Creep in reinforced concrete slabs subjected to repeated loads

Creep in Reinforced Concrete Slabs Subjected to Repeated Loads

Le fluage des dalles en béton armé soumises à des charges répétées

Kriechen in Stahlbetonplatten infolge Wechsellast

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1. Introduction

It is well-known that the creep of concrete causes the gradual change with time of deformations and stresses in concrete structures subjected to sustained loads. Both experimental and theoretical studies with respect to this phenomenon have been carried out in the past by many investigators, but most of them have dealt with the cases under the action of sustained loads of constant magnitude. Since main purpose of these works is satisfactory prediction of maximum deflection under constant sustained loads, it is difficult to obtain analytically deflections and stresses at an arbitrary time after loading. In addition, most of these analytical methods are incapable of yielding reliable results for the creep of concrete structures subjected to repeated or varying loads. Although the creep-behavior of concrete under variable stress and repeated loads has been studied by A.D.Ross[1], C.A.Miller and S.A.Guralnick[2], the subjects of their studies are restricted to such uni-axial members as plain concrete specimens and singly reinforced concrete beams, and numerical results for response of stresses are not obtained yet.

The authors published the paper[3] in 1969 with respect to the analysis of creep in flexed reinforced concrete slabs subjected to constant sustained loads. In the present paper, the previous theory is extended to the case subjected to any load, the intensity of which varies with an arbitrary time-interval, and the creep responses of deflections and stresses in reinforced concrete slabs obtained from numerical calculations are illustrated.

In order to simplify the procedure, the following assumptions are made in the subsequent development of our theory.

1) Plane sections normal to the neutral surface of the slab before bending remain plane and normal to the neutral surface after bending even though creep occurs in concrete.
2) The reinforcement behaves elastically under all conditions.
3) Modulus of elasticity in concrete is invariable with time.
4) The creep function of Arutyunyan[4]-type is used as the time-dependent law connecting stresses and strains in plain concrete.
5) The effect of shrinkage in concrete is neglected.

2. Stress-Strain Relation of Concrete Considering the Effect of Creep under Repeated Loads

Let us consider that a structure made of concrete of age τ_0 is subjected to load q_i $(i=0,1,2,\text{-------},n)$ which varies with time-interval as shown in Fig.1. Denoting the normal stresses of concrete at an instant t $(\tau_0 \leq t \leq \tau_1)$ with respect to any cartesian coordinate system o-xy by $\sigma_{x0}(t)$ and $\sigma_{y0}(t)$, the total normal strain $\varepsilon_{x0}(t)$ of concrete at an instant t $(\tau_0 \leq t \leq \tau_1)$ is expressed as follows:

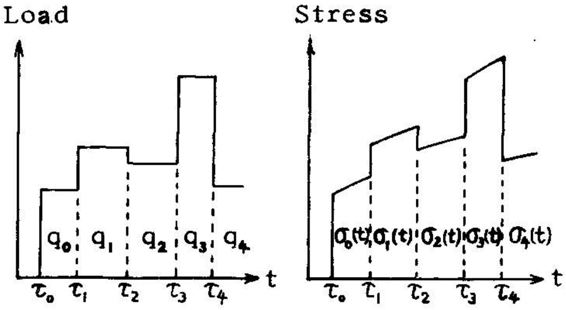

Fig. 1

$$\varepsilon_{x0}(t) = \frac{\sigma_{x0}(t)-\nu\sigma_{y0}(t)}{E_c} - \int_{\tau_0}^{t}\left[\sigma_{x0}(\tau)-\nu\sigma_{y0}(\tau)\right]\frac{\partial}{\partial\tau}C(t,\tau)\,d\tau \tag{1}$$

where E_c and ν are modulus of elasticity and Poisson's ratio of concrete, respectively, and $C(t,\tau)$ is creep function.

The form of function $C(t,\tau)$ is determined on the basis of creep tests in plain concrete, and an expression of Arutyunyan-type is used in this paper.

That is,

$$\left.\begin{aligned} C(t,\tau) &= \varphi(\tau)\left[1-e^{-\gamma(t-\tau)}\right] \\ \varphi(\tau) &= \alpha/\tau + \beta \end{aligned}\right\} \tag{2}$$

where α , β and γ are constants.

To obtain the total strain $\varepsilon_{xn}(t)$ of concrete at any time t $(\tau_n \leq t \leq \tau_{n+1})$, Eq.(1) is easily extended as follows:

$$\varepsilon_{xn}(t) = \frac{\sigma_{xn}(t)-\nu\sigma_{yn}(t)}{E_c} - \sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\left[\sigma_{xi}(\tau)-\nu\sigma_{yi}(\tau)\right]\frac{\partial}{\partial\tau}C(t,\tau)\,d\tau - \int_{\tau_n}^{t}\left[\sigma_{xn}(\tau)-\nu\sigma_{yn}(\tau)\right]\frac{\partial}{\partial\tau}C(t,\tau)\,d\tau\,. \tag{3}$$

Substituting Eq.(2) into Eq.(3) and differentiating the obtained equation with respect to t, we get, after some transformations, the following equation:

$$\dot{\varepsilon}_{xn}(t) = \frac{\dot{\sigma}_{xn}(t)-\nu\dot{\sigma}_{yn}(t)}{E_c} + \gamma\varphi(t)\left[\sigma_{xn}(t)-\nu\sigma_{yn}(t)\right] - \gamma\int_{\tau_n}^{t}\left[\sigma_{xn}(\tau)-\nu\sigma_{yn}(\tau)\right]\frac{\partial}{\partial\tau}\left\{\varphi(\tau)e^{-\gamma(t-\tau)}\right\}d\tau - \gamma\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\left[\sigma_{xi}(\tau)-\nu\sigma_{yi}(\tau)\right]\frac{\partial}{\partial\tau}\left\{\varphi(\tau)e^{-\gamma(t-\tau)}\right\}d\tau\,. \tag{4}$$

Eliminating the integral terms in Eq.(4) by using Eq.(3), we obtain the following equation:

$$\dot{\varepsilon}_{xn}(t)+\gamma\varepsilon_{xn}(t) = \frac{\dot{\sigma}_{xn}(t)-\nu\dot{\sigma}_{yn}(t)}{E_c} + \frac{\gamma}{E_c}\left\{1+E_c\varphi(t)\right\}\left[\sigma_{xn}(t)-\nu\sigma_{yn}(t)\right] - \gamma\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\left[\sigma_{xi}(\tau)-\nu\sigma_{yi}(\tau)\right]\dot{\varphi}(\tau)\,d\tau - \gamma\int_{\tau_n}^{t}\left[\sigma_{xn}(\tau)-\nu\sigma_{yn}(\tau)\right]\dot{\varphi}(\tau)\,d\tau\,. \tag{5}$$

Differentiating Eq.(5) with respect to t, we can derive the following differential equation:

$$\ddot{\sigma}_{xn}(t)-\nu\ddot{\sigma}_{yn}(t)+\gamma\left\{1+E_c\varphi(t)\right\}\left[\dot{\sigma}_{xn}(t)-\nu\dot{\sigma}_{yn}(t)\right] = E_c\left\{\ddot{\varepsilon}_{xn}(t)+\gamma\dot{\varepsilon}_{xn}(t)\right\}\,. \tag{6}$$

For determination of $\sigma_{xn}(t)$ and $\sigma_{yn}(t)$, another differential equation is necessary and it is easily derived as follows:

$$\ddot{\sigma}_{yn}(t)-\nu\ddot{\sigma}_{xn}(t)+\gamma\left\{1+E_c\varphi(t)\right\}\left[\dot{\sigma}_{yn}(t)-\nu\dot{\sigma}_{xn}(t)\right] = E_c\left\{\ddot{\varepsilon}_{yn}(t)+\gamma\dot{\varepsilon}_{yn}(t)\right\}\,. \tag{7}$$

On the other hand, by considering the total shearing strain $\gamma_{xyn}(t)$ of concrete subjected to shearing stress $\tau_{xyn}(t)$, the differential equation for determination of $\tau_{xyn}(t)$ is obtained as

$$\ddot{\tau}_{xyn}(t) + \gamma\left\{1+E_c\varphi(t)\right\}\dot{\tau}_{xyn}(t) = \frac{E_c}{2(1+\nu)}\left\{\ddot{\gamma}_{xyn}(t)+\gamma\dot{\gamma}_{xyn}(t)\right\}\,. \tag{8}$$

The initial conditions are

$$\left.\begin{aligned}
\sigma_{xn}(\tau_n) &= \frac{E_c}{1-\nu^2}\{\varepsilon_{xn}(\tau_n)+\nu\varepsilon_{yn}(\tau_n)\} + E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\sigma_{xi}(\tau)\frac{\partial}{\partial\tau}C(\tau_n,\tau)\,d\tau\,,\\
\sigma_{yn}(\tau_n) &= \frac{E_c}{1-\nu^2}\{\varepsilon_{yn}(\tau_n)+\nu\varepsilon_{xn}(\tau_n)\} + E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\sigma_{yi}(\tau)\frac{\partial}{\partial\tau}C(\tau_n,\tau)\,d\tau\,,\\
\tau_{xyn}(\tau_n) &= \frac{E_c}{2(1+\nu)}\gamma_{xyn}(\tau_n) + E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\tau_{xyi}(\tau)\frac{\partial}{\partial\tau}C(\tau_n,\tau)\,d\tau\,,\\
\dot\sigma_{xn}(\tau_n) &= \frac{E_c}{1-\nu^2}\{\dot\varepsilon_{xn}(\tau_n)+\nu\dot\varepsilon_{yn}(\tau_n)\} + \gamma E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\sigma_{xi}(\tau)\frac{\partial}{\partial\tau}\{\varphi(\tau)e^{-\gamma(\tau_n-\tau)}\}d\tau - \gamma E_c\varphi(\tau_n)\sigma_{xn}(\tau_n)\,,\\
\dot\sigma_{yn}(\tau_n) &= \frac{E_c}{1-\nu^2}\{\dot\varepsilon_{yn}(\tau_n)+\nu\dot\varepsilon_{xn}(\tau_n)\} + \gamma E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\sigma_{yi}(\tau)\frac{\partial}{\partial\tau}\{\varphi(\tau)e^{-\gamma(\tau_n-\tau)}\}d\tau - \gamma E_c\varphi(\tau_n)\sigma_{yn}(\tau_n)\,,\\
\dot\tau_{xyn}(\tau_n) &= \frac{E_c}{2(1+\nu)}\dot\gamma_{xyn}(\tau_n) + \gamma E_c\sum_{i=0}^{n-1}\int_{\tau_i}^{\tau_{i+1}}\tau_{xyi}(\tau)\frac{\partial}{\partial\tau}\{\varphi(\tau)e^{-\gamma(\tau_n-\tau)}\}d\tau - \gamma E_c\varphi(\tau_n)\tau_{xyn}(\tau_n)\,.
\end{aligned}\right\}\quad(9)$$

Solving the differential equations (6),(7) and (8) under the initial conditions of Eq.(9), we find the required stress-strain relations of concrete considering the effect of creep in the following form:

$$\left.\begin{aligned}
\sigma_{xn}(t) &= \sigma_{xn}(\tau_n) + \int_{\tau_n}^{t} e^{-\eta(\tau)}\Big[\dot\sigma_{xn}(\tau_n) + \frac{E_c}{1-\nu^2}\int_{\tau_n}^{\tau}\{\ddot\varepsilon_{xn}(\tau)+\gamma\dot\varepsilon_{xn}(\tau)+\nu(\ddot\varepsilon_{yn}(\tau)+\gamma\dot\varepsilon_{yn}(\tau))\}e^{\eta(\tau)}d\tau\Big]d\tau,\\
\sigma_{yn}(t) &= \sigma_{yn}(\tau_n) + \int_{\tau_n}^{t} e^{-\eta(\tau)}\Big[\dot\sigma_{yn}(\tau_n) + \frac{E_c}{1-\nu^2}\int_{\tau_n}^{\tau}\{\ddot\varepsilon_{yn}(\tau)+\gamma\dot\varepsilon_{yn}(\tau)+\nu(\ddot\varepsilon_{xn}(\tau)+\gamma\dot\varepsilon_{xn}(\tau))\}e^{\eta(\tau)}d\tau\Big]d\tau,\\
\tau_{xyn}(t) &= \tau_{xyn}(\tau_n) + \int_{\tau_n}^{t} e^{-\eta(\tau)}\Big[\dot\tau_{xyn}(\tau_n) + \frac{E_c}{2(1+\nu)}\int_{\tau_n}^{\tau}\{\ddot\gamma_{xyn}(\tau)+\gamma\dot\gamma_{xyn}(\tau)\}e^{\eta(\tau)}d\tau\Big]d\tau
\end{aligned}\right\}(10)$$

where $\eta(t) = \gamma\int_{\tau_n}^{t}\{1+E_c\varphi(\tau)\}d\tau$.

3. Basic Equation of Flexed Reinforced Concrete Slab

Take a cartesian coordinate system o-xy in the neutral plane of a rectangular reinforced concrete slab as shown in Fig.2, where z-axis is perpendicular to x-y plane,and assume that reinforcements are set parallel to x and y axes.

As the reinforcement is assumed to behave elastically under all conditions, its normal stresses $\sigma_{sxn}(t)$, $\sigma_{syn}(t)$ and shearing stress $\tau_{sxyn}(t)$ are expressed as

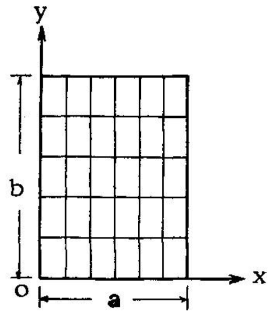

Fig. 2

$$\sigma_{sxn}(t)=E_s\,\varepsilon_{sxn}(t),\quad \sigma_{syn}(t)=E_s\,\varepsilon_{syn}(t),$$

$$\tau_{sxyn}(t)=\frac{E_s}{2(1+\nu_s)}\gamma_{sxyn}(t)\qquad(11)$$

where $\varepsilon_{sxn}(t)$, $\varepsilon_{syn}(t)$ and $\gamma_{sxyn}(t)$ are normal strains and shearing strain of reinforcement, and E_s, ν_s are modulus of elasticity and Poisson's ratio of reinforcement, respectively.

Denoting the bending moments and the twisting moments of a slab per unit width by $M_{xn}(t)$, $M_{yn}(t)$ and $M_{xyn}(t)$, $M_{yxn}(t)$, respectively, we can express them by using Eqs.(10),(11) and the deflection w(t) of a slab as follows:

$$\begin{aligned}
M_{xn}(t) &= M_{cxn}(\tau_n)+\dot M_{cxn}(\tau_n)\int_{\tau_n}^{t}e^{-\eta(\tau)}d\tau+M_{sxn}(t)-D_c\int_{\tau_n}^{t}e^{-\eta(\tau)}\Big[\int_{\tau_n}^{\tau}\Big\{\frac{\partial^2\ddot w(\tau)}{\partial x^2}+\nu\frac{\partial^2\ddot w(\tau)}{\partial y^2}+\gamma\Big(\frac{\partial^2\dot w(\tau)}{\partial x^2}+\nu\frac{\partial^2\dot w(\tau)}{\partial y^2}\Big)\Big\}e^{\eta(\tau)}d\tau\Big]d\tau,\\
M_{yn}(t) &= M_{cyn}(\tau_n)+\dot M_{cyn}(\tau_n)\int_{\tau_n}^{t}e^{-\eta(\tau)}d\tau+M_{syn}(t)-D_c\int_{\tau_n}^{t}e^{-\eta(\tau)}\Big[\int_{\tau_n}^{\tau}\Big\{\frac{\partial^2\ddot w(\tau)}{\partial y^2}+\nu\frac{\partial^2\ddot w(\tau)}{\partial x^2}+\gamma\Big(\frac{\partial^2\dot w(\tau)}{\partial y^2}+\nu\frac{\partial^2\dot w(\tau)}{\partial x^2}\Big)\Big\}e^{\eta(\tau)}d\tau\Big]d\tau,\\
M_{xyn}(t) &= M_{cxyn}(\tau_n)+\dot M_{cxyn}(\tau_n)\int_{\tau_n}^{t}e^{-\eta(\tau)}d\tau+M_{sxyn}(t)+(1-\nu)D_c\int_{\tau_n}^{t}e^{-\eta(\tau)}\Big[\int_{\tau_n}^{\tau}\Big\{\frac{\partial^2\ddot w(\tau)}{\partial x\partial y}+\gamma\frac{\partial^2\dot w(\tau)}{\partial x\partial y}\Big\}e^{\eta(\tau)}d\tau\Big]d\tau,\\
M_{yxn}(t) &= M_{cyxn}(\tau_n)+\dot M_{cyxn}(\tau_n)\int_{\tau_n}^{t}e^{-\eta(\tau)}d\tau+M_{syxn}(t)-(1-\nu)D_c\int_{\tau_n}^{t}e^{-\eta(\tau)}\Big[\int_{\tau_n}^{\tau}\Big\{\frac{\partial^2\ddot w(\tau)}{\partial x\partial y}+\gamma\frac{\partial^2\dot w(\tau)}{\partial x\partial y}\Big\}e^{\eta(\tau)}d\tau\Big]d\tau\,.
\end{aligned}\qquad(12)$$

where

$$M_{cxn}(\tau_n)=\int\sigma_{xn}(\tau_n)\,z\,dz,\quad M_{cyn}(\tau_n)=\int\sigma_{yn}(\tau_n)\,z\,dz,\quad -M_{cxyn}(\tau_n)=M_{cyxn}(\tau_n)=\int\tau_{xyn}(\tau_n)\,z\,dz,$$

$$\dot M_{cxn}(\tau_n)=\int\dot\sigma_{xn}(\tau_n)\,z\,dz,\quad \dot M_{cyn}(\tau_n)=\int\dot\sigma_{yn}(\tau_n)\,z\,dz,\quad -\dot M_{cxyn}(\tau_n)=\dot M_{cyxn}(\tau_n)=\int\dot\tau_{xyn}(\tau_n)\,z\,dz,$$

$$M_{sxn}(t)=-D_{sx}\frac{\partial^2 w(t)}{\partial x^2},\quad M_{syn}(t)=-D_{sy}\frac{\partial^2 w(t)}{\partial y^2},\quad M_{sxyn}(t)=\frac{D_{sx}}{1+\nu_s}\frac{\partial^2 w(t)}{\partial x\partial y},\quad M_{syxn}(t)=-\frac{D_{sy}}{1+\nu_s}\frac{\partial^2 w(t)}{\partial x\partial y},$$

D_c : Flexural rigidity of concrete section per unit width with respect to neutral axis,

D_{sx}, D_{sy} : Flexural rigidity of reinforcement per unit width about y and x-axis, respectively.

Denoting the intensity of the load acting on a slab at an instant t ($\tau_n \leq t \leq \tau_{n+1}$) by $q_n(x,y)$, we represent the equation of equilibrium in the following form:

$$\frac{\partial^2 M_{xn}(t)}{\partial x^2}+\frac{\partial^2 M_{yn}(t)}{\partial y^2}-\frac{\partial^2 M_{xyn}(t)}{\partial x\partial y}+\frac{\partial^2 M_{yxn}(t)}{\partial x\partial y}=-q_n(x,y). \tag{13}$$

Substituting Eq.(12) into Eq.(13), we can obtain the basic equation of a flexed reinforced concrete slab subjected to repeated loads as follows:

$$\frac{\partial^2 M_{cxn}(\tau_n)}{\partial x^2}+\frac{\partial^2 M_{cyn}(\tau_n)}{\partial y^2}-2\frac{\partial^2 M_{cxyn}(\tau_n)}{\partial x\partial y}+\left\{\frac{\partial^2 \dot{M}_{cxn}(\tau_n)}{\partial x^2}+\frac{\partial^2 \dot{M}_{cyn}(\tau_n)}{\partial y^2}-2\frac{\partial^2 \dot{M}_{cxyn}(\tau_n)}{\partial x\partial y}\right\}\int_{\tau_n}^{t}e^{-\eta(\tau)}d\tau$$
$$-D_c\int_{\tau_n}^{t}e^{-\eta(\tau)}\left[\int_{\tau_n}^{\tau}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\left\{\ddot{w}(\tau)+\gamma\dot{w}(\tau)\right\}e^{\eta(\tau)}d\tau\right]d\tau=-q_n(x,y). \tag{14}$$

Putting $t=\tau_n$ in Eq.(14), we have the initial condition as

$$\beta_1\frac{\partial^4 w(\tau_n)}{\partial x^4}+\beta_2\frac{\partial^4 w(\tau_n)}{\partial y^4}+2\beta_3\frac{\partial^4 w(\tau_n)}{\partial x^2\partial y^2}=\bar{q}_n(x,y) \tag{15}$$

where $\beta_1=D_c+D_{sx}$, $\beta_2=D_c+D_{sy}$, $2\beta_3=2D_c+\dfrac{D_{sx}+D_{sy}}{1+\nu_s}$,

$$\bar{q}_n(x,y)=q_n(x,y)+E_c\sum_{i=0}^{n-1}\left[\left\{-q_i(x,y)+D_{sx}\frac{\partial^4 w(\tau_i)}{\partial x^4}+D_{sy}\frac{\partial^4 w(\tau_i)}{\partial y^4}+\frac{D_{sx}+D_{sy}}{1+\nu_s}\frac{\partial^4 w(\tau_i)}{\partial x^2\partial y^2}\right\}\left\{C(\tau_n,\tau_{i+1})-C(\tau_n,\tau_i)\right\}\right.$$
$$\left.+\int_{\tau_i}^{\tau_{i+1}}\left\{C(\tau_n,\tau_{i+1})-C(\tau_n,\tau)\right\}e^{-\eta(\tau)}\left\{D_{sx}\frac{\partial^4 \dot{w}(\tau_i)}{\partial x^4}+D_{sy}\frac{\partial^4 \dot{w}(\tau_i)}{\partial y^4}+\frac{D_{sx}+D_{sy}}{1+\nu_s}\frac{\partial^4 \dot{w}(\tau_i)}{\partial x^2\partial y^2}-D_c\int_{\tau_i}^{\tau}\nabla^4(\ddot{w}(\tau)+\gamma\dot{w}(\tau))e^{\eta(\tau)}d\tau\right\}d\tau\right]$$

Differentiating Eq.(14) with respect to t and putting $t=\tau_n$ in the obtained eq-ation, we can get a differential equation, which gives an initial condition of w(t) as well as Eq.(15), as follows:

$$\beta_1\frac{\partial^4 \dot{w}(\tau_n)}{\partial x^4}+\beta_2\frac{\partial^4 \dot{w}(\tau_n)}{\partial y^4}+2\beta_3\frac{\partial^4 \dot{w}(\tau_n)}{\partial x^2\partial y^2}=\gamma E_c\varphi(\tau_n)\left[q_n(x,y)-\left\{D_{sx}\frac{\partial^4 w(\tau_n)}{\partial x^4}+D_{sy}\frac{\partial^4 w(\tau_n)}{\partial y^4}+\frac{D_{sx}+D_{sy}}{1+\nu_s}\frac{\partial^4 w(\tau_n)}{\partial x^2\partial y^2}\right\}\right]+\bar{\bar{q}}_n(x,y), \tag{16}$$

where

$$\bar{\bar{q}}_n(x,y)=\gamma E_c\sum_{i=0}^{n-1}\left[\left\{-q_i(x,y)+D_{sx}\frac{\partial^4 w(\tau_i)}{\partial x^4}+D_{sy}\frac{\partial^4 w(\tau_i)}{\partial y^4}+\frac{D_{sx}+D_{sy}}{1+\nu_s}\frac{\partial^4 w(\tau_i)}{\partial x^2\partial y^2}\right\}\left\{\varphi(\tau_{i+1})e^{-\gamma(\tau_n-\tau_{i+1})}-\varphi(\tau_i)e^{-\gamma(\tau_n-\tau_i)}\right\}\right.$$
$$\left.+\int_{\tau_i}^{\tau_{i+1}}\left\{\varphi(\tau_{i+1})e^{-\gamma(\tau_n-\tau_{i+1})}-\varphi(\tau)e^{-\gamma(\tau_n-\tau)}\right\}e^{-\eta(\tau)}\left\{D_{sx}\frac{\partial^4 \dot{w}(\tau_i)}{\partial x^4}+D_{sy}\frac{\partial^4 \dot{w}(\tau_i)}{\partial y^4}+\frac{D_{sx}+D_{sy}}{1+\nu_s}\frac{\partial^4 \dot{w}(\tau_i)}{\partial x^2\partial y^2}-D_c\int_{\tau_i}^{\tau}\nabla^4(\ddot{w}(\tau)+\gamma\dot{w}(\tau))e^{\eta(\tau)}d\tau\right\}d\tau\right]$$

Differentiating Eq.(14) twice with respect to t, we derive, after some transformations, the following differential equation for determination of w(t):

$$\beta_1\frac{\partial^4 \ddot{w}(t)}{\partial x^4}+\beta_2\frac{\partial^4 \ddot{w}(t)}{\partial y^4}+2\beta_3\frac{\partial^4 \ddot{w}(t)}{\partial x^2\partial y^2}+\left\{\gamma D_c+D_{sx}\dot{\eta}(t)\right\}\frac{\partial^4 \dot{w}(t)}{\partial x^4}+\left\{\gamma D_c+D_{sy}\dot{\eta}(t)\right\}\frac{\partial^4 \dot{w}(t)}{\partial y^4}+\left\{2\gamma D_c+\frac{D_{sx}+D_{sy}}{1+\nu_s}\dot{\eta}(t)\right\}\frac{\partial^4 \dot{w}(t)}{\partial x^2\partial y^2}=0 \tag{17}$$

The solution of the basic equation (14) is equivalent to that of a linear differential equation (17) under the initial conditions given by Eqs.(15) and (16).

4. Solution by Double Trigonometric Series

In this study we deal with only simply supported rectangular slabs, and then analysis by double trigonometric series is quite suitable. By taking the coordinate axes x and y as shown in Fig.2, boundary conditions for simple support are represented in the following forms:

$$w(t)=0,\quad M_{xn}(t)=0\quad \text{for } x=0,\ a\quad \text{and}\quad w(t)=0,\quad M_{yn}(t)=0\quad \text{for } y=0,b \tag{18}$$

where general expressions of $M_{xn}(t)$ and $M_{yn}(t)$ are given by Eq.(12).

Now assume that the solution of differential equation (17) takes the following form of series satisfying boundary conditions of Eq.(18):

$$w(t) = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} A_{jk}(t)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b} \tag{19}$$

where $A_{jk}(t)$ is a function of t only.

Substituting Eq.(19) into Eq.(17), we obtain the following differential equation for determination of $A_{jk}(t)$:

$$\ddot{A}_{jk}(t) + Q_{jk}(t)\dot{A}_{jk}(t) = 0 \tag{20}$$

where $$Q_{jk}(t) = \gamma\left[1 + \frac{\frac{j^4}{a^4}D_{sx} + \frac{k^4}{b^4}D_{sy} + \frac{j^2k^2}{a^2b^2}\frac{D_{sx}+D_{sy}}{1+\nu_s}}{\frac{j^4}{a^4}\beta_1 + \frac{k^4}{b^4}\beta_2 + 2\frac{j^2k^2}{a^2b^2}\beta_3}E_c\varphi(t)\right]$$

The solution of Eq.(20) can be expressed in the following form:

$$A_{jk}(t) = A_{jk}(\tau_n) + \dot{A}_{jk}(\tau_n)\int_{\tau_n}^{t} e^{-\int_{\tau_n}^{\tau}Q_{jk}(\tau)d\tau}d\tau\,. \tag{21}$$

$A_{jk}(\tau_n)$ and $\dot{A}_{jk}(\tau_n)$ in Eq.(21) are determined by the initial conditions of Eqs.(15) and (16) as follows:

$$A_{jk}(\tau_n) = \frac{\bar{q}_{njk}}{\pi^4(\frac{j^4}{a^4}\beta_1 + \frac{k^4}{b^4}\beta_2 + 2\frac{j^2k^2}{a^2b^2}\beta_3)}\,,$$

$$\dot{A}_{jk}(\tau_n) = \frac{\gamma E_c\varphi(\tau_n)\left[q_{njk} - \pi^4(\frac{j^4}{a^4}D_{sx} + \frac{k^4}{b^4}D_{sy} + \frac{j^2k^2}{a^2b^2}\frac{D_{sx}+D_{sy}}{1+\nu_s})A_{jk}(\tau_n)\right] + \bar{\bar{q}}_{njk}}{\pi^4(\frac{j^4}{a^4}\beta_1 + \frac{k^4}{b^4}\beta_2 + 2\frac{j^2k^2}{a^2b^2}\beta_3)}$$

where q_{njk} is a coefficient of load $q_n(x,y)$ in double Fourier series, namely,

$$q_{njk} = \frac{4}{ab}\int_0^a\int_0^b q_n(x,y)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b}dxdy\,,$$

and $$\bar{q}_{njk} = \frac{4}{ab}\int_0^a\int_0^b \bar{q}_n(x,y)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b}dxdy\,, \quad \bar{\bar{q}}_{njk} = \frac{4}{ab}\int_0^a\int_0^b \bar{\bar{q}}_n(x,y)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b}dxdy\,.$$

Substituting thus obtained results of Eq.(21) into Eq.(19), we finally find the deflection w(t) in a slab at any time t.

By using Eqs.(11) and (19), normal stresses of reinforcements are obtained as follows:

$$\sigma_{sxn}(t) = E_s z_s\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{j^2\pi^2}{a^2}A_{jk}(t)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b}$$

$$\sigma_{syn}(t) = E_s z_s\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\frac{k^2\pi^2}{b^2}A_{jk}(t)\sin\frac{j\pi x}{a}\sin\frac{k\pi y}{b}$$

where z_s is a distance between neutral plane and reinforcement.

Stresses of concrete are also obtained from Eq.(10), but numerical calculation for them has to make use of an iteration procedure.

5. Numerical Examples

To illustrate some practical applications of our theory, we consider singly reinforced square slabs with various percentage of reinforcements. The following characteristics are assumed for elastic constants and the creep function:

$E_c=2.1\times10^5$ kg/cm^2, $\nu=0.15$, $E_s=2.1\times10^6$ kg/cm^2, $\nu_s=0.3$,

$\alpha=4.82\times10^{-5}$, $\beta=0.9\times10^{-5}$, $\gamma=0.026$.

Numerical calculations are performed for the following two purposes:

(1) Pursuit of creep response of deflections and stresses in slabs for cyclic sustained loads.

(2) Calculation of creep recovery of deflections and stresses after the applied load is removed at any time.

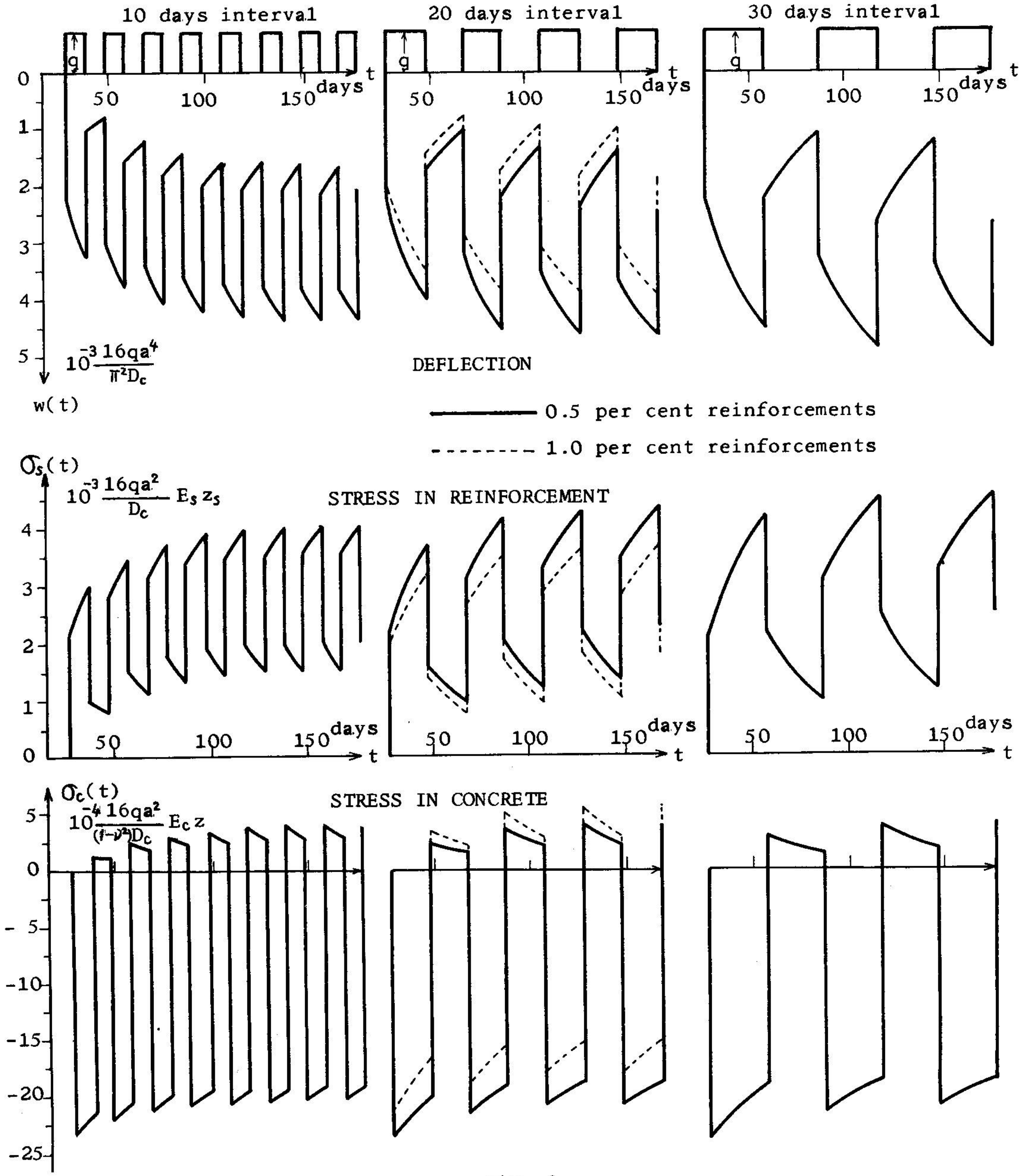

Fig. 3

Response for Cyclic Loads

Fig.3 shows the calculated responses of deflections, stresses in reinforcement and stresses in concrete at the center of square slabs, when a uniformly distributed load $q_n(x,y) = q$ is applied at the age of concrete $\tau_0 = 28$ days and thereafter unloading and reloading are repeated cyclically with time-intervals of 10 days, 20 days and 30 days, respectively. Here, solid lines are results for 0.5 per cent reinforcements in both x and y directions and dashed lines are those for 1.0 per cent. Deflections and tensile stresses in reinforcements illustrate analogous curves and it is noticed that stresses in reinforcements due to creep in concrete remain after complete removal of applied load. Stresses in concrete decrease during a period of constant sustained loading, and immediately upon removal of applied load, compressive stress changes to tensile one.

In the slab with high percentage of reinforcement, both deflections and stresses in reinforcements naturally decrease but residual stresses in concrete somewhat increase.

When a cyclic load, with a period of full loading and half loading, is applied, the calculated responses are as illustrated in Fig.4, where response of stress in reinforcement is omitted because of its similarity to that of deflection. It is interesting that during early periods of half loading deflection increases and stress in concrete decreases, namely, creep is in progress, but during later periods of half loading creep is in recovery.

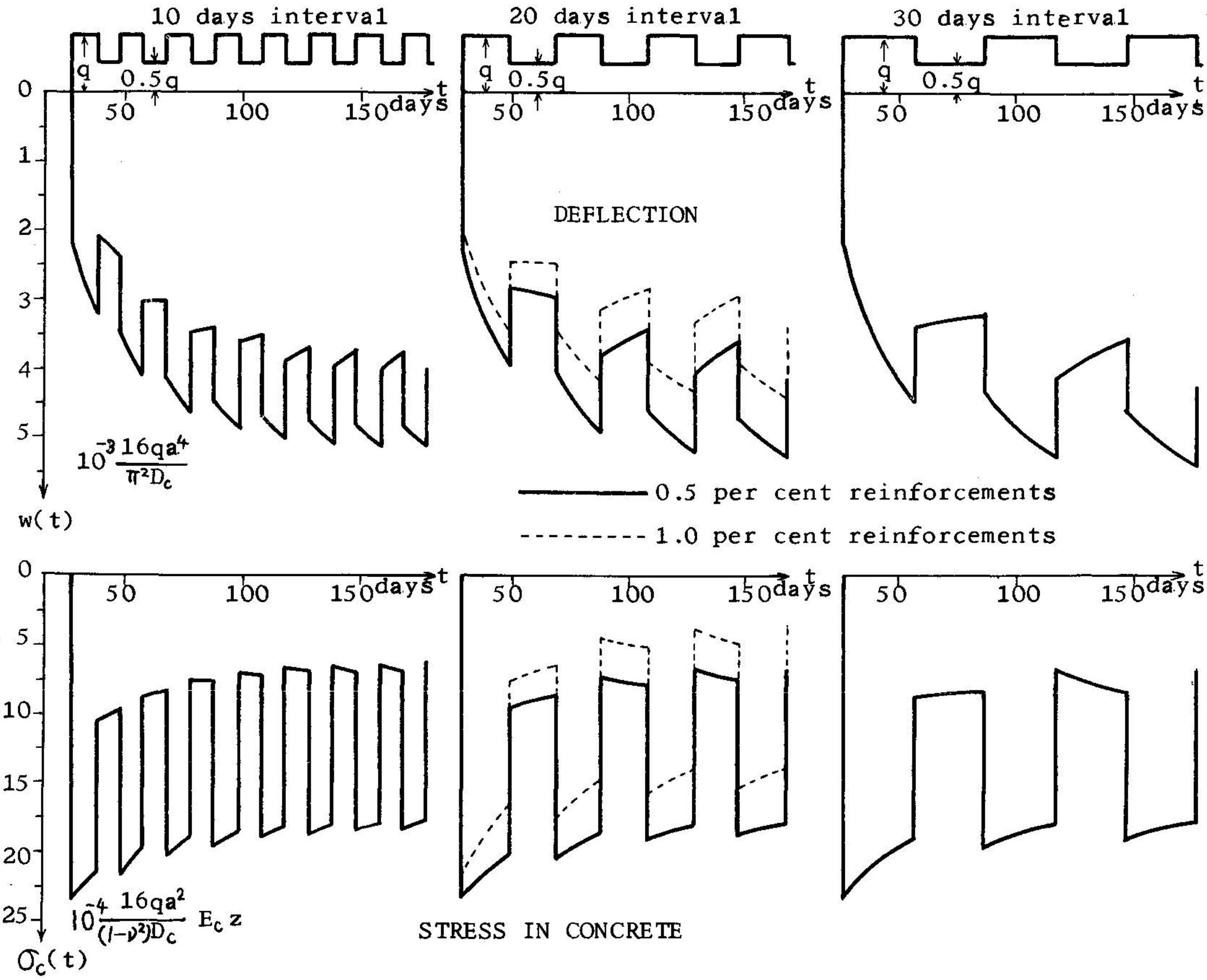

Fig. 4

Creep Recovery

Immediately upon complete removal of applied load, deflections and stresses in concrete structures do not recover to zero, but a certain deflection andstress due to creep in concrete remain. Although these residual deflection and stress are gradually reduced to zero, i.e. it is called creep recovery, some quantities remain permanently. In reinforced concrete structures this phenomenon of creep recovery becomes especially complicated, because creep recovery of concrete and elastic recovery of reinforcement are mixed in them.

By using the theory in this paper, creep recovery of reinforced concrete slabs is easily calculated, that is, we only have to set $q_n(x,y)$ equal to zero during the period of unloading.

The solid lines plotted in Fig. 5 show creep and recovery curves of deflections, stresses in reinforcements and stresses in concrete at the center of square slab of 0.5 per cent reinforcements in both x and y directions, when a uniformly distributed load $q_n(x,y)= q$ is applied at the age of concrete $\tau_0=28$ days and the load is removed after a period of 10, 20, 30, 40 and 60 days, respectively. Dashed lines in Fig.5 are results for the cases when the load is not removed completely,but left by half. It is noticed that recovery of stress in concrete is almost independent to the length of loaded period.

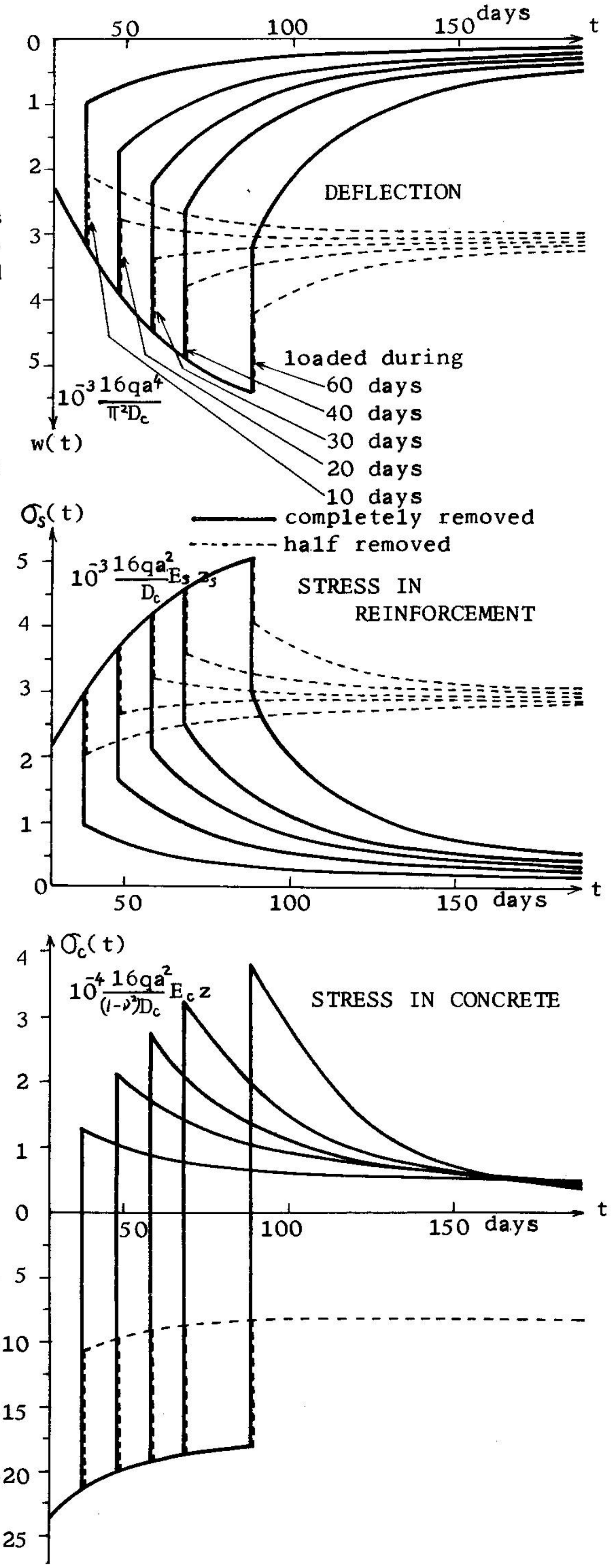

Fig. 5

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SUMMARY

Deflections and stresses in simply supported reinforced concrete slabs subjected to repeated sustained loads have been treated in this paper. A theory for calculating slab deflections and stresses in reinforcement and concrete has been developed which uses the creep function of Arutyunyan-type for plain concrete. By using electronic digital computer for the numerical work, creep response of flexed reinforced concrete slabs under the action of an arbitrary varying load can be easily calculated. Creep recovery curve of deflections and stresses at an arbitrary time after unloading can be also obtained without adding any serious complications in the procedure.

RESUME

On étudie dans le présent article les flèches et les contraintes des dalles en béton armé simplement appuyées et soumises à des charges répétées. On développe une théorie pour calculer les flèches de la dalle et les contraintes dans l'armature et dans le béton, en utilisant la fonction de fluage d'Arutyunyan. A l'aide de l'ordinateur, on peut alors facilement calculer le comportement au fluage des dalles en béton armé soumises à une charge variable quelconque. En outre, on peut obtenir sans difficulté la courbe de recouvrement du fluage pour les flèches et les contraintes à un moment quelconque après la suppression ou la diminution de la charge.

ZUSAMMENFASSUNG

In diesem Beitrag werden die Durchbiegungen und Spannungen von frei aufliegenden Stahlbetonplatten unter Wechsellast behandelt. Es wurde eine Theorie zur Berechnung der Plattendurchbiegungen und -spannungen in der Bewehrung und im Beton entwickelt, welche auf der Kriechfunktion von Arutyunyan (4) für Vollbeton fusst. Mittels digitaler Elektronenrechner kann das Kriechverhalten biegebeanspruchter Stahlbetonplatten unter beliebiger Wechsellast leicht ermittelt werden. Ebenso kann man die Kriecherholungskurve (creep recovery curve) der Durchbiegung und der Spannungen zu beliebigem Zeitpunkt nach Entlasten ohne zusätzliche Schwierigkeiten erhalten.

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