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Non-Linear Creep in Concrete Columns

Fluage non-linéaire des colonnes en béton armé

Nichtlineares Kriechen in Betonstützen

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The constitutive relation for concrete proposed by Gamble [4] takes into account non-linear effects at high stress, as well as effects of limited creep recovery and varying ambient conditions. Gamble's work is an interesting generalization of the superposition equation,

$$\epsilon_c(t) = \int_{t_o}^{t} c(t, \tau)\, d\sigma(\tau) \qquad (1)$$

Although the integral formulation has proved to be useful in theoretical studies of viscoelastic behaviour, a differential formulation is often found to be convenient when practical calculations have to be made for the analysis of structural behaviour [5] .

Details of a differential equation of state which also takes into account effects of non-linearity, limited creep recovery and time-varying ambient conditions are herein presented. The equation is, essentially, an extension of the well known Dischinger creep equation. It is at present being used by the writer in a study of creep effects in reinforced concrete columns.

A concrete fibre subjected to a stress history σ (t) is considered, and it is assumed that concrete creep is made up of three components,

$$\epsilon_c = \epsilon_d + \epsilon_v + \epsilon_n \qquad (2)$$

The first component, ϵ_d , is time-hardening and non-recoverable and is similar in all essentials to Dischinger creep. The strain rate $\dot{\epsilon}_d$ may thus be expressed as

$$\dot{\epsilon}_d(t) = \epsilon_{el}(t)\, \dot{\varphi}_d(t) = \epsilon_{el}(t)\, [\, \overset{*}{\varphi}_d - \varphi_d(t)\,]\, \frac{1}{T_d} \qquad (3)$$

in which $\varphi_d(t)$ is a creep function of the form

$$\varphi_d(t) = \varphi_d^*(1 - e^{-t/T_d}) \tag{4}$$

The second component, ϵ_v, is non-hardening and recoverable. The strain rate $\dot{\epsilon}_v$ can be expressed as

$$\dot{\epsilon}_v(t) = (\epsilon_v^*(t) - \epsilon_v(t))\frac{1}{T_v} \tag{5}$$

in which the end strain $\epsilon_v^*(t)$ is fixed by the instantaneous stress [2], ie

$$\epsilon_v^*(t) = \epsilon_{el}(t)\,\varphi_v^* \tag{6}$$

Depending on the relative magnitudes of $\epsilon_v^*(t)$ and $\epsilon_v(t)$, $\dot{\epsilon}_v(t)$ can be either positive or negative.

It will be noted that the two terms ϵ_d and ϵ_v are linear with respect to the stress, and that the total end creep value φ_n (obtained from a creep test at constant low level stress, $\varphi_n = \epsilon_c(\infty)/\epsilon_{el}$) is separated into two components which correspond to irrecoverable and recoverable creep, respectively;

$$\varphi_d^* = \alpha_d\,\varphi_n, \qquad \alpha_d < 1{\cdot}0 \tag{7}$$

$$\varphi_v^* = (1 - \alpha_d)\,\varphi_n \tag{8}$$

In Eqs. 3 and 5 the time coefficients T_d and T_v have been assumed to be constant. If greater flexibility is desired, these can be allowed to increase gradually with time : $T_d(t)$, $T_v(t)$.

The strain rate of the third creep component, ϵ_n, is assumed to be zero whenever the stress σ is less than a critical value σ_c. For $\sigma > \sigma_c$, a power function of stress is used to evaluate $\dot{\epsilon}_n$. A convenient expression for $\dot{\epsilon}_n$ is

$$\dot{\epsilon}_n(t) = (\dot{\epsilon}_d(t) + \dot{\epsilon}_v(t))\cdot f(\sigma) \tag{9}$$

in which

$$\begin{aligned} \sigma \leq \sigma_c &: \quad f(\sigma) = 0 \\ \sigma_c \leq \sigma \leq \sigma_u &: \quad f(\sigma) = \alpha_n\left[\frac{\sigma - \sigma_c}{\sigma_u - \sigma_c}\right]^n \end{aligned} \tag{10}$$

In the above relations σ_u is the ultimate strength of the concrete and α_n and n are open parameters defining the function $f(\sigma)$. Values of α_n and n must be obtained using available test data.

The total creep strain rate at time t thus becomes

$$\dot{\epsilon}_c = (\dot{\epsilon}_d + \dot{\epsilon}_v)(1 + f(\sigma)) \tag{11}$$

with $\dot{\epsilon}_d$ and $\dot{\epsilon}_v$ and $f(\sigma)$ being given by Eqs. 3, 5, and 10, respectively. An advantage of this incremental formulation is that use has not been made of the superposition principle for stresses in the non-linear range, as is the case in many integral formulations [1]. For practical calculation purposes the above Equations can be written in difference form and used in a finite step-by-step procedure for evaluating structural behaviour.

Although Eq. 11 applies only to the case of time-varying stress under constant ambient conditions, variations in relative humidity, etc., can be taken into account by regarding the "target" values φ_d^* and φ_v^* as time varying functions of the ambient history, rather than constants chosen to suit average conditions. Thus, for a sequence of varying ambient conditions H_0, H_1, ... H_i, .. ,one can write in finite form

$$\varphi_d^*(k) = g_d [\alpha_0^d H_0, \alpha_1^d H_1, .. \alpha_i^d H_i, .. \alpha_i^d H_k] \tag{12}$$

$$\varphi_v^*(k) = g_v [\alpha_0^v H_0, \alpha_1^v H_1, .. \alpha_i^v H_i, .. \alpha_k^v H_k] \tag{13}$$

in which α_i^d and α_i^v are weighting factors representing the relative importance of the i-th stage of the ambient history.

It is interesting to note in passing that the formulation used in Eqs. 12 and 13 can be extended to produce a discrete memory process model in place of the conventional state model, as represented by Eq. 11. Memory functions similar to Eqs. 12 and 13 can be introduced for stress increments, ambient conditions and strains

$$g_\sigma [\alpha_0^\sigma \Delta\sigma_0, \alpha_1^\sigma \Delta\sigma_1, .. \alpha_i^\sigma \Delta\sigma_i, .. \alpha_k^\sigma \Delta\sigma_k] \tag{14}$$

$$g_h [\alpha_0^h H_0, \alpha_1^h H_1, ... \alpha_i^h H_i, .. \alpha_k^h H_k] \tag{15}$$

$$g_\epsilon [\alpha_0^\epsilon \epsilon_0, \alpha_1^\epsilon \epsilon_1, ... \alpha_i^\epsilon \epsilon_i, ... \alpha_k^\epsilon \epsilon_k] \tag{16}$$

and expressions for either the total strain or the strain increment can be developed in terms of the memory functions,

$$\epsilon(k) = G_1 [g_\sigma, g_h, g_\epsilon] \tag{17}$$

$$\Delta\epsilon(k) = G_2 [g_\sigma, g_h, g_\epsilon] \tag{18}$$

As pointed out by Gamble and others, conditions in the early stages of the process ($i = 0,1,2$) and in the immediate past ($i = k-1, k-2$) tend to be of prime importance. Thus, provided repeated cycling of stress and of ambient conditions does not occur, the weighting factors might well be expected to approach zero for $0 << i << k$. If the ambient conditions are assumed to be constant, a suitable choice of weighting factors, together with a simple summation form for G_1, leads to a difference equation which is equivalent to Eq. 1.

Returning to the differential formulation, we note that the total strain rate in the concrete fibre is obtained by adding to $\dot{\epsilon}_c$ the elastic and shrinkage strain rates,

$$\dot{\epsilon} = \dot{\epsilon}_c + \dot{\epsilon}_{el} + \dot{\epsilon}_s \qquad (19)$$

In order to obtain an equation of state involving the total strain rate, Eq. 19 is first differentiated. After substitution and rearrangement one obtains a non-linear second order equation

$$b_2(\sigma)\ddot{\sigma} + b_1(\sigma, t)\dot{\sigma} + b_0(\sigma, t)\sigma + c(\sigma)\dot{\sigma}\dot{\epsilon}$$
$$= a_2\ddot{\epsilon} + a_1\dot{\epsilon} - \epsilon_s^* s(t) \qquad (20)$$

in which the coefficients b_1 and b_0 are functions of time, b_2 and c are functions of stress, and a_2 and a_1 are constants. The final term on the right hand side accounts for shrinkage : ϵ_s^* is the end shrinkage strain and s is a known function of time.

When $\sigma < \sigma_c$, Eq. 20 simplifies to a linear equation with time-varying coefficients,

$$\ddot{\sigma} + b_1(t)\dot{\sigma} + b_0(t)\sigma = a_2\ddot{\epsilon} + a_1\dot{\epsilon} - \epsilon_s^* s(t) \qquad (21)$$

For comparison purposes it is noted that the equation of the standard Burger's body can be expressed as [5]

$$\ddot{\sigma} + c_1\dot{\sigma} + c_0\sigma = d_2\ddot{\epsilon} + d_1\dot{\epsilon} \qquad (22)$$

in which the coefficients are constants. Eq. 22 has been described as " the simplest differential constitutive relation capable of describing complex material behaviour" [5].

If now only large values of time t are considered, the coefficient b_1 in Eq. 21 approaches a constant non-zero value, while b_0 and s approach zero, so that

$$\ddot{\sigma} + b_1\dot{\sigma} = a_2\ddot{\epsilon} + a_1\dot{\epsilon} \qquad (23)$$

Integration of Eq. 23 yields the standard equation for a Spring-Kelvin Body system in series

$$\dot{\sigma} + b_1\sigma = a_2\dot{\epsilon} + a_1\epsilon$$

If now α_d is set equal to unity in Eqs. 7 and 8, Eq. 21 reduces to the Dischinger equation

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \dot{\varphi}_d \frac{\sigma}{E} + \dot{\epsilon}_s \quad (25)$$

which is, of course, frequently used in many practical calculations. The simplifying assumption implied in Eq. 25 is that the component ϵ_v is similar in form to ϵ_d. An alternative simplification has been suggested by Nielsen [6], which has much to recommend it. Nielsen proposes that ϵ_v be regarded, for approximate calculations, as an immediate elastic strain. The strain ϵ_v is thus grouped with ϵ_{el} to give

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E'} + \dot{\varphi}_d \frac{\sigma}{E'} + \dot{\epsilon}_s \quad (26)$$

in which the effective modulus is $E' = E / (1 + \varphi_v^*)$

It is of interest, finally, to inspect the non-linear character of the creep strain rate in Eq. 11. Typical values which have been used for the coefficients in Eq. 10 are :

$$\alpha_n = 10 , \quad n = 3 , \quad \sigma_c = 0{\cdot}4\, \sigma_u .$$

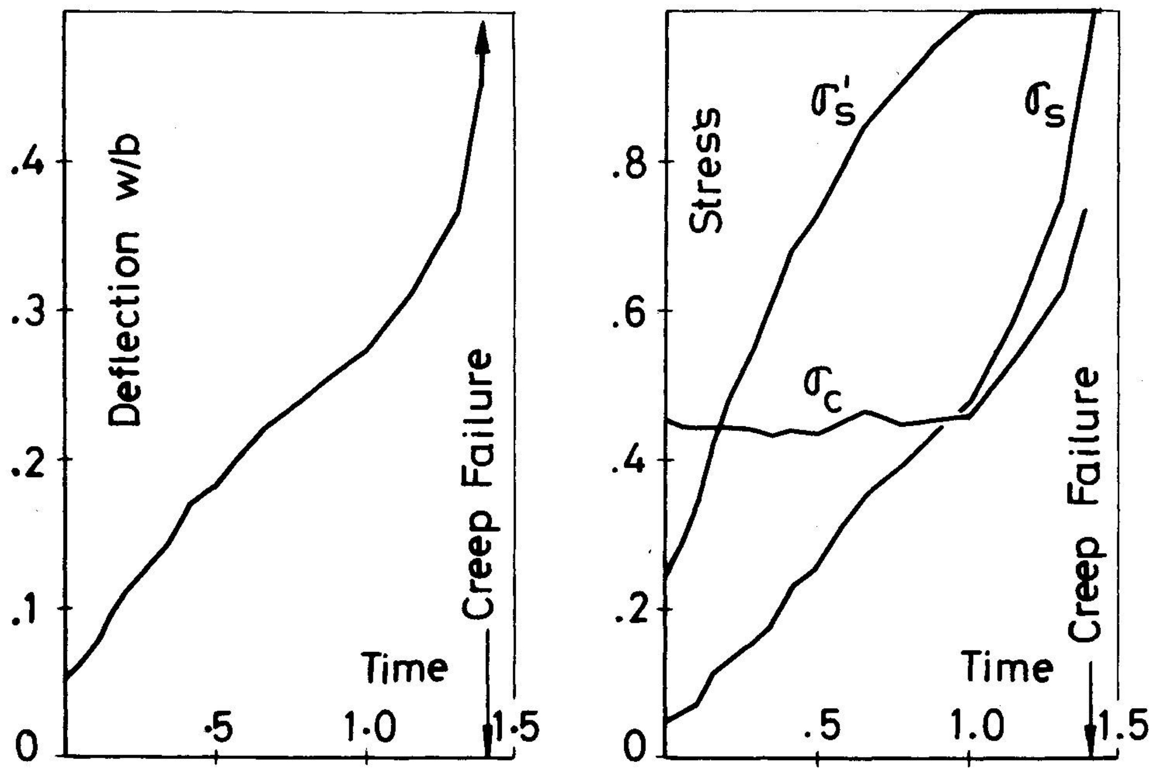
At a constant stress level $S = \sigma / \sigma_u = 0{\cdot}7$, these values produce a non-linearity factor of

$$N = 1 + f(\sigma) = 2{\cdot}25$$

which is comparable to the value of $2{\cdot}0$ quoted by Gamble. (A value of $N = 1{\cdot}0$ represents linearity.) At a stress level of $S = 0{\cdot}6$, the non-linearity factor becomes $1{\cdot}37$, which is in fair agreement with data presented by Freudenthal and Roll [3].

At high stress, eg $S = 0{\cdot}85$, Eqs. 9 and 10 predict bounded creep behaviour. This may well be in error. However, until more detailed test data become available for this final creep phase just prior to failure, a more accurate treatment is hardly warranted.

In this respect, it should be noted that a precise treatment of concrete at very high stress will not always necessarily be a prerequisite for an accurate study of creep failure in structural concrete. In preliminary calculations made by the writer for several columns failing under sustained loading, concrete compressive stresses were found to remain surprisingly small, up until a short time before failure. This was caused by the "brakeing" action of the compressive steel reinforcement. The redistribution of compressive force from the concrete to the steel was found, in these calculations, to be more than sufficient to compensate for the natural increase in stress corresponding to increased moments caused by increased deflections. Thus, only when the compressive steel had yielded was there a relatively sudden and significant increase in concrete stress. The nett result was that the concrete was subjected to high stress only in the last, late phase of the loading history. Rather large errors in predicted creep rates in this final, short phase do not affect significantly the predicted life of the element.



Owing to space limitations, details of the column calculations cannot be given here. However, typical calculated variations in mid-length deflection w, in maximum concrete compressive stress σ_c, and in the tensile steel stress σ_s and compressive steel stress σ'_s are shown in the above Figures for a pin-ended column failing under an eccentric sustained loading. Deflection is plotted as a proportion of the section width b, while stresses are plotted non-dimensionally as proportions of the ultimate or yield stress. The time unit is T_d.

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SUMMARY

A non-linear equation of state for the study of creep failure of reinforced concrete columns is described. Typical variations in concrete and steel stresses with time, up to the instant of failure, are shown for a reinforced concrete column under sustained loading.

RESUME

On décrit une relation non-linéaire pour l'analyse théorique de l'influence du fluage sur la résistance des colonnes en béton armé et on discute des variations caractéristiques des tensions de l'acier et du béton en fonction du temps pour la cas des poteaux soumis à une charge excentrique et permanente.

On donne les variations caractéristiques jusqu'à la ruine des tensions de l'acier et du béton en fonction du temps, pour les colonnes soumises à une charge permanente excentrique.

ZUSAMMENFASSUNG

Ein nicht-lineares Kriechgesetz für die Untersuchung des Versagens von Stahlbetonstützen infolge Kriechens wird beschrieben. Der typische zeitliche Verlauf der Stahl- und Betonspannungen bis zum Zeitpunkt des Bruches wird für eine unter konstanter Dauerlast beanspruchte Stahlbetonstütze dargestellt.