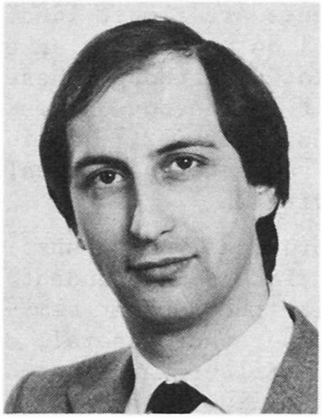
# A general basis for the selection of compliance criteria

# A General Basis for the Selection of Compliance Criteria

## Une base générale pour la sélection de critères d'acceptation

## Eine allgemeine Grundlage für die Wahl von Annahmekriterien

**Luc TAERWE**
Dr. Ir.
University of Ghent
Ghent, Belgium



Luc Taerwe, born 1952, graduated as Civil Engineer at the University of Ghent where he also received his degree of Master of Engineering in Structural Mechanics and his Doctor's degree. As university assistant he is involved in research on non-linear analysis of concrete structures, cryogenic concrete, partial prestressing, quality assurance and stochastic modelling.

**SUMMARY**

A general basis is presented for the selection of compliance criteria concerning material properties, defined by means of a characteristic value. A general expression for the filtered process curve is discussed and prior information concerning the fraction defectives is introduced. Boundary lines for unsafe and uneconomic regions for OC-lines are derived, and some reasonable assumptions concerning rejected lots are introduced in order to extend the justification of the proposed unsafe region.

**RÉSUMÉ**

Une base générale est proposée pour la sélection de critères d'acceptation valables pour les propriétés de matériaux définies par des valeurs caractéristiques. L'effet de filtration du contrôle d'acceptation sur la distribution de la production offerte est discuté. L'information préliminaire concernant la fraction de défectueux est introduite, et des limites du domaine d'insécurité pour des courbes d'acceptation et du domaine non économique en sont dérivées. L'introduction de quelques hypothèses raisonnables concernant les lots rejetés permet d'étendre la justification du domaine d'insécurité proposé.

**ZUSAMMENFASSUNG**

Der Beitrag enthält eine allgemeine Grundlage für die Wahl von Annahmekriterien bezüglich Materialeigenschaften, die mittels eines charakteristischen Wertes definiert sind. Zuerst wird ein allgemeiner Ausdruck für das gefilterte Produkt diskutiert und a priori Informationen bezüglich des Schlechtanteils eingeführt. Weiter werden Grenzlinien für die Abgrenzung der unsicheren und der unwirtschaftlichen Gebiete für Annahmekennlinien vorgeschlagen. Um das unsichere Gebiet zu erweitern, werden einige vernünftige Annahmen für abgelehnte Lose eingeführt.

## 1. INTRODUCTION

Compliance control of materials is only one of the many stages in the quality assurance program of a whole building project. However, it is a step where control can be performed in a rather easy way, once the appropriate criteria are defined, this in contrast with e.g. prevention of human errors. Although universally valid concepts probably will never be available, it is however important that the background of the whole compliance control plan consists of a fair and transparent system of rules and procedures.

As compliance control of concrete concerns, the actual situation is such that in every country different criteria are in use, although the procedures for the design of structural elements are almost identical. In the CEB-FIP Model Code for Concrete Structures [1] an unsafe and an uneconomic region, for the OC-lines not to fall in, are stipulated. Although these regions are frequently mentioned in literature, it appears that the boundaries of these regions were fixed in a rather arbitrary way and no clear justification of the followed procedure is available. This situation reflects that actually no common basis exists for the selection of compliance criteria.

In this paper, a general proposal for an unsafe and uneconomic region for OC-lines is made. The formulation is independent of any particular design situation and is fully consistent with the current semi-probabilistic design methods. The approach that is followed is rather general, due to the fact that use is made of process curves and that the filtering effect of compliance control is considered. For readers who are less familiar with these concepts, a comprehensive introduction to each aspect is given in the different sections.

Although the basic principles outlined in this paper are illustrated for only one material, namely concrete, their field of application is quite general.

## 2. GENERAL FORMULATION OF THE FILTERING EFFECT

a) For the practical application of level I safety formats, a material property X is represented by its characteristic value, defined as a fractile in the global distribution.

This global distribution represents the variation of X in a broad sense as e.g. obtained for the strength of concrete of a given class produced in one country. The total variation then includes within-batch variation, between-batch variation for a given plant and given class, variation between plants etc..

For concrete strength, the 5% fractile is generally used as the characteristic value. In practice, each supplied lot includes a random fraction $\theta$ of items, the relevant property of which has a value lower than the *required* characteristic value $X_k$. This fraction defectives is denoted by $\theta$.

We call $f_{\Theta,i}$ the probability density function of $\theta$ corresponding to the offered lots (*input*) and $f_{\Theta,o}$ the PDF of the fraction defectives of the accepted lots (*output*). As the distribution $f_{\Theta,i}$ characterizes the production process, it is often called *process curve* (PC) whereas $f_{\Theta,o}$ is called the *filtered process curve* (FPC). The function $f_{\Theta,i}$ is transformed into $f_{\Theta,o}$, because certain lots are rejected and disappear from the global population. In the following, $f_{\Theta,o}$ is defined with respect to the *accepted lots* and results from a selective filtering by means of compliance control (fig.1). Indeed, the probability to pass trough the filter is lower for lots with a high $\theta$ value than for lots with a low $\theta$ value. Analytically, $f_{\Theta,o}$ is obtained by multiplying $f_{\Theta,i}(\theta)$ by the probability of acceptance $P_a(\theta)$ and normalizing the distribution function :

$$f_{\Theta,o}(\theta) = \frac{f_{\Theta,i}(\theta) \cdot P_a(\theta)}{\int f_{\Theta,i}(\theta) \cdot P_a(\theta) d\theta} = f_{\Theta,i}(\theta) \cdot \frac{P_a(\theta)}{P_a^*} \qquad (1)$$

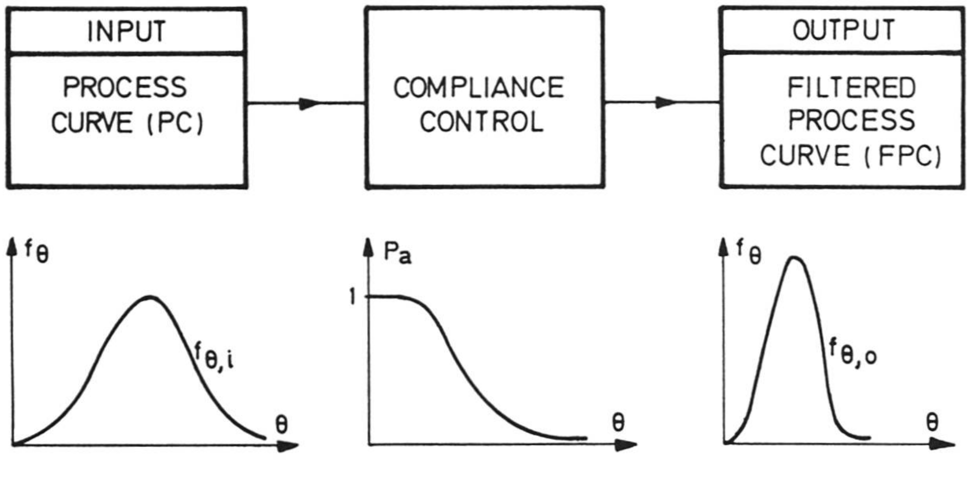



Fig.1 General representation of the filtering effect

$P_a^*$ denotes the global probability of acceptance, taking into account the distribution of offered lots and hence is a function of the parameters of $f_{\Theta,i}$.

Some cases where the variation of the parameters of the distribution function is considered, instead of the variation of a fractile, are discussed in [2].

b) For low $\theta$ values $P_a(\theta) > P_a^*$ and hence $f_{\Theta,o} > f_{\Theta,i}$. The inverse holds for high $\theta$ values and results in $f_{\Theta,o} < f_{\Theta,i}$. Thus it is clear that $f_\Theta$ is submitted to a beneficial shift towards low $\theta$ values as a result of compliance control (fig.1). This effect is illustrated by the following example (fig.2).

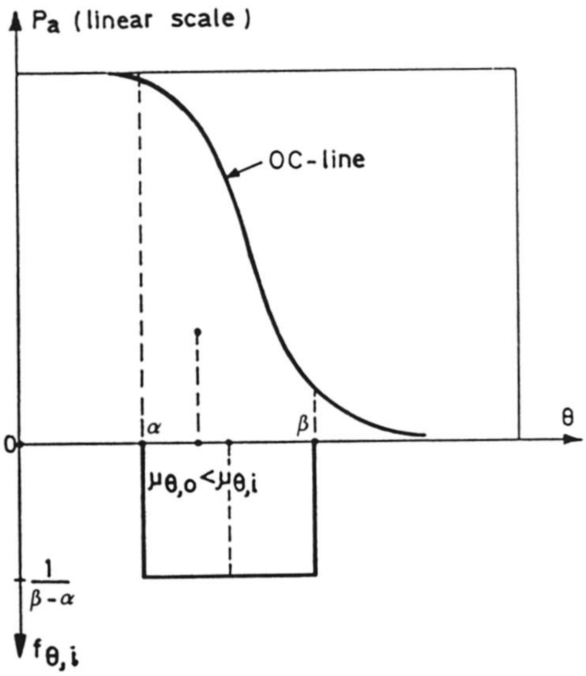



Fig.2 Shift of the mean value of θ

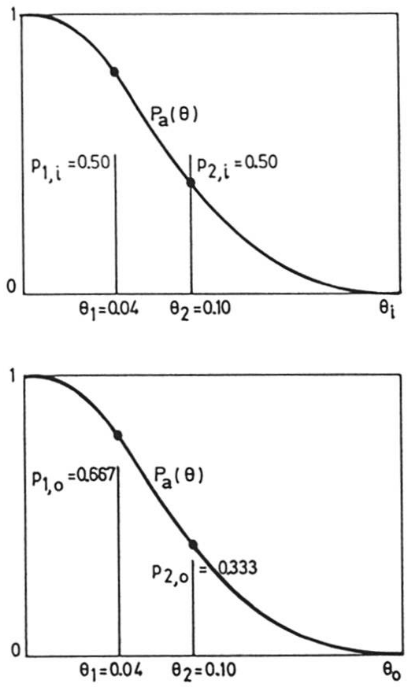



Fig.3 Numerical example of the filtering effect

Suppose that $\theta$ is uniformly distributed in the interval $[\alpha,\beta]$, then $\mu_{\Theta,i}=(\alpha+\beta)/2$. From (1) it follows that

$$\mu_{\Theta,o} = \frac{\int \theta \, P_a \, d\theta}{\int P_a \, d\theta} \tag{2}$$

which means that $\mu_{\Theta,o}$ is the abscissa of the centroid of the surface between the straight lines $\theta = \alpha$, $\theta = \beta$, $P_a = 0$ and the curve $P_a(\theta)$ . From the shape of $P_a(\theta)$ it follows that $\mu_{\Theta,o} < \mu_{\Theta,i}$ .

When the variance of $f_{\Theta,i}$ tends to 0, i.e. when always the same value of $\theta$ occurs, the increase of the quality level does not take place.

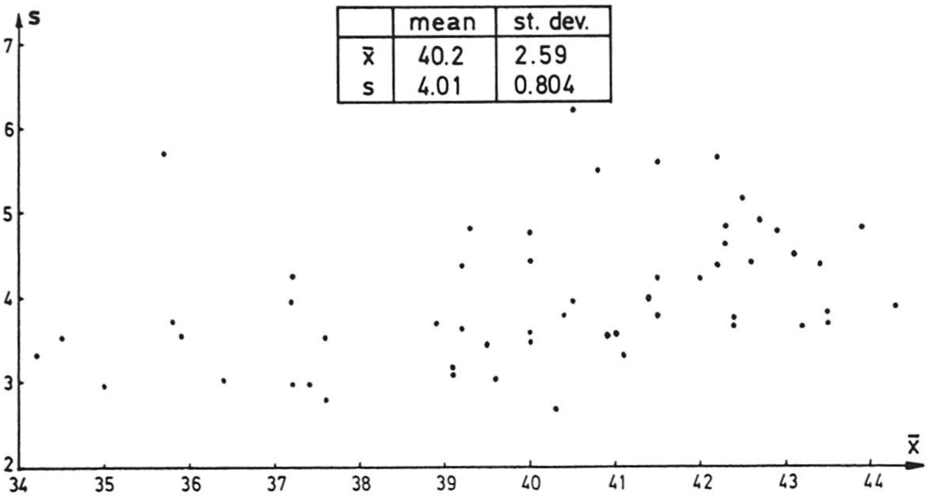



Fig.4 Characteristics of the production of Belgian ready-mixed concrete plants ($f_{ck,cub}$ = 27.5 N/mm$^2$)

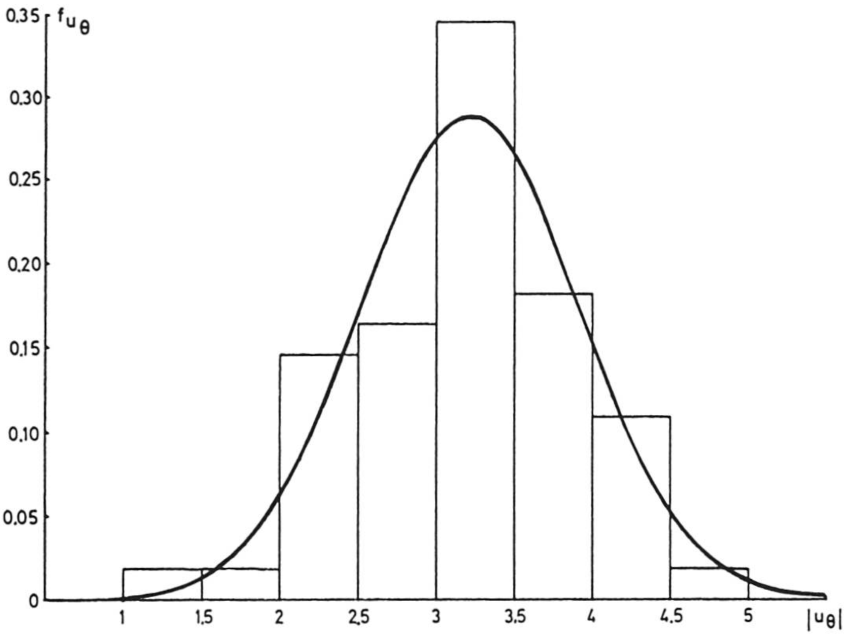



Fig.5 Histogram of the empirical values of $|u_\theta|$ and fitted normal distribution

c) A second example concerns the rather hypothetical case of a discrete distribution for $\theta$, depicted in fig.3. Suppose $\theta_1 = 0.04$ and $\theta_2 = 0.10$ are equally likely to occur ($p_{1,i} = p_{2,i} = 0.5$) and that $P_a(\theta_1) = 0.8$ and $P_a(\theta_2) = 0.4$. It follows that

$$p_{1,o} = \frac{0.5 \times 0.8}{0.4 + 0.2} = 0.667 \qquad p_{2,o} = \frac{0.5 \times 0.4}{0.4 + 0.2} = 0.333$$

The initial value $\mu_{\Theta,i} = 0.07$ is reduced to

$$\mu_{\Theta,o} = 0.667 \times 0.04 + 0.333 \times 0.10 = 0.06$$

The global probability of acceptance equals 60%. Thus, for the 60% accepted lots, the mean value of the fraction defective has moved from 7% to 6%.

d) The filtered distribution of X, the material property we are interested in, is given by

$$f_X(x) = \int_\theta f_X(x|\theta).f_{\Theta,o}(\theta).d\theta = \frac{\int_\theta f_X(x|\theta).f_{\Theta,i}(\theta).P_a(\theta)d\theta}{P_a^*} \tag{3}$$

The fraction of values lower than the characteristic value $X_k$ in the global distribution of X, is denoted $\theta^*$ and calculated as follows :

$$\theta^* = \int_{-\infty}^{X_k} f_X(x)dx = \int_{-\infty}^{X_k} \int_\theta f_X(x|\theta)\, f_{\Theta,o}(\theta)\, d\theta\, dx = \int_\theta \theta f_{\Theta,o}\, d\theta = \mu_{\Theta,o} \tag{4}$$

This value is the global fraction defective and is a generalization of the fraction defective defined in relation to one lot.

## 3. A PROCESS CURVE FOR CONCRETE STRENGTH

The points in fig.4 represent values of $\bar{x}_n$ and $s_n$ corresponding to the concrete production of Belgian ready-mixed concrete plants during different periods of six months ($f_{ck,cub} = 27.5$ N/mm$^2$). In fig.5 a histogram of the 55 values

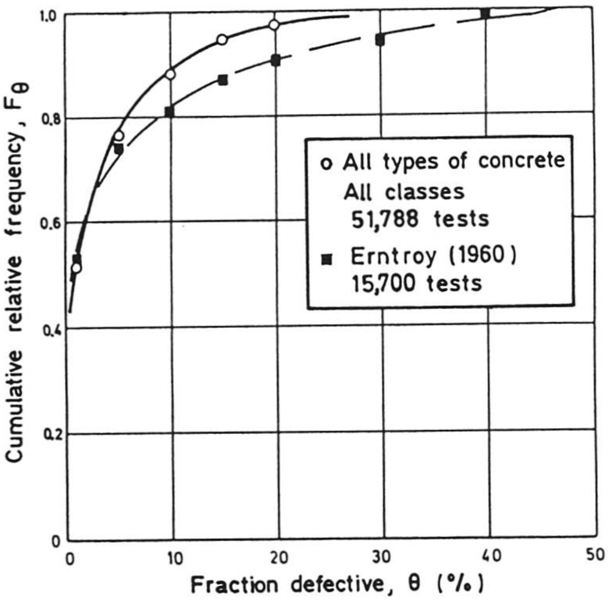



Fig.6 Empirical distribution of $\theta$ according to METCALF ([3])

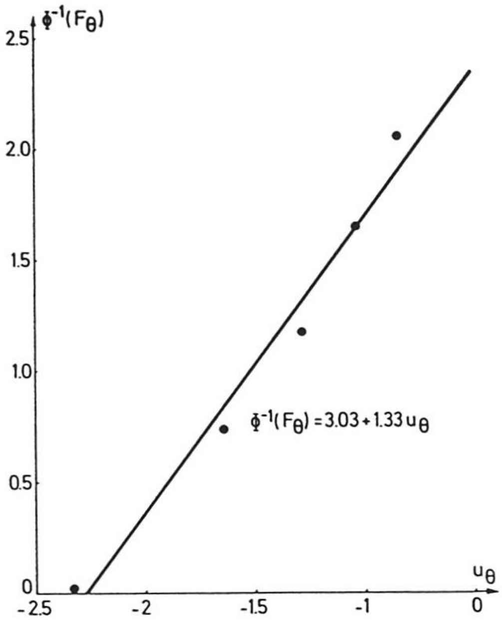



Fig.7 Representation of the points from fig.6 in a diagram with transformed scales

$$\hat{u}_\theta = (f_{ck} - \bar{x}_n) / s_n \qquad (5)$$

is given. The following notations are used :

- $\bar{x}_n$, $s_n$ : mean and standard deviation of the available strength values in the considered periods
- $f_{ck}$ : specified characteristic concrete strength
- $\hat{u}_\theta$ : estimate of $u_\theta = \Phi^{-1}(\theta)$
- $\Phi(.)$ : standardized normal CDF

The mean value of $u_\theta$ equals -3.215 which corresponds to the 0.065% fractile in the normal distribution fitted to the empirical data.

In fig.6 an investigation by METCALF [3] is summarized. The five upper empirical points $(F_\Theta,\theta)$ indicated in fig.6, are plotted again in fig.7, but this time in a diagram with the scale of both axes transformed according to the function $\Phi^{-1}(.)$. Again it appears that a normal distribution represents the variation of $u_\theta$ fairly well.

In both cases the standard deviation $u_\theta$ is approximately equal to $1/\sqrt{2} \simeq 0.71$. Assuming that this figure also holds for higher mean values of $u_\theta$, we propose as process curve for concrete strength, a normal distribution for $u_\theta$, with standard deviation equal to $1/\sqrt{2}$.

## 4. DERIVATION OF AN UNSAFE REGION FOR OC-LINES

### 4.1 Introduction

Once the stochastic model for X is accepted and the type of compliance criterion is fixed, the numerical values of the parameters occuring in the criterion have to be determined taking into account certain boundary conditions. In any case, conditions related to safety have to be considered at the first stage and, within the class of criteria that are judged to be sufficiently safe, it is reasonable to select those which yield the most economical solution.

The way of introducing the safety aspect obviously depends on the level of analysis that is used. We assume that a level I safety format, such as outlined in [4] for the general case and in [1] for concrete structures, is applied. We take into account the definition of the characteristic value as the 5% fractile on the basis of the generalized formulation given by equation (4). In a higher level safety format one can relate the probability of failure to the fraction defective $\theta$ for a particular design situation and derive a more optimal solution for that case. This aspect is considered in [5].

The unsafe region that is given in the CEB-FIP Model Code for Concrete Structures [1], was fixed as an envelope of the OC-lines corresponding to the compliance criteria in use at the time the document was edited. Hence, the need has arisen to establish an unsafe region, the boundary curve of which can be derived issuing from some reasonable assumptions.

Other proposals can be found in [6] and [7], and a similar one in [8] but the formulation in this paper is more general and not dependent on any particular design situation.

### 4.2 Definition

As equation of the boundary curve for the unsafe region for OC-lines we propose :

$$\boxed{\theta . P_a = 0.05} \qquad \text{for} \quad \theta > 0.05 \qquad (6)$$

It is represented in fig.8. When the characteristic value is defined as the p - fractile with $p \neq 0.05$, the right hand side of (6) has to be changed accordingly.

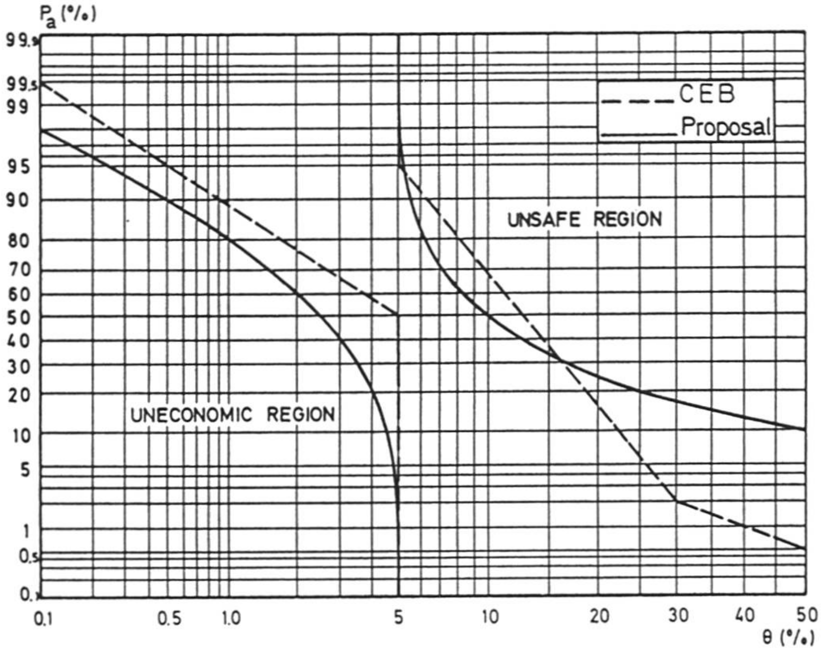



Fig.8 Comparison of proposed boundaries and those given in [1]

### 4.3 Interpretation

When a compliance criterion is applied, the OC-line of which is situated below the curve (6), at long term, the global fraction defective in all the lots is not more than 5%. This implies that rejected lots are completely "transformed to a perfect state", which means mathematically that $\theta \to 0$. We use the formulation "transformed to a perfect state" rather than "replaced by perfect lots", as the latter action is generally impossible to realize in practice for most materials used in the construction process. Although the formulation could appear to be rather unrealistic at first sight, it means that in practice structural measures, e.g. strengthening, are taken to improve the structural performance of the elements in which the defective lots are used. These measures could even prove to be unnecessary when e.g. a low level of concrete strength does not influence the safety level significantly as is the case for reinforced concrete beams where failure is introduced by yielding of the steel. Nevertheless, the rather rigid basic assumption is relaxed in one of the next sections.

The function $\theta.P_a(\theta)$ has the general shape that is depicted in fig.9. The slope of the curve at the origin always equals 45° since

$$\frac{d}{d\theta}(\theta.P_a) = \theta\frac{d\,P_a}{d\theta} + P_a \tag{7}$$

and $P_a = 1$ for $\theta = 0$. As $P_a$ is a strictly decreasing function of $\theta$, the product $\theta\,.P_a$ reaches a maximum. For a given OC-line this maximum can be calculated and if it is lower than 0.05, the OC-line will not intersect the unsafe region defined by (6).

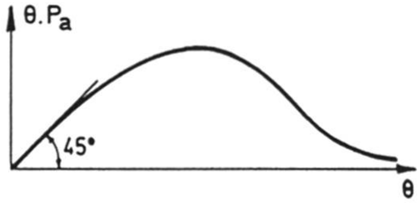



Fig.9 General shape of the function $\theta.P_a$

### 4.4 Analytical derivation

Assuming that $\theta$ becomes 0 for initially rejected lots, the distribution $f_{\Theta,o}$ consists of a continuous part $f_{\Theta,i}.P_a$ and a discrete value at $\theta = 0$ equal to

$$\int f_{\Theta,i}.(1 - P_a)\,d\theta \tag{8}$$

It follows that

$$\theta^* = \mu_{\Theta,o} = \int \theta .\, f_{\Theta,i} . P_a \, d\theta + 0 . \int f_{\Theta,i} . (1 - P_a)\, d\theta = \int \theta . f_{\Theta,i} . P_a d\theta \qquad (9)$$

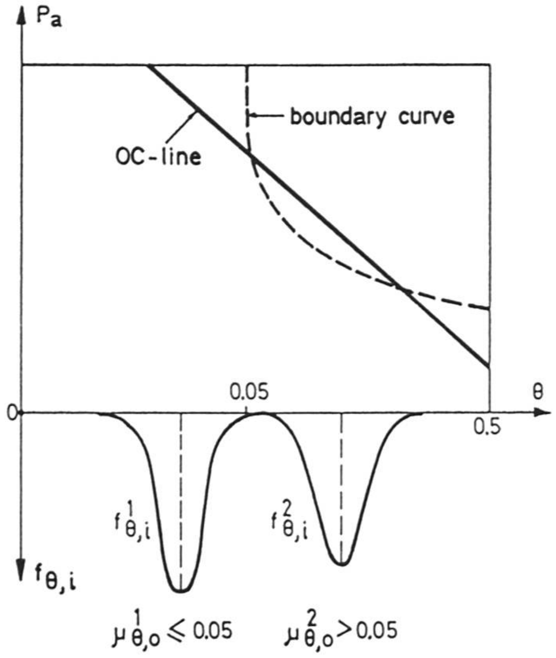



Fig.10 Influence of the location of the process curve

Introducing (6) we find that $\theta^* = 0.05$. In practice this means that the global fraction defective is lower than 0.05, since an OC-line never has the shape of curve (6), but is a straight line or a curve with downward concavity. Consequently an OC-line may intersect the proposed unsafe region and still yield $\theta^* \leqslant 0.05$, depending on the variation of $\theta$ and hence on the location and the shape of $f_{\Theta,i}$ as illustrated in fig.10. It results that the boundary line (6) is a sufficient but not a necessary condition with respect to safety. However, we propose to use compliance criteria the OC-line of which falls outside the unsafe region defined above because use shall be made of this favourable aspect in one of the next sections, in order to relax the rigid assumptions concerning rejected lots.

## 5. DERIVATION OF AN UNECONOMIC REGION FOR OC-LINES

### 5.1 Introduction

As, by definition, we can only consider an unsafe region for $\theta > 0.05$, an uneconomic region only makes sense for $\theta < 0.05$.

The uneconomic region mentioned in [1] and represented in fig.8 has only an informative character and is not intended as an absolute economic limit. A fully generally valid basis for the elaboration of such a limit is not available because the economic aspect depends on various exterior constraints, some of them time dependent, others much affected by the specific characteristics of the project. Although apparently this aspect is not clearly definable, a reasonable proposal is made in the sequel.

### 5.2 Definition and derivation

Let us denote by $f_{X,i}$ the global distribution of X for lots having a fraction defective $\theta$ (fig.11). Consider that the producer makes an effort to produce less than 5% defectives. A certain fraction of the supplementary good items is rejected. This fraction of the offered distribution between its 5% value and $X_k$ which is not accepted equals $(0.05 - \theta)(1 - P_a)$. The fraction of defective lots that is accepted equals $\theta.P_a$. Both quantities are represented by the shaded areas in fig.11. When we express that it is reasonable that these fractions compensate each other, this yields

$$(0.05 - \theta)(1 - P_a) = \theta.P_a \qquad (10)$$

and finally

$$\boxed{\theta/(1 - P_a) = 0.05} \qquad (11)$$

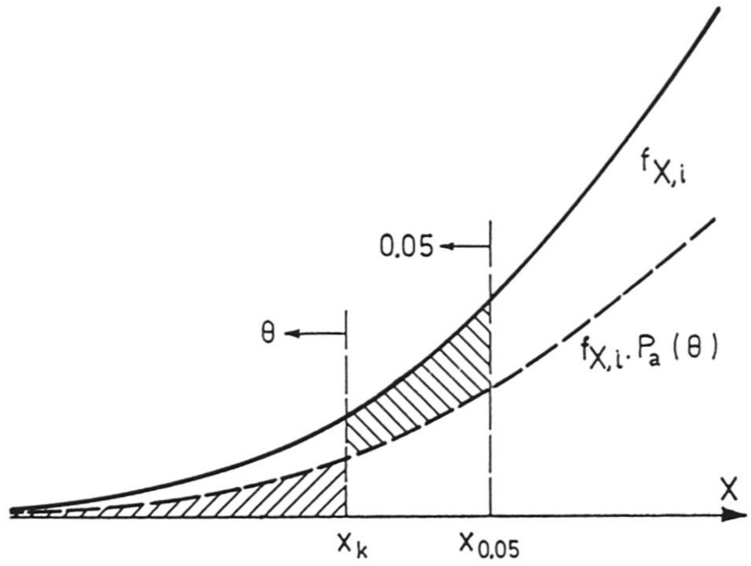



Fig.11 Representation of different fractions used in the derivation of a boundary line for the uneconomic region

This curve is represented in fig.8 as a proposed boundary for the uneconomic region. In this way we obtain an uneconomic region that is not widely different from the one tentatively defined by CEB, but that has the advantages of being continuous and having an origin that can easily be explained.

## 6. PRACTICAL APPLICATION

a) Given a type of acceptance criterion, we shall derive the numerical values of its parameters in such a way that the corresponding OC-line is tangent to one of the boundary curves presented above.

b) When $\sigma$, the standard deviation of the underlying population, is known a criterion of the type

$$\bar{x}_n \geqslant f_{ck} + \lambda\sigma \tag{12}$$

is applied. In the case of independent observations, the probability of acceptance is given by

$$P_a = \Phi\,[\,-\sqrt{n}\,(u_\theta + \lambda)] \tag{13}$$

For given values of n and $\lambda$ , we first determine the value of $\theta$ corresponding to the maximum of $\theta . P_a$. It is found by solving the following equation

$$\frac{d}{d\theta}\,(\theta . P_a) = P_a + \theta\,\frac{dP_a}{du_\theta}\cdot\frac{du_\theta}{d\theta}$$

$$= \Phi[\,-\sqrt{n}\,(u_\theta+\lambda)] - \theta\,\sqrt{n}\,\exp\,[-\frac{n}{2}\,(u_\theta+\lambda)^2 + \frac{u_\theta^2}{2}\,] = 0 \tag{14}$$

This equation can be solved by successive iterations.

In the next step we suppose that only n is fixed and we determine $\lambda$ in such a way that the OC-line is tangent to the unsafe region or, in other words, the condition

$$\max_\theta\,[\theta . P_a] \;=\; 0.05 \tag{15}$$

has to be fulfilled. This is obtained by enclosing the first iteration cycle by a second one, built up as follows.

A reasonable starting value $\lambda_1$ is chosen and a first value $\theta_1$ is found by solving eq.(14). This yields a value $P_{a,1} = 0.05/\theta_1$ and allows one to calculate a second value $\lambda_2$ from

$$\Phi^{-1}(P_{a,1}) = -\sqrt{n}\,(u_{\theta,1} + \lambda_2) \tag{16}$$

Successive iterations yield the final value of $\lambda$. Some numerical values are mentioned in table 1 ($\lambda_s$).

c) To determine OC-lines tangent to the uneconomic region, eq.(15) is replaced by

$$\min_{\theta}\,[\theta/(1 - P_a)] = 0.05 \tag{17}$$

and (14) has to be changed appropriately. Some numerical values of $\lambda$ are mentioned in table 1 ($\lambda_e$).

d) If the mean as well as the standard deviation are estimated from the sample, criteria of the type $\overline{x}_n \geqslant f_{ck} + \lambda\, s_n$ are applied and a similar procedure can be followed. However, the probability of acceptance has to be calculated by making use of the non-central t-distribution for the case when the underlying population is normally distributed. Some values of $\lambda_s$ are mentioned in table 2. For n = 5, 10 and 15 the corresponding OC-lines are depicted in fig.12.

e) In the case of serial correlation between successive observations, we consider, by way of example, the following AR(2)-model that is proposed in [9] for concrete strength

$$x_i = \phi_1\, x_{i-1} + \phi_2\, x_{i-2} + \varepsilon_i \tag{18}$$

where : - $\phi_1 = 0.40$ and $\phi_2 = 0.20$
- $\varepsilon_i$ : $N(o, \sigma_\varepsilon)$ a series of independent normal variables.

The values of $\lambda_s$ corresponding to an OC-line tangent to the unsafe region were determined by numerical simulation by means of random numbers, since no exact analytical calculation of $P_a$ is possible. Some results are mentioned in table 2 and for n = 15, the OC-lines valid for independent and dependent observations are compared in fig.13. It appears that the influence of correlation is not negligible. This aspect is discussed more thoroughly in [9]. In the case of the mentioned AR(2)-model it appears that $n \geqslant 7$ is necessary when the uneconomic region has to be respected whereas $n \geqslant 5$ is sufficient in the case of independent observations.

| n | $\lambda_s$ (unsafe region) | $\lambda_e$ (uneconomic region) |
|---|---|---|
| 3 | 1.297 | 1.833 |
| 4 | 1.284 | 1.904 |
| 5 | 1.282 | 1.935 |
| 6 | 1.284 | 1.950 |
| 7 | 1.288 | 1.957 |
| 8 | 1.294 | 1.960 |
| 9 | 1.299 | 1.960 |
| 10 | 1.305 | 1.959 |

Table 1 Criterion $\overline{x}_n \geqslant f_{ck} + \lambda\sigma$
Values for the parameter $\lambda$

| n | $\lambda_s$ | |
|---|---|---|
| | independent observations | AR(2)-model |
| 3 | 1.753 | 2.67 |
| 4 | 1.513 | 2.20 |
| 5 | 1.424 | 1.99 |
| 6 | 1.379 | 1.87 |
| 7 | 1.353 | 1.77 |
| 8 | 1.339 | 1.72 |
| 9 | 1.330 | 1.67 |
| 10 | 1.325 | 1.62 |
| 11 | 1.321 | 1.58 |
| 12 | 1.320 | 1.55 |
| 13 | 1.319 | 1.52 |
| 14 | 1.319 | 1.50 |
| 15 | 1.318 | 1.48 |

Table 2 Criterion $\overline{x}_n \geqslant f_{ck} + \lambda\, s_n$
Values for the parameter $\lambda$

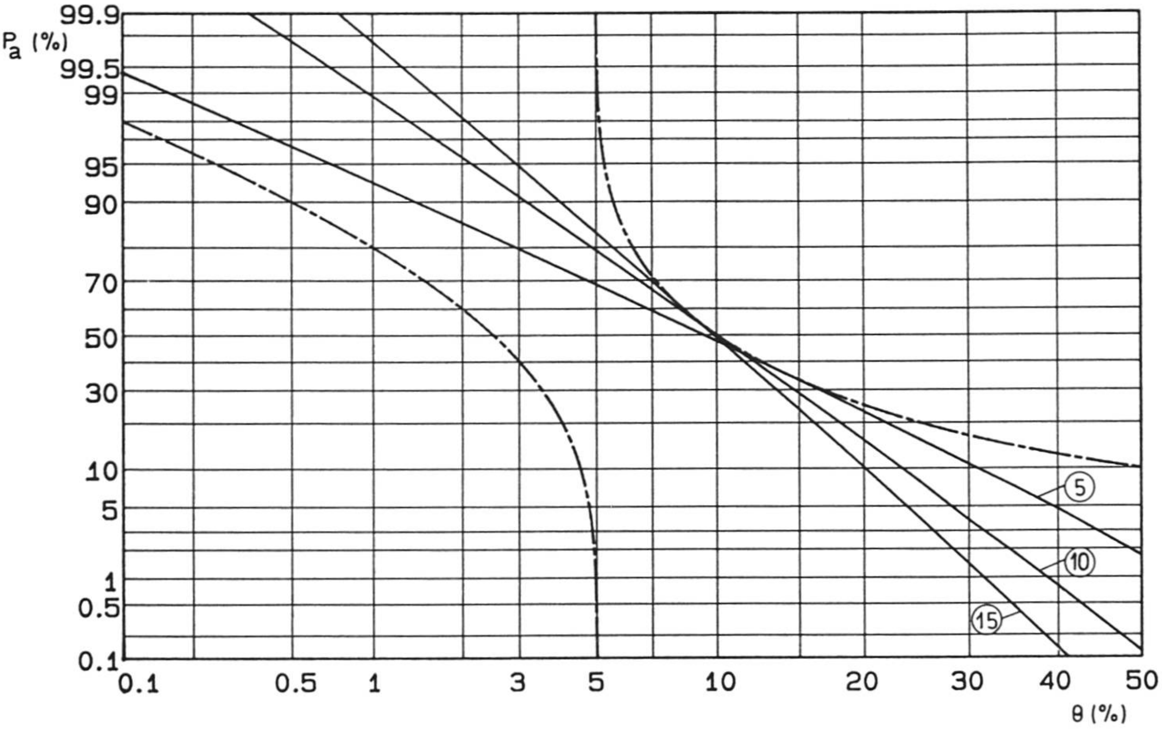



Fig.12 Criterion $\overline{x}_n \geqslant f_{ck} + \lambda s_n$ (n=5, 10, 15). OC-lines tangent to the unsafe region

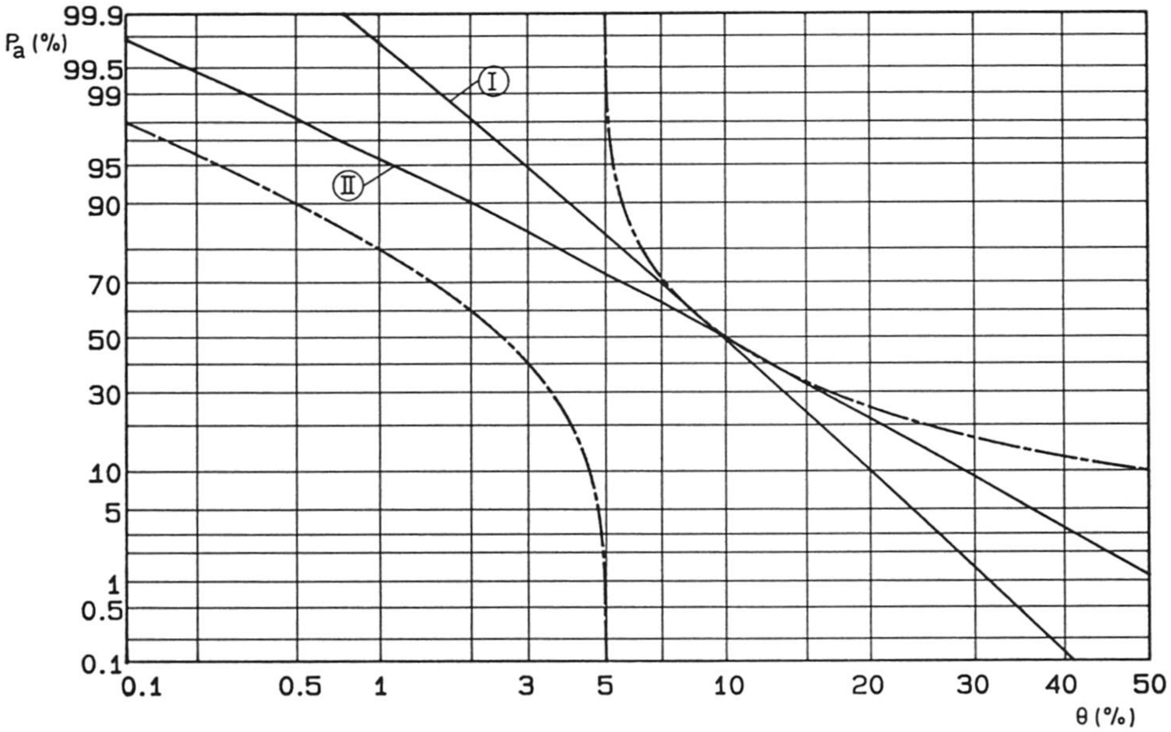



Fig.13 Criterion $\overline{x}_{15} \geqslant f_{ck} + \lambda s_{15}$ ($\lambda$ according to table 2). OC-lines tangent to the unsafe region. Independent observations (I); AR(2)-model (II)

## 7. CONSIDERATION OF REJECTED LOTS

### 7.1. Transformed process curve

In the case of products that can easily be replaced, consideration of the accepted lots only is sufficient. In other cases, such as cast in place concrete, the latter cannot be replaced without considerable effort, and consideration of both accepted and rejected lots is necessary. In section 4.3 we supposed that rejected lots were transformed to a perfect state. This hypothesis will be weakened in the following way (fig.14).

We make use of $u_\theta$ instead of $\theta$ and omit the subscript $\theta$. The fraction of wrongly rejected lots ($\theta < 0.05$) is given by

$$\int_{-\infty}^{-1.645} f_{u,i}\,(1 - P_a)\,du \tag{19}$$

These lots remain unaffected after a closer investigation that leads to the establishment that they correspond to $\theta < 0.05$. This situation occurs when lack of compliance is not due to real changes in the properties of the material being tested, but to the fact that the judgment is made on the basis of a limited number of observations. The fraction rejected lots with $\theta > 0.05$ is denoted by $p_o$

$$p_o = \int_{-1.645}^{\infty} f_{u,i}(1 - P_a)\,du \tag{20}$$

This quantity is represented by the shaded area B in fig.14. We now suppose that the rejected lots are "transformed" (in the sense outlined in section 4.3), in such a way that the corresponding value of $u_\theta$ is lower than -1.645 and moreover that the area B can be shifted and transformed into the area A in such a way that the shape of a normal density function PDF is maintained. The exact way in

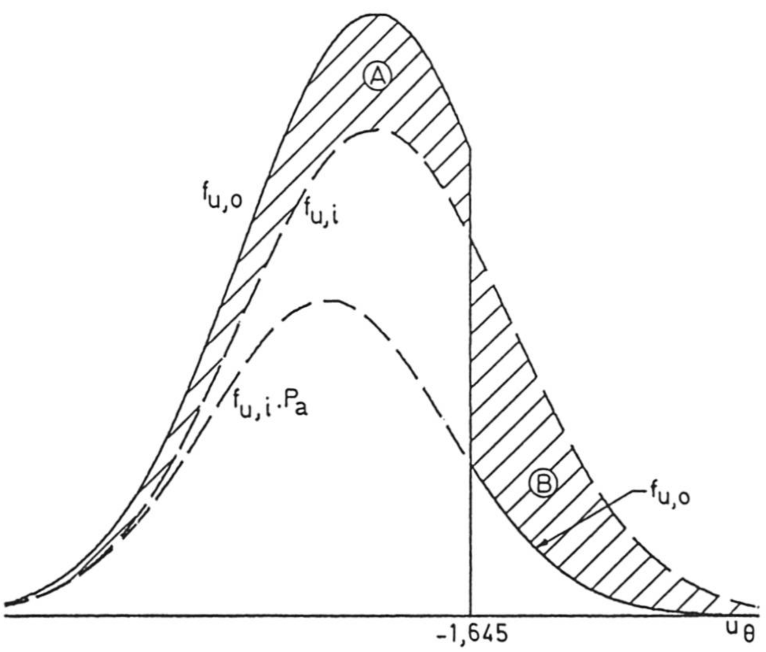



Fig.14 Filtered process curve taking into account rejected lots

which the lots are transformed is not considered here because we only want to show that it really is not necessary to suppose that rejected lots are transformed to a perfect state for the boundary curve (6) to be valid, but that some weak assumptions are largely sufficient. These assumptions, that are rather conventional, need to be expressed in an analytical way in order to make it possible to quantify the resulting effect. With the mentioned assumptions we obtain the following equation for $f_{u,o}$ :

$$\begin{cases} u_\theta < -1.645 \quad : \quad f_{u,o} = f_{u,i} \left[1 + \dfrac{p_o}{\int_{-\infty}^{-1.645} f_{u,i} du}\right] \\ u_\theta \geqslant -1.645 \quad : \quad f_{u,o} = f_{u,i} \cdot P_a \end{cases} \tag{21}$$

7.2 Calculation of the global fraction defectives

We now suppose that a compliance criterion of the type $\overline{x}_n \geqslant f_{ck} + \lambda s_n$ is applied in which the parameter $\lambda$ takes on the values mentioned in the first part of table 2. For $f_{u,i}$ we take the process curve mentioned in section 3, i.e.a normal distribution with standard deviation equal to $1/\sqrt{2}$. The mean value of $u_\theta$, i.e. $\mu_{u,i}$, is supposed to coincide with the abscissa of the tangent point of the OC-line to the boundary line (fig.15), which is approximately the most unfavourable situation.

On the basis of (21) we calculate the mean value of $f_{u,o}$ and denote it by $\mu_{u,o}$. The results are mentioned in table 3 for three typical cases.

| n | $\lambda$ | $\mu_{u,i}$ | $\Phi(\mu_{u,i})$ | $\mu_{u,o}$ | $\Phi(\mu_{u,o})$ |
|---|---|---|---|---|---|
| 5 | 1.424 | - 1.080 | 14,0 % | - 1.853 | 3.2 % |
| 10 | 1.325 | - 1.293 | 9.8 % | - 1.881 | 3.0 % |
| 15 | 1.318 | - 1.379 | 8.4 % | - 1.964 | 2.5 % |

Table 3 Numerical results

From the last two columns it follows that $\theta^* < 0.05$ for OC-lines that are tangent to the unsafe region. Consequently, the global fraction defectives is se-

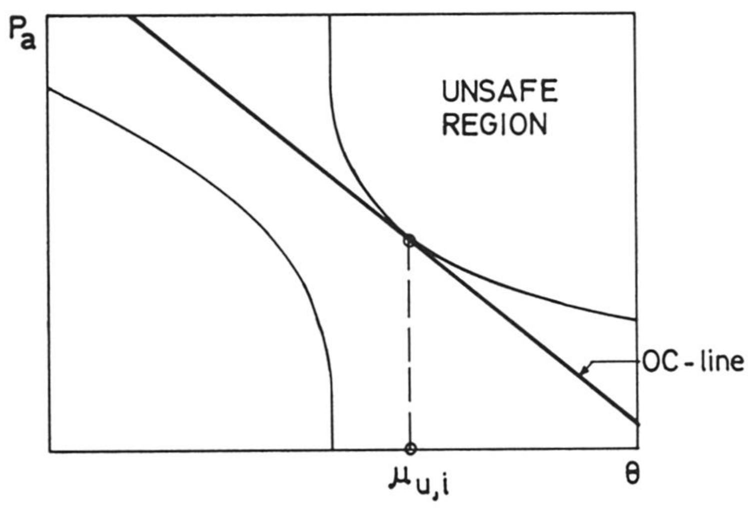



Fig.15 Location of the mean value of the process curve in the most unfavourable case

riously reduced by the filtering effect of compliance control (e.g. from 14.0% to 3.2% in the case n = 5).

Hence we have shown that, when OC-lines are tangent to the boundary curve of the proposed unsafe region, the resulting global fraction defective is lower than 5% even when rejected lots are not transformed to a perfect state. The results are obtained on the basis of some reasonable assumptions concerning accepted and rejected lots and by considering the filtering effect of compliance control. In this way, the assumptions on which the unsafe region is based are extended and generalized. Moreover, this region is reasonable with respect to current practice since it is situated in the same zone as the corresponding CEB region.

## 8. CONCLUSIONS

In the paper, that summarizes a part of the contents of reference [9], the following results with respect to compliance control are derived.

- The general expression for the filtered process curve is given.
- An empirical process curve for concrete strength is presented.
- An unsafe and an uneconomic region for OC-lines are proposed on the basis of some reasonable assumptions.
- OC-lines tangent to these regions are calculated.
- The case of rejected lots is also taken into consideration and some weak assumptions concerning a transformation of these lots allow to extend the justification of the proposed unsafe region.

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