# Analysis of asymmetric structures by Galerkin technique

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Objekttyp: **Article**

Zeitschrift: **IABSE proceedings = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **8 (1984)**

Heft P-76: **Analysis of asymmetric structures by Galerkin technique**

PDF erstellt am: **16.08.2024**

Persistenter Link: https://doi.org/10.5169/seals-38338

# Analysis of Asymmetric Structures by Galerkin Technique

## Analyse de structures asymétriques par la méthode Galerkine

## Anwendung des Verfahrens von Galerkin auf hohe Bauten

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### SUMMARY
The Galerkin method of weighted residuals is employed for the analysis of non-uniform asymmetric tall structures. The governing equations are formulated through the continuum approach idealizing the structure as a shear-flexure cantilever. Transcendental shape functions are used as an approximation to the true displacement field. The method is simple yet powerful. The validity and versatility of the method are exemplifield through several numerical examples where the results are compared with those obtained from the existing methods.

### RÉSUMÉ
La méthode Galerkine des résidus pondérés est utilisée pour analyser des grandes structures asymétriques. Les équations de base découlent de l'idéalisation par une console fléchie composée. Des équations transcendantales servent à approcher l'état de déformation réel. La méthode est simple et néanmoins puissante. Sa validité est vérifiée par des méthodes existantes pour certains exemples.

### ZUSAMMENFASSUNG
Bei der Berechnung hoher Bauten, auch asymmetrischer und ungleichförmiger, kann die ganze Tragstruktur als Kontinuum betrachtet und als auf Biegung und Schub beanspruchter Kragarm behandelt werden. Zur Lösung des sich ergebenden Systems gekoppelter Differentialgleichungen wird das Verfahren der gewichteten Residuen von Galerkin angewendet. Sogenannte Formfunktionen werden dabei als Näherungen für das wirkliche Verschiebungsfeld benutzt. Diese Methode ist einfach aber effektvoll. Die Gültigkeit und die Vielseitigkeit dieses Verfahrens werden mittels mehrerer numerischer Beispiele veranschaulicht. Die Resultate werden mit jenen anderer Verfahren verglichen.

## 1. INTRODUCTION

Tall buildings comprising frames and shear walls properly coupled together is one of the most efficient and economical structural system. The analysis of such structures subjected to lateral load can be grouped into two categories. The first reduces the walls to wide columns and treat each part as a discrete element[1]. This results in large core storage requirement and long computing time. The second approach pioneered by CHITTI[2] considers the frame and the spandrel beams as shear contributing continua and solve the whole structure as one problem. The latter is simpler, less expensive and yet provides reasonable results good enough for preliminary design purposes. In this paper, the governing equations using the continuum approach are adopted and the Galerkin method of weighted residuals is employed in the solution of general shear wall frame structures.

## 2. ASSUMPTIONS

In order to simplify the analysis certain basic assumptions are made.

(a) The floor slabs of the building are assumed to act as diaphragms which are infinitely rigid in-plane but very flexible out-of-plane. Rigid body displacements in the horizontal planes of the whole structure are assumed.

(b) The materials are elastic and homogeneous.

(c) Vlasov's thin-walled beam theory is valid for each individual wall; that is, the warping stresses are considered, the cross-sectional shape of the wall remains undistorted and the shear strain in the middle surface of the section is negligible.

(d) Shear and axial deformations are neglected.

(e) Points of contraflexure are assumed to be at the mid-length of the connecting beams and columns when the equivalent distributed shear properties for the frames are computed.

## 3. GOVERNING EQUATIONS

It is a well-known fact that tall buildings comprising frames and shear walls coupled together will deform as a shear-flexure cantilever [3-7]. The governing equations for each element at its principal axis in its principal directions can be expressed as

$$(EI_{yy})_e u_e^{iv} - (GA_{xx})_e u_e'' = (f_x)_e$$

$$(EI_{xx})_e v_e^{iv} - (GA_{yy})_e v_e'' = (f_y)_e$$

and

$$(EI_{ww})_e \theta_e^{iv} - (GJ_{ww})_e \theta_e'' = (f_w)_e$$

or in a more comprehensive form as

$$(EI_{ii})_e \delta_{ij} (u_j^{iv})_e - (GA_{ii})_e \delta_{ij} (u_j'')_e = (f_i)_e \qquad i,j = 1,2,3 \tag{1}$$

where

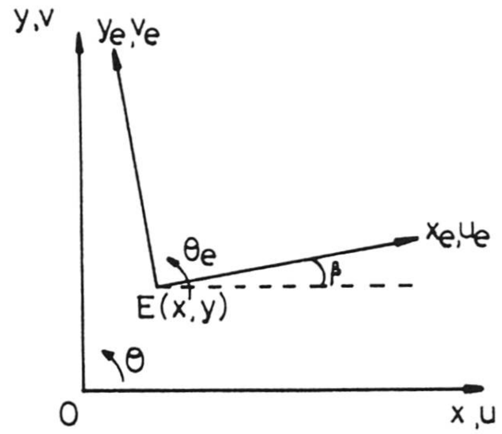
$(u_j)_e = (u_e v_e \theta_e)^T$, the displacements and twist in the element principal directions;

$(f_i)_e = [(f_x)_e \ (f_y)_e \ (f_w)_e]^T$, the distributed loads and torque corresponding to $u_e$, $v_e$ and $\theta_e$ respectively;

$\delta_{ij}$ = Kronecker-delta function

$(EI_{ii})_e = D[(EI_{yy})_e \; (EI_{xx})_e \; (EI_{ww})_e]$, the flexural and warping resistance diagonal matrix;

$(GA_{ii})_e = D[(GA_{xx})_e \; (GA_{yy})_e \; (GJ_{ww})_e]$, the shear and torsional constant diagonal matrix;




| | |
|---|---|
| O | REFERENCE POINT |
| Ox,Oy | REFERENCE AXES |
| $Ex_e, Ey_e$ | ELEMENT PRINCIPAL AXES |

Fig. 1 Coordinate systems

Note that $EI_{ii}$ are contributed mainly from the walls and the contribution of frames on $EI_{ii}$ is negligible and usually neglected.

If the local principal system is at the location (x,y) with an orientation angle β with respect to a common global system as shown in Fig. 1, the transformation of the displacement and force vectors in the two systems are related as follows:

$$(u_i)_e = R_{ij}T_{jk}u_k \qquad i,j,k = 1,2,3 \qquad (2)$$

and
$$f_k = T_{jk}R_{ij}(f_i)_e \qquad i,j,k = 1,2,3 \qquad (3)$$

where $u_k$ and $f_k$ are the global displacement and force vectors;

$$R_{ij} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ , the rotational transformation matrix; (4)

and
$$T_{jk} = \begin{bmatrix} 1 & 0 & y \\ 0 & 0 & -x \\ 0 & 0 & 1 \end{bmatrix}$$ , the translational transformation matrix (5)

Matrices $EI_{ij}$ and $GA_{ij}$ in global system can thus be shown as

$$EI_{ij} = T_{ki}R_{mk}\,(EI_{mm})_e R_{ml}T_{lj} \qquad i,j,k,l,m = 1,2,3 \qquad (6)$$

and
$$GA_{ij} = T_{ki}R_{mk}\,(GA_{mm})_e R_{ml}T_{lj} \qquad i,j,k,l,m = 1,2,3 \qquad (7)$$

Assembling all n elements comprising the whole building, the final governing equation becomes

$$EI_{ij}u_j^{iv} - GA_{ij}u_j'' = f_i \qquad i,j = 1,2,3 \qquad (8)$$

## 4. BOUNDARY CONDITIONS

For foundation with translational, rocking and torsional flexibilities, the static equilibrium at the base requires that

$$K^t_{ij}u_j(0) = -EI_{ij}u_j'''(0)$$

$$K^r_{ij}u_j'(0) = EI_{ij}u_j''(0) \qquad i,j = 1,2,3 \qquad (9)$$

where $K^t_{ij}$ and $K^r_{ij}$ represent the equivalent foundation stiffness matrices with respect to the global reference axes. They are formulated from each individual footing, transformed to the same global system and assembled in the same manner as the formulation of $EI_{ij}$'s matrix.

If $S^o_i$ and $M^o_i$ are the applied forces and moments at the top of the building, the mechanical boundary conditions are

$$GA_{ij}[u_j'(h)-u_j'(0)] - EI_{ij}u_j'''(h) = S^o_i$$

$$EI_{ij}u_j''(h) = M^o_i \qquad i,j = 1,2,3 \qquad (10)$$

If the building is fixed on rigid foundation, the geometric boundary conditions at its base can be expressed as

$$u_j(0) = 0$$

$$u_j'(0) = 0 \qquad j = 1,2,3 \qquad (11)$$

If only the rocking flexibility of the ground is considered, then the geometric boundary conditions become

$$u_j(0) = 0 \qquad j = 1,2,3$$

$$u_3'(0) = 0 \qquad (12)$$

The mechanical boundary conditions at the top of the structure for the last two cases are the same as those given in Eq. (10).

## 5. METHOD OF SOLUTION

In the Galerkin's method of weighted residuals, a set of displacement functions is assumed for each of the independent variables. The errors incurred when the assumed functions are substituted into the governing equations are minimized in the averaged sense by using the assumed shape functions as the weighting functions[8]. Let the assumed set of continuous displacement functions be

$$u_j = \delta_{sj}a_{sr}\phi_{jr} \qquad j,s = 1,2,3; \quad r = 1,2,...m \quad (13)$$

where $a_{sr}$ are the unknown parameters, and $\phi_{jr}$ the shape functions. Applying Galerkin procedure leads to

$$\int_o^h (EI_{ij}u_j^{iv} - GA_{ij}u_j'' - f_i)\,\delta_{ik}\phi_{kr}dz = 0$$

$$i,j,k = 1,2,3; \quad r = 1,2,...m \quad (14)$$

Integrating by parts and substituting the appropriate boundary conditions yield

$$[\int_o^h(EI_{ij}\phi_{js}''\phi_{kr}'' + GA_{ij}\phi_{js}'\phi_{kr}')\,dz - GA_{ij}\phi_{js}'(0)(\phi_{kr}(h)-\phi_{kr}(0))$$

$$K^r_{ij}\phi_{js}'(0)\,\phi_{kr}'(0) + K^t_{ij}\phi_{js}(0)\,\phi_{kr}(0)]\,\delta_{ik}\delta_{qj}a_{qs}$$

$$= \int_o^h w_i\phi_{kr}\delta_{ik}dz + M^o_i\phi_{kr}'(h)\delta_{ik} + S^o_i\phi_{kr}(h)\delta_{ik}$$

$$i,j,k,q = 1,2,3; \quad r,s = 1,2,..m \quad (15)$$

for flexible foundation and

$$\int_0^h (EI_{ij}\phi''_{js}\phi''_{kr} + GA_{ij}\phi'_{js}\phi'_{kr})\, dz\delta_{ik}\delta_{qj}a_{qs} = \int_0^h w_i\phi_{kr}\delta_{ik}dz$$

$$+ (S_i^o\phi_{kr}(hr) + M_i^o\phi'_{kr}(h))\,\delta_{ik}$$

$$i,j,k,q = 1,2,3;\; r,s = 1,2,..m \quad (16)$$

for rigid foundation. Eqs. (15) or (16) constitutes a set of simultaneous equations from which the unknown parameters $a_{qs}$ can be determined.

## 6. NON-UNIFORM BUILDINGS

For non-uniform buildings, the integration is carried out in each sub-regions and the results are summed up to yield

$$[\int_0^h (EI_{ij}\phi''_{js}\phi''_{kr} + GA_{ij}\phi'_{js}\phi'_{kr})\, dz - GA_{ij}\phi'_{js}(0)(\phi_{kr}(h)-\phi_{kr}(0))$$

$$+ K^r_{ij}\phi'_{js}(0)\,\phi'_{kr}(0) + K^t_{ij}\phi_{js}(0)\,\phi_{kr}(0)]\,\delta_{ik}\delta_{qj}a_{qs}$$

$$+ \sum_{p=1}^{ns-1} [(\{(EI_{ij})_{p+1} - (EI_{ij})_p\}\,\phi''_{js}(h_p)\phi'_{kr}(h_p))$$

$$+ (\{(GA_{ij})_{p+1} - (GA_{ij})_p\}\,\phi'_{js}(h_p)$$

$$- \{(EI_{ij})_{p+1} - (EI_{ij})_p\}\,\phi'''_{js}(h_p))\,\phi_{kr}(h_p)]\,\delta_{ik}\delta_{qj}a_{qs}$$

$$= \int_0^h w_i\phi_{kr}\delta_{ik}dz + M_i^o\phi'_{kr}(h)\,\delta_{ik} + S_i^o\phi_{kr}(h)\,\delta_{ik}$$

$$i,j,k,q = 1,2,3;\quad r,s = 1,2,..m \quad (17)$$

where ns is the number of the segments. The last summation on the left hand side of Eq. (17) is due to the continuity conditions at the juncture of the segments. The displacement functions in various segments of different properties should actually be conceived differently so that this extra term vanishes. If the continuous function defined in Eq. (13) is used over the entire domain, the correction is necessary. The effect of this term on the general results is however insignificant and usually ignored as adopted herein.

## 7. STRESS RESULTANTS

Once the displacements of the elements are found, the stress resultants for each element can be determined by taking the appropriate differentials.

For the core and the shear walls, the bending moments in their own local set of principal axes are given as

$$(M_x)_e = (EI_{yy})_e u''_e$$

$$(M_y)_e = (EI_{xx})_e v''_e$$

and the bimoments in the core wall as

$$(M_w)_e = (EI_{ww})_e \theta''_e \quad (18)$$

The shear forces in the walls are

$$(S_x)_e = -(EI_{yy})_e u_e'''$$

$$(S_y)_e = -(EI_{xx})_e v_e''' \qquad (19)$$

and the corresponding warping and St. Venant torque as

$$(T_w)_e = -(EI_{ww})_e \theta_e'''$$

$$(T_s)_e = (GJ_w)_e \theta_e' \qquad (20)$$

The shear forces in the frame at the kth storey are given by

$$(S_{xk})_e = (GA_x)_e[\{(u_k)_e - (u_{k-1})_e\}/h_k - u_e'(0)]$$

$$(S_{yk})_e = (GA_y)_e[\{(v_k)_e - (v_{k-1})_e\}/h_k - v_e'(0)] \qquad (21)$$

where $h_k$ is the height of the storey concerned; $u_k$ and $u_{k-1}$ are the deflections at the kth and (k-1)th floor respectively.

The moments in the frame are found by assuming that the points of contraflexure occur at mid-height of the columns. At the kth storey, they are

$$(M_{xk})_e = (S_{xk})_e h_k/2$$

$$(M_{yk})_e = (S_{yk})_e h_k/2 \qquad (22)$$

## 8. SHAPE FUNCTIONS

A necessary and sufficient condition in choosing the shape function is that the function must satisfy the geometric boundary condition. This will lead to a class of admissible functions. If in addition to this condition, the mechanical boundary conditions are satisfied, then a subset of the admissible functions, namely the comparison functions are formed. Due to the difficulty in satisfying the mechanical boundary conditions, the set of comparison functions is usually limited and normally avoided, despite its advantage of producing rapid convergent results. A compromise solution between the two is to introduce the functions which satisfy completely the geometric boundary conditions and approximately the mechanical boundary conditions.

Considering Eq. (10), if the matrix $EI_{ij}$ is made diagonal by transforming to its principal axes, the homogeneous boundary conditions at the top of the structure are

$$u_i''(h) = 0$$

$$GA_{ij}u_j'(h) - EI_{ii}u_j'''(h)\ \delta_{ij} = 0 \qquad i,j = 1,2,3 \qquad (23)$$

It is difficult to satisfy Eq. (23b) completely, thus a simpler condition is adopted by dropping all the off-diagonal terms in matrix GA from which Eq. (23b) becomes

$$GA_{ii}u_j'(h)\ \delta_{ij} - EI_{ii}u_j'''(h)\ \delta_{ij} = 0 \qquad i,j = 1,2,3 \qquad (23c)$$

Eqs. (23a) and (23c) are the approximate mechanical boundary conditions to be satisfied by the assumed functions.

In the present study, the mode shapes of a shear-flexure cantilever beam under free vibration are chosen. For rigid foundation case, they are

$$\phi_{jr} = (\sin \lambda_{jr} z/h - \sinh \lambda_{jr} z/h) + C_{jr}(\cos \lambda_{jr} z/h - \cosh \lambda_{jr} z/h) \qquad (24)$$

where $C_{jr}$ is determined from equation (23a) as

$$C_{jr} = - \frac{\sin \lambda_{jr} + \sinh \lambda_{jr}}{\cos \lambda_{jr} + \cosh \lambda_{jr}} \qquad j = 1,2,3; \quad r = 1,2,...m$$

Substituting Eq. (24) into Eq. (23c) leads to three uncoupled equations of the form

$$EI_{jj}(1+\cos \lambda_{jr} \cdot \cosh \lambda_{jr}) + GA_{jj} \sin \lambda_{jr} \cdot \sinh \lambda_{jr} = 0$$
$$j = 1,2,3; \quad r = 1,2,...m \qquad (25)$$

from which $\lambda_{jr}$ can be determined. The values of $\lambda_{jr}$ against $\alpha_j h$ (= $\sqrt{(EI_{jj}/GA_{jj})}$) are plotted in Fig. 2. It is noted that $\lambda_{jr}$ corresponding to higher modes do not vary significantly with $\alpha_j h$.

If the mode shapes of a pure flexure cantilever are chosen, the values of $GA_{jj}$ in Eq. (25) will be set to zero and we obtain

$$(1+\cos \lambda_{jr} \cdot \cosh \lambda_{jr}) = 0 \qquad (26)$$

The well-known values of $\lambda_{jr}$ in this case are 1.875, 4.694, 7.855, 10,996, 14.137...etc.

For flexible foundation, if rocking is allowed an extra term z should be added for displacements u and v, and if translation and twisting at the base are present, the extra rigid body displacement constant $(u_i)_o$ must be included.

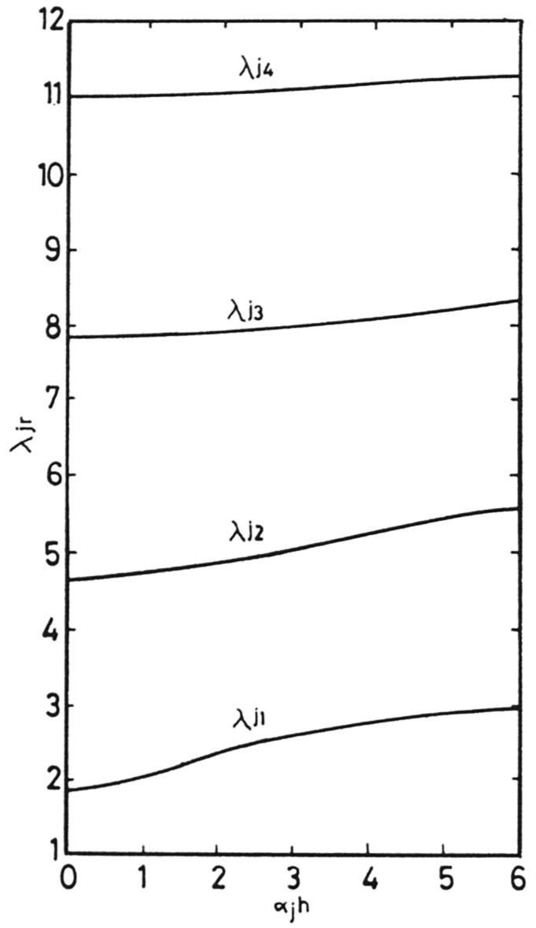



Fig. 2 Shape function parameter

## 9. ACCURACY AND CONVERGENCE

By using the admissible functions, uniform convergence can be obtained for zero and first derivatives and convergence in the mean sense for the second differentials. The convergence of the third differentials especially at the boundary is not assured [9].

Using the proposed approximate comparison functions may assist convergence to an approximate value at the boundary for the third differential. Convergence in the domain is also better with the improved functions. The degree of accuracy is dependent on the choice of the functions, the number of terms, the layout and properties of the building and the complexity of the loadings. It is found that generally, five terms are sufficient to ensure good results for preliminary design purposes.

## 10. NUMERICAL EXAMPLES

Several examples are presented to demonstrate the accuracy and simplicity of the proposed method for the static analysis of frame shear wall structures.

### 10.1 Example 1

A 16-storey building used by several authors [1,3] in the study of asymmetrical multi-storey building is solved by the method presented. The structural plan is shown in Fig. 3 and the floor-to-floor height is 3 metres. The base is fixed and the structure has uniform properties along the height with an assumed modulus of elasticity of 20000 $MN/m^2$. The lateral load which acts on the structure is uniformly distributed with a magnitude of 40 kN per metre of height acting in the positive y direction.

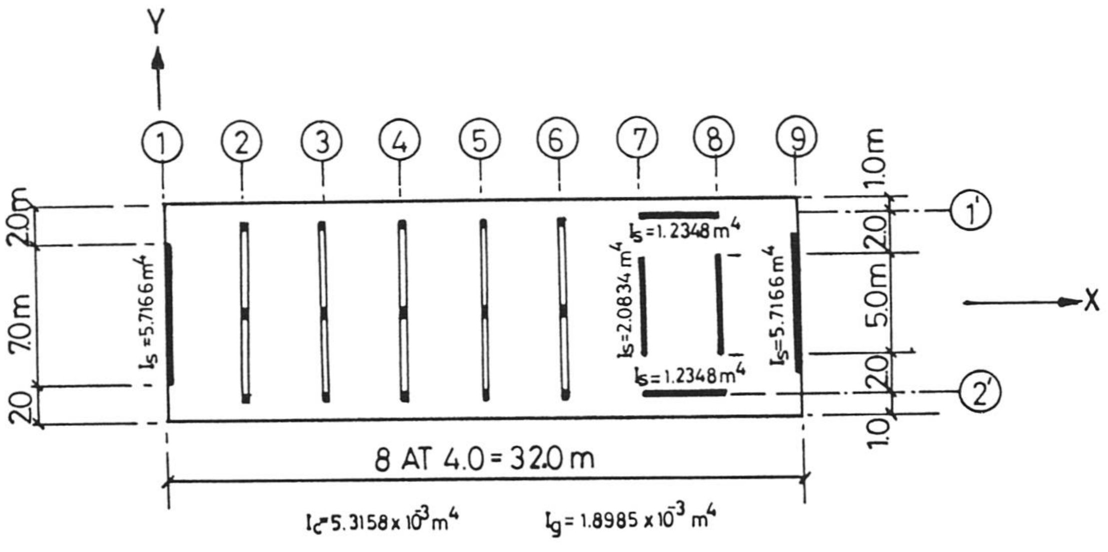



Fig. 3 Structural plan of example 1

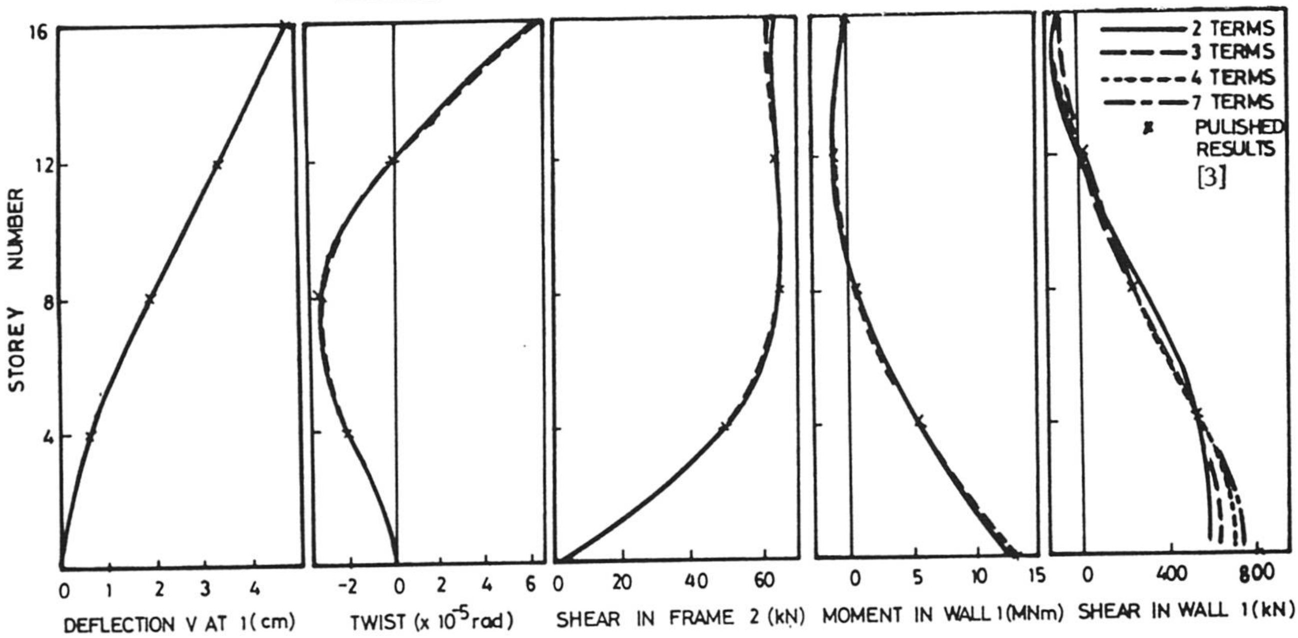



Fig. 4 Convergence study for uniform loading (Eq. 25)

Figs. 4 and 5 show that for uniform load, the results converge fast and four terms for each set of displacement functions are sufficient for practical purposes. The discrepancies between the two types of functions used is only obvious in the shear force at the top of the wall where the third differential is involved. In Fig. 5e the shear force at the top of the wall is zero as it is forced to be so when Eq. (26) is satisfied. However, only the top one or two storeys are affected by the inaccuracies.

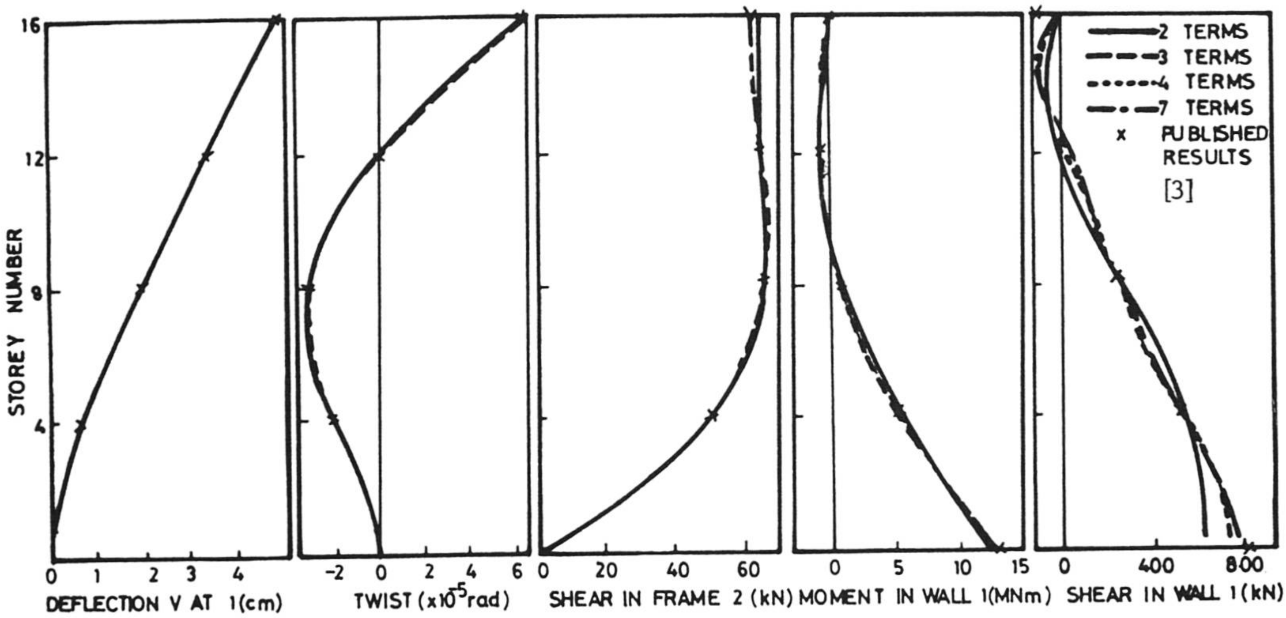



Fig. 5 Convergence study for uniform loading (Eq. 26)

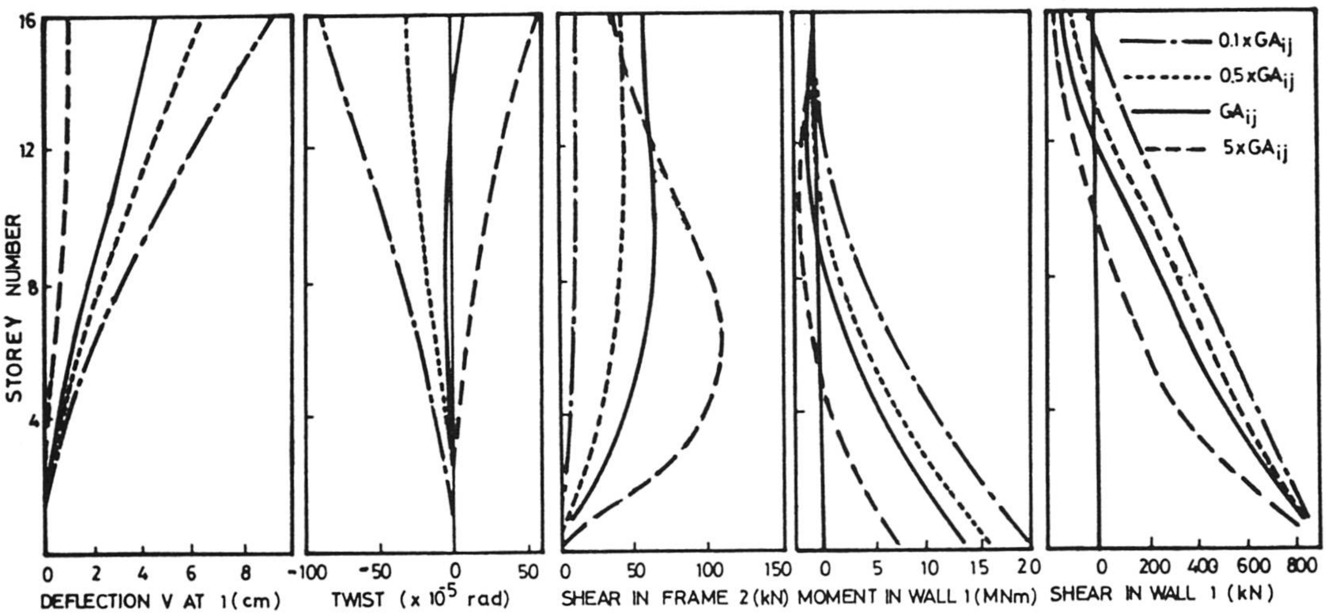



Fig. 6 Wall frame interaction effect

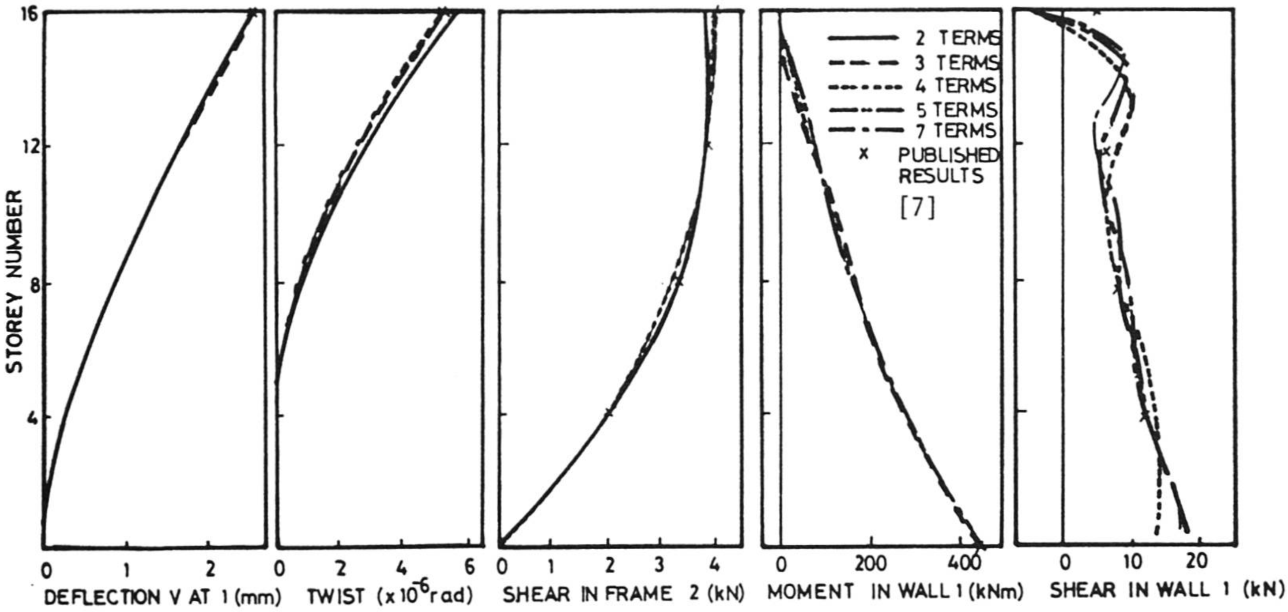



Fig. 7 Convergence study for point loading (Eq. 25)

The rotation of the structure changes sign along the height. This is due to the different behaviour of the wall and the frame and their non-symmetrical layout of the structural elements. At the lower storeys, the frames deflect more whereas the group of walls on the right deflect less, giving rise to a tendency towards negative rotation. The reverse behaviour at the upper storeys results in a positive rotation. If the group of frames are sufficiently stiff, then positive rotation throughout the height of the building will be observed. The opposite also holds true. These observations are verified by analysing the problem using different relative stiffnesses. As shown in Fig. 6, the attraction of forces depends on the relative location and stiffnesses of the walls and frames. When the frames are relatively more flexible than the walls, the walls will attract more forces whereas if the frames are stiffer, greater frame action is exhibited. Hence by adopting an optimal layout and using an appropriate ratio of wall and frame stiffnesses, an efficient structural system can be obtained.

The convergence study was also carried out for structure under point load of 40 kN acting at the top of the structure in the positive y-direction. The results are compared with the values obtained from the computation using tables proposed in reference [7] and depicted in Fig. 7. It is shown that only five terms are necessary to produce reasonably accurate results.

## 10.2 Example 2

A 16 storey building completely asymmetric in plan with uniform properties along the height shown in Fig. 8 is investigated. The base is assumed to be fixed and the floor-to-floor height is 3 metres. A uniformly distributed lateral load of 1 kN per metre square acts in the positive y-direction and the modulus of elasticity is 20000 $MN/m^2$. The sectional properties and the location of the elements are given in Table 1.

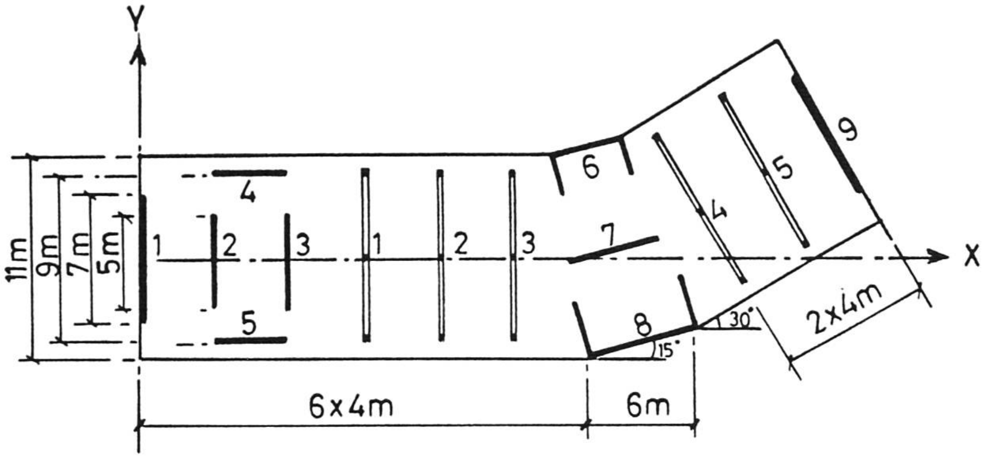



Fig. 8 Structural plan for example 2

| No. | Wall Properties | | | | | | |
|---|---|---|---|---|---|---|---|
| | x(m) | y(m) | β(degrees) | $I_{xx}(m^4)$ | $I_{yy}(m^4)$ | $I_{ww}(m^6)$ | $GJ/E(m^4)$ |
| 1 | 0.0 | 0.0 | 0.0 | - | 5.72 | - | - |
| 2 | 4.0 | 0.0 | 0.0 | - | 2.08 | - | - |
| 3 | 8.0 | 0.0 | 0.0 | - | 2.08 | - | - |
| 4 | 6.0 | 4.5 | 0.0 | 1.23 | - | - | - |
| 5 | 6.0 | -4.5 | 0.0 | 1.23 | - | - | - |
| 6 | 23.74 | 6.74 | 15.0 | 4.28 | 1.24 | 1.87 | 0.010 |
| 7 | 25.50 | 0.5 | 15.0 | 2.08 | - | - | - |
| 8 | 27.19 | -5.81 | 15.0 | 21.7 | 3.40 | 21.30 | 0.051 |
| 9 | 36.93 | 6.5 | 30.0 | - | 5.72 | - | - |

| Frame Properties | | | |
|---|---|---|---|
| No. | x(m) | y(m) | β(degrees) |
| 1 | 12.0 | 0.0 | 0.0 |
| 2 | 16.0 | 0.0 | 0.0 |
| 3 | 20.0 | 0.0 | 0.0 |
| 4 | 30.0 | 2.5 | 30.0 |
| 5 | 33.46 | 4.5 | 30.0 |

For all frames $GJ/E = 0.0273\ m^2$

Table 1 Data for example 2

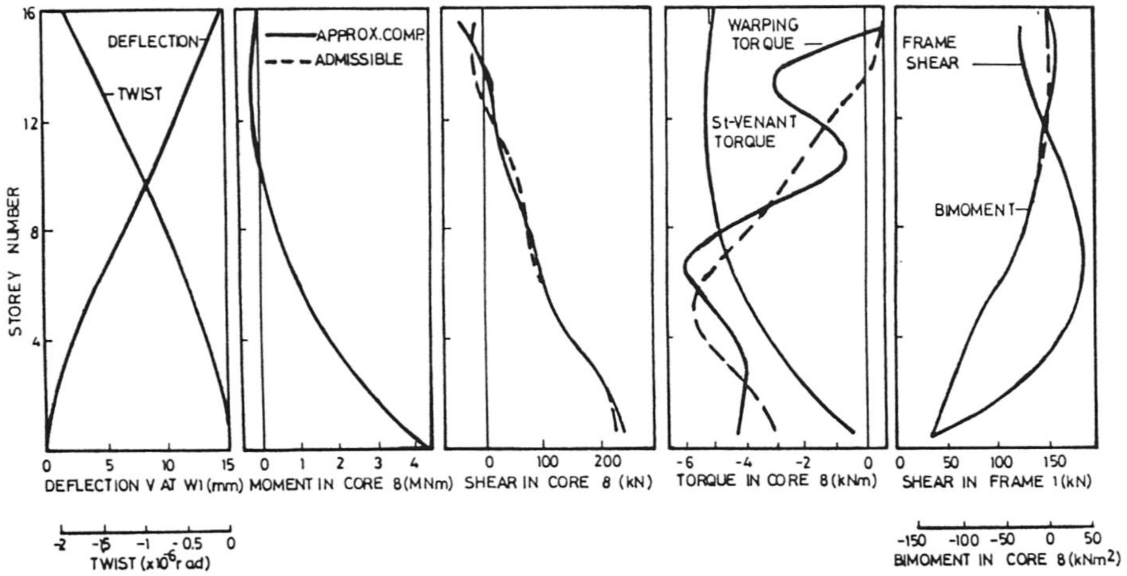



Fig. 9 Results of example 2

The results using five terms in displacement field as shown in Fig. 9 again show that functions satisfying Eq. (25) perform better than those satisfying Eq. (26). The core-frame interaction effect is again exemplified in the direction of twist which will change along the height of the building depending on the relative stiffnesses and the location of the elements. If the three frames on the left of the building are conceived to be stiffer than those on the right, the rotation will increase and depending on the flexibility of the right group, a positive rotation may be obtained at the top. These observations have already been verified in example 1.

## 10.3 Example 3

A building of non-uniform properties along the height investigated earlier by the discrete approach [1] is re-analysed. The height of the first storey is 3.5 metres and the other 19 storeys are 3 metres each. The properties of the structural components are given in Table 2 and the uniformly distributed load of intensity 150 kg/m$^2$ is assumed to act on the structure in the positive y direction. The structural plan is shown in Fig. 10.

The building is treated as a three segment problem. The first covers only the first storey to take care of the difference in storey height. The second and third portions arise from the change in sectional properties at the tenth storey. The convergence study shows that five terms are sufficient. The results are plotted and compared with those presented in reference [1] in Fig. 11. They show good agreement.

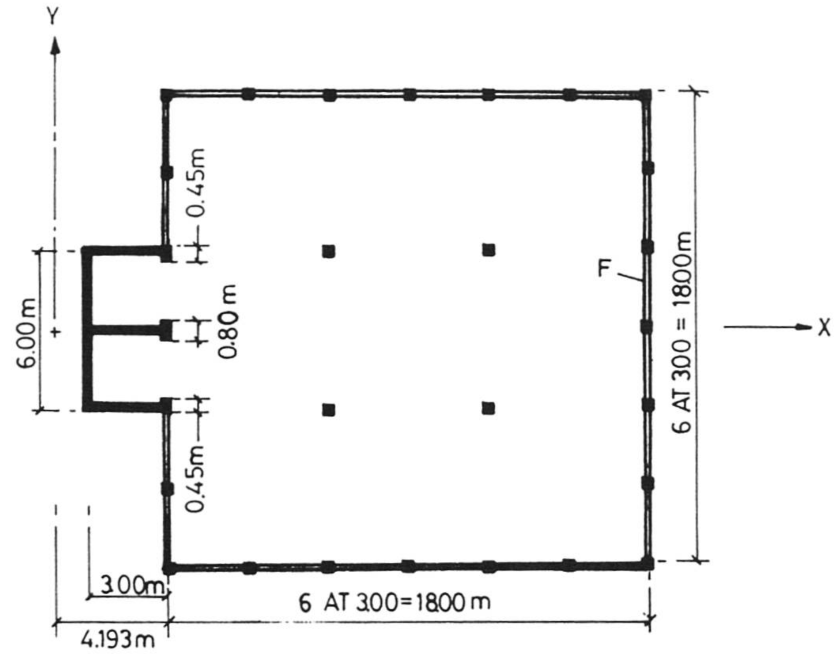



Fig. 10 Structural plan of example 3

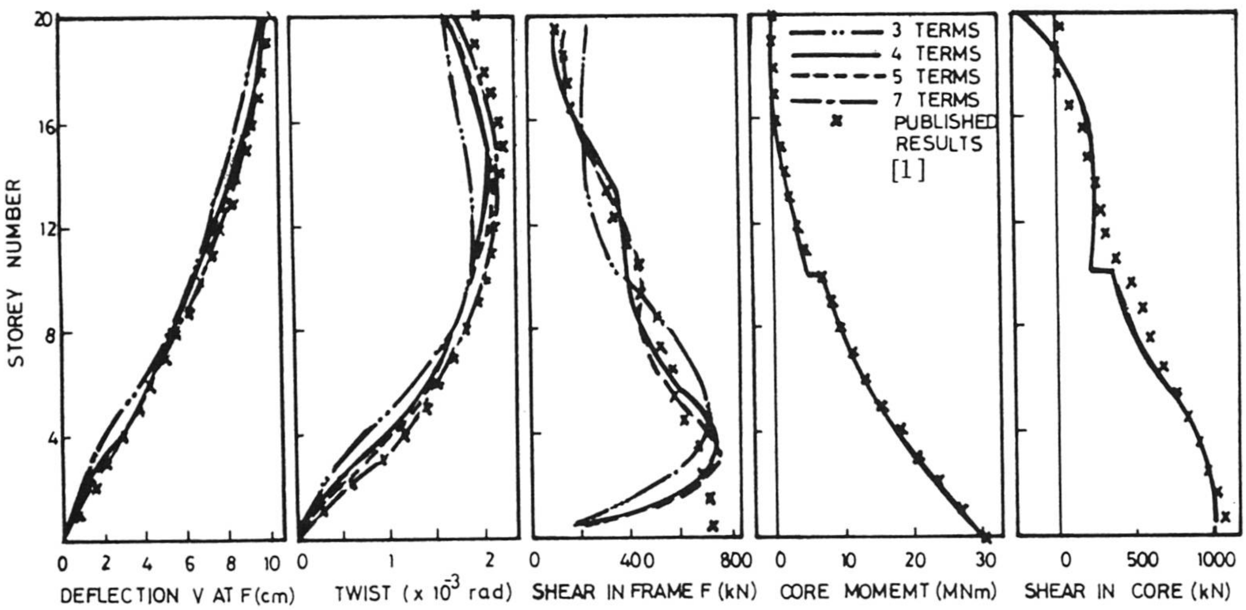



Fig. 11 Results of example 3 using approximate comparison function

| Storey | 1-10 | | 11-20 | |
|---|---|---|---|---|
| Frame | $I_c(m^4)$<br>$1.08 \times 10^{-2}$ | $I_g(m^4)$<br>$1.9 \times 10^{-3}$ | $I_g(m^4)$<br>$5.22 \times 10^{-3}$ | $I_g(m^4)$<br>$1.9 \times 10^{-3}$ |
| Shear Wall | t(m)<br>0.3 | $I_s(m^4)$<br>21.22 | t(m)<br>0.2 | $I_s(m^4)$<br>14.15 |

Table 2 Structural properties of building of example 3

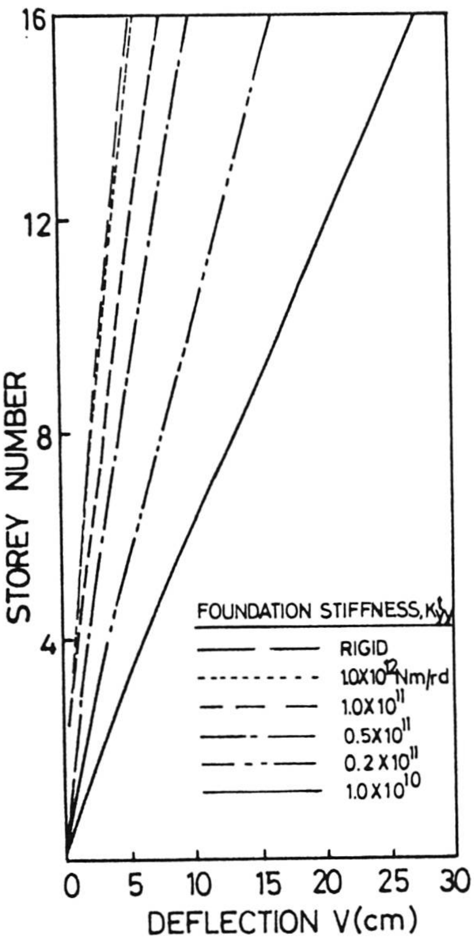



Fig. 12 Results of example 4

10.4 Example 4

The building described in Example 1 is now assumed to be built on elastic foundation with rocking flexibility. A range of rocking stiffnesses are investigated to study the effect on the displacements and stresses for this particular structure. The results obtained by using five terms in displacement field are plotted in Fig. 12. As the foundation becomes more flexible, the deflection in the y-direction increases due to the rigid body rotation at the base. However, the stresses in the structure are not affected.

## 11. CONCLUSIONS

The Galerkin method of weighted residuals used here for solving the governing coupled differential equations of non-uniform asymmetric shear wall frame buildings proves to be powerful. The method is simple and easy to apply. The effects of ground flexibility and buildings with complex loadings and properties that vary with height can be easily incorporated, indicating its versatility. In this study, the free vibration mode shapes of a cantilever are chosen as the assumed displacement functions. Numerical comparisons with existing methods show the results to be sufficently accurate for practical purposes. As the amount of core storage and computing time required is minimal, the method is suitable for preliminary design purposes using a micro-computer.

## NOTATIONS

| | |
|---|---|
| $a$ | displacement parameters |
| $EI_{ij}$ | global flexural stiffness matrix |
| $(EI_{xx})_e$, $(EI_{yy})_e$ | flexural rigidity about local centroidal |
| $(EI_{ww})_e$ | warping stiffness about local principal pole |
| $f$ | distributed load vector |
| $GA_{ij}$ | global shear stiffness matrix |
| $(GA_{xx})_e$, $(GA_{yy})_e$ | shear stiffness of element in the local x and y axes respectively |
| $(GJ_{ww})_e$ | torsional stiffness about local principal pole |
| $h$ | total height of building |
| $h_k$ | height of the kth storey |
| $K^t_{ij}$ | translational stiffness matrix of the foundation mass |
| $K^r_{ij}$ | rocking stiffness matrix of the foundation mass |
| $M^o_i$ | applied moment vector at the top |
| $(M_x)_e$, $(M_y)_e$ | bending moments about local y and x directions respectively |
| $(M_w)_e$ | bimoments in each wall |
| $(M_{xk})_e$, $(M_{yk})_e$ | moments in the frame about the local y and x directions at the kth storey respectively |
| $S^o_i$ | applied shear vector at the top |
| $(S_{xk})_e$, $(S_{yk})_e$ | shear force in each frame in the local x and y directions at the kth storey respectively |
| $(S_x)_e$, $(S_y)_e$ | shear force in each frame in the local x and y directions |
| $(T_w)_e$, $(T_s)_e$ | warping and St-Venant torques in the wall |
| $u_j$ | global displacement vector |

| | |
|---|---|
| u,v,θ | displacement in the x,y and about the z directions respectively |
| x,y | coordinates of the shear centre of the element with respect to the global axes |
| α | sqrt(GA/EI) |
| β | orientation of the local axes to the global axes |
| δ | Kronecker-delta function |
| φ | displacement shape function |
| λ | frequency of mode shape of transcedental function |
| $'$,$''$,$'''$,$^{iv}$ | first, second, third and fourth derivatives with respect to z respectively |

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