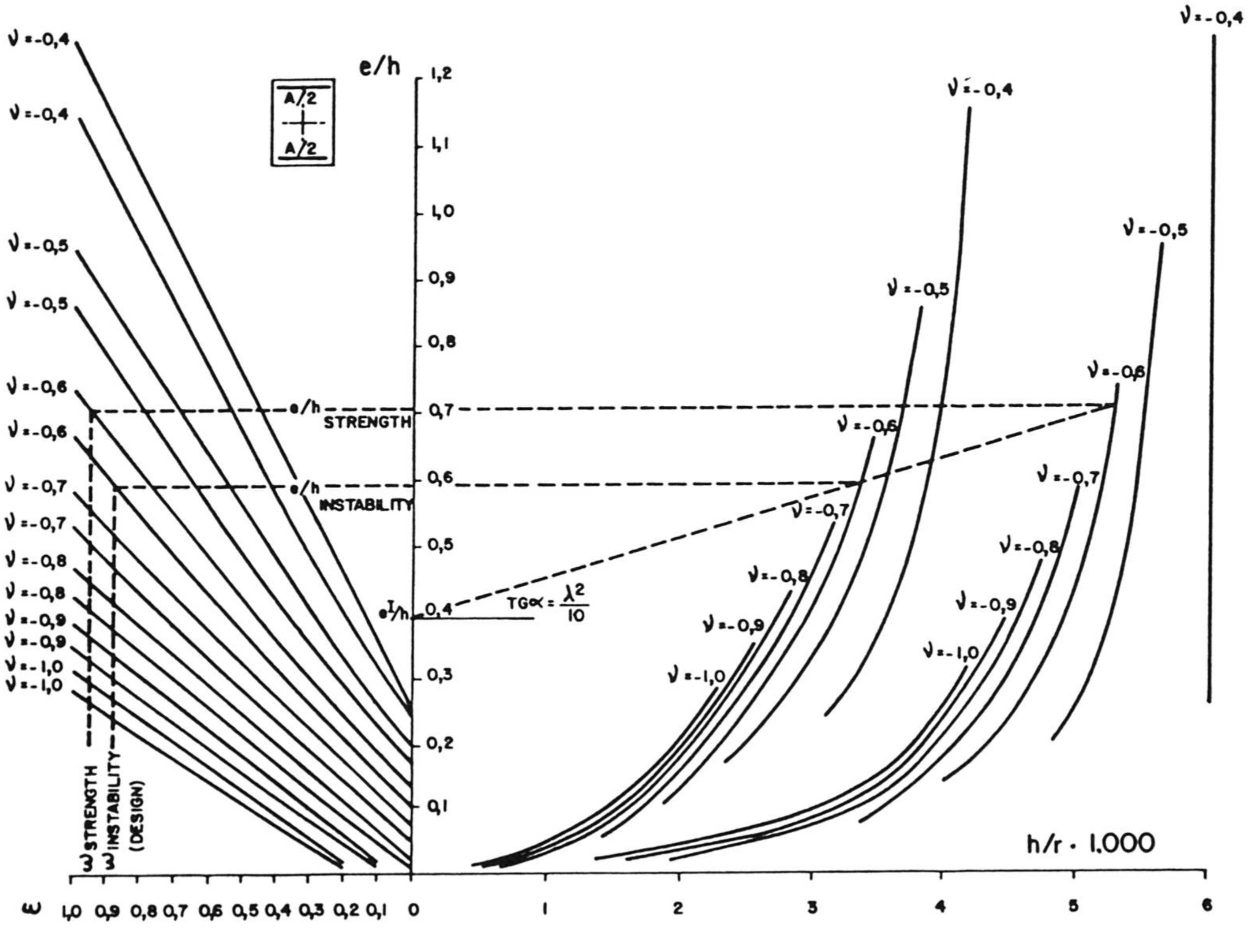


Fig. 5 Design chart for the Reference Curvatures Method $\nu \leq -0,4$

- λ = 0; 10; 15; 20; 25 .
- ω = .1; .2; .3; .4; .5; .6; .7; .8; .9; 1.0 .

For each interaction diagram 9 to 19 points have been checked, corresponding to values of the axial force ν = 0; .1; .2; ...

For each point the error of the Design Charts has been computed as:

$$e\ (\%) = \frac{\nu_p - \nu_b}{\nu_b} \cdot 100 \quad ,$$

with ν_p = reduced axial force resisted by the column, according to the proposed method (RCM based Design Charts);

ν_b = reduced axial force, acting with the same first order eccentricity, resisted by the column according to the base method (MCM or SDM).

The results of the comparison are given on Table 1. The proposed method never gives unsafe designs; therefore the maximum positive error is always 0. The average error and the standard deviation are small. Its maximum values are -0,8% and 2,7% respectively. The maximum values of the error on the safe side correspond to low reinforcement ratios and high axial forces.

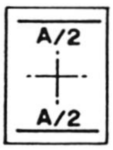
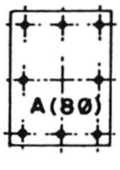
e_1/e_2	1				0				-1				
CROSS - SECTION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	BASE METHOD
	0,0	-12	-0,2	0,8	0,0	-13	-0,5	1,5	0,0	-12	-0,5	1,5	SDM
	0,0	-9	-0,3	0,7	0,0	-13	-0,5	1,5	0,0	-12	-0,5	1,5	MCM
	0,0	-20	-0,5	1,5	0,0	-22	-0,7	2,4	0,0	-22	-0,7	2,2	SDM
	0,0	-16	-0,5	1,5	0,0	-22	-0,7	2,3	0,0	-22	-0,6	2,1	MCM
	0,0	-24	-0,8	1,9	0,0	-24	-0,8	2,7	0,0	-22	-0,8	2,6	SDM
	0,0	-20	-0,8	1,8	0,0	-23	-0,8	2,6	0,0	-22	-0,8	2,5	MCM

Table 1 Comparison between Design Charts and Model Code Method and Sinusoidal Deflection Method

6. APPROXIMATE FORMULAE

Approximate formulae for the numerical application of the Reference Curvatures Method have been derived with the aim of obtaining:

- an adequate representation of the problem;
- sufficiently accurate results;
- a simple a quick design.

To derive these formulae, the following hypothesis have been made:

1. The geometrical behaviour line of the Model Column Method is adopted.

2. The reference curvatures curves of strength and instability are replaced by their least square linear fits. These are of the form:

$$h/r = \beta_1 + \beta_2 \cdot e/h$$

$$\omega = \alpha_1 + \alpha_2 \cdot e/h$$

where α_1, α_2, β_1, β_2 are numerical coefficients which are given on Table 2 as a function of the reduced axial force ν for different cross-section types.

The use of the formulae is as follows:

a) The intersection points of the geometric behaviour curve and the reference curvatures curves of strength and instability are obtained by:

$$e/h = \frac{e^I/h + \beta_1 \cdot \beta}{1 - \beta_2 \cdot \beta} \qquad \text{with} \quad \beta = \lambda^2 . 10^{-4}$$

using the β_1 and β_2 coefficients for strength and instability cases.

b) The reinforcement ratios for the ultimate limit states of strength and instability are obtained by:

$$\omega = \alpha_1 + \alpha_2 \cdot \frac{e}{h}$$

using the α_1 and α_2 coefficients corresponding to the strength and instability cases respectively.

c) The lower of the two values of ω above obtained will be the reinforcement ratio corresponding to the strict design, and will indicate whether the failure of the column will correspond to an ultimate limit state of strength or instability.

The creep effect due to permanent loads can be taken into account as in the Design Charts, namely by means of an additional eccentricity e_c due to creep deformation (linear approach method, [2] , paragraph 3.2.1).

To check the accuracy of the approximate formulae an analytical comparison has been made with the Model Column Method. This comparison has been made following the same procedure as for the Design Charts ($ 5 above). The results are given on Table 3. The agreement is good.

Also in this table are shown, for comparison, the agreements of the approximate design methods given in several reinforced concrete Codes, as compared with

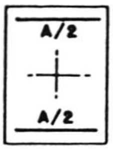
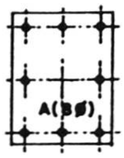
CROSS SECTION	ν	INSTABILITY				STRENGTH			
		α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2
	-0,1	-0,10	0,26	3,11	0,22	-0,11	0,25	13,89	0,19
	-0,2	-0,18	0,51	3,73	0,25	-0,19	0,50	12,04	0,00
	-0,3	-0,23	0,75	4,54	0,00	-0,24	0,75	8,03	0,00
	-0,4	-0,23	1,08	3,14	1,03	-0,26	1,00	6,02	0,00
	-0,5	-0,22	1,43	2,27	2,01	-0,24	1,32	4,84	0,93
	-0,6	-0,17	1,77	1,76	2,81	-0,21	1,65	4,01	1,99
	-0,7	-0,10	2,08	1,42	3,53	-0,14	1,96	3,43	3,01
	-0,8	-0,02	2,37	1,06	4,58	-0,05	2,22	2,72	4,90
	-0,9	0,08	2,63	0,88	5,28	0,04	2,46	2,33	6,34
	-1,0	0,17	2,90	0,72	6,11	0,15	2,68	1,96	8,14
	-0,1	-0,14	0,33	3,08	0,28	-0,15	0,29	15,77	-1,83
	-0,2	-0,24	0,68	3,70	0,32	-0,26	0,63	11,65	-1,94
	-0,3	-0,31	1,00	4,54	0,00	-0,32	0,99	8,08	-0,58
	-0,4	-0,32	1,44	3,06	1,37	-0,35	1,34	5,91	0,67
	-0,5	-0,30	1,90	2,15	2,67	-0,31	1,75	4,90	1,50
	-0,6	-0,24	2,30	1,64	3,67	-0,27	2,04	3,93	2,90
	-0,7	-0,14	2,63	1,33	4,48	-0,19	2,37	3,35	4,19
	-0,8	-0,04	2,93	1,00	5,69	-0,08	2,61	2,67	6,36
	-0,9	0,06	3,22	0,85	6,48	0,02	2,85	2,35	7,81
	-1,0	0,16	3,52	0,70	7,45	0,12	3,06	2,05	9,56
	-0,1	-0,15	0,37	3,06	0,32	-0,16	0,31	15,54	-1,81
	-0,2	-0,27	0,76	3,69	0,36	-0,26	0,66	11,81	-2,15
	-0,3	-0,35	1,13	4,54	0,00	-0,33	1,03	8,08	-0,65
	-0,4	-0,37	1,63	3,01	1,55	-0,36	1,42	5,93	0,78
	-0,5	-0,35	2,13	2,09	3,00	-0,33	1,75	4,75	1,85
	-0,6	-0,28	2,56	1,59	4,09	-0,28	2,15	3,89	3,31
	-0,7	-0,17	2,89	1,30	4,93	-0,20	2,48	3,31	4,69
	-0,8	-0,05	3,17	0,98	6,19	-0,09	2,72	2,64	6,93
	-0,9	0,06	3,48	0,83	7,02	0,02	2,96	2,33	8,40
	-1,0	0,16	3,80	0,69	8,05	0,12	3,17	2,05	10,16

Table 2 Numerical coefficients for approximate formulae

the Model Column Method. The selected Codes are the English Code (CP 110-72), the German Code (DIN 1045-72), the American Code (ACI 318-77) and the Spanish Code (EH-80). It must be borne in mind, on the other hand, that these approximate design methods do not allow a direct design (that is to say, a direct obtention of the reinforcement ratio ω) as the proposed approximate formulae do.

7. NUMERICAL EXAMPLE

Design the column in fig. 6 with the following data:

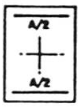
e_1/e_2	1				0				-1				
CROSS-SECTION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	MAXIMUM POSITIVE ERROR %	MAXIMUM NEGATIVE ERROR %	AVERAGE ERROR %	STANDARD DEVIATION	BASE METHOD
 A/2 A/2	5,9	-30	- 5,0	7,4	18,2	-29	-2,8	7,2	6,9	-32	-4,6	8,5	CP110-72
	12,3	-34	-10,0	6,9	2,1	-38	-8,2	8,8	6,9	-22	-2,6	5,4	DIN1045-72
	48,9	-33	- 2,0	14,4	35,3	-36	-2,6	10,1	30,7	-35	-1,7	7,3	ACI 318-78
	1,4	-40	-11,0	8,6	2,6	-40	-7,9	10,1	2,9	-38	-5,9	9,5	EH-80
	3,6	-18	- 0,0	1,8	3,6	-18	-0,5	2,0	3,4	-15	-0,5	1,9	PROPOSED FORMULAE
 A(6ϕ)	16,0	-34	- 3,6	8,0	19,1	-29	-2,1	7,2	7,5	-36	-4,4	8,2	CP110-72
	13,5	-28	- 9,2	6,7	4,5	-31	-8,2	8,3	12,2	-19	-2,1	4,7	DIN 1045-72
	61,1	-28	2,3	16,7	43,4	-30	-0,5	10,6	32,0	-31	-0,5	7,5	ACI 318-78
	4,1	-42	- 9,8	8,8	3,9	-42	-7,9	9,9	3,4	-40	-5,8	9,4	EH-80
	2,9	-22	- 0,4	2,1	2,9	-22	-0,6	2,6	2,9	-20	-0,5	2,6	PROPOSED FORMULAE
 A/4 A/4 A/4 A/4	13,3	-34	- 2,5	7,9	19,8	-30	-1,7	7,5	6,3	-34	-4,2	7,9	CP 110-72
	12,1	-26	- 8,3	6,3	3,4	-31	-7,9	7,9	12,2	-17	-1,8	4,4	DIN 1045-72
	69,1	-26	4,2	17,3	47,0	-28	0,4	11,0	36,8	-30	-0,3	7,9	ACI 318-78
	5,9	-43	- 8,9	8,4	5,8	-43	-7,7	9,5	6,3	-41	-5,6	9,1	EH-80
	3,5	-24	- 0,8	2,4	3,5	-24	-1,1	3,1	2,6	-22	-1,0	2,9	PROPOSED FORMULAE

Table 3 Comparison between approximate formulae and Model Code Method

l_o = 7,35 m
b = 0,30 m
h = 0,30 m; h' = 0,03 m
N = -675 kN
$e^I_1 = e^I_2 = 0,12$ m
f_{ck} = 30 MPa ; $\gamma_c = 1,5$
f_{yk} = 420 MPa; $\gamma_s = 1,1$
$\gamma_f = 1,6$

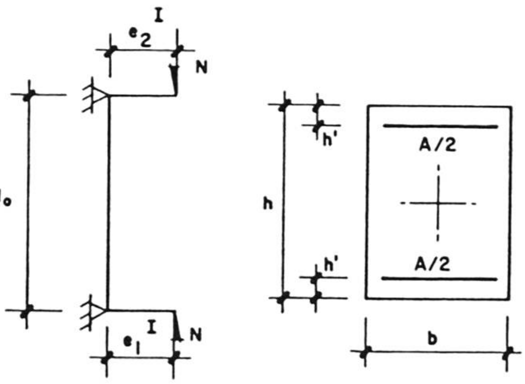


Fig. 6 Design example

7.1 Design with the Design Charts

The following reduced values must be computed:

$$\nu = \frac{-675 \cdot 1,6}{0,30 \cdot 0,30 \cdot \frac{30000}{1,5}} = -0,6$$

$$e^I_1/h = e^I_2/h = 0,12/0,30 = 0,4$$

$$\lambda = 7,35/0,30 = 24,5$$

The hypothesis of column behaviour of the Model Column Method is adopted. The figure 5 shows the geometrical behaviour line (a straight line) and the reinforcements ratios corresponding to this example:

$$\omega_{strength} = 0,960$$
$$\omega_{instabil.} = 0,885$$

In this case the strict design of the column is $\omega_{instabil} = 0,885$ and the column will reach instability failure. The reinforcement area must be:

$$A_{tot} = \frac{0,885 \cdot 0,30 \cdot 0,30 \cdot 30000/1,5}{420000/1,1} = 0,00417\ m^2 = 41,7\ cm^2$$

7.2 Design with the Approximate Formulae

From Table 2 for $\nu = -0,6$ and the first cross-section type:

Instability	Strength
$\alpha_1 = -0,17$	$\alpha_1 = -0,21$
$\alpha_2 = 1,77$	$\alpha_2 = 1,65$
$\beta_1 = 1,76$	$\beta_1 = 4,01$
$\beta_2 = 2,81$	$\beta_2 = 1,99$

$$\beta = 24,5^2 . 10^{-4} = 0,06$$

$$e/h = \frac{0,4 + 1,76 \cdot 0,06}{1 - 2,81 \cdot 0,06} = 0,61\ ; \qquad e/h = \frac{0,4 + 4,01 \cdot 0,06}{1 - 1,99 \cdot 0,06} = 0,73\ ;$$

$$\omega = -0,17 + 1,77 \cdot 0,61 = 0,91\ ; \qquad \omega = -0,21 + 1,65 \cdot 0,73 = 0,99\ .$$

The strict design of the column is $\omega_{instabil.} = 0,91$ and the column will reach an instability failure. The reinforcement area must be:

$$A_{tot} = \frac{0,91 \cdot 0,30 \cdot 0,30 \cdot 30000/1,5}{420000/1,1} = 0,00429\ m^2 = 42,9\ cm^2$$

8. CONCLUSIONS

8.1 The Reference Curvatures Method is a general method for the direct design of reinforced concrete columns. It gives the strict reinforcement quantity required for a slender as well as for a short column.

For a short column the result is, up to a good degree of accuracy, the same as that given by an interaction diagram for the column cross-section.

For a slender column which reaches an ultimate limit state of strength failure at the critical cross-section, the accuracy is the same as with the Model Column Method or the Sinusoidal Deflection Method.

For a slender column which reaches an ultimate limit state of instability failure, the results are the same as with the Model Column Method or the Sinusoidal Deflection Method, or lie with a small error on the safe side.

8.2 The Reference Curvatures Method allows an adequate representation of the behaviour at failure of slender columns, distinguishing between the ultimate limit states of member instability and of strength at the critical cross-section.

8.3 As regards its application, the Reference Curvatures Method has been implemented by means of design charts and approximate formulae. It can also be used on a computer, as it is a direct algorithm for the design.

The design charts and approximate formulae obtained are easy to use and give a solution to the design problem with good accuracy.

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