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II

Influence of Creep on Column Instability

Influence du fluage sur l'instabilité des colonnes

Der Kriecheinflüß auf das Knicken von Stützen

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Whenever a structural element is required to resist compressive stresses, the possibility of failure due to instability has to be taken into account. Moreover, if the element is made of a visco-elastic material, the analysis should include the consideration of delayed instability phenomena (creep buckling).

Let's have a slightly bent concrete column under a constant compressive load. In the presence of creep, lateral deflexions increase with time, the action being triggered by the bending moments resulting from the simultaneous existence of initial deformation and end load. After a certain time, collapse may occur or, alternatively, the column may reach a new equilibrium position.

An usual definition states that a system in equilibrium is stable when, subjected to a small disturbance, it returns to its original position after the disturbance is removed. This definition is not of much use in the situations discussed in this work. A non-conservative system, when disturbed, may never return to its original position, but if a small disturbance causses only small displacement, it is for practical purposes as safe as a stable conserative system.

A stability concept introduced by Distéfano [1] seems more appropriate as a practical criterion: a column is considered stable if, for times $t \to \infty$ its deflexions approach a finite limit, that is

$$\lim_{t \to \infty} y(x,t) = y(x,\infty) < \infty$$

Naturally, the deflexions at $t=\infty$, although finite, could result larger than the material's allowable strain and stresses. In these circunstances, other considerations have to be used in addition to the stability criterion.

The simplest rigorous analysis of creep instability in concrete columns has been pionnered by the works of J. N. Distéfano. In the present discussion, the main results of this approach are outlined, and compared with Mr. Faessel's criteria and with other developments.

Linear case: the most general representation of linear viscoelastic behavior may be obtained with a stress-strain relation of the form

$$\varepsilon(t) = \frac{\sigma(t)}{E} + \int_0^t \sigma(\tau) f(t,\tau)\, d\tau \tag{1}$$

in which

$$f(t,\tau) = -\frac{\partial \varepsilon_o(t,\tau)}{\partial \tau} \tag{2}$$

and $\bar{\varepsilon}_o(t,\tau)$ is the specific creep strain function.

There are several different expressions currently used to represent the ageing of concrete. According to Aroutiounian [2] the specific deformation may be written:

$$\bar{\varepsilon}_o(t,\tau) = \left(\gamma_o + \frac{c}{\tau}\right)\left(1 - e^{-\delta(t-\tau)}\right) \tag{3}$$

When the stress-strain relation (1) is introduced into the classical formulation for the equilibrium of a slightly bent bar, the following equation is obtained [1] :

$$\frac{\partial^2 y(x,t)}{\partial x^2} + \frac{M(x,t)}{EI} + \frac{1}{I}\int_0^t M(\tau) f(t,\tau)\, d\tau = 0 \tag{4}$$

In equation (4), $y(x,t)$ is the lateral deflexion, I the moment of inertia of the cross section and $M(t)$ the bending moment due to lateral loads and/or initial excentricity.

In adition to the terms present in the classical buckling analysis, equation (4) has the additional term representing the creep effects. Replacing equation (1) into the equation (4) the following integro-differential equation is obtained:

$$\frac{\partial^2 y(x,t)}{\partial x^2} + \frac{P}{EI} y(x,t) + \frac{P}{I}\int_0^t y(x,\tau) f(t,\tau)\, d\tau = \tag{5}$$

$$= -\frac{\varkappa(x,t)}{EI} - \frac{1}{I}\int_0^t \varkappa(x,t) f(t,\tau)\, d\tau$$

Equation (5) may be solved by expanding the functions $y(x,t)$ and $\varkappa(x,t)$ is series of orthogonal functions. Thus, the behavior of deflexions for times $t\to\infty$ may be analyzed. This analysis shows that deflexions remain bounded provided the axial load P satisfies the inequality

$$P < \frac{P_E}{1 + E\gamma_o} = P^* \tag{6}$$

in which P_E is Euler's critical load and γ_o represents the asymptotic value of specific creep for the aged concrete. Similar results have been obtained by Ostlund [3] assuming expressions for the ageing for concrete other than (3).

It should be remarked at this point that, when Dischinger's formulation for ageing creep is considered, the asymptotic value of specific creep vanishes for aged concrete. Accordingly a limit loas is obtained which coincides with Euler's load [4] . Of course this result greatly overestimates the column's actual limit load.

Equation (6) applies to non-reinforced concrete elements. For reinforced members, this expression must be modified. In the particular case of a column with symmetrical cross section, the condition for asymptotic stability is written [5]

$$P < \frac{P_E}{1 + E_b\gamma_o}(1 + E_b\gamma_o n) \quad ; \; n = \frac{E_e I_e}{E_b I_b} \tag{7}$$

in which I_b, E_b and I_e, E_e are respectively the moments of inertia and modulus of elasticity of concrete and steel.

References [1] and [5] give the exact expressions for the deflexions. A close approximation for the final values is obtained if a reduced modulus

$$\frac{E(\infty)}{1 + E(\infty)\gamma_o} \tag{8}$$

is used, together with the equations for instantaneous deflexions.

<u>Non-linear case</u>: when concrete stresses exceed 30-40 % of its compressive strength, non-linear behavior becomes important. In order to take into account non-linear creep effects, a procedure similar to that one used for the linear theory may be followed, provided some simplifying assumptions are made [6]

First, stress-strain relations are expressed in the form [2]

$$\varepsilon(t) = \frac{\sigma(t)}{E} + \int_0^t F(\sigma)\, f(t,\tau)\, d\tau \tag{9}$$

with

$$F(\sigma) = \sigma + \beta\sigma^2 \tag{10}$$

Second, the actual column cross section is replaced by an ideal I cross section.

The analysis indicates that asymptotic deflexions and buckling load are governed by a reduced modulus

$$\frac{E(\infty)}{1+E(\infty)\gamma_o(1+2\beta\sigma_o)} \tag{11}$$

in which

$$\sigma_o = P/A \tag{12}$$

is the average stress on the cross section.

For the non-linear case, Mr. Faessel [14] proposes the use of a reduced modulus that in our notation may be written

$$\frac{E_t}{1+E_t\gamma_o} \tag{13}$$

E_t being the concrete tangent modulus which corresponds to the buckling stresses. This modulus applies to the case in which the same function represents the non-linear effect in both instantaneous and creep deformations, namely, when we may write

$$\varepsilon(t) = \frac{F(\sigma)}{E} + \int_0^t F(\sigma)\, f(t,\tau)\, d\tau \tag{14}$$

The above considerations show the great generality of stability criteria based on the concept of reduced modulus. As this concept has been greatly abused, it ought to be remarked that the mentioned results are exact under the assumed hypothesis and asymptotic behavior (i. E. for $t \rightarrow \infty$).

On the other hand, these conditions were found to be independent of excentricity or the existence of lateral loads. For practical cases, this appears to be not true, except for small excentricities and/or large slenderness ratios. The main reason for this discrepancy is the indefinitely elastic behavior assumed for the instantaneous strains. S nce elastic behavior is limited, it is; possible that for a given exentricity and a load $P<P^*$ the asymptotic deflexions predicted by the theory could never be reached because of premature plastic failure.

That is to say, a column whose material is characterized by a non-linear stress-strain curve for instantaneous loading, has a critical deflexion at which the applied load is the critical load. This critical deflexion can be calculated from the instantaneous material. This must be accompanied by a computation of the critical time necessary to reach the critical deflexion or, alternatively, the load which can be sustained during an infinite time; the latter approach being the more usual in concrete structures analysis [7]

Several works (besides the one under discussion) follow the above mentioned approach. By using digital computers they are able to introduce close representations of actual material / characteristics, producing very reliable results. Mauch and Holley [8]for instance, perform an analysis similar to Naerlovic's [9] using experimental creep data. Warner and Thurlimann[7] consider the diminution in strength of concrete due to sustained load. Manuel and McGregor [10] introduce an analysis suitable for restrained / columns and frames.

A simplified procedure has been introduced in ref. [6]. In order to evaluate the actual limiting loads, the non-linear theory in combination with a suitable P-M interaction curve for the column cross section appears to be a reasonable approach.

Assume a given P-M interaction curve as shown in Fig. 1 and let $y(\infty,P,y_0)$ be the asymptotic deflexion. The resulting bending moment $P[y(\infty,P,y_0)+y_0]$ can be plotted and the load corresponding to the intersection of both curves will give the desired limit load. Deflexions may be calculated by using the reduced modulus (8).

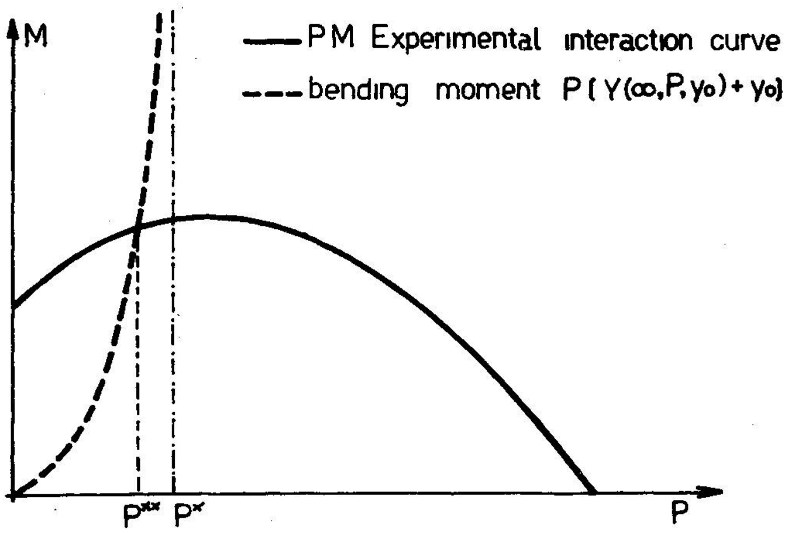

Fig. 1

Experimental results: an experimental program on plain and reinforced concrete columns has been carried out at I.M.A.E. in the period 1963-1965. A description of test procedures may be found in ref. [11]. Results have been published in refs. [12], [13].

The experiments were performed in a controlled environment with temperature 20°C ± 1°C, and humidity 50% ± 1%. Standard deviation in material properties were estimated in 5-6 %.

Columns with several different initial excentricities were tested in compression, under loads amounting to 0.95, 0.90, 0.85...

etc. of the instantaneous buckling value. The applied loads were maintained constant up to the columns failure; time dependent increments of deflexions and strains were registered.

A typical set of results, plotted as load versus critical

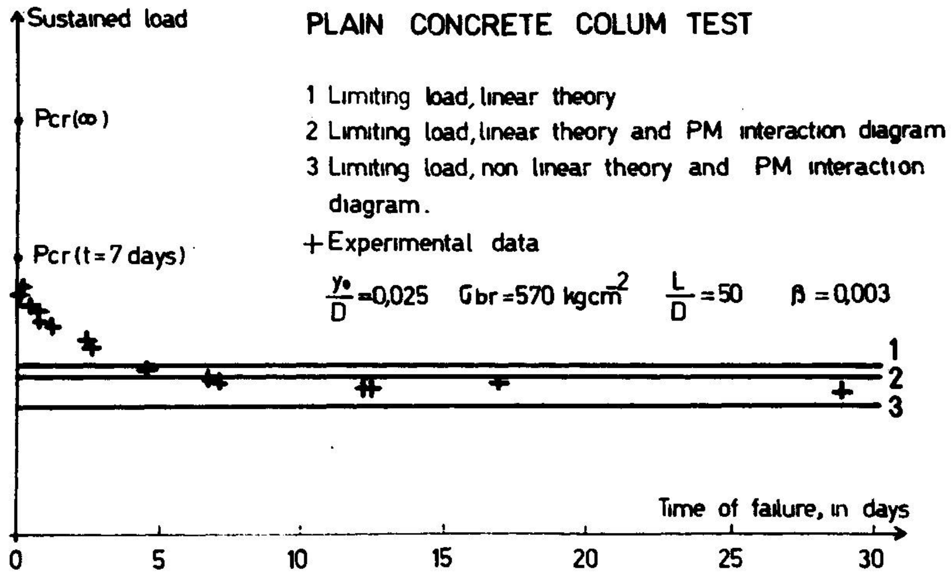

Fig. 2

time may be seen in Fig.2. The results are compared with some of the above mentioned theoretical conclusions. Line l indicates the

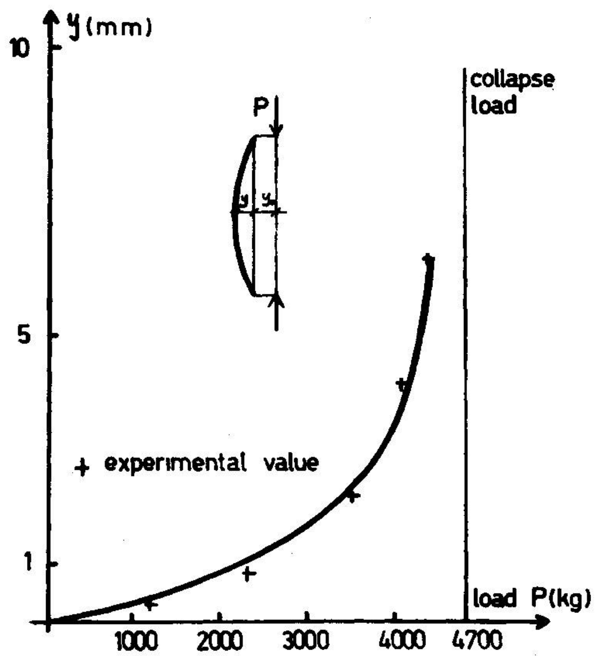

Fig.3

limit load predicted by the linear theory. Line 2 corresponds to the consideration of linear creep and the P-M interaction curve, and line 3 takes account of non-linear creep and the P-M curve.

Deflexions were calculated using the non-linear reduced modulus. In Fig. 3 a comparison between theoretical and experimental results for instantaneous deflexions may be seen.

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SUMMARY

A short review of some simplified methods for the calculation of creep buckling loads is given.

Theoretical predictions are found in good agreement with experimental results.

RESUME

On donne un bref rappel des quelques méthodes simplifiées du calcul de la charge de flambement avec effet du fluage. Les résultats théoriques présentent un accord satisfaisant avec les valeurs expérimentales.

ZUSAMMENFASSUNG

Es wird ein kurzer Rückblick auf einige vereinfachte Verfahren zur Berechnung des Knickens beim Kriechen gegeben.

Theoretische Knicklasten stimmen gut mit Versuchsergebnissen überein.