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Analytical Models of Tubular Beam-Columns

Modèles d'analyse de poutres-colonnes tubulaires

Analytische Modelle für die Berechnung von Stahlrohrstützen

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SUMMARY

Four analytical models describing the behaviour of fabricated tubular steel columns are developed based on an assumed deflection approach. The moment-curvature relationship is idealized in various manners. It has been found that the method of combining the exact moment-curvature relation with an elastic deflection for elastic-plastic columns strikes the good balance between the requirement for realistic representation of tubular column behaviour and the requirement for simplicity in use. Other conclusions are also made.

RÉSUMÉ

Quatre modèles, utilisant une hypothèse de déformation, ont été développés pour l'analyse du comportement de colonnes tubulaires fabriquées en acier. Divers cas de relations moment-courbure sont considérés. L'article montre que la méthode combinant la fonction exacte moment-courbure avec une flèche élastique, dans le cas d'un comportement élasto-plastique, donne une représentation réaliste — et simple à l'emploi — du comportement d'une colonne tubulaire. D'autres conclusions sont également présentées.

ZUSAMMENFASSUNG

Das Verhalten von Stahlrohrstützen wird mit vier verschiedenen Berechnungsmodellen beschrieben. Sie basieren auf angenommenen Verformungskurven. Die Momenten-Krümmungs-Beziehung wird in verschiedener Weise idealisiert. Es wurde gefunden, dass die Kombination der exakten Momenten-Krümmungs-Beziehung mit der elastischen Mittenauslenkung für die Beschreibung des elastoplastischen Verhaltens von Rohrstützen eine gute Übereinstimmung und einen einfachen Bemessungsansatz ergibt. Weitere Ergebnisse der Studie werden ebenfalls aufgezeigt.

1. INTRODUCTION

The ultimate strength study of structures has been a subject of research in structural engineering for many years. The huge structures such as offshore oil platforms in deep water are designed against extreme environmental conditions rather than normal loadings. For example, the design of an offshore platform is controlled by earthquake or wave force rather than by its own weight and working loads which must be sustained by the structure during normal operation. Since these design extreme environmental loads occur infrequently during the life of the structure, it is therefore important for the designer to ensure that the structure is adequately designed against collapse under extreme loading conditions for the planned location. Thus, offshore structures are often designed by their ultimate strength.

To investigate the ultimate strength of structures, one must know first the precise behavior of its components. This includes the member behavior under axial and/or lateral loads, and the connection behavior under various load conditions. In modelling the load deformation behavior of members, simplified methods are frequently used. Among various simplifications, the so-called assumed deflection method is found most popular. This method of analysis is found not only simple to use but also gives a reasonably accurate solution for practical use [1].

In this paper, four simple mathematical models for fabricated tubular steel columns are developed based on three types of assumed deflected shape of column combined with three types of moment-curvature idealization for tubular cross section, ranging from an almost exact solution to a rather crude approximation. These four simple analytical models are based on different combinations of assumed deflection shape with moment-curvature relationships. The basic features of the four models considered are summarized in Table 1, consisting of (1) plastic hinge method, (2) modified plastic hinge method, (3) average flow moment method; and (4) exact moment-curvature method.

The behavior of a beam-column can be described either in terms of axial load-lateral deflection relation or axial load-axial shortening relation. A precise load-deflection relation is required for the development and study of load-shortening behavior. These will be described in the present paper.

Numerical studies based on these four models are carried out for a pipe having dimensions and material properties shown in Table 2.

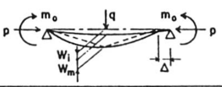
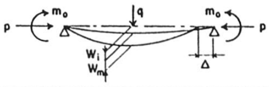
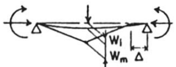
Table 2. Pipe Used in Numerical Studies

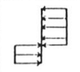
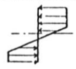
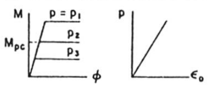
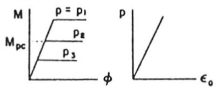
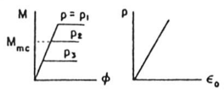
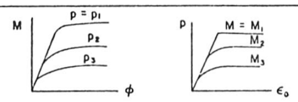
Pipe Dimensions	Material Properties
Diameter = 4.5 in.	Yield Stress = 36 ksi
Thickness = 0.09375 in.	Young's Modulus = 30000 ksi
Area = 1.30 in.^{2}	
Radius of Gyration = 1.56 in.	

2. ASSUMED DEFLECTION METHOD

The deflected shape of an elastic-plastic beam-column is generally quite complex, and requires numerical procedure for a solution. However, if we assume the deflected shape of a column to be a certain function and this general function or shape is assumed not to alter during further loading

Table 1. Comparison of Various Assumed Deflection Methods.

<table>
<tr><th></th><th>1. PLASTIC HINGE METHOD</th><th>2. MODIFIED PLASTIC HINGE METHOD</th><th>3. AVERAGE FLOW MOMENT METHOD</th><th>4. EXACT MOMENT-CURVATURE METHOD</th></tr>
<tr><td>DEFLECTED SHAPE</td><td>
</td><td colspan="3">
</td></tr>
<tr><td rowspan="2">ELASTIC RANGE:
</td><td colspan="4">DEFLECTION : $W_m = (W_i + W_o)\dfrac{p/p_{cr}}{1 - p/p_{cr}}$ W_o = DEFLECTION DUE TO q and m_o</td></tr>
<tr><td>$\Delta = \dfrac{pL}{AE} + \left(\cos\dfrac{2W_i}{L} - \cos\dfrac{2(W_i+W_m)}{L}\right)L$</td><td colspan="2">$\Delta = \dfrac{pL}{AE} + \dfrac{\pi^2 W_m^2}{4L}$</td><td>$\Delta = \dfrac{pL}{AE} + \Delta_G$
Δ_G = GEOMETRICAL SHORTENING CALCULATED BY NUMERICAL INTEGRATION</td></tr>
<tr><td>PRIMARY YIELDED RANGE:
</td><td colspan="2">SAME AS ABOVE</td><td>SAME AS ABOVE OR BELOW</td><td>$W_m = f_{Ep}(m_o, q, p, W_i)$
$\Delta = \Delta_S + \Delta_G$
Δ_S = ELASTIC-PLASTIC AXIAL STRAIN SHORTENING</td></tr>
<tr><td>SECONDARY YIELDED RANGE</td><td>

$W_m = \dfrac{M_{pc}}{p} - W_i$
$\Delta = \dfrac{pL}{AE} + \left(\cos\dfrac{2W_i}{L} - \cos\dfrac{2(W_i+W_m)}{L}\right)L$</td><td colspan="2">
$W_m = \dfrac{M_{pc}}{p} - W_i$ or $W_m = \dfrac{M_{mc}}{p} - W_i$
$\Delta = \dfrac{pL}{AE} + \dfrac{\pi^2 W_m^2}{4L}$</td><td>
$W_m = f_p(m_o, q, p, W_i)$
$\Delta = \Delta_S + \Delta_G$
(f_{Ep}, f_p = CUBIC EQUATION)</td></tr>
<tr><td>STRESS-STRAIN RELATIONSHIPS</td><td>
</td><td>
</td><td>
</td><td>
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</table>

but merely changes its magnitude as the axial load increases, then, the beam-column problem is simplified drastically to a one-degree-of-freedom problem [3].

In this approach, we need to consider the equilibrium between external loads and internal resistance of a member only at one critical section of the column. For a symmetrically loaded column, this critical section is at the mid-length of the column. This simplifies drastically the beam-column analysis and is found to be most efficient for parametric studies and analytical modelling of the behavior and strength of beam-column problems among many analysis methods available. Further, most of these existing methods can only trace the behavior of the column up to ultimate load (buckling), excluding the post-buckling branch of the load-deflection curve.

2.1 Deflection Functions

It is obvious that a proper choice of deflection shape is one of the key factors in the present analysis. The assumed deflection function should be as close to actual deflected shape as possible. Its closeness will reflect the accuracy of the solution. In the present work, reasonable functions are sinusoidal function [1], linear function [5,6] and polynomial function.

Since sinusoidal function is the exact shape for an axially loaded column with pin-ends, it gives the exact solution for an axially loaded beam-column in the elastic range. Hence, this function is chosen for elastic as well as elastic-plastic analysis of beam-column up to ultimate load. Near and beyond the ultimate load range, plastic hinges will form at critical sections, and a two-bar linkage type of mechanism will develop. This mechanism which consists of two straight lines knuckled at the plastic hinge locations, contributes mainly to the additional deflection of a beam-column in the fully plastic range. Hence, the two-straight-line deflection shape will also be chosen in the present analysis, and its results will be compared with those of sinusoidal shape.

The polynomial function is the type of deflection shape for an elastic beam subjected to lateral loads or end moments. Hence, it is suitable for beam-columns with large lateral loads or end moments. Herein, this function is used only for the calculation of initial shape of a beam-column resulted from the application of lateral loads or end moments but before the axial load is applied.

3. GENERAL BEHAVIOR OF BEAM-COLUMN

To investigate the general behavior of an isolated beam-column with initial imperfection w_i, let us consider a simply supported beam-column subjected to axial load P (see Fig. 1a). The deflection at mid-span w_m will be amplified as the axial load increases. Since we consider only the overall equilibrium of the column at a single point, the equilibrium condition for this column is now reduced to the finding of intersections of two curves resulted from external equilibrium consideration of the member and the internal resistance consideration of the cross section (Fig. 1b). The moment at mid-length induced by external force has the linear form:

$$M_{ext} = P\,(w_i + w_m) \tag{1}$$

Since a deflection function is assumed in the present analysis, the curvature at mid-length Φ_m is therefore directly related to the deflection at mid-length w_m. Hence, a linear relation between M_{ext} and Φ_m also exists. This linear relation is shown in Fig. 1b.

The internal resistance of a member follows of course the M-P-Φ curve. This nonlinear relation is also shown in Fig. 1b. In general, there are two intersecting points A and B in Fig. 1b, and they correspond to two points in the (P-w_m) curve; one in the pre-buckling branch of the curve and the other in the post-buckling branch respectively, as shown in Fig. 1c.

When the applied load is increased, the equilibrium line corresponding to external force P in Fig. 1b will move upward, while the moment-curvature curve for the internal resistance of the member moves downward. As a result, there exists a critical value of P at which the external force line will be tangent to the M-P-Φ curve. This critical value of P defines the ultimate strength of a column.

If we simplify the moment-curvature relation to the elastic-perfectly plastic type as shown by the dotted lines in Fig. 1b, then, it is obvious that such an idealization will result in a much higher prediction of the ultimate strength of a column. This is further illustrated in Fig. 1c where a sharp peak of the load-deflection curve (dotted line) is apparent. Since the exact M-P-Φ relation is smooth, its load-deflection behavior for column tends to have a round peak. In the elastic and full plastic regions, however, exact and idealized moment-curvature relations give essentially the same result.

To obtain these equilibrium points A and B between external force and internal resistance, the following numerical procedures may generally be followed: first, a trial deflection (or curvature Φ_1) marked as point "O" in Fig. 2 is assumed, and corresponding moment M_1 at the mid-length, the corresponding internal curvature Φ_2 can be obtained. If Φ_1 and Φ_2 are not close enough, new cycle of iteration is necessary. Through this procedure, the elastic deflection point "A" in Fig. 2 corresponding to a point in the pre-buckling branch of the load-deflection curve is determined (Fig. 1c). To locate the point "B" in the post-buckling branch of the curve, we calculate the internal moment M_2 from the trial curvature Φ_1 (Fig. 2). If M_1 and M_2 are not sufficiently close, a new deflection (or curvature) can be calculated from M_2, from which further cycle of iteration can be made.

It should be noted, however, that if the initial point for the curvature Φ_1 is assumed outside the range of the two intersection points A and B, then the iteration procedure will only converge to one of the two solutions, and the other can not be obtained.

If we approximate the M-P-Φ curve by closed form expressions, then, this iterative procedure is not necessary. Instead, analytical expressions for lateral deflection and axial shortening of a beam-column can be derived.

4. ELASTIC ANALYSIS - BEFORE BUCKLING

4.1 Load-Deflection Relation (P-w Curve)

In this paper, the total deflection is considered to have three components as shown in Fig. 3. Firstly, the initial imperfection w_i produces no internal moment and therefore should be excluded from M-P-Φ calculations. Secondly, the deflection due to end moment and/or lateral load w_0 can be expressed in terms of polynomial functions based on conventional elastic beam theory.

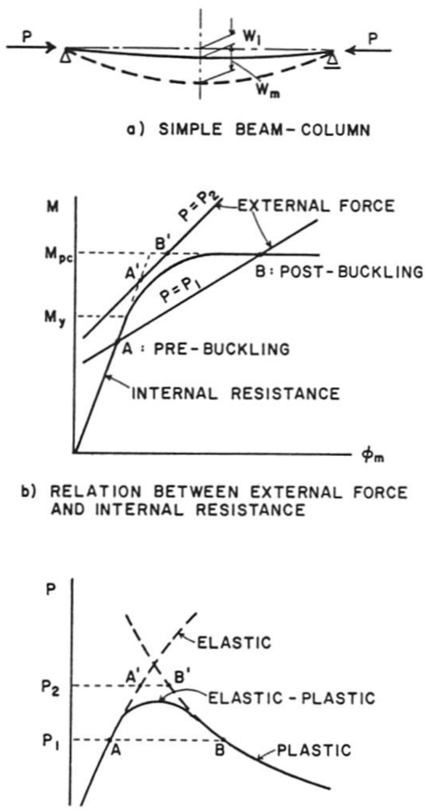

Fig.1 General behavior of a Beam-Column

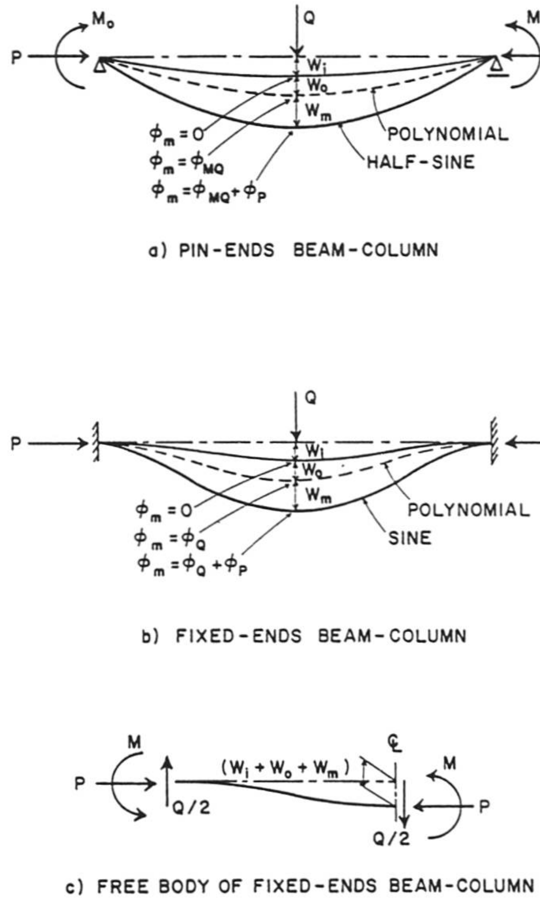

Fig.3 Deflection Shape of a Beam-Column in Elastic Range

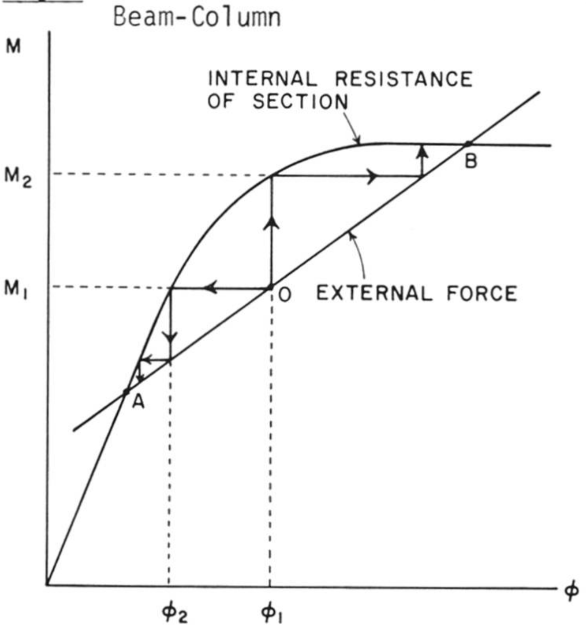

Fig.2 Iterative Procedure Diagram for Equilibrium

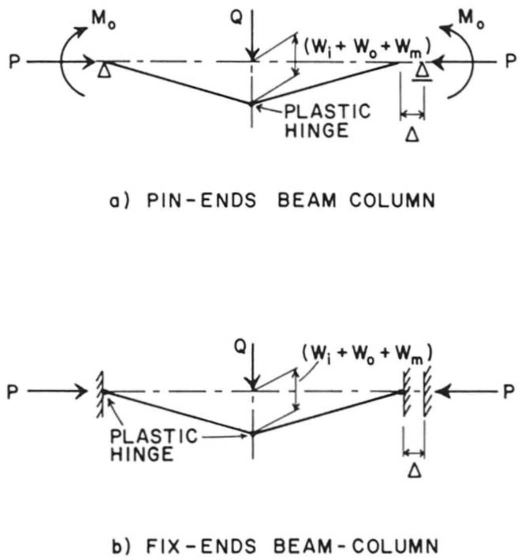

Fig.4 Plastic Hinge Method

Finally, the deflection w_m amplified by the axial force P is assumed to be a sinusoidal function.

The term "elastic" implies that the internal resisting moment must remain in the straight line portion of the M-P-Φ curve (see Fig. 1b). Therefore, in the elastic range, the solution to the problem is reduced to simply obtaining the intersection of two straight lines representing the moment equilibrium condition of the column. For an elastic-perfectly plastic type of idealization for M-P-Φ relation, the present analysis is valid until the internal resisting moment reaches the flow moment. This elastic limit condition will be discussed further in each of the four models to be described later.

Taking the equilibrium condition at mid-span between the external and internal moments, the deflections due to axial load for both pin-ended and fix ended beam-columns can be solved as follows [3]:

$$(W_m)_{EL} = \frac{M_i + M_{op}}{P_{cr} - P} \tag{2}$$

$$\text{where } P_{cr} = \frac{\pi^2 EI}{(KL)^2}, \quad M_i = PW_i, \; M_{op} = PW_0$$

$$\text{and } W_o = \frac{M_o L^2}{8EI} + \frac{QL^3}{48EI} \quad \text{for pin-end}$$

$$= \frac{QL^3}{192} \quad \text{for fix-end}$$

4.2 Load-Shortening Relation (P-Δ Curve)

The axial shortening of a beam-column consists of two parts: the axial shortening due to axial compressive strain and the axial shortening due to lateral deflection. The axial shortening due to lateral deflection can be derived directly from the assumed shape of the column which here is assumed to be a sine shape. Thus, in the elastic range, the total shortening can be expressed by

$$(\Delta)_{EL} = \frac{PL}{EA} + \frac{\pi^2 (W_0 + W_m)^2}{4L} \tag{3}$$

A more detailed description of the elastic analysis is given in Ref. 3.

5. INELASTIC ANALYSIS - POST-BUCKLING

5.1 Plastic Hinge Method

Here, the moment-curvature relation is idealized as elastic-perfectly plastic with plastic hinge moment M_{pc} at which the curvature increases indefinitely. The value M_{pc} is known as the full plastic moment including the effect of axial load P.

In the elastic range, the solutions presented previously are applicable. In the elastic-plastic range, plastic hinges will be formed successively at critical sections of the member. When sufficient number of plastic hinges are formed, a collapse mechanism will be developed, and the segment between plastic hinges will now behave as a rigid body. The additional deflection beyond this stage of loading consists of two straight lines knuckled at center (Fig. 4). This mechanism approach for deriving load-deflection relation is good when the deflection becomes large [5,6]. However, in the elastic-plastic transition range, appreciable errors must be expected for columns near initial elastic-plastic range of loading where the peak or maximum load usually occurs. Nevertheless, this type of simplification is very appealing and worth pursuing. This will be described here.

Load-Deflection Relation (P-w Curve)

A. Pin-Ended Beam-Column

The column will be capable of resisting external loads until the maximum moment at the center of a column is equal to the full plastic moment M_{pc}. Solving the equilibrium equation for mid-span deflection, we have

$$(w_m)_{PL} = \frac{1}{P}\left(M_{pc} - M_o - \frac{QL}{4}\right) - w_i - w_o \tag{4}$$

The definition for each term is given in Figs. 3 and 4.

The full plastic moment M_{pc} of a tubular section with axial load has been developed in Reference 8 in the form non-dimensionalized by the yield values, M_y and P_y, which is

$$\begin{aligned} m_{pc} &= 1.273\,(1 - 1.18\,p^2) \quad \text{for } 0 \le p \le 0.65 \\ &= 1.82\,(1 - p) \qquad\qquad \text{for } 0.65 \le p \le 1.0 \end{aligned} \tag{5}$$

where $m_{pc} = \frac{M_{pc}}{M_y}$, $p = \frac{P}{P_y}$ (P_y = yield axial force = $\sigma_y A$)

The mid-span deflection $(w_m)_{PL}$ in the plastic range can be calculated from Eq. (4) using Eq. (5). Equation (4) describes the post-peak branch of load-deflection relation where for given loads the elastic deflection as calculated from Eq. (2) is smaller than that of the plastic one:

$$(w_m)_{EL} \le (w_m)_{PL} \tag{6}$$

B. Fix-Ended Beam-Column

For a fix-ended beam-column, a mechanism will be developed when plastic hinges are formed at ends and at the center of a beam-column. In this case, the equilibrium at the center of the beam-column gives

$$(w_m)_{PL} = \frac{1}{P}\left(2\,M_{pc} - \frac{QL}{4}\right) - w_i - w_o \tag{7}$$

Using Eqs. (5) and (7), the mid-span deflection for a fix-ended beam-column in the plastic range can be calculated. Equation (6) determines the range of validity of Eqs. (2) and (7).

Load-Shortening Relation (P-Δ Curve)

For an elastic-perfectly plastic M-P-Φ relation, it needs to consider only elastic axial strain for the axial shortening of a beam-column throughout the entire range of loading including post-peak unloading. Further, the axial shortening due to lateral deflection can be calculated directly from the two-straight-line deflection shape in the plastic as well as the elastic range. This is probably a good approximation since the lateral deflection in the elastic range is generally small.

Based on these simplifications, the total axial shortening of the beam-column can be expressed in the form (see Fig. 4).

$$\Delta = \frac{PL}{EA} + [\cos \frac{2w_i}{L} - \cos \frac{2(w_i + w_o + w_m)}{L}] L \qquad (8)$$

The first term in the right-hand side of this equation is the contribution to axial shortening due to axial strain, and the second represents the contribution from lateral deflection.

Numerical results of the present method are shown in Figs. 6 through 8. Also shown are the results using sine curve for deflection shape. This later method is called modified plastic hinge method and will be discussed further in the forthcoming (Fig. 5).

For beam-columns with a bi-linear type of moment-curvature relation, it can be seen that the maximum or peak load corresponds to the attainment of plastic hinge moment at the center of the pin-ended case or plastic hinge moments at the center and two ends of the fix-ended case. There is a sharp drop of load corresponding to the formation of mechanism. It will be seen that the more pronounced the nonlinearity of the moment-curvature relation is, the more smooth is the load-deflection curve in the region close to the peak of the curve. The above is, of course, an extremely simplified approach to the problem; nevertheless, the general behavior illustrated in the load-deflection or -shortening diagrams of Figs. 6 to 8 is valid. A more refined method of analysis to be described later uses the exact moment-curvature relation.

5.2 Modified Plastic Hinge Method

The deflection shape in the plastic range is assumed here to be a sine or a cosine type of curve as shown in Fig. 5 [1]. Here, we extend the usage of the shape of an elastic deflection into the plastic range. As in the plastic hinge method, the elastic-perfectly plastic type of M-P-Φ curve is used here, and the plastic hinges are assumed to develop at mid-span for the pin-ended case, and at mid-span and ends for the fix-ended case.

Load-Deflection Relation (P-w Curve)

For an elastic-perfectly plastic M-P-Φ relation, the maximum internal bending moment at the critical sections is always equal to the full plastic moment M_{pc}. The moment induced by external loads at the critical section is, of course, not affected by the particular type of deflection shape assumed. Therefore, the same load-deflection equations as developed for the plastic hinge method can be used here (Eqs. 4 and 7).

Load-Shortening Relation (P-Δ Curve)

Here, as in the plastic hinge method, only the elastic axial strain is needed to compute the axial strain shortening of a beam-column. The shortening due to lateral deflection can be calculated on the basis of the assumed deflection shape as shown in Fig. 5. Thus, the total axial shortening due to applied forces is approximated by the same equation for the elastic range, Eq. 3. It has the same form for both the pin-ended case and fix-ended case.

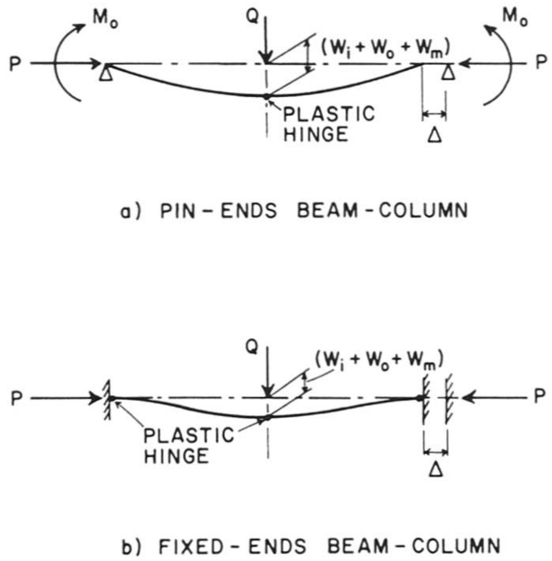

Fig.5 Modified Plastic Hinge Method

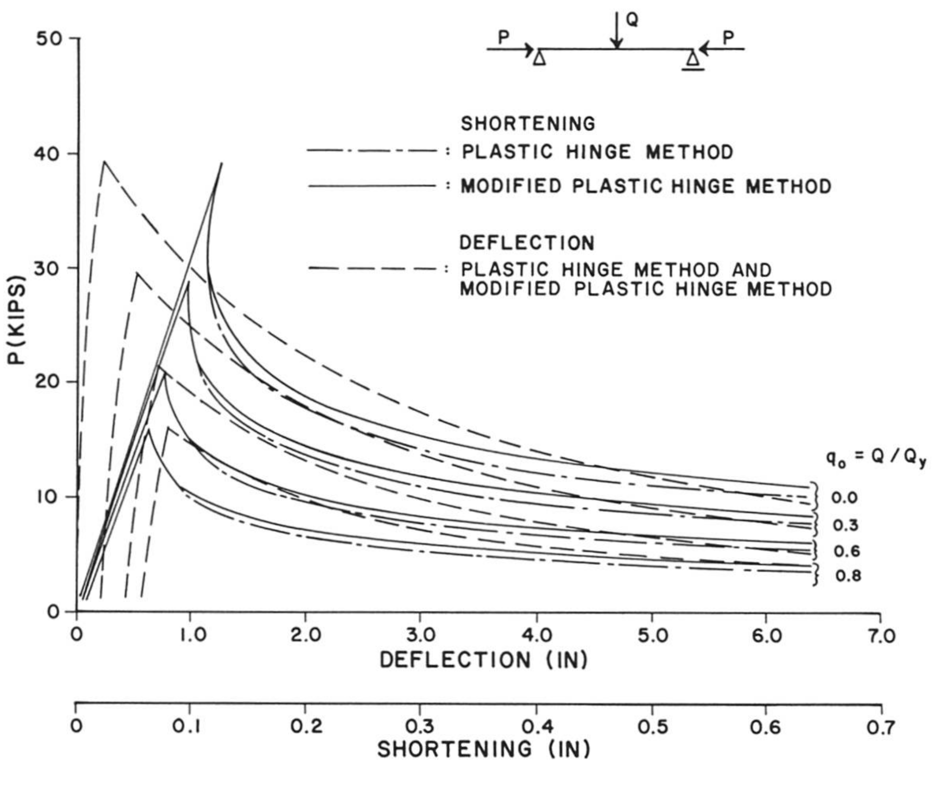

Fig.6 Comparison between Plastic Hinge Method and Modified Plastic Hinge Method, Pin-Ends, L/r=80

Numerical Results

Numerical results using the modified plastic hinge method are also shown in Figs. 6 through 8 in comparison with the results obtained by the plastic hinge method. Column lengths of L/r=80, 120 and support conditions with pin-ended and fix-ended supports are used in the calculations. As expected, the load-deflection relations in these figures are same for both methods. Further, the difference in load-axial shortening relation between these two analyses is not significant.

It should be noted that the effective column length for the fix-ended beam-column of Fig. 8 is the same as that of pin-ended case in Fig. 6. The general behavior in these two figures is seen to be quite similar, but for a given load, the fix-ended case has a much larger value of lateral deflection and axial shortening than that of pin-ended case. This is because the fix-ended case has a much longer actual column length.

5.3 Exact Moment-Curvature Method

The method of analysis described herein is intended to trace out the load-deflection response of a beam-column from zero load upwards as exactly as possible within the limitation of using an assumed deflection function. The present analysis is essentially the same as that of the elastic analysis in that polynomial function is used for the deflection induced by lateral load or bending moment, and sinusoidal function is used for the additional deflection amplified by the axial load. Here, unlike the earlier analysis, we take an accurate account of the moment-curvature relation and moment-axial strain relation.

In order to save computer time, closed form expressions are employed to approximate closely the exact M-P-Φ and M-P-ε_0 relations. The present method is rigorous and can be used to assess the consequences of simplification made previously for moment-curvature relation and moment-axial strain relation. Thus, if this simplified relations are acceptable, analysis and design of tubular member can be based on a consideration of the elastic deformed shape with either elastic or plastic hinge condition at critical sections.

The general pattern of curvature distribution for an elastic-plastic pin-ended beam-column is shown as the solid line in Fig. 9a. Formal mathematical treatment of this problem by integrating the exact curvature distribution along the length will yield an exact equation for the bent shape. Taking the conservative triangular curvature distribution with the maximum curvature at center Φ_m and integrating it, we have the mid-span deflection $w_m = \Phi_m L^2/12$. This value may be considered as a lower bound. If we take the unconservative rectangular curvature distribution with the constant curvature Φ_m, the corresponding w_m will be $(\Phi_m L^2/8)$. This may be considered as an upper bound. It is plain, therefore, that a reasonable estimate of w_m, certainly suitable for design purposes, would be to take it as an average value of $\Phi_m L^2/10$. Reference 4 reported that this simplification results in a good agreement with the test results in the case of reinforced concrete columns. Consider now the assumed deflection shape here, a half sine curve for a pin-ended beam-column. The deflection at mid-span is $w_m = \Phi_m L^2/\pi^2$ (Fig. 9b), which is very close to the average deflection described above.

Formulations of the exact moment-curvature method based on assumptions described above are given in Reference 3.

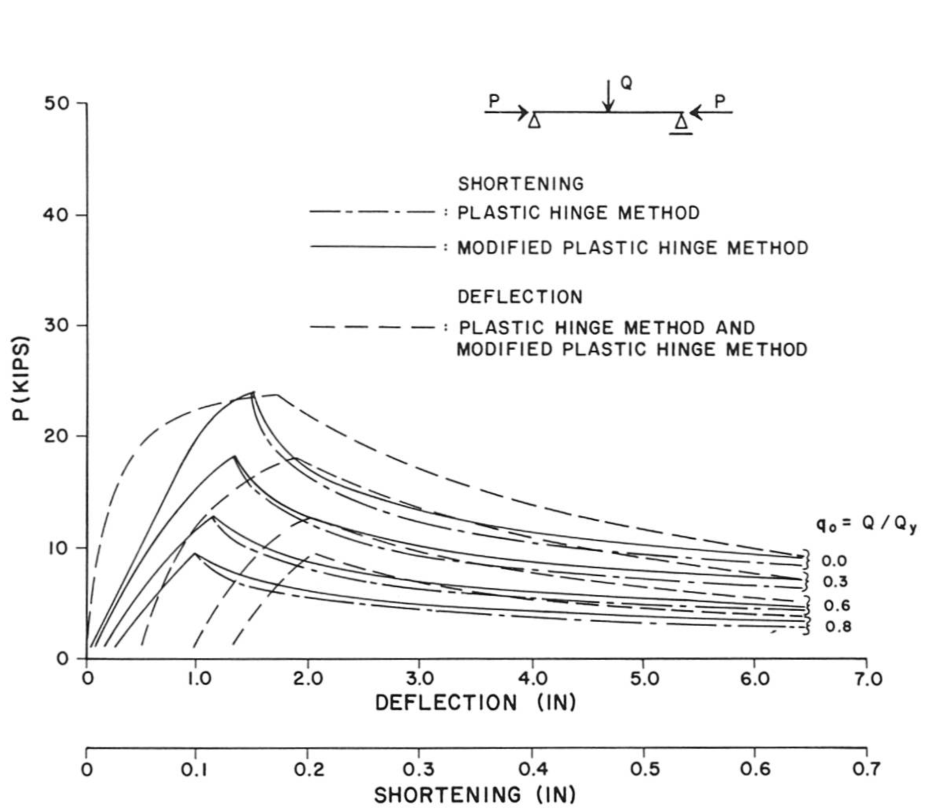

Fig.7 Comparison between Plastic Hinge Method and Modified Plastic Hinge Method, Pin-Ends, L/r=120

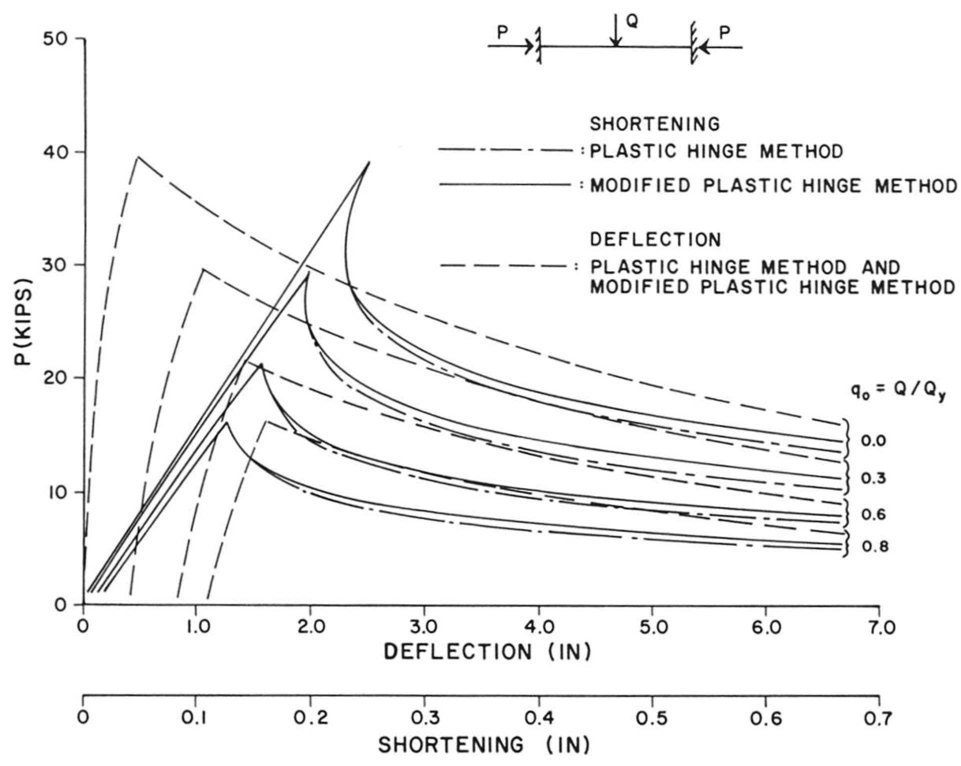

Fig.8 Comparison between Plastic Hinge Method and Modified Plastic Hinge Method, Fix-Ends, KL/r=80

Numerical Results

Numerical results for pin-ended columns using the exact moment-curvature method are shown in Figs. 10 and 11 for two column lengths, L/r=80 and 120. Also shown are the results by the modified plastic hinge method. Initial imperfection is assumed to be 0.001L in both analyses. It can be seen from the figures that the ultimate strengths of beam-columns by the exact moment-curvature method are significantly lower than those obtained by the modified plastic hinge method. This is because the nonlinear part of the M-P-Φ curve results in a smooth transition curve near the peak portion of P-w curve. It is plain that the more pronounced the nonlinearity of the moment-curvature relation, the smoother, and thus the less peak load, the load-deflection curve is likely. Thus, if bi-linear moment-curvature relation is accepted as a simplification for practical analysis, the ultimate strength of a tubular member may be considerably overestimated.

Figures 12 through 14 show the comparison of the exact moment-curvature method with Newmark's method (exact solution) [3,8] for L/r=80 and 120 varying the end moment or the lateral load. Both initial imperfection of 0.001L and the residual stresses are considered here. It can be seen that the exact moment-curvature method gives a little conservative values for the ultimate strength but generally agrees well with the exact solution. Also, it is found that the column behavior with end moment is closer to the exact solution than that of lateral load (Fig. 14). This is because the curvature distribution due to the end moment is closer to the assumed curvature distribution.

5.4 Average Flow Moment Method

The bi-linear moment-curvature relation has been used in both the plastic hinge method and the modified plastic hinge method. The solutions based on the exact moment-curvature method indicate that this simplified relation is a reasonable idealization for predicting the response of beam-columns in the initial elastic region and in large post-buckling region, but it grossly overestimates the intermediate transition region which is probably the most important region for the ultimate strength of structures. In order to improve this prediction near the peak load but still using the bi-linear moment-curvature simplification, the concept of an average flow moment may be applied. The application of this concept to beam-columns of wide flange cross section has resulted in an accurate prediction of the maximum load carrying capacity of all beam-columns studied [2].

Average Flow Moment

Turning now to the elastic-perfectly plastic moment-curvature relation (Fig. 15), it is obvious that the elastic-fully plastic moment-curvature relation will give an upper bound solution (see Fig. 15), and the elastic-initial yield moment-curvature relation will result in a lower bound. Therefore, the true response of a beam-column must lie between these two extreme idealizations. Hence, if the elastic-average flow moment-curvature relation is introduced, plastic hinge type of analysis based on a consideration of an elastic deformed shape corresponding to an average flow moment condition being presented at the critical cross sections will be much improved.

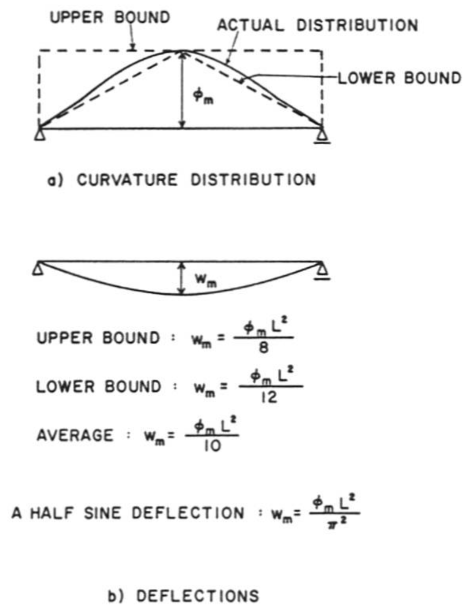

Fig.9 Relationship between Curvature Distribution and Deflection at Center

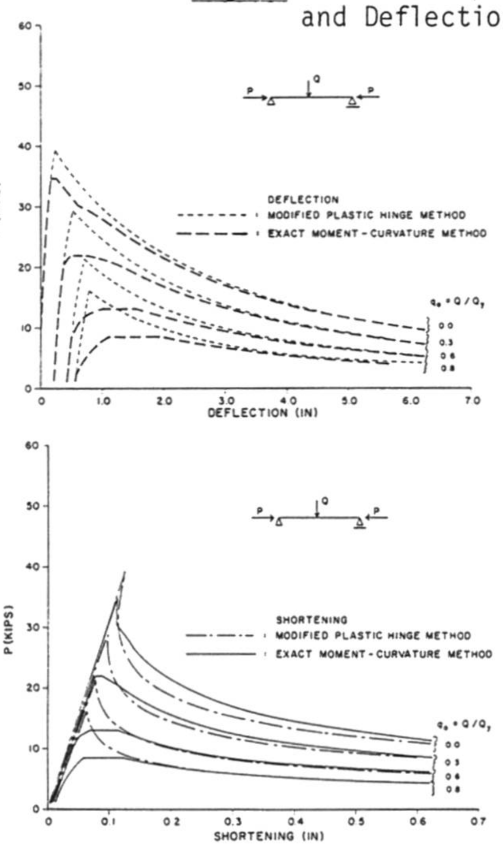

Fig.10 Comparison between Exact Moment-Curvature Method and Modified Plastic Hinge Method, Pin-Ends, L/r=80

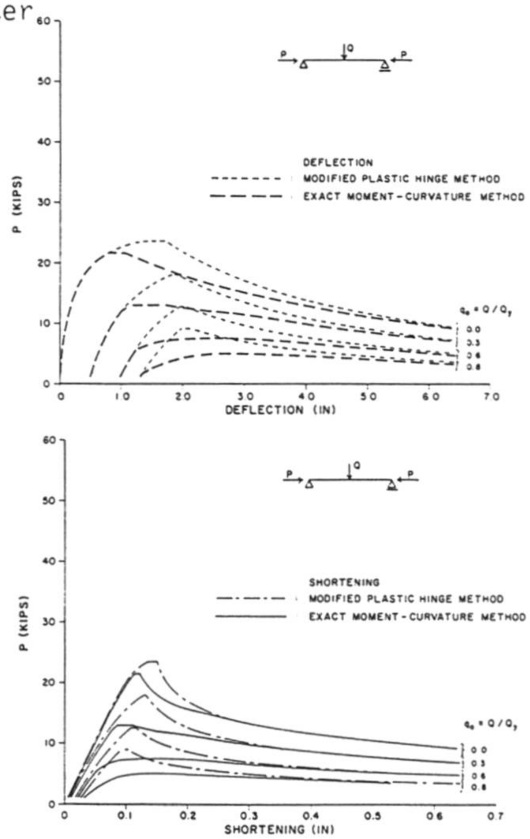

Fig.11 Comparison between Exact Moment-Curvature Method and Modified Plastic Hinge Method, Pin-Ends, L/r=120

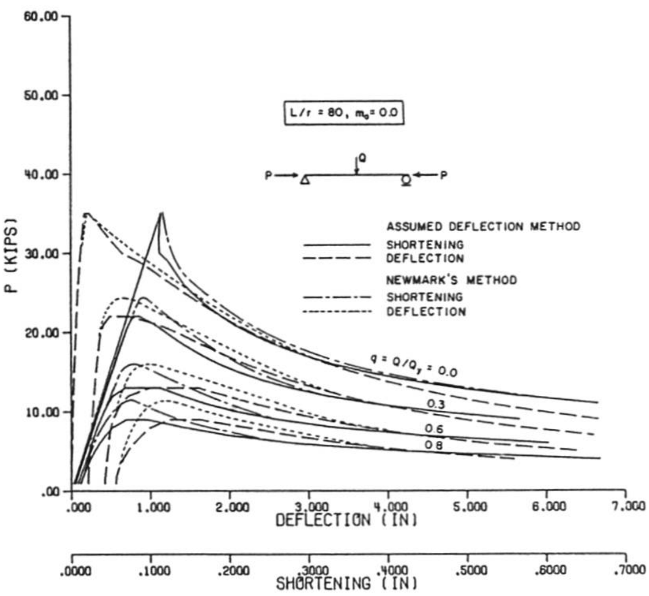

Fig.12 Comparison of Fxact Moment-Curvature Method with Newmark's Method, L/r=80, m_o=0.0.

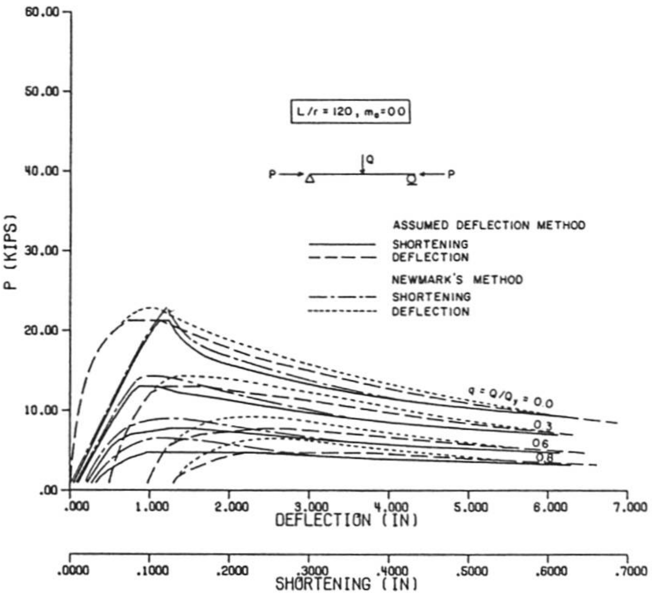

Fig.13 Comparison of Exact Moment-Curvature Method with Newmark's Method, L/r=120, m_o=0.0.

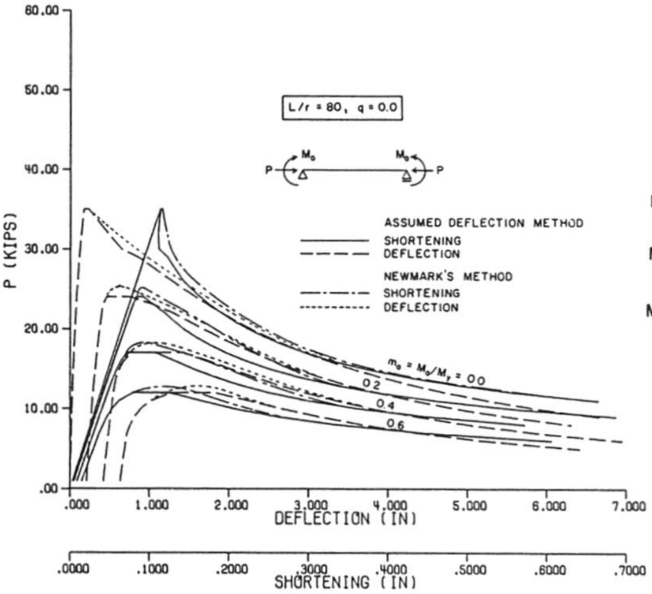

Fig.14 Comparison of Exact Moment-Curvature Method with Newmark's Method, L/r=80, q=0.0.

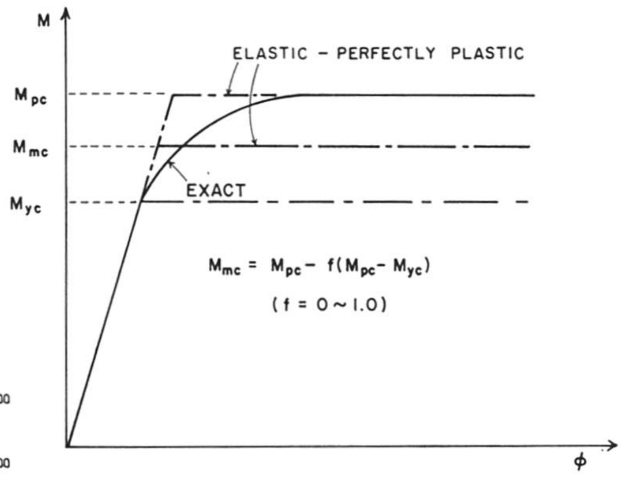

Fig.15 Average Flow Moment

The normalized average flow moment $m_{mc} = M_{mc} M_y$ can be written in the form [2]

$$m_{mc} = m_{pc} - f\,(m_{pc} - m_{yc}) \tag{9}$$

where $m_{yc} = \frac{M_{yc}}{M_y}$ = normalized initial yield moment, and f = parameter function.

It can be seen that when the parameter function f takes the extreme values, zero and one, the corresponding average flow moment is reduced to m_{pc} and m_{yc}, respectively. The parameter function may be a function of thrust $p = P/P_y$, slenderness ratio L/r and the boundary conditions of a beam-column. Herein, the parameter function is assumed to have the form

$$f = f_1\left(\frac{P}{P_{BUCK}}\right)\; f_2\left(\frac{L}{r}\right)\; f_3(B.C.) \tag{10}$$

where $f_1\left(\frac{P}{P_{BUCK}}\right)$ = the parameter depending on axial force

$f_2\left(\frac{L}{r}\right)$ = the parameter depending on slenderness ratio

f_3 (B.C.) = the parameter depending on boundary conditions

For a tubular beam-column, the parameters f_1, f_2 and f_3 are assumed to have the simple forms

$$f_1 = \left(\frac{P}{P_{BUCK}}\right)^n \;, \left(n = \frac{4}{1 + m_o + q}\right) \tag{11}$$

$$f_2 = \frac{1}{70}\;\frac{L}{r} \tag{12}$$

$$f_3 = \begin{cases} 1.0 \text{ for pin-ends} \\ 0.5 \text{ for fix-ends} \end{cases} \tag{13}$$

where P_{BUCK} = the ultimate strength (buckling load) of a column by the plastic hinge type of analysis using the full plastic moment M_{pc}

$m_o = \frac{M_o}{M_y}$ = normalized applied moment at ends,

$q = \frac{Q}{Q_y}$ = normalized lateral load at center, and r = radius of gyration.

P_{BUCK} is the maximum load of a beam-column when the full plastic moment M_{pc} is attained at critical sections. This value can be determined simply by equating the lateral deflection from the elastic branch of the load-deflection curve to that of the plastic branch and solving for the maximum axial force. For example, for the case of pin-ended beam-column, equate Eq. (2) to Eq. (4) and solve for P. For the fix-ended case, equate Eq. (2) to Eq. (7) and solve for P.

Load-Deflection Relation (P-w Curve)

Since the present analysis is identical to that of the modified plastic hinge method except that we use the average flow moment, it follows that we can use the same equations by simply substituting the full plastic moment M_{pc} by the average flow moment M_{mc}.

For a pin-ended beam-column, we have

$$(w_m)_{PL} = \frac{1}{P}\left(M_{mc} - M_o - \frac{QL}{4}\right) - w_i - w_o \tag{14}$$

and for a fix-ended beam-column, we have

$$(w_m)_{PL} = \frac{1}{P}\left(2\, M_{mc} - \frac{QL}{4}\right) - w_i - w_o \tag{15}$$

valid for $(w_m)_{EL} \leq (w_m)_{PL}$ (6)

Load-Shortening Relation (P-Δ Curve)

To take account of the effect of the development of plastic strain near critical sections along the length of a column throughout most of the loading range on the column axial stiffness, the concept of effective axial stiffness is introduced here for the calculation of axial shortening due to axial strain. The effective axial stiffness may take the following form by dividing the column into elastic and plastic portions:

$$E_{eff} = h\, E + g\, E \tag{16}$$

where h = a constant for elastic portion of the column (=0.5), and g = plastic reduction factor depending on axial force. The following function for the plastic reduction factor g was found suitable for tubular columns:

$$g = 0.5\left(\frac{P}{P_{BUCK}}\right) \tag{17}$$

Thus, the axial shortening due to axial strain can be calculated using the effective axial stiffness AE_{eff} in Eq. (3).

Numerical Results

Typical numerical results using an average flow moment are shown in Figs. 16 and 17 for L/r=80 and 120, respectively. Also shown are the results by the exact moment-curvature method. It can be seen that for a more slender column (say, L/r over 90), the average flow moment method agrees well with the exact moment-curvature method (Fig. 17). However, when the beam-column is relatively short while lateral load is large, the average flow moment method shows some errors. It should be noted that where a bi-linear moment-curvature relation is used, the load-deflection response of a beam-column will always show a sharp peak at the ultimate strength. This is unavoidable for this type of idealized moment-curvature relation.

6. SUMMARY AND CONCLUSIONS

Four analytical methods based on assumed deflection shapes and M-P-Φ relations have been studied in this paper. This includes (1) the plastic hinge method; (2) the modified plastic hinge method; (3) the exact moment-curvature method; and (4) the average flow moment method. In the elastic range, all these methods use the same conventional elastic beam-column analysis [7].

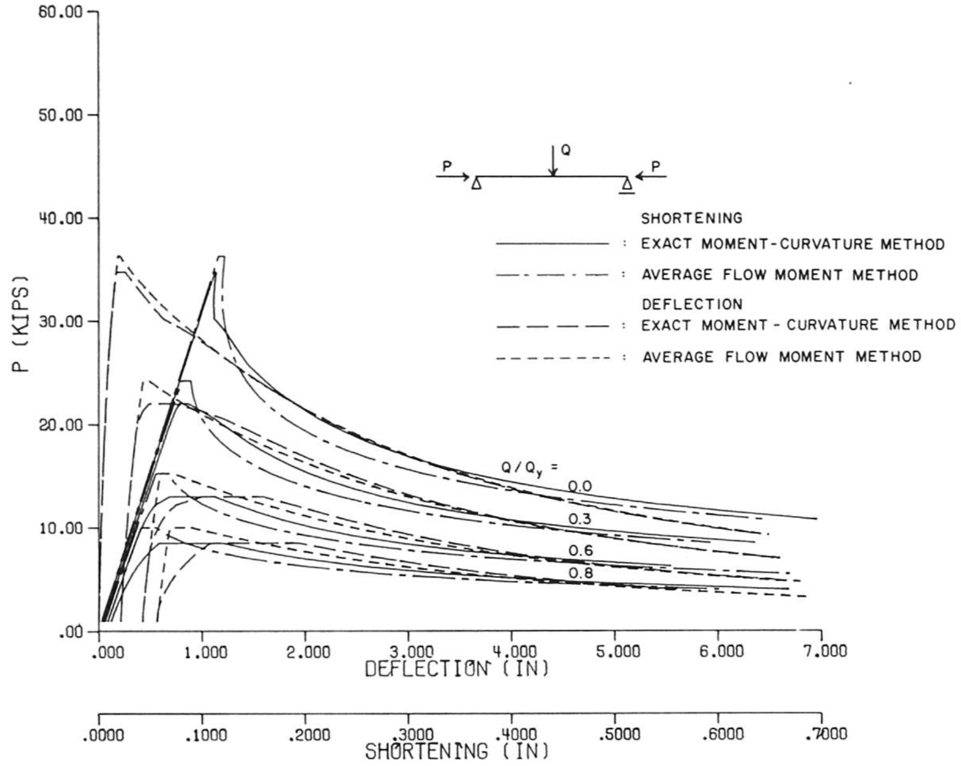

Fig.16 Comparison between Exact Moment-Curvature Method and Average Flow Moment Method, Pin-Ends, L/r=80

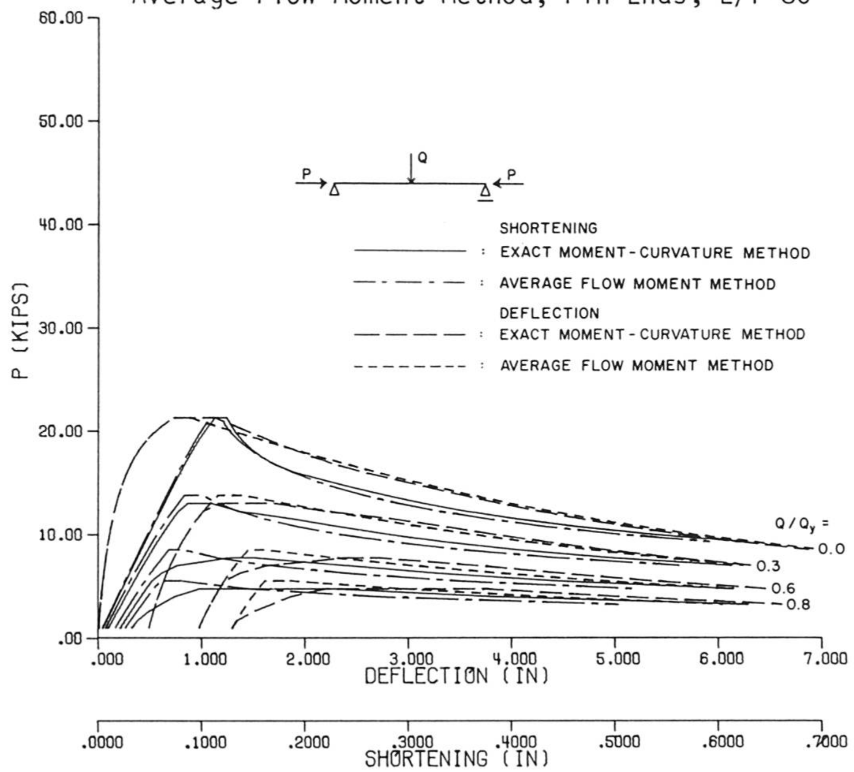

Fig.17 Comparison between Exact Moment-Curvature Method and Average Flow Moment Method, Pin-Ends, L/r=120

In the plastic range, only the plastic hinge method used the two-straight-line deflected shape (Fig. 4), and the other analyses assumed the deflection shape to be a sinusoidal function throughout the entire loading range (Fig. 3). As for the M-P-Φ relation, only the exact moment-curvature method used the exact M-P-Φ curves, and the others used the elastic-perfectly plastic type of bi-linear relation (Fig. 15) where the average flow moment method modified the full plastic moment to a somewhat averaged value.

The conclusions made from these studies are as follows:

1) The exact moment-curvature mehtod is most suitable among the assumed deflection methods studied in this paper. This method enables designers to assess in a simple manner the behavior of beam-columns up to ultimate load including post-buckling behavior.
2) The general validity of the exact moment-curvature method has been demonstrated by comparisons with other analytical procedures as well as available test data [3].
3) Although solving a cubic equation is required in the exact moment-curvature method, computational time required by a computer is found to be very reasonable [3].
4) The exact moment-curvature method strikes the balance between the requirement of realistic representation of column behavior and the requirement for simplicity in use. It is considered that in both these respect, the method is most satisfactory.
5) The elastic-perfectly plastic type of M-P-Φ simplification with full plastic flow moment always overestimates the ultimate strength of a beam-column.
6) The average flow moment method improves the ultimate strength prediction for a beam-column using the elastic-perfectly plastic type of moment-curvature relation, especially for the case of long columns.
7) Formal mathematical treatment of the beam-column problem will yield an exact equation for the bent shape of a beam-column. However, the exactness of this bent shape is found to be not significant in affecting the overall behavior and strength of tubular beam-columns. The shape of the moment-curvature relation is found to play the key role in the present study.

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