Evaluation of Load Combination Criteria in Structural Codes

Evaluation des critères de combinaisons de charges dans les normes de projet

Beurteilung der Lastkombinationskriterien in Bemessungsnormen

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SUMMARY

The combination of stochastic load processes in codified design is discussed in this contribution. Several sections for improvements are identified and modifications for future developments are proposed. A comparison of different load combination factors with exact results is included.

RÉSUMÉ

La contribution traite de la combinaison de charges aléatoires dans les normes de projet. Plusieurs domaines où une amélioration est possible sont identifiés et des modifications sont proposées. Une comparaison de différents facteurs de charges avec les résultats exacts est incluse.

ZUSAMMENFASSUNG

Der Beitrag diskutiert die Behandlung von Lastkombinationsproblemen in Bemessungsnormen. Verbesserungsvorschläge für die Bemessung sind angegeben. Ein Vergleich von Lastkombinationsfaktoren in derzeit gültigen Normen mit theoretisch abgeleiteten Ergebnissen wird durchgeführt.

1. INTRODUCTION

The combination of time varying stochastic loads acting upon a structure has been researched in great depth during the last decade. The most important results are reviewed and their applicability in code formats are discussed in this contribution. The main objective is to hereby determine whether or not the load combination formats in present codes (such as the CEB Model Code [1]) are leading to satisfactory results. The following tasks are therefore relevant:

- Revision of the existing format, if necessary.
- Extension of the existing format to other fields, such as nonlinear combination, combination of dynamic loads, combination of dependent loads.

The purpose of a change of load combination format is to achieve a more unified level of reliability which may lead to a somewhat reduced required safety level. This can only be achieved if individual load models reflect the real loading patterns and magnitudes reasonably well.

Several sections for improvements in codified load combination are identified and modifications for future developments are proposed. A comparison of different load combination factors with "exact" results is also included.

2. MATHEMATICAL PROCEDURES FOR LOAD COMBINATION

When only one time varying load acts on a structure and when failure is defined as the crossing of a given level by the load effect process, the distribution of the maximum load contains sufficient information on the load process for design purposes. The analysis of stochastic load combination is necessary in situations where a structure is subjected to two or more time varying loads acting simultaneously. The loads can be components of the same load process or components of different load processes. To evaluate the reliability of the structure, loads can no longer be characterized by their extreme value distribution alone, but stochastic process presentations are necessary, since the loads in general do not achieve their extreme values at the same time.

A structure subjected to loads modeled as a vector valued load process $\mathbf{Q}(t)$ is considered. Failure of the structure is defined to occur at the time of the first exceedence of a deterministic function $\xi(t)$ by the random function $b(\mathbf{Q}(t))$. Here $\xi(t)$ represents a strength threshold and the b-function converts the load processes to the load effect process under consideration. By repeating the analysis for several arguments of a time independent threshold ξ the distribution function for the combined loading can be obtained. A *linear load combination* corresponds to the case when the b-function is linear. Otherwise the load combination is *non-linear*. The failure event is illustrated geometrically in Fig.1 for a combination of two loads and for a constant threshold $\xi(t)$. The figure shows that failure can be thought of as either the first upcrossing of $\xi(t)$ by the process $b(\mathbf{Q}(t))$, i.e. in the load effect space, or as the first outcrossing of the set $B(t) = \{\mathbf{q} \mid b(\mathbf{q}) \leqslant \xi(t)\}$ by the vector process $\mathbf{Q}(t)$., i.e. in the load space. In both cases, of course, the condition that failure does not occur at time zero, is assumed. The two representations are directly generalized to combinations of more loads.

Exact solutions to the load combination problems are in general difficult to obtain. Upper bound solutions given in terms of the mean number of crossings in general provide a good approximation. This is discussed in [2].

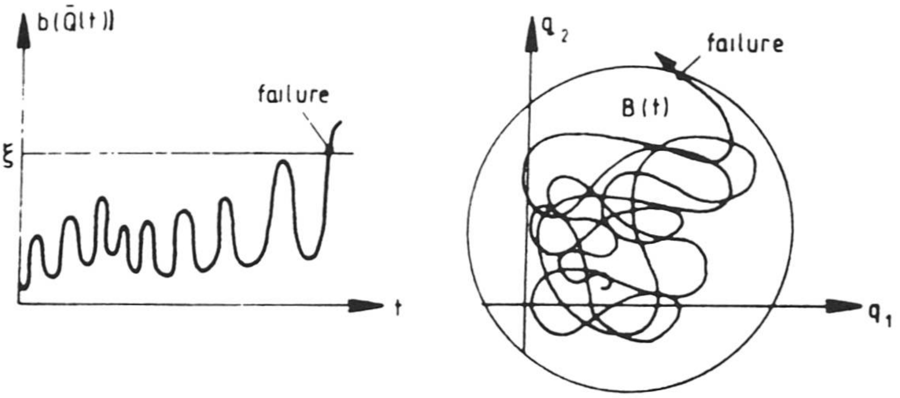


Fig.1 Geometrical illustration of failure event from combined load

3. LOAD COMBINATION FORMATS IN CODIFIED DESIGN

The philosophy of implementation of load combination in structural design is described within the hazard-scenario concept in [3]. In general, load combination requirements in structural codes should include the following characteristics:

a) simplicity (to be applicable in routine design)

b) completeness (to be able to treat all design cases)

c) accuracy (to result in a safety level close to the target)

d) compatibility (with the analysis of single actions)

In general such load combination formats provide a list of the combinations to be considered and a set of appropriate load factors to be applied to the nominal (eg. characteristic) values of the individual loads. To provide for the many different situations which can arise in design, most codes have found it useful to categorize loads as being either permanent (e.g. self weight) or variable. Variable loads can be further decomposed into those with long term variation (e.g. sustained live load) or short term variation (e.g. transient live load,wind load,etc.). For each type of load, codes specify characteristic or representative values, normally corresponding to a specified probability of being exceeded in a given period. As an example the 50-year wind speed commonly used in building codes corresponds to the wind speed with a probability of 2% of being exceeded in one year.

To describe the basic design formats, permanent loads are denoted D and variable loads L, further decomposed into long-term components LL and short-term components LS. With this basic notation, a first subscript k is used to denote a specified characteristic value, and a second subscript $j=1,2,\cdots$ to denote a particular load type, such as wind or earthquake. In the following the most common code formats are given:

a) Companion Action Factor Method (CAF)

When combining two design loads, one takes the sum of one of the design loads (i.e. lifetime maximum value) and the other design load multiplied by a factor (i.e. arbitary-point in time value). The format proposed by the CEB [1] and Eurocode [4] employs this method and is of the general form

$$\gamma_D D_k + \gamma_i L_{ki} + \sum_j \gamma_j \psi_j L_{kj} \tag{1}$$

for the ultimate state, in which γ-values are load factors. The products $\gamma_j \psi_{ij} L_{kj}$ may be called companion values of the loads. The format involves the factored or design value of one load plus factored companion values of the others. There are at least as many such equations as there are loads.

b) Load Reduction Factor Method

Many building codes recommend a reduction factor when different loads are combined. The format proposed in ACI [5] for example is of the form

$$\gamma_D D_k + \phi (\sum_j \gamma_j L_{kj}) \tag{2}$$

in which ϕ is a reduction factor to account for the fact that extreme values of different loads are unlikely to occur together. When only one load acts $\phi=1$, otherwise $\phi<1$.

The Sovjet Union has adopted a slightly different ultimate state format which can be written as, [6]:

$$\gamma_D D_k + \gamma_{LL} LL_k + \phi (\sum_j \gamma_j LS_{kj}) \tag{3}$$

The long-term loads are considered at their full design values and short-term actions are considered at reduced companion values by means of a common reduction factor.

The essential difference between the basic code formats is whether they multiply design loads $\gamma_i L_{ki}$ by combination factors before summation or after. In all cases, serviceability loads are obtained directly from the characteristic values.

Within a geographic region and a specific class of intended use, the physical effect of loads vary from structure to structure and between elements in a structure. The total variable load effect $S(t)$ in a linear or quasi-linear analysis can be written in the form

$$S(t) = c_1\gamma_1 L_1(t) + c_2\gamma_2 L_2(t) + c_3\gamma_3 L_3(t) \tag{4}$$

in which $L_i(t)$ are the random time dependent variable loads; c_i are deterministic influence coefficients; and γ_i are deterministic load factors. Within a single structure, c_i may be zero for one load type at one element and dominant at another element. The relative magnitudes of the random loads $L_i(t)$ depend on geography and intended use.

Note that any load $L_i(t)$ in a combination can appear alone if $c_j=0$, $j \neq i$. A feasible criterion for determining the design load combination is: *establish a set of companion action factors* ψ_{ij} *in Eq.(1) or load reduction factors* ϕ *in Eq.(2) such that the probability of exceeding design loads is approximately constant for all situations involving one or more loads, the spectrum of influence coefficients* c_i*, all geographic areas and intended structural uses, and all materials and types of structural form covered by a code.*

Given the wide range of design situations and practical limitations on the number of factors permissible in any design procedure, it is evident that great precision cannot be expected.

Before proceeding to the determination of the load combination factors it is of some interest to view the various load combination formats in the light of the results obtained for linear load combinations. In this context the so-called *Turkstra's rule* [7] plays a central role. The rule states that the maximum value of the sum of two independent random processes occurs when one of the processes reaches its maximum value. The rule is an approximation and corresponds to the assumption that the distribution functions of the two random variables

$$Z_1 = \max_{0 \leq t \leq T} (Q_1(t) + Q_2(t)) \tag{5}$$

and

$$Z_2 = \max \begin{cases} \max\limits_{0 \leq t \leq T} Q_1(t) + Q_2(t) \\ Q_1(t) + \max\limits_{0 \leq t \leq T} Q_2(t) \end{cases} \tag{6}$$

are the same.

For Z_2 the complementary cumulative distribution function is

$$\begin{aligned} P(Z_2 > \xi) = {} & P(\max_{0 \leq t \leq T} Q_1(t) + Q_2(t) > \xi) + P(Q_1(t) + \max_{0 \leq t \leq T} Q_2(t) > \xi) \\ & - P(\max_{0 \leq t \leq T} Q_1(t) + Q_2(t) > \xi \text{ and } Q_1 + \max_{0 \leq t \leq T} Q_2(t) > \xi) \end{aligned} \tag{7}$$

The extreme value distribution of a load can be bounded by

$$P(\max_{0 \leq t \leq T} Q(t) > \xi) \leq 1 - F_Q(\xi) + \nu_Q(\xi) T \tag{8}$$

where $F_Q(\,)$ is the distribution function for the arbitrary point time value of $Q(t)$ and $\nu_Q(\xi)$ is the mean up-crossing rate of level ξ. Neglect of the negative term in eq.(7) and use of the bound (8) without $1-F_Q(\xi)$ leads to

$$P(Z_2 > \xi) \leq T \int_{q=-\infty}^{\infty} \nu_{Q_1}(q) f_{Q_2}(\xi - q) dq + T \int_{q=-\infty}^{\infty} \nu_{Q_2}(q) f_{Q_1}(\xi - q) dq \tag{9}$$

For the combined load the mean up-crossing rate is bounded by the so-called point crossing terms

$$\nu_{Q_1+Q_2}(\xi) \leq \int_{q=-\infty}^{\infty} \nu_{Q_1}(q) f_{Q_2}(\xi - q) dq + \int_{q=-\infty}^{\infty} \nu_{Q_2}(q) f_{Q_1}(\xi - q) dq \tag{10}$$

This bound is almost always a very good approximation. Use of the bound (8) without $1-F_Q(\xi)$ together with (10) leads to the same upper bound for $P(Z_1 > \xi)$ as in (9). Based on these results it can be concluded, that when eq. (9) is a good approximation for both $P(Z_1 > \xi)$ and $P(Z_2 > \xi)$ then Turkstra's rule is also a good approximation. The conditions for the applicability of the upper bound given here are, however, not necessary conditions for a good accuracy of Turkstra's rule.

Example 1: Modified Turkstra's rule

A modified version of the original Turkstra's rule is illustrated here. This modification also takes the load duration into consideration. In this case the arbitary-point-in-time values of the companion loads are replaced by the maximum values over a period of constant load value for the leading load. The rule is illustrated in a simple example, in which a sustained live load is combined with a transient live load.

The sustained live load $Q_s(t)$ is modeled by a Poisson square-wave process and the transient live load $Q_t(t)$ is modeled by a stationary Poisson "spike" process.The random variable Z_2 in (6) is replaced by:

$$Z_2 = \max \begin{cases} \max\limits_{[0,T]} Q_s(t) + \max\limits_{[0,d]} Q_t(t) \\ Q_s(t) + \max\limits_{[0,T]} Q_t(t) \end{cases} \tag{11}$$

with d the duration of each square-wave. The extreme value distribution of type I is applied for both load types. The computation of Z_2 has been carried out for office and appartment buildings. The corresponding statistics (mean value, standard deviation, mean rate of occurrence) have been taken from the *Basic Note A-02 (Live Loads in Buildings)*, [8] and are approximately valid for areas between $8m^2$ and $20m^2$. Fig. 2 illustrates the two load processes with their statistical data (arbitary-point-in-time mean value m and standard deviation σ).

For the computations the program *PROBAN* [9] has been used. Table 1 summarizes the results in terms of z_2 (given in KN/m^2) for T=50 years and for several fractile values q. "Exact" results have been also computed by applying the technique proposed in [10] and are included in Table 1 as z_1. We can conclude that the modified Turkstra's rule gives good (slightly conservative) results.

Table 1: Combination of sustained and transient live load

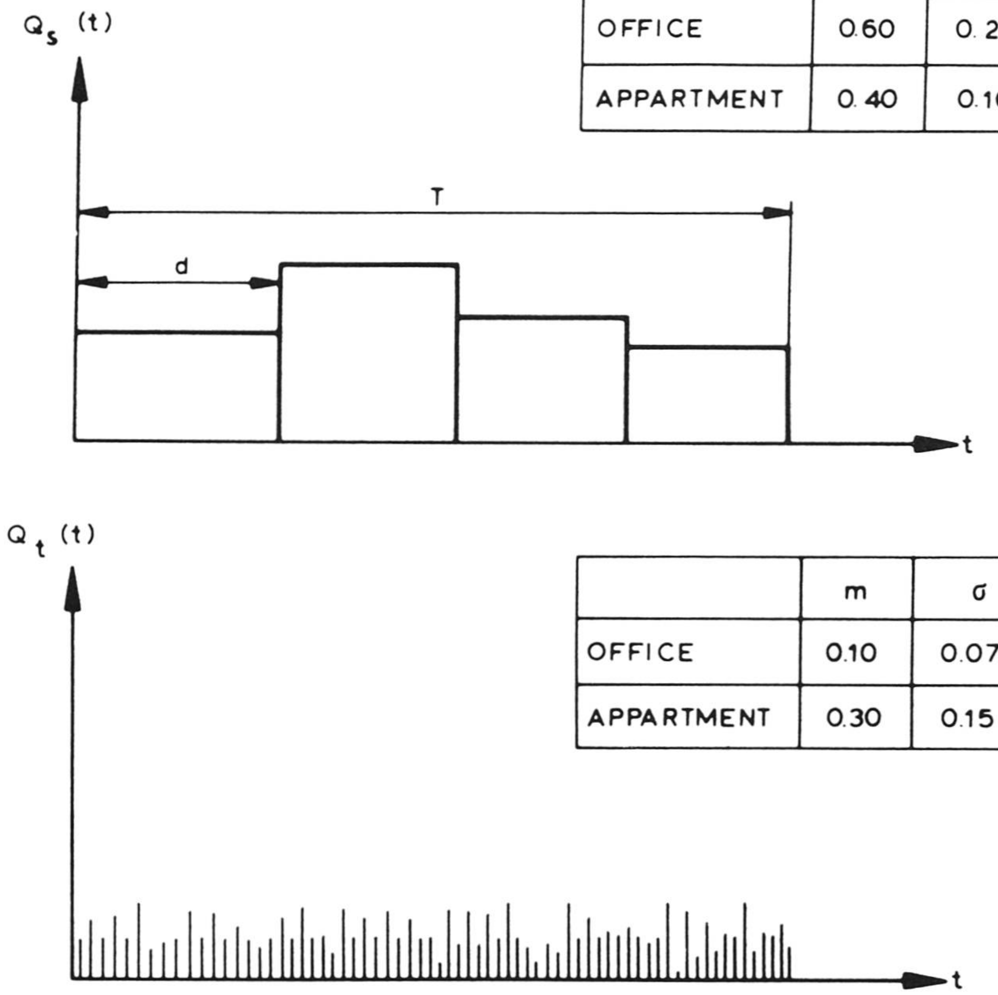
Fractile	Office Buildings		Appartment Buildings	
q	z_2	z_1	z_2	z_1
0.900	1.56	1.55	1.59	1.56
0.950	1.71	1.68	1.71	1.67
0.980	1.89	1.85	1.82	1.77
0.990	2.04	1.99	1.93	1.88
0.995	2.10	2.05	1.99	1.94

The modified Turkstra's rule can be also applied to the combination of other and more types of loads. For the computation of the maximum design load first- or second order reliability methods are useful.

Turkstra's rule indicates, that a natural code format for a combination of two loads is

$$\max \begin{cases} \gamma_1 q_{1k} + \gamma_2 \psi_{21} q_{2k} \\ \gamma_1 \psi_{12} q_{1k} + \gamma_2 q_{2k} \end{cases} \tag{12}$$

where the ψ-factors express the ratio between fractiles in the extreme value distributions and the marginal distributions. It should be emphasized that the ψ- factors depend on both loads to be combined. This important point is discussed further in the next section.



	m	σ	v
OFFICE	0.60	0.24	1/8
APPARTMENT	0.40	0.16	1/10

	m	σ	v
OFFICE	0.10	0.07	4
APPARTMENT	0.30	0.15	2

Fig. 2: Sustained and transient load models (statistical values μ and σ in KN/m^2).

4. COMPARISON OF LOAD COMBINATION FORMATS

General aspects

The ψ-factors proposed in different codes or other technical committees for the ultimate limit state are given in Table 2.

Table 2: ψ-factors in technical standards

load	CEB [1]	JCSS [11]	Eurocode [4]	Revised Eurocode [12]	NaBau [13]
live	0.3/0.6*)	0.50	0.70	0.50	0.60
snow	0.50	0.55	0.70	0.50	0.70
wind	0.50	0.55	0.70	0.50	0.70

*): The factor 0.3 corresponds to live loads in dwellings and the factor 0.6 to live loads in offices and retail stores.

The following general remarks can be made:

a) None of the proposed formats treat sustained live loads SL and transient live loads TL separately.

b) Snow load in continental regions with long periods of snow cover should be seperated from those in temperate areas with little snow accumulation.

c) The load combination factors ψ depend only on the load itself and not on the types of loads to be combined with in each specific combination. This is a simplification which can lead to unconservative results [14].

d) Earthquake loads are not included in the present load combination format.

The first two remarks are also important for the analysis of creep phenomena and serviceability limit states. In order to fulfil the four requirements for the load combination rule the above considerations should be included.

Comparison with 'exact' results

A comprehensive study aimed at determining the load combination factors ψ_{ij} has been presented by *Turkstra and Madsen* in [15]. The analysis is restricted to cases where loads do not act in opposite senses leading to stress reversal. The load combination factors are aimed at being the same for all materials. The criteria of probability of exceedance at the levels of 0.01, 0.001 and 0.0001 in one year are used for individual loads as well as combined loads. A linear combination is used with the complete range of influence factors being covered. The major conclusions of the study are

- The uncertainty in load models is of major importance in the study of individual loads. However, results for the combination of loads are relatively insensitive to the load models used.
- Design combination rules depend on the probability level at which comparisons are made. In general, the less likely the exceedance of the design values of individual loads, the less important the combination problem.
- Simple addition of design loads can lead to very conservative results. Ignoring load superposition can lead to extremely nonconservative results.
- No combinations of transient loads with very short duration need to be made at the fractile levels used in conventional structural design.
- The load reduction factor approach leads to significant errors in a number of cases.
- The companion factor approach coupled with a simple model for the ψ- factors (see Table 3) leads to design values almost always within 10% and normally within 5% of calculated values based on random process analysis.

Based on the results presented in [15] and [16] the following load combination factors given in Table 3 have been proposed for the ultimate limit state.

Table 3: Load combination factors						
Companion Actions	SL	TL	CS	TS	W	E
Sustained live SL	-	0.5	0.5	0.5	0.5	0.5
Transient live TL	0.6	-	0.3	0.0	0.0	0.0
Continental Snow CS	0.6	0.3	-	-	0.3	0.3
Temperate Snow TS	0.6	0.0	-	-	0.2	0.2
Wind W	0.7	0.0	0.3	0.0	-	0.0
Earthquake E	0.2	0.0	0.0	0.0	0.0	-

The values of Table 3 correspond to a probability level $p=\Phi(-\alpha_L\beta)=0.001$ which for a load sensitivity factor $\alpha_L=0.75$ is approximatelly equal to an annual reliability index of $\beta=4.2$. For example, for a factored sustained design live load of 120 in combination with a factored design wind load of 150, total loads to be considered would be $120+0.7\times150=225$ and $0.5\times120+150=210$. The largest value of 225 would be used. If three loads are combined, four combinations must be considered.

The alternative design formats of Table 1 have been evaluated. To compare design formats, the relative influence coefficients in eq. (4) were assigned values of 0, 0.2, 0.5, and 1.0. The exact result is obtained by a random process calculatio, [14] and an average relative error for each specific load combination has then been calculated (equal weighting of each specific design situation described by the varying influence coefficients). The average relative errors in percent in the design approaches (codified load combination factors of Table 2) are shown in Table 4. The 'exact' results are based on the load models described in [15] and in [16] and correspond again to a probability level $p=0.001$. In case that snow load is combined two errors are given; the first for continental snow and the second for temperate.

Table 4: Errors (in %) in recent (codified) design approaches					
load combination	CEB	JCSS	Eurocode	revised Eurocode	NaBau
Comb. live + snow	+9/+15	+7/+12	+13/+19	+5/+11	+12/+18
Comb. live + wind	+14	+11	+18	+10	+17
Snow + wind	+2/+8	+4/+11	+9/+16	+2/+8	+9/+16

Conclusions which may be drawn from the study are:

- All the investigated design formats overestimate the total combined design load.
- The ψ-factors proposed in Eurocode and NaBau are very conservative.

A simplified load combination proposal

Although a matrix in the form of Table 3 is readily used in computerized analysis, simplified results are required for conventional design. To reduce the dimensions of the problem, one can restrict attention to cases involving equal values of factored design loads. Shown in Table 5 are approximate factors for load combinations which consider all types of loads in a simplified form. Two possible load conditions, the ordinary and the extreme, must be checked in each design situation.

Table 5: Proposed load combination factors					
load condition	SL	TL	CS/TS	W	E
ordinary	1.0	0.6	0.6	0.6	0.2
extraordinary	0.6	1.0/0.0	0.3/0.2	1.0/0.0	1.0/0.0

In the ordinary condition the sustained live load is governing the design while in the extraordinary condition one of the transient loads TL (transient live load), W (wind load) or E (earthquake load) is the most significant one (having a factor of 1.0 while the other two transient loads have factors of 0.0). The load combination factor for the snow load S in the extraordinary condition is 0.3 for continental snow and 0.2 for temporary. For the example considered previously (combination of sustained live load and wind load) the total loads to be considered would be $120+0.6\times150=210$ and $0.6\times120+150=222$. The value of 222 would be used. Table 6 illustrates the average relative error in the simplified proposal, compared to the use of values from Table 3 for the combinations of three loads considered in [15.16].

Table 6: Average errors (in %) for the proposed simpl. format	
load combination	average error
SL + TL + CS	-2
SL + TL + W	+2
SL + CS + E	0

5. EXTENSION OF THE PRESENT FORMAT TO OTHER DESIGN SITUATIONS

The load combination formats discussed above are valid only for the *linear* combination of *independent* loads. In fact the pre-suppositions, that actions must be uncorrelated in time and that only linear combinations are dealt with, impose a limitation to the applicability of the proposed format. Therefore there is a need for extension of this formulation to other design situations.

a) Combination of dynamic effects

In structural dynamics the following simple rules for load combination (combination of the design values) are commonly used:

- SRSS-rule: The square-root-of-sum-of-squares law is generally used for the combination of independent dynamic effects.
- CQC-rule: This law is an improvement of the SRSS-rule because it takes also the correlation between the combined loads into consideration and is therefore suggested for the combination of dynamic effects [17].

Example 2: Combined earthquake acceleration

For the combination of the horizontal a_H and vertical acceleration a_V due to earthquake motion the following formula can be used:

$$a_{\max} = \sqrt{a_H^2 + a_V^2 + 2\rho a_H a_V} \tag{21}$$

where ρ is the correlation coefficient between the horizontal and the vertical acceleration; ρ must be estimated in each specific design case and can be neglected only if it is very small.

In special cases stochastic dependencies in load combinations can be considered by applying the so-called load coincidence method developed by *Wen* [18], in which occurrence time, intensity and duration are allowed to be correlated (within each process and between processes). This procedure is most suitable for sparse load pulses.

b) General non-linear combination

Non-linear combinations are not uncommon in design. The combination of bending moment and normal force in the stability limit state offers a good example. A general solution to this problem can be reached by applying the modified Rice's formula given through eq. (8). Applications can be found in [3]. Another possibility is to linearize the nonlinear safe domain in order to obtain an approximation to the upper bound [19].

It should be further noted, that in almost all cases load combination factors are calibrated from a linear combination rule. The load combination factors are, however, also often used for nonlinear combinations which can lead to incorrect results. A more realistic formulation is therefore desired for nonlinear combinations.

6. SUMMARY AND CONCLUSIONS

The combination of load processes in codified design is addressed in this contribution. The following conclusions can be drawn:

- Exact solutions to the load combination problem are in general difficult to obtain but an upper bound solution in terms of the mean number of crossings is generally a good approximation.
- Present load combination rules are based on the so-called Turkstra's rule and load combination factors are calibrated considering linear combinations of independent actions.
- The load combination factors proposed in several code formats are discussed. A more refined classification of actions and ψ-factors dependent on all the combined actions are suggested. Two alternative load combination factor matrices for the ultimate limit state are proposed.
- Areas for extending the present format are identified and discussed.

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