

# Strength of webs containing circular cut-outs

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## Strength of Webs Containing Circular Cut-Outs

Résistance d'âmes présentant des ouvertures circulaires

Festigkeit von Stegen mit runden Löchern

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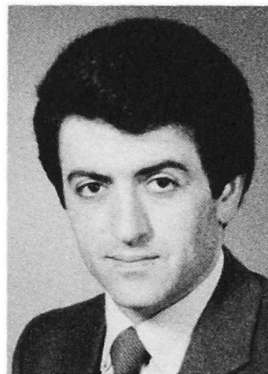
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### SUMMARY

A theoretical method of predicting the ultimate capacity of slender webs containing circular holes, and subjected to shear is presented. The solution is obtained by considering the equilibrium of two tension bands, one above and the other below the cut-outs. These bands have been chosen to conform to the failure pattern observed in the plate girders with holes tested at Cardiff. Experimental results show that the method gives satisfactory and safe predictions.

### RÉSUMÉ

Une méthode théorique permet de déterminer la charge de rupture d'âmes élancées présentant des ouvertures circulaires et soumises à l'effort tranchant. La résolution est basée sur l'équilibre de deux bielles de traction, au-dessus et au-dessous des ouvertures. Ces bielles ont été choisies sur la base de l'observation du développement de la rupture telle qu'observée à Cardiff sur des poutres à âme pleine avec des ouvertures circulaires. Les résultats d'essais montrent que la méthode fournit des prédictions satisfaisantes.

### ZUSAMMENFASSUNG

Eine theoretische Methode zur Berechnung der Bruchlast unter Schubkraft von schmalen Stegen mit runden Löchern wird vorgestellt. Zwei oberhalb und unterhalb der Löcher theoretisch fixierte Zugfelder werden ins Gleichgewicht gestellt, um die Lösung zu erhalten. Die Zugfelder wurden gewählt, um dem Versagensablauf von Vollwandträgern mit Löchern zu entsprechen, die in Cardiff als Versuche durchgeführt wurden. Die Versuchsergebnisse zeigen, dass die Methode befriedigende und sichere Voraussagen gewährleistet.



## 1. INTRODUCTION AND REVIEW OF PREVIOUS WORK

Thin webs containing inspection openings are frequently encountered in the webs of plate and box girders. These openings may be circular, rectangular or elliptical.

Current methods of estimating the stress levels around such openings are based on elastic analysis. Since slender webs buckle at relatively low stress levels and have considerable post-buckling strengths, a reliable method of assessing the ultimate capacity of these webs is necessary.

While the behaviour of "thick" walled webs, having web slenderness ( $\frac{h}{t}$ ) values below 80 and containing various forms of openings has been studied extensively by various investigators [1-3], available literature of direct relevance to the behaviour of thin webs appears to be limited. In 1971, Hoglund [4] reported on tests on four plate girders made of thin webs ( $\frac{h}{t}$  values in the range 200-300) containing rectangular and circular perforations. On the basis of a limited number of tests, he proposed a theoretical model for the calculation of the ultimate capacity of girders containing holes. The model consisted of tension fields, with the stress equal to the tensile yield stress, and compression fields, with the stress levels estimated from tests.

A more accurate analytical model is developed in this paper and is based on the theoretical method for estimating the ultimate capacity of plate girders suggested by Porter et al [5]. This method consists of evaluating the strength of the plate girder as the sum of three contributions :

1. Critical load on the web,
2. The load carried by the membrane stress  $\sigma_t^y$  developed in the post-critical stages and
3. The load carried by the flange, when the collapse is about to occur due to the formation of plastic hinges.

The ultimate shear is given by :

$$V_u = \frac{4M_p}{c} + c \cdot t \sigma_t^y \sin^2 \theta + \sigma_t^y t \cdot h (\cot \theta - \cot \theta_d) \sin^2 \theta + \tau_{cr} ht \quad (1)$$

where

- $M_p$  = fully plastic moment of flange
- $c$  = distance between the hinges
- $t$  = thickness of web
- $h$  = depth of web
- $\tau_{cr}$  = elastic critical shear stress in the web
- $\sigma_t^y$  = membrane stress in the post-critical stage
- $\theta$  = angle of inclination of the tensile membrane stress  $\sigma_t^y$
- $\theta_d$  = the angle of inclination of the panel diagonal

The values of  $c$  and  $\sigma_t^y$  in the above equation can be evaluated in terms of known quantities (see below); however,  $\theta$  is unknown. As this is an equilibrium solution,  $\theta$  is evaluated by trial and error to give a maximum value for  $V_u$ .

By considering the equilibrium of the flange at the collapse limit state, the value of  $c$  can be obtained, and is given by :

$$c = \frac{2}{\sin \theta} \sqrt{\frac{M_p}{\sigma_t^y \cdot t}} \quad (2)$$

$\sigma_t^y$  can be calculated by applying the Von Mises criterion, if the yield stress,  $\sigma_{yw}$ , and critical stress of the web are known :

$$\sigma_t^y = -\frac{3}{2} \tau_{cr} \sin 2\theta + \sqrt{\sigma_{yw}^2 + (\tau_{cr})^2 \left| \left(\frac{3}{2} \sin 2\theta\right)^2 - 3 \right|} \quad (3)$$

The elastic critical stress in shear ( $\tau_{cr}$ ) for a rectangular web is given by :

$$\tau_{cr} = \kappa \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2 \quad (4)$$

where  $\kappa$  = a non-dimensional coefficient given by

$$\begin{aligned} \kappa &= 5.35 + 4 \left(\frac{h}{b}\right)^2 \text{ for } \frac{b}{h} > 1.0 \\ \kappa &= 5.35 \left(\frac{h}{b}\right)^2 + 4 \text{ for } \frac{b}{h} < 1.0 \end{aligned}$$

An approximate method of assessing the ultimate capacity of webs containing circular holes has been suggested by Narayanan and Rockey [6] and is based on a large number of tests carried out by them at Cardiff. In this method it has been assumed that the ultimate capacity  $V_{ult}$ , can be estimated by linear interpolation between the Vierendeel load,  $V_v$ , and  $V_u$  for the unperforated web assessed as described above. When  $d=h$ , the failure would be essentially due to a Vierendeel mechanism and the corresponding collapse load is given by :

$$V_v = \frac{8M_p}{b} \quad (5)$$

The ultimate strength of a plate girder with a web opening is given by :

$$V_{ult} = V_v + \left(\frac{V_u - V_v}{h}\right) (h-d) \quad (6)$$

Even though the above method is somewhat empirical, it has been shown that it gives satisfactory predictions with a reasonable degree of accuracy in comparison with the test results.

## 2. A SUGGESTED EQUILIBRIUM SOLUTION

The proposed method also consists of evaluating the strength of the girder as the sum of three contributions as in the case of an unperforated web.

When the load on the girder is increased, the web plate reaches its elastic critical value first. Rockey, Anderson and Cheung [7] have shown that the critical stress decays linearly when the diameter of the cut-out is increased. In other words, the coefficient  $\kappa_1$  appropriate to the perforated web is given by:

$$\kappa_1 = \kappa \left(1 - \frac{d}{h}\right) \quad (7)$$

where  $\kappa$  = coefficient for unperforated web [8]  
 $d$  = diameter of cut-out  
 $h$  = depth of web

The reduced value for elastic critical stress in shear is given by :

$$(\tau_{cr})_{red} = \kappa \left(1 - \frac{d}{h}\right) \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2 \quad (8)$$

In the post critical stage, additional load is carried by the membrane stress ( $\sigma_t^y$ ) developed in the web. These form two tension bands, one above and the other below the cut-out. (See Figs. 1(a) and 1(b)). This phenomenon has been observed in the pattern of buckles seen in the girders [6] tested in the experimental programme.

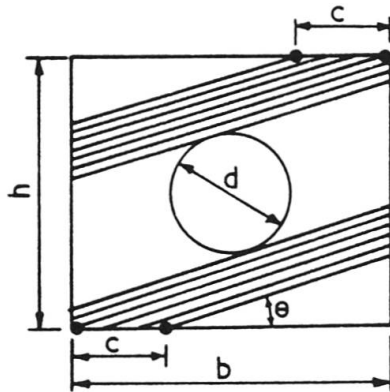


FIG. 1(a) "SMALL" CUT-OUTS

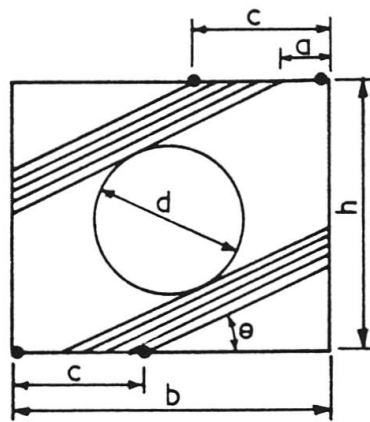


FIG. 1(b) "LARGE" CUT-OUTS

A part of the load is carried by the flanges. When the collapse is about to occur, the moment capacity of the flanges is equal to the plastic moment ( $M_p$ ). The ultimate load of the girder is obtained by adding the contribution due to the flange stiffness to the load taken by the web.

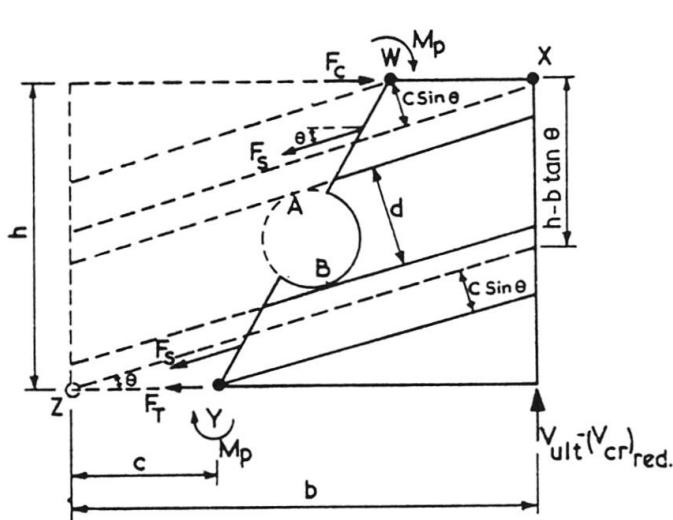


FIG. 2(a)

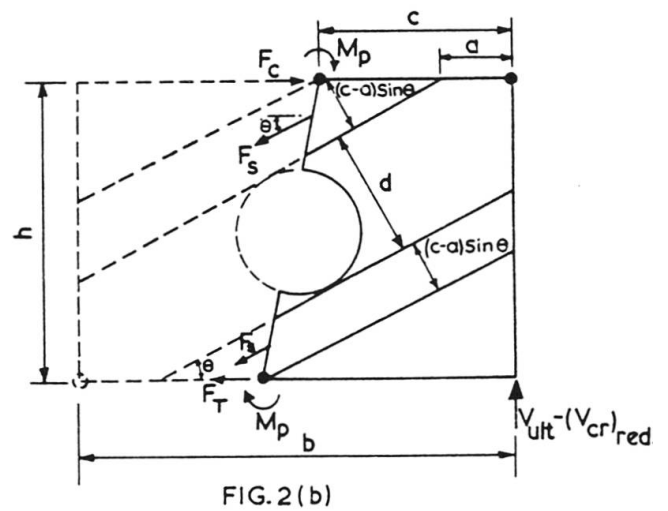


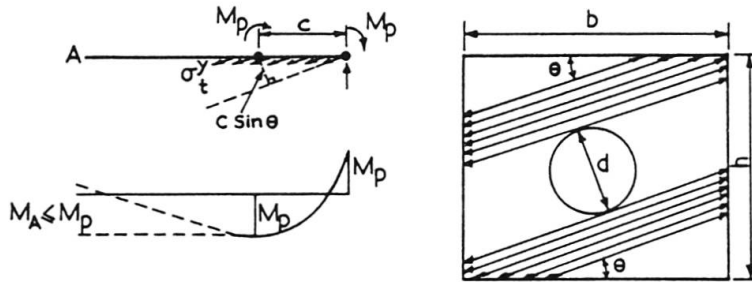
FIG. 2(b)

Figure 2(a) shows the position of hinges (W,X,Y,Z) at the instant of failure. Consider the element to the right of WY. Across AB, there are no forces, as at the boundary of the perforation, the stresses are zero. The tensile membrane forces on the two tension bands are replaced by two forces  $F_S$  as shown. The vertical components of these two forces is  $2 F_S \sin\theta$ .

From the geometry of the structure it can be seen that :

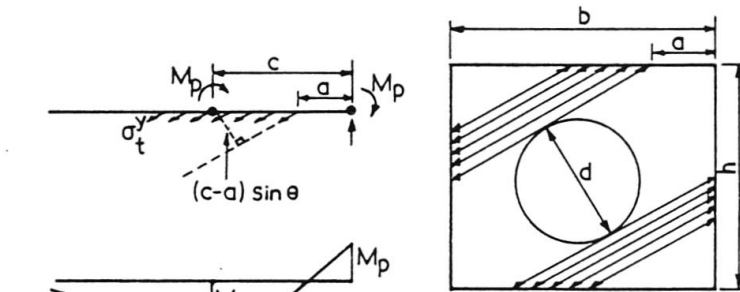
$$2F_S = \sigma_t^y \cdot t \cdot |2c \sin\theta + (h-b \tan\theta) \cos\theta - d|$$

The internal plastic hinge will form at the position of maximum bending moment, where the shear force is zero. Hence the hinge distance,  $c$ , can be evaluated by considering the equilibrium of the flange (see Fig. 3(a)).



$$c = \frac{2}{\sin\theta} \sqrt{\frac{M_p}{\sigma_t^y \cdot t}} \quad \text{valid for } d < (h \cos\theta - b \sin\theta)$$

FIG. 3(a)



$$a = \frac{1}{2} \left[ b - \left( h - \frac{d}{\cos\theta} \right) \cot\theta \right]$$

$$c = \sqrt{a^2 + \frac{4M_p}{\sigma_t^y \cdot t \sin^2\theta}} \quad \text{valid for } d > (h \cos\theta - b \sin\theta)$$

FIG. 3(b)

$\sigma_t^y$  can be evaluated by applying the Von Mises yield criterion :

$$\sigma_t^y = -\frac{3}{2} (\tau_{cr})_{red} \sin 2\theta + \sqrt{\sigma_w^2 + (\tau_{cr})_{red}^2 \left| \left( \frac{3}{2} \sin 2\theta \right)^2 - 3 \right|}$$

In calculating  $(\tau_{cr})_{red}$  the value of  $\kappa$  appropriate to a web fixed at its edges

should be used [8]. This is justified as, when there is a cut-out, the stiffness of the flange is much higher than that of the web and the behaviour of the web plate is nearer that of encastred supports.



Since there is no shear at the internal hinge position, we obtain from vertical equilibrium of the forces :

$$2F_s \sin\theta = V_{ult} - (V_{cr})_{red}$$

$$V_{ult} = 2F_s \sin\theta + (V_{cr})_{red}$$

Substituting for  $F_s$  :

$$V_{ult} = \sigma_t^y \cdot t \sin\theta (2c \sin\theta) + \sigma_t^y t | (h - b \tan\theta) \cos\theta - d | \sin\theta + V_{cr} (red)$$

$$\begin{aligned} \therefore V_{ult} &= 2c \sigma_t^y \cdot t \sin^2\theta + \sigma_t^y \cdot t h (\cot\theta - \cot\theta_d) \sin^2\theta \\ &\quad - \sigma_t^y \cdot t \cdot d \sin\theta + (\tau_{cr})_{red} \cdot h \cdot t \end{aligned} \quad (9)$$

This expression is valid for all small holes, i.e. when  $0 < d < h \cos\theta - b \sin\theta$ .

If the diameter of the cut-out is greater than the limits given above, the tension band is assumed to commence at a distance "a" from the corner hinge as shown in Fig. 1(b), "a" can be calculated from geometry and is given by :

$$a = 0.5 \left( b - \left( h - \frac{d}{\cos\theta} \right) \cot\theta \right) \quad (10)$$

c can be computed from equilibrium of the flange at the instant of collapse (see Fig. 3(b)).

$$c = \sqrt{a^2 + \frac{4M_p}{\sigma_t^y \cdot t \sin^2\theta}} \quad (11)$$

As before, we can obtain from Fig. 2(b) :

$$\begin{aligned} V_{ult} &= 2F_s \sin\theta + (V_{cr})_{red} \\ &= 2\sigma_t^y \cdot t (c-a) \sin^2\theta + (\tau_{cr})_{red} \cdot h \cdot t \end{aligned} \quad (12)$$

It can be seen that when the diameter d, is zero the equation (9) reduces to the solution suggested for an unperforated web (equation 1). For a given size of girder, the only variable in equations (9) and (12) is the angle of inclination of membrane stress  $\theta$ . It only remains to find that value of  $\theta_m$  which will provide maximum value of  $V_{ult}$ . A parametric study on plate girders without perforation was carried out by Rockey, Evans and Porter [9], who showed that the optimum angle  $\theta_m$  producing maximum  $V_u$  for unperforated webs is very close to  $(2 \theta_d/3)$ . Figure 4 shows a similar study carried out for girders with central circular cut-outs, wherein the variation of  $V_{ult}$  with the angle  $\theta$ , for various diameters of cut-out in a square panel having  $h/t=360$  is plotted. Similar studies have been carried out for the other values of  $h/t$ .

In Fig. 5, the variation of the optimum angle,  $\theta_m$ , with the diameter of the cut-out for a square panel and a rectangular panel (aspect ratio 1.5:1) is shown. It will be seen that for girders representative of normal sizes, the optimum angle  $\theta_m$  reduces linearly as the diameter d, increases up to about 0.8 of the depth of panel (h).

Equation (9) is therefore valid for all girders having cut-outs of practical proportions.

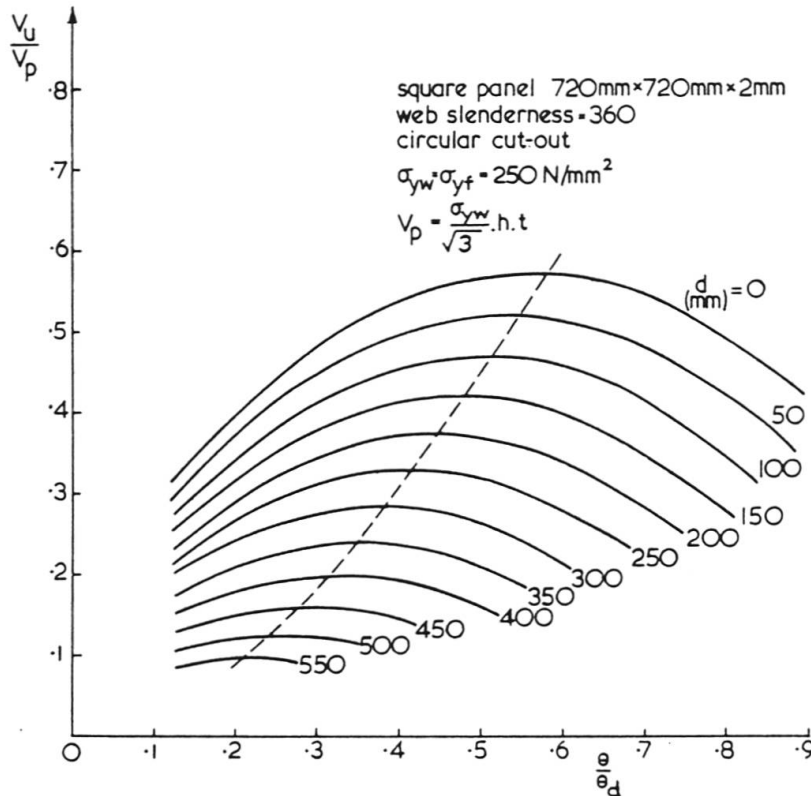


FIG. 4 VARIATION OF  $V_{ult}$  WITH  $\theta$

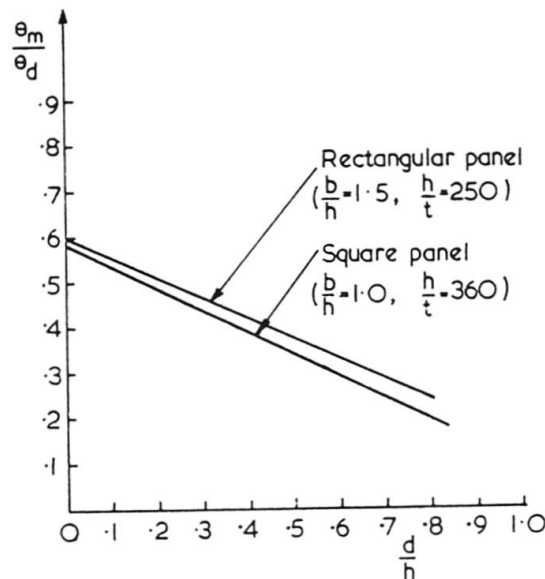


FIG. 5 VARIATION OF OPTIMUM ANGLE  $\theta_m$  WITH THE DIAMETER OF CUT-OUT

Equation (10) applies for larger diameters of cut-outs, with  $d > h \cos\theta - b \sin\theta$ ; such large cut-outs are generally unlikely to be met in practice; this equation is only therefore included for a complete discussion of the theory.

### 3. COMPARISON WITH TEST RESULTS

The values of the ultimate loads observed in the laboratory on a number of test girders containing perforated webs will now be compared with the corresponding values predicted by using the theoretical treatment described in the foregoing pages; the important details of these test girders are set out in the first four columns of Table 1 and Fig. 6. Full details of these tests are given in Ref. [6].



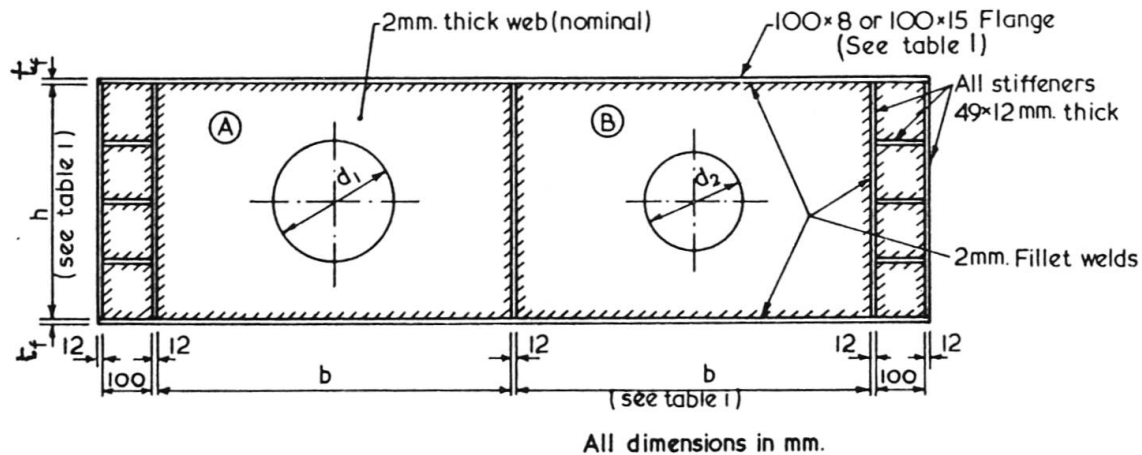


FIG. 6 DETAILS OF THE EXPERIMENTAL GIRDERS:- (SERIES 1 & 2)

Table 1 also shows the predicted values of  $c$  and  $V_{ult}$  compared with the measured values. In obtaining these values, the measured dimensions of the test girders and the measured yield stresses have been used. It will be seen that the predicted values of the hinge distances and the ultimate shear are in close agreement with the corresponding observed values.

Any equilibrium solution of the type discussed above will give predictions which are below the true values. Thus a safe estimate of the collapse load is possible by the above method. From the comparisons presented in Table 1, it is observed that the mean predicted value of the strength of the webs for the 20 tests is 0.84 with a standard deviation of 0.067.

#### 4. CONCLUSIONS

A satisfactory equilibrium solution for predicting the ultimate shear capacity of plate girders with circular openings has been presented.

The method is based on the actual failure mechanism developed in test girders. The predictions obtained by this method are shown to be safe and satisfactory in comparison with the test results.

#### 5. ACKNOWLEDGEMENT

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#### NOTATION

$b$	width of web plate
$b_f$	width of flange plate
$c$	distance between the hinges (in compression and tension flange)
$d$	diameter of the hole
$h$	depth of web plate
$M_p$	plastic moment of resistance of flange plate $(\frac{b_f t_f^2}{4} \sigma_{yf})$
$t$	thickness of web plate
$t_f$	thickness of flange plate
$\kappa$	non-dimensional coefficient for shear buckling stress
$\sigma_{yf}$	yield stress of flange member
$\sigma_{yw}$	yield stress of web plate
$\sigma_t^y$	membrane stress in the post-critical stage
$\tau_{cr}$	elastic critical shear stress in the web
$\theta$	angle of inclination of the tensile membrane stress $\sigma_t^y$
$\theta_d$	the angle of inclination of the panel diagonal

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TABLE 1

Experimental values of ultimate load and hinge positions compared with predicted values.

GIRDER NO.	NOMINAL WEB DIMENSIONS b x h x t	NOMINAL FLANGE SIZES b <sub>f</sub> x t <sub>f</sub>	NOMINAL DIAMETER OF HOLE d	h t	b h	d h	COMPARISON OF HINGE LOCATIONS				ULTIMATE LOAD COMPARISON			
							PREDICTED HINGE DISTANCE FOR UNPERF. WEB (mm)	PREDICTED HINGE DISTANCE FOR PERF. GIRDER (mm)	MEASURED HINGE DISTANCE FOR PERF. GIRDER (mm)	RATIO OF PRED. HINGE DISTANCE TO MEASURED HINGE DISTANCE FOR PERF. WEB	PREDICTED ULTIMATE LOAD FOR UNPERF. WEB (kN)	PREDICTED ULTIMATE LOAD FOR PERF. WEB (kN)	OBSERVED ULTIMATE LOAD FOR PERF. WEB (kN)	RATIO OF PREDICTED TO OBSERVED ULTIMATE LOAD FOR PERF. WEB.
<b>SERIES 1</b>														
CP1 (0)	750x500x2	100x8	0	250	1.5	0	188	188	217	0.87	166.5	166.5	176.0	0.95
CP1 (50)	750x500x2	100x8	50	250	1.5	0.10	190	195	232	0.84	166.5	147.5	168.0	0.88
CP2 (100)	750x500x2	100x8	100	250	1.5	0.20	185	217	247	0.88	181.8	140.2	160.0	0.88
CP2 (125)	750x500x2	100x8	125	250	1.5	0.25	185	236	262	0.90	181.7	130.0	142.0	0.92
CP3 (150)	750x500x2	100x8	150	250	1.5	0.30	183	248	285	0.87	167.6	112.0	125.0	0.90
CP3 (175)	750x500x2	100x8	175	250	1.5	0.35	183	259	307	0.84	167.7	105.0	115.0	0.91
CP4 (200)	750x500x2	100x8	200	250	1.5	0.40	181	285	322	0.88	171.7	93.5	105.0	0.89
CP4 (250)	750x500x2	100x8	250	250	1.5	0.50	181	304	382	0.79	171.7	76.0	86.0	0.88
CP5 (275)	750x500x2	100x8	275	250	1.5	0.55	192	311	397	0.78	168.0	67.0	79.0	0.85
CP5 (300)	750x500x2	100x8	300	250	1.5	0.60	192	334	382	0.87	168.3	59.0	65.0	0.91
CP6 (350)	750x500x2	100x8	350	250	1.5	0.70	184	356	382	0.93	167.1	43.0	54.5	0.80
CP6 (400)	750x500x2	100x8	400	250	1.5	0.80	185	382	360	1.06	166.6	28.2	33.5	0.84
<b>SERIES 2</b>														
CP7 (180)	720x720x2	100x8	180	360	1.0	0.25	157	187	187	1.00	215.1	150.0	182.0	0.82
CP7 (270)	720x720x2	100x8	270	360	1.0	0.38	157	201	245	0.82	215.0	118.0	150.0	0.79
CP8 (360)	720x720x2	100x8	360	360	1.0	0.50	122	209	295	0.71	283.0	110.0	157.0	0.70
CP8 (480)	720x720x2	100x8	480	360	1.0	0.67	122	252	356	0.71	283.0	64.0	88.0	0.74
CP9 (180)	720x720x2	100x15	180	360	1.0	0.25	212	245	223	1.09	330.8	229.0	270.0	0.85
CP9 (270)	720x720x2	100x15	270	360	1.0	0.38	212	274	317	0.86	330.8	183.0	220.0	0.83
CP10 (360)	720x720x2	100x15	360	360	1.0	0.50	210	303	345	0.88	324.4	138.0	180.0	0.77
CP10 (480)	720x720x2	100x15	480	360	1.0	0.67	210	351	363	0.91	324.6	89.0	122.0	0.73

mean  $\left(\frac{\text{predicted hinge distance}}{\text{measured hinge distance}}\right) = 0.875$   
 standard deviation = 0.094

mean  $\left(\frac{\text{predicted ultimate load}}{\text{observed ultimate load}}\right) = 0.842$   
 standard deviation = 0.067

## APPENDIX

A design method based on the equilibrium solution outlined in the paper for predicting the ultimate shear load of a plate girder containing a circular opening of practical proportions is suggested below :

Step 1 : From the known dimensions  $t$ ,  $d$  and  $h$ , calculate the value of the elastic critical shear stress, appropriate to the perforated web :

$$(\tau_{cr})_{red} = \kappa \left(1 - \frac{d}{h}\right) \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2$$

Step 2 : An approximate value for the angle of inclination of the tensile membrane stress ( $\theta$ ) appropriate to the perforated web may be calculated from

$$\theta \approx \frac{2}{3} \theta_d \left(1 - \frac{d}{h}\right)$$

Step 3 : Calculate the tensile membrane stress in the web,  $\sigma_t^y$ , using

$$\sigma_t^y = -\frac{3}{2} (\tau_{cr})_{red} \sin 2\theta + \sqrt{\sigma_{yw}^2 + (\tau_{cr})_{red}^2 \left[\left(\frac{3}{2} \sin 2\theta\right)^2 - 3\right]}$$

Step 4 : Calculate the fully plastic moment of flange,  $M_p = \frac{b_f \cdot t_f^2}{4} \cdot \sigma_{yf}$

Step 5 : Calculate the distance between the plastic hinges in the flange

$$c = \frac{2}{\sin \theta} \sqrt{\frac{M_p}{\sigma_t^y \cdot t}}$$

Step 6 : Calculate the ultimate shear load,  $V_{ult}$ , using the calculated values for  $\sigma_t^y$ ,  $c$ ,  $\theta$  and  $(\tau_{cr})_{red}$  in the following equation :

$$V_{ult} = 2 c \sigma_t^y \cdot t \sin^2 \theta + \sigma_t^y \cdot t \cdot h (\cot \theta - \cot \theta_d) \sin^2 \theta - \sigma_t^y \cdot t \cdot d \sin \theta + (\tau_{cr})_{red} \cdot h \cdot t$$

Step 7 : If a more accurate value of  $V_{ult}$  is required, it can be obtained by trying a few values of  $\theta$  in the region of the trial value suggested in Step 2, and computing the maximum value of  $V_{ult}$ .

The validity of the above method will be illustrated using the measured dimensions of a test girder having a realistic size of opening. For this purpose, web CP2 having a hole of diameter 125 mm is chosen. The measured details of CP2(125) are reproduced below from reference 6.

Web	Flange	Cut-out
$b = 747 \text{ mm}$	$b_f = 100 \text{ mm}$	$d = 125 \text{ mm}$
$h = 500 \text{ mm}$	$t_f = 8 \text{ mm}$	
$t = 2.10 \text{ mm}$		
$\sigma_{yw} = 255 \text{ N/mm}^2$	$\sigma_{yf} = 263 \text{ N/mm}^2$	

Collapse load(experimental value)= 142 kN

Step 1 :  $(\tau_{cr})_{red} = 36.1 \text{ N/mm}^2$   
(assuming that  $E = 205,000 \text{ N/mm}^2$  and  $\nu = 0.3$ )



Step 2 : An approximation to the optimum value of  $\theta$  is given by

$$\frac{2}{3} \theta_d (1 - \frac{d}{h}) \approx 17^\circ$$

$$\therefore \theta \approx 17^\circ$$

Step 3 :

$$\sigma_t^y = - \frac{3}{2} (\tau_{cr})_{red} \sin 2\theta + \sqrt{\sigma_{yw}^2 + (\tau_{cr})_{red}^2 \left[ \left( \frac{3}{2} \sin 2\theta \right)^2 - 3 \right]}$$

$$\sigma_t^y = 219 \text{ N/mm}^2$$

Step 4 :

$$M_p = \frac{b_f t_f^2}{4} \cdot \sigma_{yf} = 421 \times 10^3 \text{ N mm}$$

Step 5 :

$$c = \frac{2}{\sin \theta} \sqrt{\frac{M_p}{\sigma_t^y \cdot t}} = 207 \text{ mm}$$

Step 6 :

$$V_{ult} = 2 c \sigma_t^y \sin^2 \theta + \sigma_t^y \cdot t \cdot h (\cot \theta - \cot \theta_d) \sin^2 \theta - \sigma_t^y \cdot t \cdot d \sin \theta + (\tau_{cr})_{red} \cdot h \cdot t$$

$$= 63.8 \text{ kN}$$

$V_{ult}$  is the ultimate shear load for a single panel.

The predicted central load =  $2 \times 63.8 = 127.6 \text{ kN}$ .

It can be seen that this value of the predicted load is 90% of experimentally observed load. A more accurate prediction can be obtained by trying several values of  $\theta$  and picking out the maximum  $V_{ult}$ . The value of the predicted load obtained by maximizing  $V_{ult}$  with respect to  $\theta$ , by trial and error is 92% of the observed load (see Table 1). It can be seen that there is no significant loss in accuracy by choosing the approximate value of  $\theta$  suggested in Step 2.

The design method suggested above is satisfactory and is generally adequate for practical purposes.