Cracks and crack control at concrete structures

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Fissures et contrôle des fissures dans les structures en béton

Risse und Risskontrolle bei Betonbauwerken

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SUMMARY

Cracks are almost unavoidable in large concrete structures. Their causes and their meaning for the serviceability and durability of the structures are treated. Simple rules for the design and sizing of reinforcement or prestressing are given in order to keep the crack width in admissible limits.

RÉSUMÉ

Il est pratiquement impossible d'éviter les fissures dans les grands ouvrages en béton. Leurs causes et leurs conséquences sur l'aptitude au service et la durabilité des structures sont traitées. De simples règles pour le projet et le dimensionnement de l'armature passive et de précontrainte sont proposées, afin de maintenir les fissures dans des limites acceptables.

ZUSAMMENFASSUNG

Risse im Beton sind in grossen Bauwerken fast unvermeidlich. Ihre Ursachen und ihre Bedeutung für die Gebrauchsfähigkeit und Dauerhaftigkeit der Bauwerke werden behandelt. Einfache Regeln für die Bemessung der Bewehrung oder Vorspannung werden angegeben, um zulässige Grenzen der Rissbreiten einzuhalten.

1. FOREWORD

We wish to achieve concrete structures without cracks, because laymen as clients or users consider cracks as damage or as beginning deterioration, they make the engineer or contractor liable and demand repair. On the other side we assume in the design analysis that the tensile zone of the concrete member is cracked - what a contradiction! Prestressing of concrete structures was invented and applied in order to eliminate tensile stresses and hereby to prevent cracks. But soon we found cracks also in prestressed concrete structures. Why? Are these cracks harmful or harmless? More than 30 years of research and observations referring to the causes and consequences of cracks allow helpful answers.

2. CAUSES OF CRACKING

2.1 Tensile strength of concrete

The main cause of cracking is the very low and widely scattering tensile strength of concrete. A statistical evaluation of laboratory tests by H. Rüsch [1] gave the following values for axial tension, related to the 28 day compression cube strength $f_{c,W}$

$$5\ \%\ \text{fractile}\ f_{c,t} = 0{,}18\ f_{c,W}^{2/3}$$

$$95\ \%\ \text{fractile}\ f_{c,t} = 0{,}36\ f_{c,W}^{2/3} \quad \text{N/mm}^2$$

In structures the tensile strength may even be lower for reasons which are described in section 2.2.

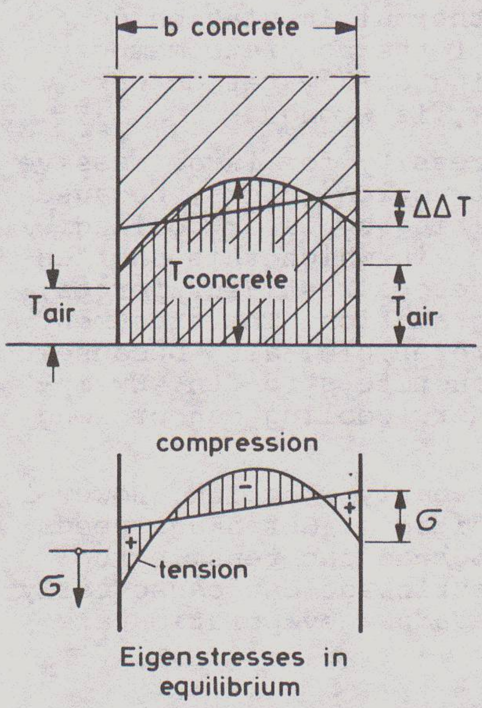
The flexural tensile strength is slightly higher in beams with a depth between d = 15 to 30 cm, however, it is better to neglect this in practical work.

Concrete members crack if the tensile strain ε_{ct} exceeds 0,01 % to 0,012 %. This rupture strain is almost independent of the concrete strength.

The 5 % fractile of f_{ct} has to be assumed in design analysis in order to find those zones in the structure which may be affected by cracks. The 95 % fractile of f_{ct} must be considered for the calculation of the maxima of restraint forces and the necessary amount of reinforcement for the crack width limitation.

2.2 Causes of cracking during the hardening period of the concrete

In numerous cases it could be proven that the cracks occurred already during the first days after placing the concrete before any loads acted on the structure. They are caused by "Eigenstresses" (self equilibrating stresses) due to differential temperatures ΔT (Fig. 1) which are higher than the slowly developing tensile strength f_{ct} of the concrete (Fig. 2). These T must mainly be traced to the heat of hydration which the cement produces during the hardening period and which so far was usually neglected (with the exception of massive structures like concrete dams, see for example [2]). Depending on the type and the quantity of cement, concrete members 20 to 30 cm thick can warm up by about 20 °C, 1 m thick up to 60 °C during the first two days. If the heated member cools down too quickly by cold air, mainly at night, then the stresses σ_{ct} get easily higher than the still low tensile strength f_{ct} and the concrete must crack. Even if only micro-cracks form, they will reduce the final tensile strength of the hardened concrete. However, quite often wide cracks show up due to these effects, even when much reinforcement was placed, because the young concrete gives not sufficient bond strength for making rebars effective to limit the crack width.



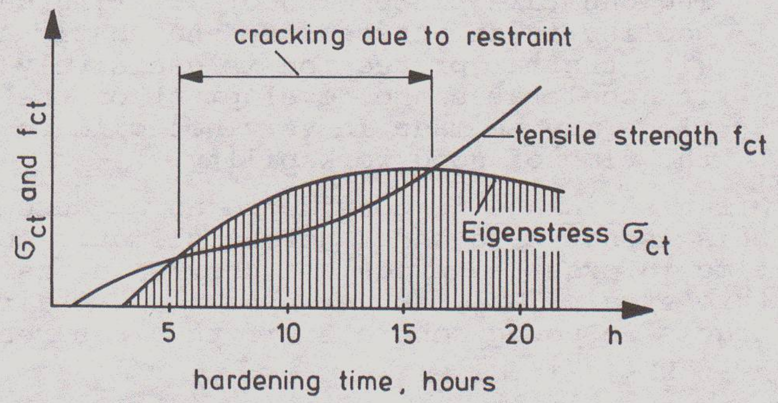


Fig. 1 Heat of hydration gives high temperature T. Cooling from outside causes "Eigenstresses"

Fig. 2 Development of the tensile strength of concrete f_{ct} and of "Eigenstresses" due to ΔT caused by early cooling

It is necessary to prevent such early cracks by keeping the ΔT so low that the σ_{ct} remain smaller than the f_{ct} (Fig. 2). This can be reached by the following measures, single or in combination:

- Choice of a cement with low initial heat of hydration. Table 1 shows how different the heat development of German cements is, given in Joule per gram cement at 20^{o} C initial temperature. The quantity of the cement per m³ of concrete should be kept as low as possible by good grading of the aggregates. The heat development can be slowed down by adding fly ash or using slag furnace cement.

Table 1: Heat of hydration of German cements in J/g

cement class	1 day	3 days	7 days	28 days
Z 25 Z 35 L	60 to 170	125 to 250	150 to 300	210 to 380
Z 35 F Z 45 L	125 to 210	210 to 340	275 to 380	300 to 420
Z 45 F Z 55	210 to 275	300 to 360	340 to 380	380 to 420

- Curing. First, evaporation of water must be prevented at all open surfaces of the concrete structure by spraying a vapour barrier or by covering the concrete with a dense membrane.
- Curing by thermal insulation. Too quick cooling of exterior zones must be prevented. The degree of thermal insulation depends on the climate and the thickness of the concrete member, but also upon the type of cement. Spraying cold water on warm young concrete, as it was done for years, is wrong.
- Cooling of young concrete. This is a necessity for large massive concrete structures like dams with construction joints, because the shortening of the concrete after joining by later cooling must be prevented. For normal structures, in which this shortening can take place without creating dangerous restraint forces, cooling is an unnecessary and costly aggravation. The treatment with thermal protection is decisively preferable, also because it accelerates the development of the concrete strength. Exemptions may be made in very hot climate where cooling can prolong the time of good workability.

Often shrinkage is considered as a cause of early cracking. However, this is not true under normal climatic conditions. Shrinkage needs time in order to produce a shortening as high as the tensile rupture strain. Only in very hot and dry air shrinkage can cause early cracks in young concrete, if the measures against evaporation are not applied.

2.3 Causes of cracks after the hardening of the concrete

The tensile stresses σ_{ct} due to dead loads DL and live loads LL, producing action forces M,N,V may first be mentioned. The necessary amount of reinforcement or prestressing must be calculated to satisfy ultimate limit state capacity and simultaneously to keep crack widths in admissible limits in the serviceability limit state. These tensile stresses due to service loads can fully or partially be suppressed by prestressing. The degree of prestressing $\varkappa = M_D/M_{DL+LL}$ can be chosen $\varkappa \leqq 1{,}0$ (M_D = moment of decompression) along structural or economic criteria. Normally $\varkappa = 0{,}4$ to $0{,}6$ lead to better serviceability than full prestressing if the reinforcement is designed, following the rules given in section 5.

Cracks can also occur by tensile stresses which are produced by restraining deformations caused by strains due to rising or falling temperatures or due to shrinkage and creep of the concrete. Imposed deformations like differential settlement between foundations can also cause cracks.

We speak of restraint forces - there is internal restraint causing "Eigenstresses" as shown in Fig. 1, and external restraint in hyperstatic (redundant) structures, as shown in Fig. 3.

Cracks due to these causes in prestressed concrete briges have taught us that they were mainly due to temperature differences produced by sunshine and following cooling by rain or night. Extreme weather conditions must be considered as they may come every 20 to 50 years. The possible maxima of ΔT depend much upon the local climate. The highest ΔT were found in continental climate and in high mountains in zones of moderate or cold climate. In several countries measurements of ΔT at bridges have been made - see [3,4,5].

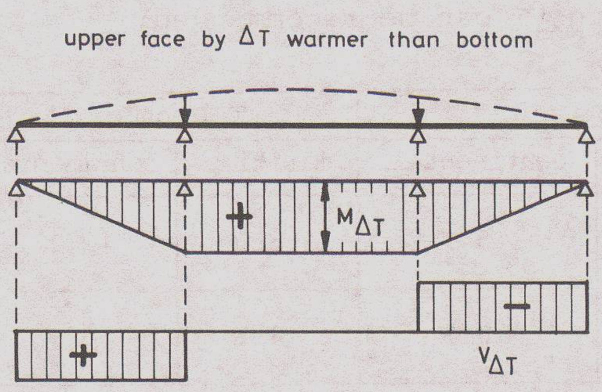


Fig. 3 Restraint action forces $M_{\Delta T}$ and $V_{\Delta T}$ at a continous beam due to ΔT

Lately the Transportation Research Board of USA has published the Report 276 on "Thermal effects in concrete bridge superstructures" (September 1985).

These ΔT have to be superimposed to the mean temperature changes T_m which must be assumed for calculating the max or min changes of the lengths of the structures. In central Europe these T_m are specified for concrete bridges with + 20° C and - 30° C from a mean of + 10° C.

The extreme temperature diagram can be subdivided into three parts (Fig. 4). The linear part of ΔT causes restraint forces in hyper-

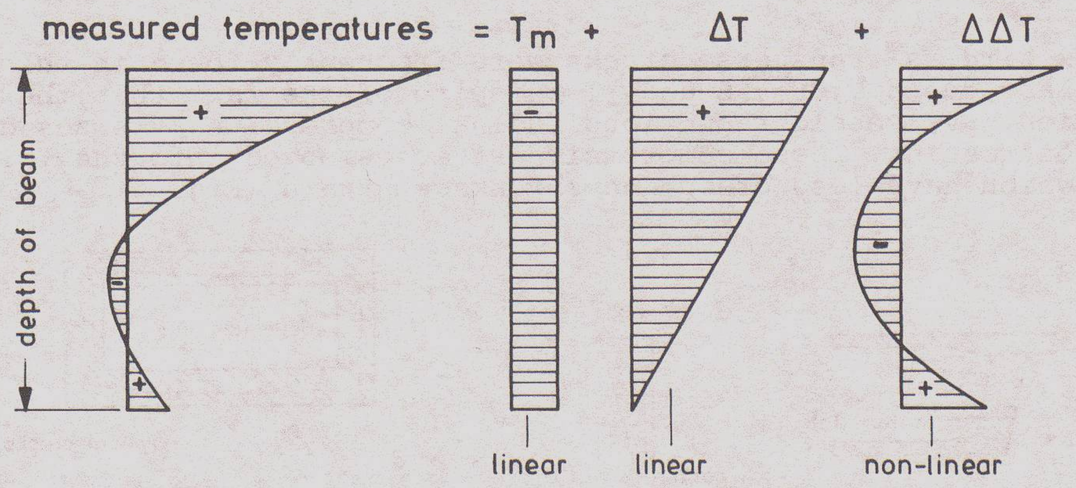


Fig. 4 Division of a temperature diagram into linear ΔT and non-linear $\Delta\Delta T$

static structures, e.g. $M_{\Delta T}$ and $V_{\Delta T}$ in a three span continuous beam as shown in Fig. 3. The non-linear part causes Eigenstresses, which are in equilibrium over the cross-section and produce no action forces, but exist also in statically determinate structures. These Eigenstresses due to $\Delta\Delta T$ can simply be calculated:

$$\sigma_{c,T} = \Delta\Delta T \cdot \alpha_T \cdot E_c$$

α_T = thermal expansion factor, 10^{-5} per 1 K for normal concrete.

Only cooling causes tensile stresses at the edge zones.

For bridges in Europe, the following ΔT can be recommended:

Type of structure	box girder		T beams	
climate	maritime	continental	maritime	continental
upper face warmer than bottom	ΔT = 10 K	15 K	8 K	12 K
bottom edge warmer than upper face	ΔT = 5 K	8 K	4 K	6 K

Differential shrinkage ΔS can in addition to ΔT cause such stresses if the shortening of the concrete is restrained. ΔS often lead to cracks if thin members are connected to thick members. Also differential creep ΔCr can cause cracks like those found in construction joints of some German bridges, built spanwise, if all tendons were coupled in the web. This was not the case when the incremental launching method was used with tendon couplers distributed over the whole cross section.

In box girders transverse cracks were frequently found in thin bottom slabs due to ΔCr, ΔS and ΔT in spite of the fact that the calculation gave considerable longitudinal compressive stresses due to prestressing. These compressive stresses moved into the thick webs which have less creep and shrinkage strain (Fig. 5).

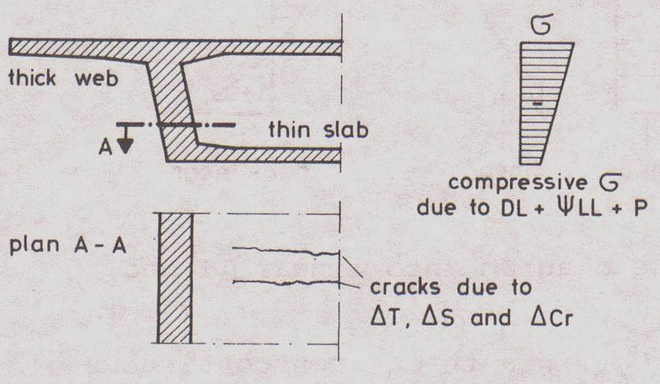


Fig. 5 Transverse cracks in thin bottom slab due to ΔS, ΔCr, ΔT in spite of high prestressing

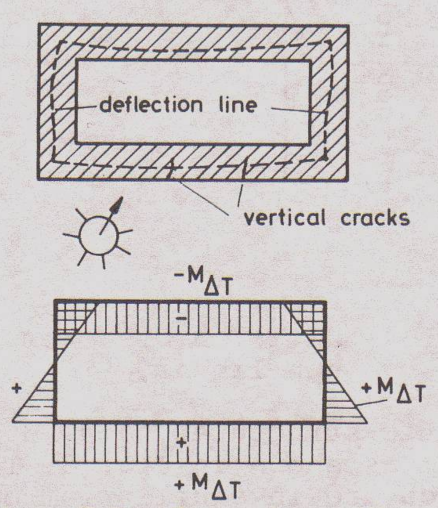


Fig. 6 Bridge pier, $M_{\Delta T}$ due to sunshine by restrained deformations

Box sections are redundant frames and therefore they are affected by restraint moments if they are heated on one side, e.g. by sunshine. This leads to vertical cracks in bridge piers or tower shafts (Fig. 6).

Examples of temperature cracks at p.c. bridges are published in [6] with additional references.

3. DETERMINATION OF ZONES ENDANGERED BY CRACKS AND TREATMENT OF ACTION FORCES DUE TO RESTRAINT

Cracks occur in zones of the structures in which the principal tensile stresses σ_{ct} due to loads or due to restraint forces or due to the addition of both in service condition exceed the tensile strength of the concrete f_{ct}. The σ_{ct} are normally calculated for the uncracked state I with the linear theory of elasticity. The 5 % fractile of f_{ct} should be assumed as the limit strength.

The tension flange of beams under bending is crack-endangered over the length in which $M_{load+restr} > M_{crack}$, where this cracking moment is defined by $\sigma_{ct} = f_{ct,5\%}$ in the edge fibre.

When the flange zone cracks, then the crack tends to continue into the web. The upper limit of the crack-endangered zone in the web has to be found by calculating the strain diagram for the cracked state II under max M. The limit is given by $\varepsilon = 0{,}015$ % (Fig. 7).

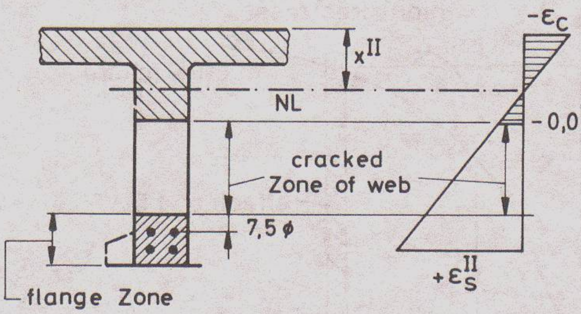


Fig. 7 Cracking zone in webs of beams under max M_{load} or $M_{DL+\ LL+restraint}$

The max possible action forces caused by restraint, preferably bending moments, have to be calculated with the maxima of the causing forces, like ΔT, assuming that the 95 % fractile of the tensile strength f_{ct} has to be overcome in the tension flange.

This M_{restr} has to be added to the moments due to loads, at least for the frequent ones, and it hereby lengthens the zones in which $\sigma_{ct} > f_{ct,5\%}$ occurs in the flange (Fig. 8).

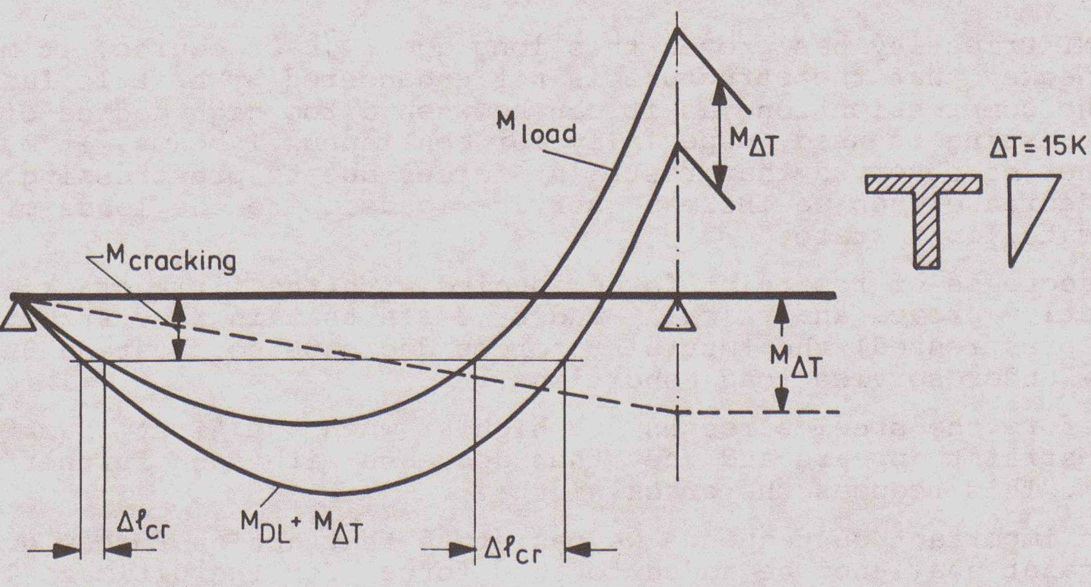


Fig. 8 Additional length $\Delta\ell_{cr}$ of the crack endangered zone of the bottom flange of a continous beam due to ΔT

Favourable live load moments, like negative moments, can of course not be superimposed onto positive moments due to restraint forces.

The sectional forces due to restraint define only location and quantity of the reinforcement or of prestressing forces necessary to limit the crack width in the serviceability state. They do not decrease the ultimate carrying capacity because these M_{restr} are reduced and finally disappear by cracking and plastic deformation when we increase the loads with the required safety factor to reach the ultimate limit state, which defines the necessary quantity of steel (rc + pc) for the carrying capacity (Fig. 9).

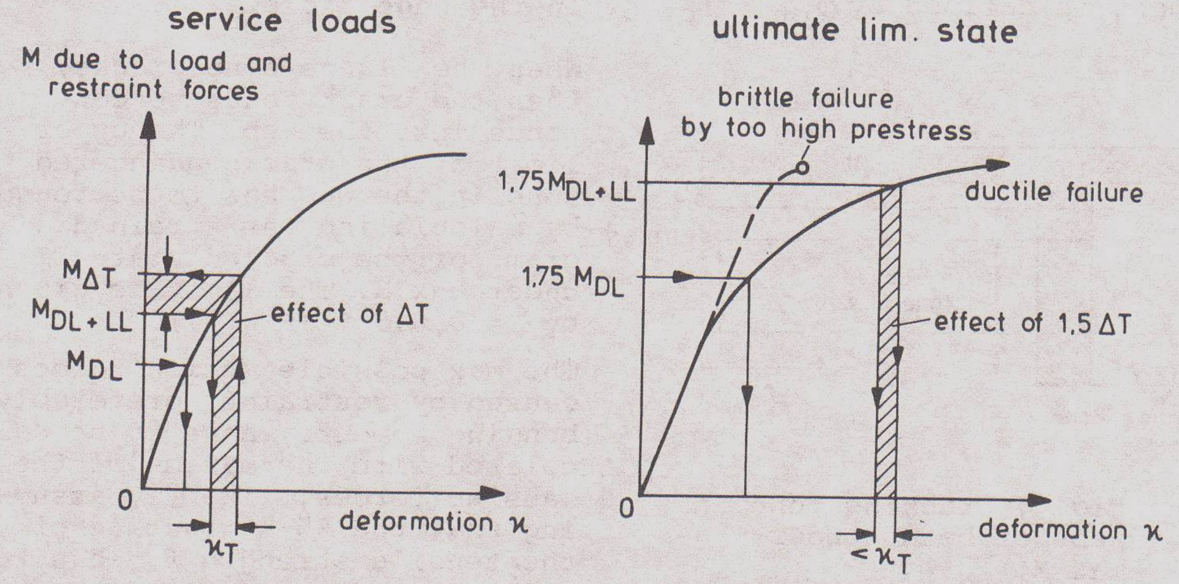


Fig. 9 Priestley's display how restraint forces $M_{\Delta T}$ disappear in hyperstatic structures due to cracking and plastic deformation if loads are increased to the ultimate limit state, here full prestressing for load moments.

M.J.N. Priestley has proven this long ago [4]. Of course, it must be checked that the structure is not endangered by brittle failure of the compression zone as it can be when a too high degree of prestressing is used, especially for continuous T beams. It must further be observed that restraint forces due to prestressing do not decrease when we increase service loads up to the loads of the ultimate limit state.

The decrease of restraint forces begins with the first crack. Priestley proved analytically and by tests that in r.c. structures (not prestressed) the restraint forces decrease to about 50 % already under service load conditions.

Therefore the steel stresses are highest when the first crack due to restraint appears and they then decrease with each further crack. This reduces the crack widths.

As an important consequence we can state that action forces due to restraint shall not be added to load forces for the ultimate limit state which defines the sizing of the steel $A_s + A_p$ in tension flange members. For the serviceability limit state they must be added to define mainly A_s for crack width limitation.

4. VALUATION OF CRACKS

Cracks are judged by the crack width w at the surface of the concrete (Fig. 10) which decreases towards deformed rebars. Long years of research [7] and [8] and experience showed that crack widths up to w = 0,4 mm do not significantly harm the corrosion protection of the rebars, if the concrete cover is sufficiently thick and dense.

Polluted air, especially CO_2 causing carbonation, and SO_2 forming acids, or chlorides from deicing salts, damage the concrete independent of cracks. Structures must be protected against such attacks, having cracks or no cracks.

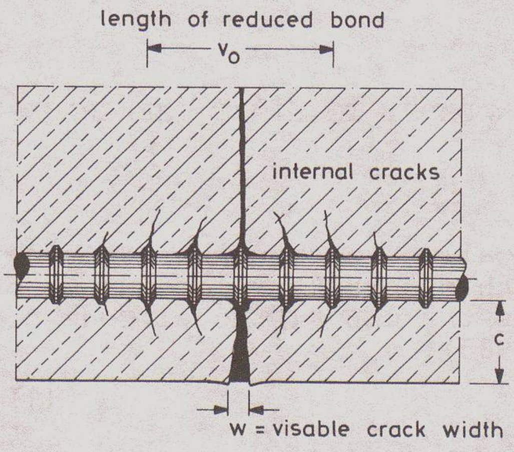


Fig. 10 The crack width w at the surface serves as a scale

Cracks are harmful for the image of the engineers if they are easily visible, because laymen consider them a damage. Therefore at concrete faces which are often seen from a short distance, crack widths w > 0,2 mm should be avoided just for appearance or image sake.

Different grades of environmental aggression and different sensibility of steel types against corrosion led to different requirements for the concrete cover. It makes sense to scale also the admissible limits of the crack width for different environmental conditions. Herefore the limit values should be defined with the 90 % fractile w_{90} in order to keep a sufficient margin for occasionally surpassing crack widths, which should prevent claims for repair liability be raised too quickly.

On the other side, a max w should be given and when this will be surpassed, then a damage must be admitted.

For the environmental criteria of CEB and Eurocode No. 2, we can define the following crack widths:

Table 2: Allowable crack widths

environment	w_{90}	max w	appearance
a low aggressivity	0,3 mm	0,5 mm	easily visible
b medium aggressivity	0,2 mm	0,4 mm	scarcely visible for the unarmed eye
c high aggressivity	0,1 mm	0,3 mm	

These values are valid for a normal concrete cover c = 30 mm and hereby for bar diameters $\emptyset \lesssim c/1{,}2 \lesssim 25$ mm. For a larger cover, the allowable crack width should increase with c/30 (c in mm). For $c > 60$ mm and bar $\emptyset > 32$ mm an anchored skin reinforcement with thin bars inside the concrete cover must be recommended in order to prevent cracks to open too wide.

5. SIMPLE METHODS FOR SIZING REINFORCEMENT TO LIMIT THE CRACK WIDTH

5.1 Basic analysis

The sizing must be based on theoretically and experimentally derived formulae for calculating the width of cracks which can be displayed as follows (The author follows the CEB-FIP Model Code of 1978 and the CEB Manual of October 1983).

The mean crack width is $w_m = s_{rm} \cdot \varepsilon_m$ (1)

The strain ε_m is found in the stress - strain diagram of an axially tensioned r.c. bar according to Fig. 11:

$$\varepsilon_m = \varepsilon_s^{II} - \Delta\varepsilon_s \text{ and here is } \Delta\varepsilon_s = \frac{1}{E_s} \frac{\sigma^2_{s,1.cr.}}{\sigma_s^{II}} \text{ (see [9])} \qquad (2)$$

$\Delta\varepsilon_s$ corresponds to the strain reduction by concrete in tension between cracks, the so-called tension stiffening.

ε_s^{II} and $\Delta\varepsilon_s$ include the considerable influence of the concrete strength and of the relative amount of reinforcement $\rho_r = \frac{A_s}{A_c}$

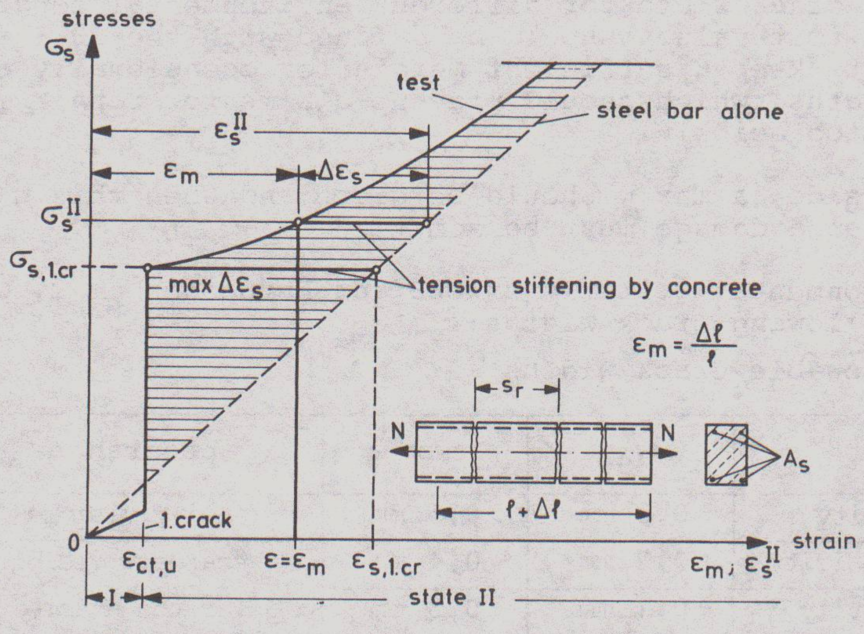


Fig. 11 Stress-strain diagram of a reinforced concrete bar under axial tension. Definition of the ε values for crack width formulae

The mean spacing of cracks can be written

$$s_{cr,m} = 2\,(c + \frac{s}{10}) + k_1\,k_2\,\frac{\emptyset}{\rho_r}\;[mm] \qquad (3)$$

herein is

c = concrete cover in mm
s = transverse bar spacing in mm
k_1 = 0,4 for normal ribbed bars, factor to consider the bond strength
k_2 = 0,125 for bending, factor to consider shape of ε diagram
k_2 = 0,25 for centric tension
k_2 $<$ 0,125 for bending + axial compression (M with $- N_p$)
$\emptyset$ = diameter of rebar in mm
ρ_r = degree of reinforcment $A_s/A_{c,eff}$ related to the effective zone, see Fig. 13.

With these formulae, the mean width of cracks can be calculated. The characteristic value $w_{90} = k_4\, w_m$ depends on the k_4 factor for the width of scatter which was found to be as low as $k_4 = 1,3$ in tests with restraint forces because the steel stress decreases at cracking. Values of k_4 up to 1,7 were found by evaluation of crack measurements at structures. The Eurocode gives $k_4 = 1,3$ for restraint forces and $k_4 = 1,7$ for load actions. This differentiation is too complicated for practical design. The author recommends to use generally $k_4 = 1,5$.

The effect of repeated loads can be considered by a reduction of $\Delta\varepsilon_s$ in equation (2) with the factor k_s

$$\Delta\varepsilon_{s,rep.} = k_5\,\frac{\sigma^2_{s,1.cr.}}{\sigma^{II}_s\;E_s} \quad \text{with } k_5 = 0,4 \text{ to } 0,8$$

depending on the severeness of the dynamic loading (see [9]).

If the direction of the rebars is not rectangular to the crack, like in shear and torsion, then the crack width increases with k_α which can be assumed to

k_α = 1,0 for angles up to $\alpha = 15°$

k_α = 2,0 for angles of $\alpha = 45°$

for intermediate angles, k_α can be linearly interpolated.

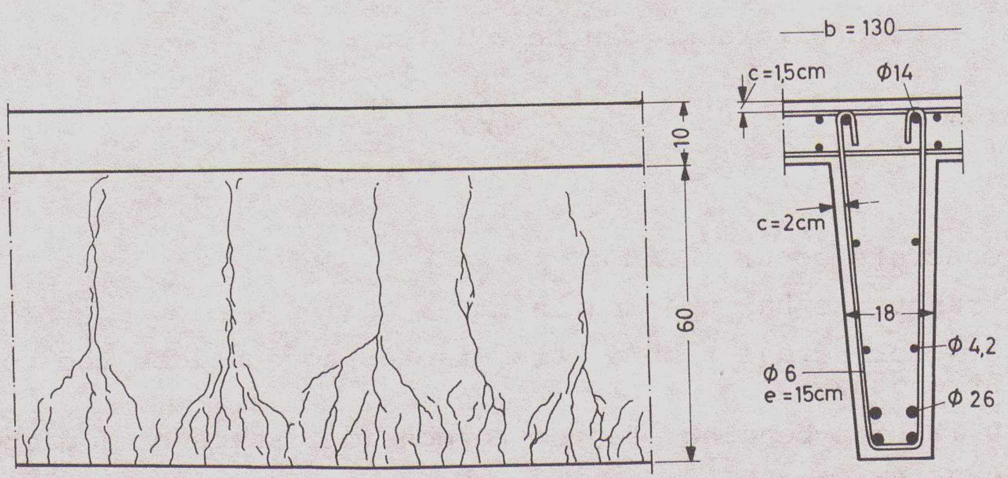
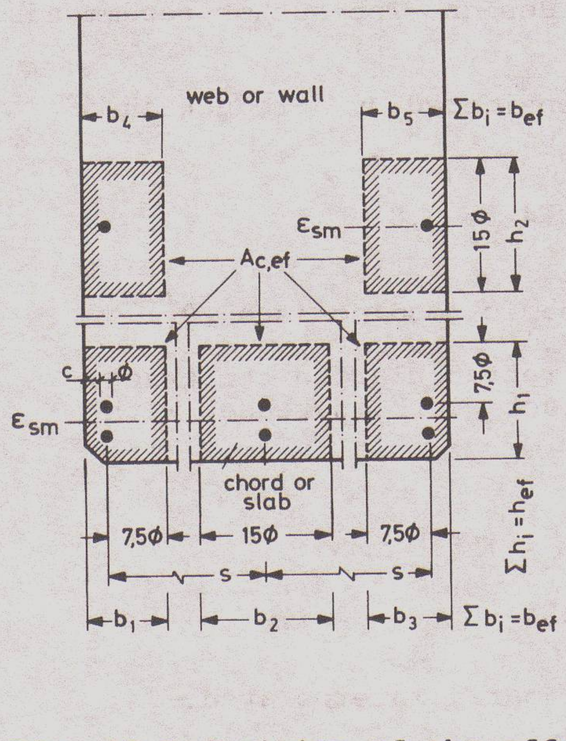


Fig. 12 Cracks of a T beam prove that the small crack spaces and the corresponding small crack widths obtained by the four bars Ø 26 mm in the bottom flange are restricted to a small zone around the bars. Outside this zone, the web reinforcement was too weak to prevent wide cracks

Fig. 12 shows that the reinforcement limits the crack width only within a small zone around the bars which was defined in the CEB-FIP Model Code as the effective zone $A_{c,eff}$ as shown in Fig. 13.



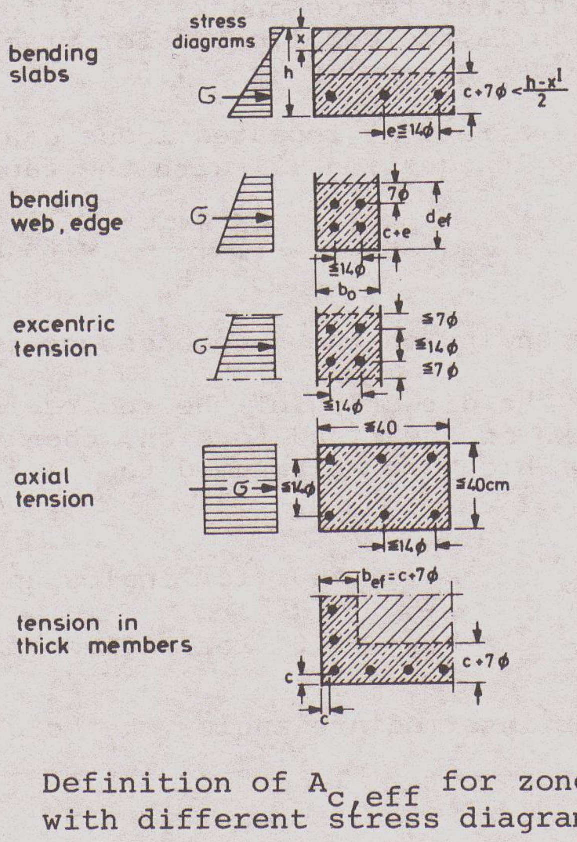


Fig. 13 Definition of the effective zone $A_{c,eff}$ according to CEB

Definition of $A_{c,eff}$ for zones with different stress diagrams

The degree of reinforcement must be related to the rather small effective area. Outside this area, wide cracks can form which are harmless for the carrying capacity but should be avoided by additional reinforcement, if appearance counts. Such wide cracks inside massive structures must also be avoided if the structure must be tight against water pressure.

For practical design work it was not intended to calculate crack widths for an assumed amount of rebars with these theoretically based formulae. As early as 1969 it was recommended to use simple charts for sizing the necessary reinforcement (see [10]) and such charts have been published in the CEB Manual of October 1983 in section 2.42. Their use will be explained in the following chapter.

5.2 Sizing reinforcement for crack control under axial tension

The ϱ_r - Ø diagram in Fig. 14 allows to read the necessary amount of deformed bars A_s related to the effective concrete area $A_{c,eff}$ with $\varrho_r = A_s/A_{c,eff}$ for a chosen bar diameter Ø and for a specified limit of crack width $w_{90} = 1,5\ w_m$. The diagram is valid for axial tension due to loads or restraint forces under free elongation conditions.

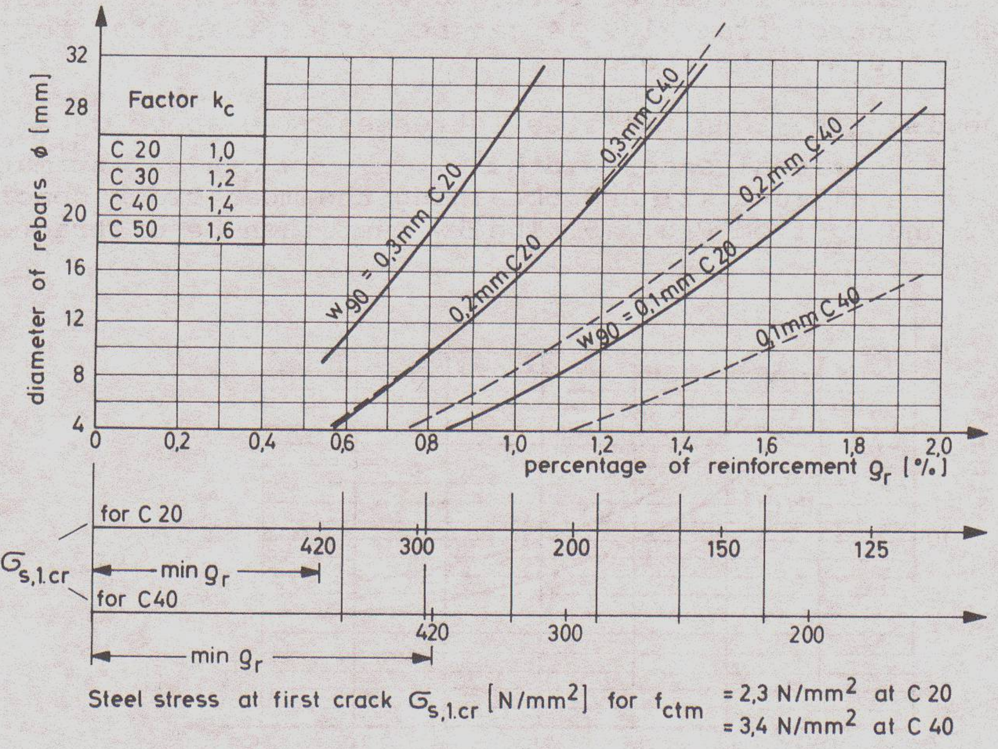


Fig. 14 ϱ_r - Ø diagram for axial tension, see text

The full lines refer to a characteristic cube strength of the concrete C 20, the dotted lines to C 40. For other strengths, the factor k_c has to be used. For crack control one should always choose the concrete class above the one specified for ultimate strength of the structure.

The bar Ø should be chosen for getting small bar spacings, see section 5.5.

Below this diagram, there are the steel stresses $\sigma_{s,1.cr.}$ given which exist at the first crack, they are

$$\sigma_{s,cr} = \frac{f_{ctm}}{\rho_r} = \frac{0,27\ f_{ck}^{2/3}}{\rho_r}$$

This stress shall not exceed the yield strength of the steel and therefore a min ρ_r is noted, assuming a steel quality of St 420/500. For C 20 we get min ρ_r = 0,6 %, for C 40 min ρ_r = 0,8 %.

The steel stresses at cracking are in a wide range higher than allowable stresses in former times for service conditions. This is acceptable for restraint forces because they decrease by further cracks. For loads, however, such high stresses are prevented by the dimensioning for ultimate limit state with loads being multiplied with the safety factor γ leading to

$$A_s = \frac{\gamma S}{f_{sy}}$$

Normally this A_s due to loads is sufficiently large to satisfy crack control requirements in the effective area. Is the load small, then ρ_r for crack control from Fig. 14 can be larger than that for carrying the load and must be chosen.

If the load is high, then the steel stresses rise above $\sigma_{s,1.cr}$ and cause an additional crack width Δw. This Δw can be estimated, using equations (1) and (2) and obtaining the mean crack spacing for given Ø and ρ_r from Fig. 15. The Δw must then be distracted

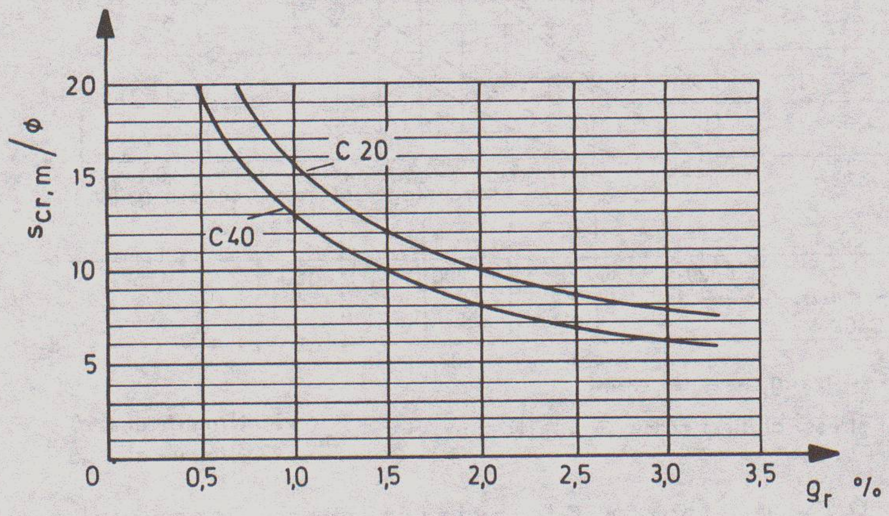


Fig. 15 Mean crack spaces $s_{cr,m}$ at a r.c.bar under tension, related to bar Ø and ρ_r

from the specified w_{90} in order to read the higher ρ_r from Fig. 14 along a line for $w_{90} - \Delta w$. Rough estimations are sufficient.

5.3 Sizing reinforcement for crack control for bending and bending with normal force due to prestressing

In a member stressed by bending or bending plus longitudinal compression, a much smaller quantity of reinforcement is sufficient for crack control than for axial tension. This is easily understood if we consider the jump of steel stress at cracking in Fig. 16 and Fig. 17.

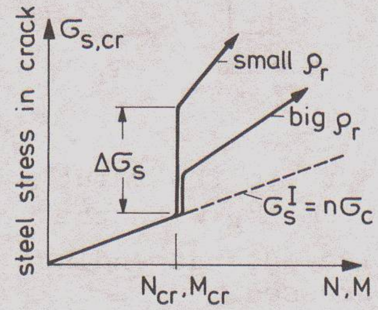


Fig. 16 Jump of steel stress at cracking for different ρ_r

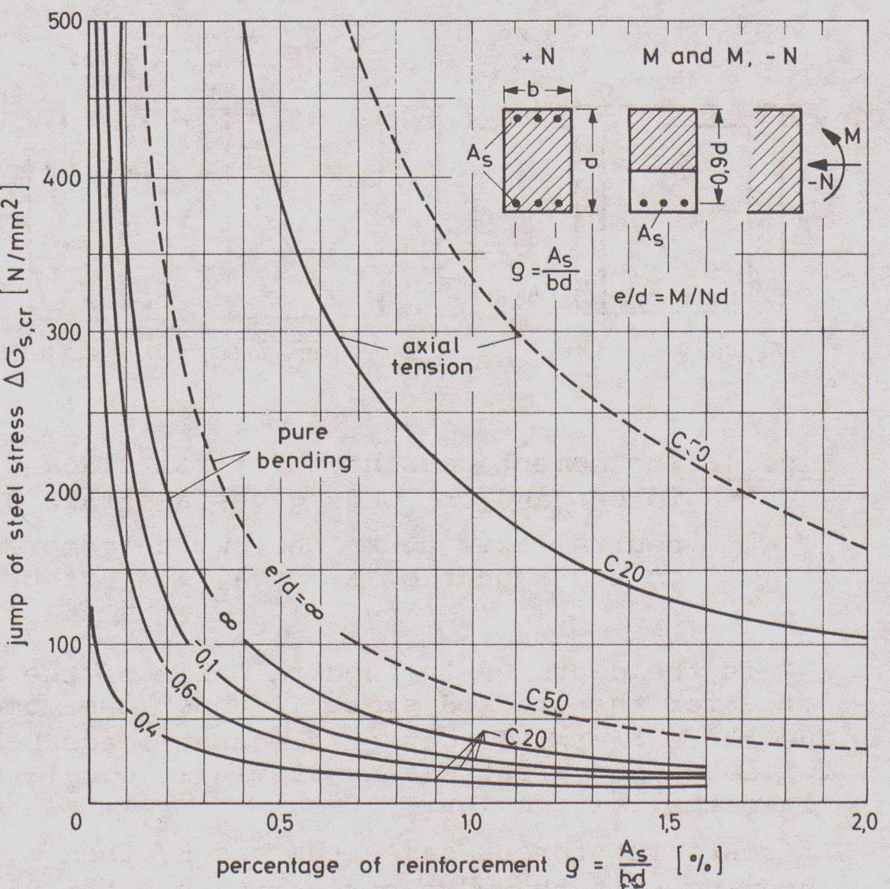


Fig. 17 Jump of steel stress $\Delta\sigma_{s,cr}$ at cracking of concrete in a r.c.bar under tension + N or under bending by M or bending with longitudinal compression - N with different excentricity e/d, for $f_{ct,m} = 0{,}19\ f_{ck}^{2/3}$ [N/mm²].

This jump of steel stress depends on the concrete quality f_{ct}, the percentage of reinforcement ρ_r and the stress characteristic: tension or bending or bending with axial compression of varying excentricity as for example by prestressing. It must be noted that in Fig. 17 ρ_r is always related to $A_c = b\,d$.

The big difference of $\Delta\sigma_s$ between tension and bending is obvious. For p.c. structures it is important to see how small $\Delta\sigma_s$ is getting by the axial compression due to prestressing. The range e/d - 1,0 corresponds to a moderate prestressing degree, $e/d < -0{,}4$ corresponds to "limited" prestressing and $e/d = -0{,}17$ would be full prestressing. A moderate prestressing ($\varkappa = 0{,}3 - 0{,}5$) leads already to low steel stresses at cracking in the service state and therefore small ρ are sufficient for crack control.

Also for bending and for M plus - N we can use the ρ_r - Ø diagram of Fig. 14 for finding the necessary ρ_r if we apply the correction factor

$$k_B = \frac{h - x^{II}}{h} \quad \text{as explained in Fig. 18}$$

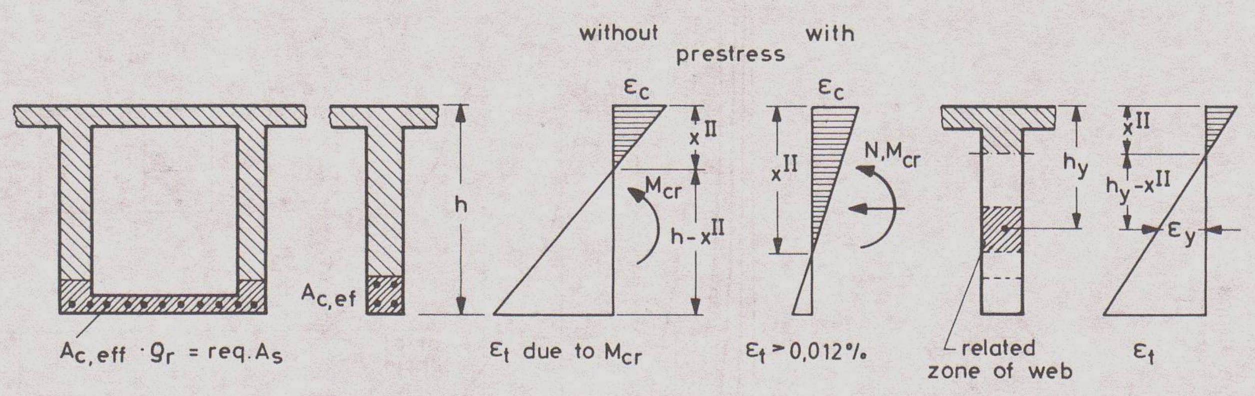


Fig. 18 The moment causing the first crack M_{cr} due to restraint forces and/or loads gives a strain diagram ε_c with the neutral axis at X^{II} below the compressive edge depending on the amount of A_s or $A_s + A_p$ if prestressed.

X^{II} is the depth of the neutral axis of the considered beam calculated for the cracked state II under the moment at cracking M_{cr} due to restraint forces or frequent loads DL + LL with the reinforcement and prestressing steel (if p.c.) necessary to satisfy ultimate limit design.

If restraint forces cause the crack, then M_{cr} has to be calculated assuming that the edge stress reaches the 95 % fractile of f_{ct}. For calculating X^{II} we assume as usual that the cross section remains plane (straight strain diagram!). In fact, this is not true if shear forces act simultaneously which reduce X^{II}, but so far there is no simple method to consider this correctly.

If partial prestressing is applied, then k_B can easily be as low as 0,2 or 0,3 leading to small ρ_r for satisfying crack control. Here again we have to be aware that this ρ_r read from the diagram Fig. 14 is related to cracking load. Should higher loads later cause stresses considerably above σ_{cr}, then a correction is necessary for

$$\Delta\sigma_s = \sigma_{loads} - \sigma_{cr} \quad \text{with} \quad \Delta w = s_{cr,m} \cdot \varepsilon_m \approx s_{cr,m} \cdot 0{,}7\Delta\varepsilon_s$$

($s_{cr,m}$ from Fig. 15).

In box girders with thin bottom slabs the strain ε^{II} is restrained by the connection to the webs (Fig. 18). The slab is almost under axial tension - but not unrestrained, as assumed for Fig. 14. However, at such slabs we have to think also of restraint stresses due to differential shrinkage Δ S, therefore a supplement to $k_B \rho_r$ is recommended. This supplement can be roughly calculated assuming ΔS with ε_S = 0,01 % which increases the crack width by $\Delta w_S = s_{cr,m} \cdot 0,6\ \varepsilon_S$ and $k_B \varrho_r$ has to be read in Fig. 14 for $w_{90} - \Delta w_S$.

For sizing the reinforcement needed for crack control in the webs above the flange zone

$$k_B = \frac{h_y - x^{II}}{h_y} \quad \text{as shown in Fig. 18}$$

has to be used. The depth of the web should be subdivided into several portions.

For crack control in members stresses by shear or torsion, the formulae given in [9] should be used.

5.4 Crack control without reinforcement

In massive concrete structures or in moderately prestressed structures which get tensile stresses due to $\Delta\Delta T$ or ΔT (see Fig. 1 and 4) it can occur that cracks remain fine hair cracks with widths below w_{90} even without reinforcement. This is so because the tensile strain ε_{ct} is restraint by the adjoining zone under compression (Fig. 19).

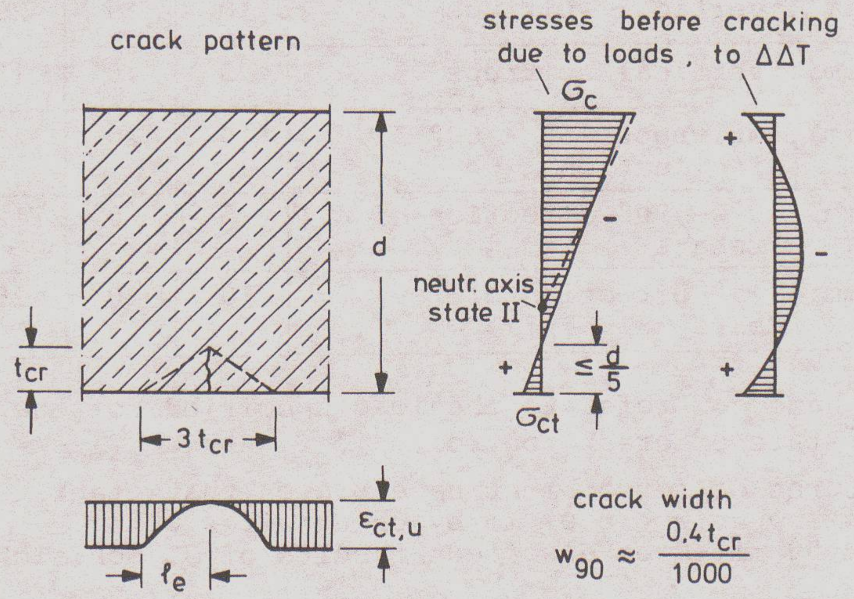


Fig. 19
If ε_{ct} < 0,015 % then crack width remains small without reinforcement

The width of such cracks depends upon the possible depth t_{cr} of the crack and can be calculated from the max tensile strain of concrete $\varepsilon_{ct,u} \lesssim 0,012$ % with $k_4 = 1,6$

$$w_{90} = 1,6 \cdot 2\ t_{cr} \cdot \varepsilon_{ct,u} \approx 0,4\ t_{cr} \cdot 10^{-3}\ [mm]$$

In dry climate, shrinkage of the cracked zone should be considered with $\Delta S \approx 0,01$ %, then we get

$$w_{90} = 1,6 \cdot 2\ t_{cr}\ (\varepsilon_{ct,u} + \varepsilon_S) = 0,6\ t_{cr} \cdot 10^{-3}$$

For w_{90} = 0,1 mm the depth of the cracks can be as large as 25 cm, resp. 17 cm. For restraint bending (e.g. unreinforced but moderately prestressed slabs or beams) the depth should remain below $t_{cr} \lesssim d/5$.

5.5 Recommendations for spacing and diameters of rebars

The small effective area of 7,5 Ø around the rebars requires small spaces between bars s $\lesssim$ 15 Ø. The crack width is further almost linearly depending on the bar diameter. Therefore, optimal crack control is obtained by choosing small Ø and small spacing which lead also to the lowest steel quantities. The following table gives

Recommended upper limits of bar spacings
measured rectangularly to the bars, in cm

Allowable crack width w_{90} in mm	0,1	0,2	0,3
tension	10	15	20
tension by bending with σ_s^{II} 240 N/mm²	10	15	20
tension by bending with σ_s^{II} 120 N/mm²	15	20	30
shear with $\tau_o \approx 2$ N/mm², vertical stirrups	10	15	20
shear with $\tau_o \gtrsim 3$ N/mm², vertical stirrups	5	10	15
shear with $\tau_o \gtrsim 3$ N/mm², stirrups 45° - 60° inclination	10	20	25
torsion for $\tau_T > 2$ N/mm², 0° - 90° direction of rebars	5	8	12
torsion for $\tau_T > 2$ N/mm², 45° direction of rebars	10	20	25

The stresses σ_s^{II}, τ_o and τ_T refer to the load specified for the serviceability limit state of crack control.

Nervi's famous structures of ferrocement have proven that crack widths can be kept as low as w < 0,01 mm by using wires with Ø = 2 mm spaced 30 to 50 mm - see also test results of J. Schlaich in [11].

6. MINIMUM REINFORCEMENT

The minimum reinforcement has to fulfil two requirements:

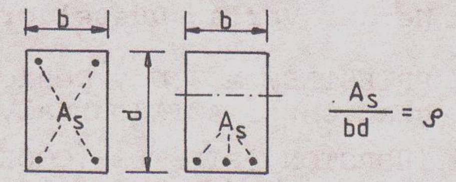
a) To secure the load-carrying capacity which has to be calculated for max γ(DL + LL) , (γ= safety factor, global or split according to codes) but without restraint forces. The rebars with A_s and prestressed steel with A_p must remain within the ultimate limits of strain ε_s and ε_p.

In order to prevent sudden failure at cracking, which can be caused by restraint forces, the minimum amount of reinforcement must be for pure tension:

$$\min \rho = \frac{f_{ct,95\%}}{f_{sy}} = \frac{0{,}36\ f_{ck}^{2/3}}{f_{sy}} \quad \left[N/mm^2\right]$$

for bending:

$$\min \rho = 0{,}2\ \frac{f_{ct,95\%}}{f_{sy}} \quad \left[N/mm^2\right]$$



in both cases ρ must be related to the full cross section $A_c = b\,d$. If cracking is primarily caused by restraint forces due to ΔT or ΔS or differential settlement, then the related area A_c can be limited to two or three times the $A_{c,eff}$ according to Fig. 13.

This requirement leads to the following min ρ in [%]:

concrete strength f_{ck} N/mm²	20	30	40	50	related area
min ρ for tension %	0,75	0,93	1,10	1,26	$A_c = b\,d$
min ρ for bending %	0,15	0,18	0,22	0,25	$A_c = b\,d$

b) For the serviceability limit state the min ρ must secure to keep the crack widths below the required limit. The min ρ_{cr}, therefore, depends on the allowed w_{90}, the concrete strength and the chosen bar Ø and must be related to the effective area $A_{c,eff}$. This min ρ_{cr} must be built into all zones where the concrete tensile stresses, calculated for the uncracked state I due to loads or restraint forces become higher than the 5 % fractile of the tensile strength of concrete, this is where

$$\sigma_{ct}^{I} \geq 0{,}18\ f_{ck}^{2/3} \quad \left[N/mm^2\right]$$

In these zones, min ρ is found from fig. 14 together with the k_B factor according to Fig. 18 if bending or prestressing is involved.

In zones without this cracking danger, the min reinforcement can be chosen along constructional criteria.

REFERENCES

1. RÜSCH H., Die Ableitung der charakteristischen Werte der Betonzugfestigkeit. Beton, Heft 2/1975.
2. WIDMANN R., Massenbetonprobleme beim Bau der Gewölbemauer Kölnbrein. Lectures of Betontag 1977. Deutscher Beton-Verein.
3. KEHLBECK F., Einfluß der Sonnenstrahlung bei Brückenbauwerken. Werner Verlag Düsseldorf, 1975.
4. PRIESTLEY M.J.N., Design of concrete bridges for temperature gradients. ACI Journal, 1978, pp. 209-217.

   THURSTON,PRIESTLEY,COOKE, Influence of cracking for thermal response of reinforced concrete bridges. Concrete International, August 1984.
5. DILGER W., GHALI A., Temperature stresses in composite box girder bridges. Journal of Structural Engineering, ASCE, June 1983, p. 1460.
6. LEONHARDT F., Rißschäden an Betonbrücken, Ursachen und Abhilfe. Beton- und Stahlbetonbau, Heft 2/1979.
7. SCHIESSL P., Admissible crack width in reinforced concrete structures. Preliminary Report Vol. 2 of IABSE-FIP-CEB-RILEM-IASS, Colloquium Liège, June 1975.
8. Concrete International, May 1985, p. 24, A.W. BEEBY's contribution.
9. LEONHARDT F., Vorlesungen über Massivbau, 4. Teil, Springer Verlag, 1977.
10. FALKNER H., Zur Frage der Rißbildung durch Eigen- und Zwangsspannungen infolge Temperatur in Stahlbetonbauteilen. Deutscher Ausschuß für Stahlbeton, Heft 208.
11. SCHLAICH J., DIETERLE H., Versuche mit Ferrozement. 15. Forschungskolloquium des DAfStB in Stuttgart, April 1984.