Stresses in thin cylindrical webs of curved plate girders

III

RAPPORTS INTRODUCTIFS / EINFÜHRUNGSBERICHTE / INTRODUCTORY REPORTS

Stresses in Thin Cylindrical Webs of Curved Plate Girders

Contraintes dans les âmes minces cylindriques de poutres courbes à âme pleine

Spannungen in dünnen, zylindrischen Stegen von gekrümmten Vollwandträgern

RYSZARD DABROWSKI
Institute of Civil Engineering
Technical University
Gdańsk, Poland

JERZY WACHOWIAK
Polytechnic Institute
Koszalin, Poland

1. Introduction

The title subject falls beyond the scope of the present Colloquium on limit design of plane plate girders. Stresses and displacements in thin cylindrical webs of curved plate girders under design loads are analysed herein. However, in both analyses one approaches the problem as a stress problem - without bifurcation of equilibrium - on the basis of a geometrically nonlinear theory of elastic plates and shells, respectively. Whereas investigation of postcritical behaviour of plane webs is rather well advanced, the present paper ought to be considered as a first step toward a more comprehensive investigation of the title problem.

The analysis of stresses and displacements in thin cylindrical webs of curved plate girders is of practical interest to designers of horizontally curved bridge girders in multi-girder or box-type bridge structures. Curved girders are subjected to stresses and displacemets under given dead and live loads. These stresses and displacements can be calculated e.g. according to the theory of torsion and bending of thin-walled girders with nondeformable or deformable cross-section [1] . Free transverse displacements of a cylindrical web panel within its supporting edges - which on two opposite sides are formed by curved flanges and by vertical stiffeners (Fig.1) - give rise to a redistribution of stresses and, consequently, to a deviation of the final stress pattern from the original one calculated on the basis of torsion bending theory.

The problem is treated as a so-called second-order-theory stress problem within elastic range of material properties. Small deflections, say, not exceeding half web thickness, are assumed. Donnell-type equations describing bending of shallow cylindrical shells are employed. Thus second-order effects due to the original membrane stresses only are accounted for. Linearized relations do not, however, constitute a serious limitation of the present solution. More refined results can be obtained by a step-by-step procedure.

11.2

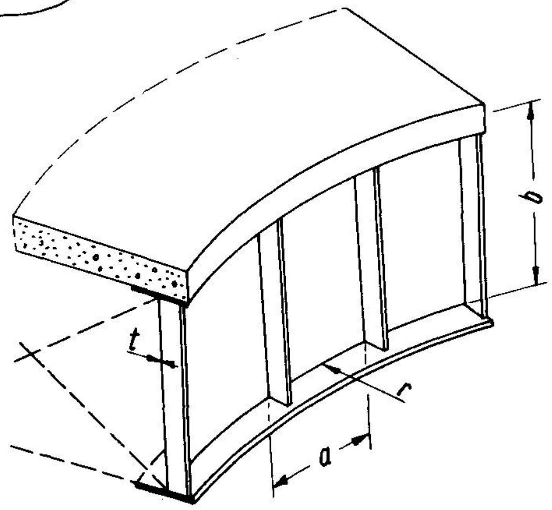


Fig. 1

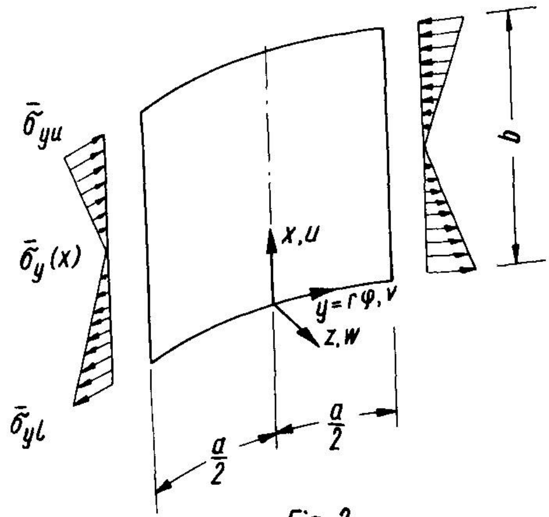


Fig. 2

2. Differential equations of the problem

A cylindrical panel (Fig.2) rigidly supported along curved edges at the junction with flanges and fixed along straight edges at vertical stiffeners is considered. The assumption of absolutely rigid stiffeners means a simplification of analysis and an oversimplification of the problem in many situations of practical design. It allows, however, to expose more clearly the relative importance of other parameters. Two kinds of support along curved edges are considered: (1) simple (hinged) support, and (2) fixed support with unrestricted displacement in x-direction (Fig.2).

Load acting upon cylindrical web panel is formed by original longitudinal stresses $\bar{\sigma}_y(x) = \bar{n}_y(x)/t$ uniformly distributed over web thickness t and varying linearly over web depth b (a bar is placed over the symbol σ for distinction of the original stress pattern from the final one which is denoted simply by σ without a bar). The assumed stress pattern, constant in y-direction, corresponds realistically with the performance of a web panel at midspan sections of the girder where maximum bending and warping moments due to continuous load occur and, accordingly, shear forces and secondary torsion moments disappear. (This is, of course, not the case with panels adjacent to intermediate supports of continuous girders where large shear forces are present, and should be accounted for, and, besides, the stresses $\bar{\sigma}_y(x)$ vary markedly in y-direction as well.)

Pertinent equations based on large deflections theory [2], [3], relating normal displacement w, stress function F with the original membrane forces $\bar{n}_x = \bar{\sigma}_x t$, $\bar{n}_y = \bar{\sigma}_y t$ and $\bar{n}_{xy} = \bar{\tau}_{xy} t$ read as follows

$$\left.\begin{aligned} K\nabla^4 w &= (\bar{n}_y + F'')\left(\frac{1}{r} + w^{\cdot\cdot}\right) + \\ &\quad + 2(\bar{n}_{xy} - F'^{\cdot}_{xy})\,w'^{\cdot} + (\bar{n}_x + F^{\cdot\cdot})\,w'', \\ \frac{1}{Et}\nabla^4 F &= -\frac{1}{r}w'' + (w'^{\cdot})^2 - w''w^{\cdot\cdot}, \end{aligned}\right\} \quad (1)$$

in which $K = Et^3/12\,(1-\nu^2)$ is the plate bending stiffness and r denotes the radius of curvature of web panel. Derivatives with respect to x and y are denoted as follows:

$$(\ )' = \frac{\partial(\ )}{\partial x}, \qquad (\ )^{\cdot} = \frac{\partial(\ )}{\partial y},$$

and, furthermore,

$$\nabla^2(\ ) = (\ )'' + (\ )^{\cdot\cdot}.$$

Final membrane forces are given by the relations

$$n_x = \bar{n}_x + F^{\cdot\cdot}; \quad n_y = \bar{n}_y + F'', \quad n_{xy} = \bar{n}_{xy} - F'^{\cdot}. \quad (2)$$

For a preliminary research pursued in this paper Eqs.(1) are to much involved. Linearized Donnell-type equations of small deflections theory are deduced directly from Eqs.(1) by deleting products of the unknowns w and F. Thus one obtains for the case under consideration, with $\bar{n}_x = \bar{n}_{xy} = 0$, the equations

$$\left.\begin{aligned} K\nabla^4 w - \frac{1}{r}F'' &= \bar{n}_y\left(\frac{1}{r} + w^{\cdot\cdot}\right), \\ \frac{1}{Et}\nabla^4 F + \frac{1}{r}w'' &= 0. \end{aligned}\right\} \qquad (3)$$

Second-order effect is accounted for by a single load term $\bar{n}_y w^{\cdot\cdot}$ on the right-hand side of the first Eq.(3).

For convenience in dealing with boundary conditions an equivalent set of three differential equations with respect to displacement components u, v, w (Fig.2) has been used - in conjunction with Galerkin's method of solution. These equations are as follows [4]:

$$\left.\begin{aligned} & u'' + \frac{1-\nu}{2}u^{\cdot\cdot} + \frac{1+\nu}{2}v'^{\cdot} - \frac{\nu}{r}w' = 0, \\ & \frac{1+\nu}{2}u'^{\cdot} + v^{\cdot\cdot} + \frac{1-\nu}{2}v'' - \frac{1}{r}w^{\cdot} = 0, \\ & -\frac{1}{r}\left(\nu u' + v^{\cdot} - \frac{1}{r}w\right) + \frac{t^2}{12}\nabla^4 w - \bar{n}_y\frac{1-\nu^2}{Et}\left(\frac{1}{r} + w^{\cdot\cdot}\right) = 0. \end{aligned}\right\} (4)$$

Accordingly, membrane forces n_x, n_y and n_{xy} and bending moments m_x, m_y and m_{xy} are given by the relations [4]

$$\left.\begin{aligned} n_x &= \frac{Et}{1-\nu^2}\left[u' + \nu\left(v^{\cdot} - \frac{w}{r}\right)\right], \\ n_y &= \bar{n}_y + \frac{Et}{1-\nu^2}\left(v^{\cdot} - \frac{w}{r} + \nu u'\right), \\ n_{xy} &= \frac{Et}{2(1+\nu)}\left(u^{\cdot} + v'\right) \end{aligned}\right\} \qquad (5)$$

and

$$\left.\begin{aligned} m_x &= -K(w'' + \nu w^{\cdot\cdot}), \\ m_y &= -K(w^{\cdot\cdot} + \nu w''), \\ m_{xy} &= -(1-\nu)K w^{\prime\cdot}. \end{aligned}\right\} \tag{6}$$

The most significant stress component, σ_y, on the inward and outward (with relation to the centre of curvature) web surfaces is equal to

$$(\sigma_y)_{z=\pm t/2} = \frac{n_y}{t} \pm \frac{6 m_y}{t^2}, \tag{7}$$

in which n_y is given by the second Eq.(5).

3. Galerkin's method of solution

The unknown displacement components u, v, w of Eq.(4) are assumed in form of double series with unknown coefficients u_{mn}, v_{mn}, w_{mn} as follows:

$$\left.\begin{aligned} u(x,y) &= \sum_m \sum_n u_{mn} u_m(y) u_n(x), \\ v(x,y) &= \sum_m \sum_n v_{mn} v_m(y) v_n(x), \\ w(x,y) &= \sum_m \sum_n w_{mn} w_m(y) w_n(x). \\ (m &= 1,2\ldots,\ n = 1,2\ldots) \end{aligned}\right\} \tag{8}$$

Shape functions $u_m(y)$, $u_n(x)$, $v_m(y)$, $v_n(x)$ are the sine and cosine functions satisfying appropriate boundary conditions at $y = \pm a/2$ and $x = 0, b$ (Fig.2). Shape functions $w_m(y)$ and $w_n(x)$ are assumed in form of eigenfunctions of transverse vibrations of a beam with fixed or simply supported ends, respectively, which comply with corresponding boundary conditions of the web panel.

Taking $m = 1, 2, 3, 4$ and $n = 1, 2, 3, 4$ one has to determine 48 unknown coefficients of the series, Eqs.(8), from a set of 48 linear equations obtained by means of Galerkin's method. The equations written down in general terms are as follows [5]:

$$\left.\begin{aligned} \iint_A R_u\,(\varphi_u)_{mn}\,dx\,dy &= 0, \\ \iint_A R_v\,(\varphi_v)_{mn}\,dx\,dy &= 0, \\ \iint_A R_w\,(\varphi_w)_{mn}\,dx\,dy &= 0. \end{aligned}\right\} \qquad (9)$$

in which R_u , R_v and R_w denote left-hand sides of equilibrium equations (4) expressed by the series, Eqs.(8), and $(\varphi_u)_{mn}$, $(\varphi_v)_{mn}$ and $(\varphi_w)_{mn}$ are virtual displacements in x, y and z-direction, complying with given boundary conditions. Clearly, these displacements are selected as products of assumed shape functions: $u_m(y)\,u_n(x)$, $v_m(y)\,v_n(x)$ and $w_m(y)\,w_n(x)$, respectively.

All calculations involved in determination of stresses and displacements have been programmed for a digital computer [6]. Some numerical results are presented subsequently in Section 3.3.

3.1 Cylindrical panel fixed at vertical stiffeners and simply supported along curved edges

Boundary conditions at $y = \pm a/2$ are as follows: $u = v = w = w' = 0$, and boundary conditions at $x = 0, b$: $u' = v = w = w'' = 0$. With newly introduced notations

$$\xi = x/b, \qquad \eta = y/a$$

the above conditions are satisfied by the following shape functions:

$$\left.\begin{aligned} u_m(y) &= \cos(2m-1)\pi\eta, \\ u_n(x) &= \cos n\pi\xi, \\ v_m(y) &= \sin 2m\pi\eta, \\ v_n(x) &= \sin n\pi\xi, \\ (m = 1,2,3,4;&\quad n = 1,2,3,4) \end{aligned}\right\} \qquad (10)$$

$$\left.\begin{aligned} w_m(y) &= \cos 2m^*\pi\eta - \frac{\cos m^*\pi}{\cosh m^*\pi}\cosh 2m^*\pi\eta, \\ w_n(x) &= \sin n\pi\xi, \end{aligned}\right\} \qquad (10a)$$

in which

$$m^* = 0{,}7528 \text{ for } m = 1,$$

$$m^* \approx m - 0{,}25 \text{ for } m = 2,\ 3,\ 4.$$

3.2 Cylindrical panel fixed at vertical stiffeners and fixed along curved edges while free to move in x-direction

Boundary conditions at $y = \pm a/2$ stated in Section 3.1 do apply again, and boundary conditions at $x = 0, b$ taking the form $u' = v = w = w' = 0$ are satisfied by the shape functions, Eqs. 10 , and by the following functions $w_m(y)$ and $w_n(x)$:

$$\left.\begin{aligned} w_m(y) &= \cos 2m^*\pi\eta - \frac{\cos m^*\pi}{\cosh m^*\pi}\cosh 2m^*\pi\eta, \\ w_n(x) &= \sin n^*\pi\xi - \sinh n^*\pi\xi - \\ &\quad - \frac{\sin n^*\pi - \sinh n^*\pi}{\cos n^*\pi - \cosh n^*\pi}(\cos n^*\pi\xi - \cosh n^*\pi\xi), \end{aligned}\right\} \qquad (11)$$

in which

$$m^* = 0{,}7528 \text{ for } m = 1,$$

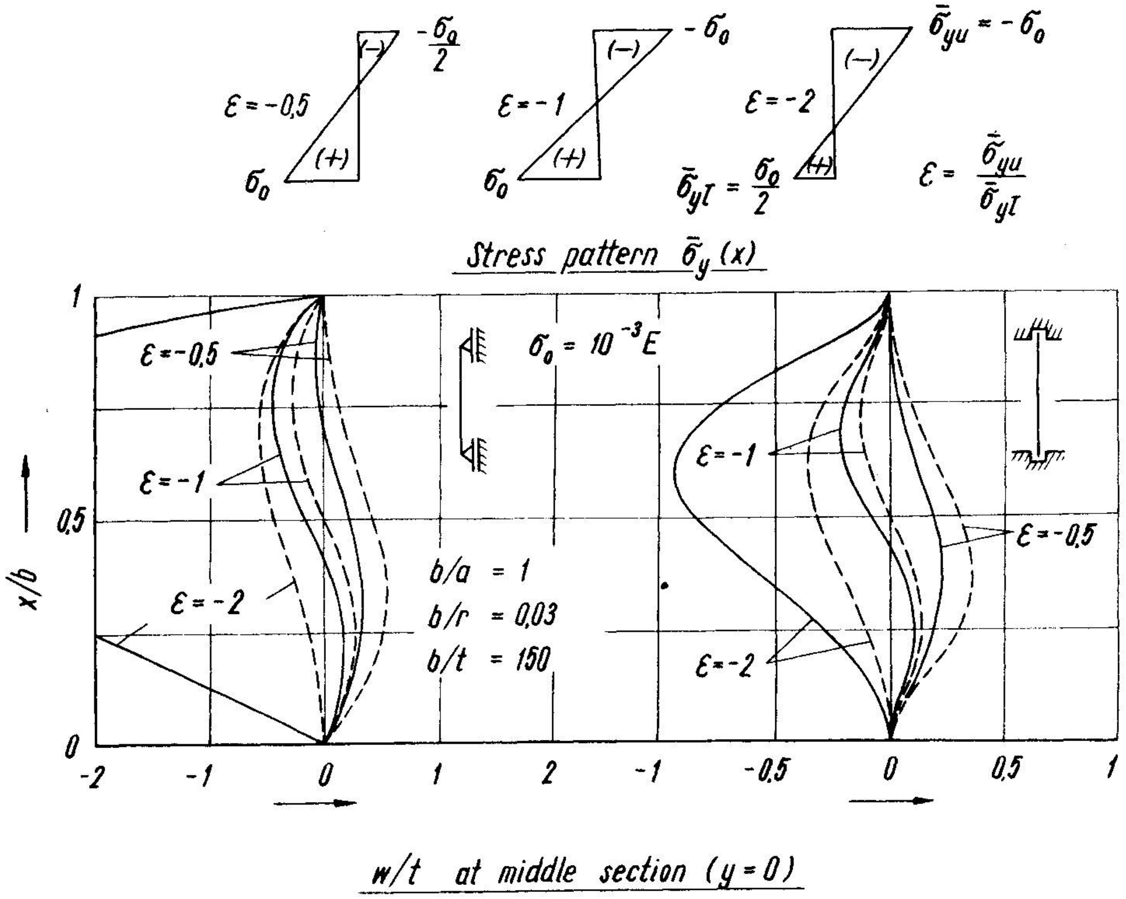
$$m^* \approx m - 0{,}25 \text{ for } m = 2,\ 3,\ 4,$$

$$n^* = 1{,}5056 \text{ for } n = 1,$$

$$n^* \approx n + 0{,}5 \text{ for } n = 2,\ 3,\ 4.$$

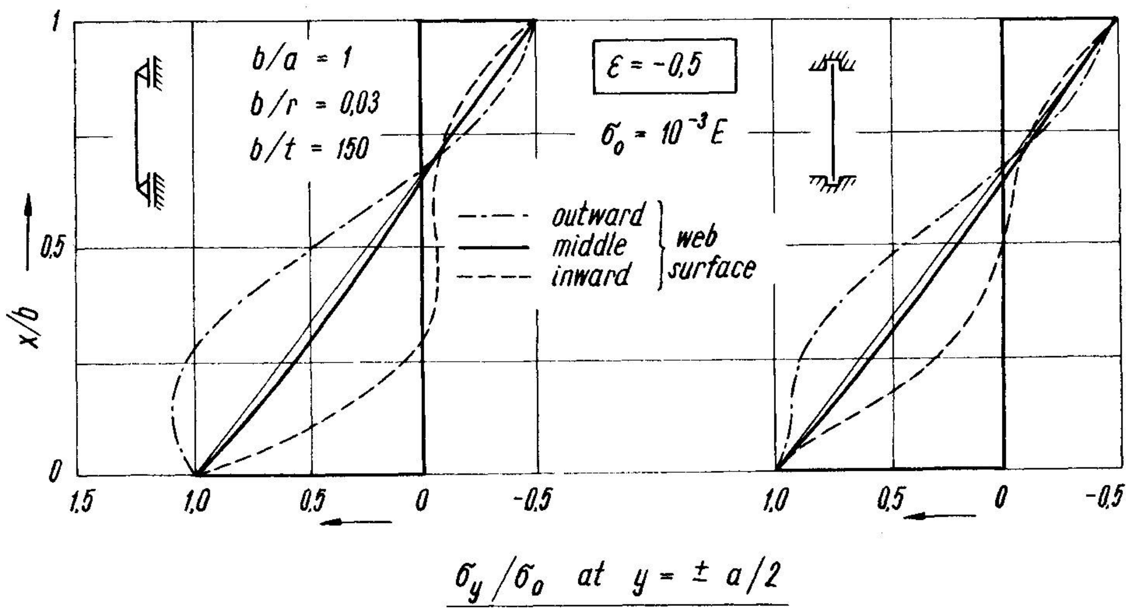
3.3 Numerical results expressed in terms of nondimensional parameters of web panel geometry

In Fig.3 there are shown normal displacements w at middle section of a square web panel $(a = b)$ for both sets of boundary conditions stated in Sections 3.1 and 3.2 and three different stress patterns characterized by the ratio of the upper edge stress to the lower edge stress, i.e. by $\varepsilon = \bar{\sigma}_{yu} / \bar{\sigma}_{yl}$, equal



$-\frac{\sigma_0}{2}$
(−)
(+)
$\varepsilon = -0{,}5$
σ_0
$-\sigma_0$
$\varepsilon = -1$
$\bar{\sigma}_{yu} = -\sigma_0$
$\varepsilon = -2$
$\bar{\sigma}_{yl} = \frac{\sigma_0}{2}$
$\varepsilon = \frac{\bar{\sigma}_{yu}}{\bar{\sigma}_{yl}}$
Stress pattern $\bar{\sigma}_y(x)$
$\sigma_0 = 10^{-3}E$
$b/a = 1$
$b/r = 0{,}03$
$b/t = 150$
x/b
w/t at middle section (y = 0)

Fig. 3



$b/a = 1$
$b/r = 0{,}03$
$b/t = 150$
$\varepsilon = -0{,}5$
$\sigma_0 = 10^{-3}E$
outward
middle
inward
web surface
x/b
σ_y/σ_0 at $y = \pm a/2$

Fig. 4

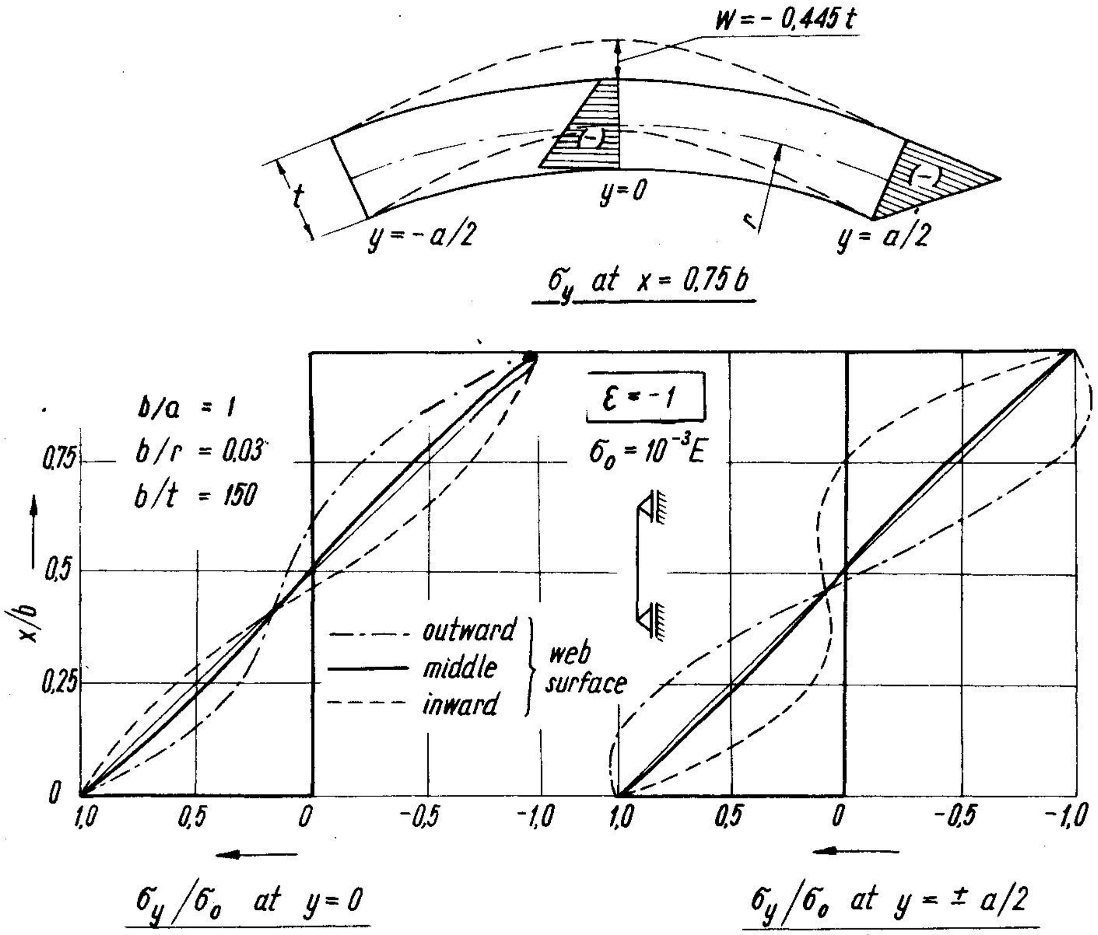

W = - 0,445 t
t
y = - a/2
y = 0
y = a/2
r
(-)
(-)
σy at x = 0,75 b
b/a = 1
b/r = 0,03
b/t = 150
ε = -1
σ0 = 10^-3 E
outward
middle
inward
web surface
x/b
0,75
0,5
0,25
0
1,0
0,5
0
-0,5
-1,0
σy/σ0 at y = 0
σy/σ0 at y = ± a/2

Fig. 5

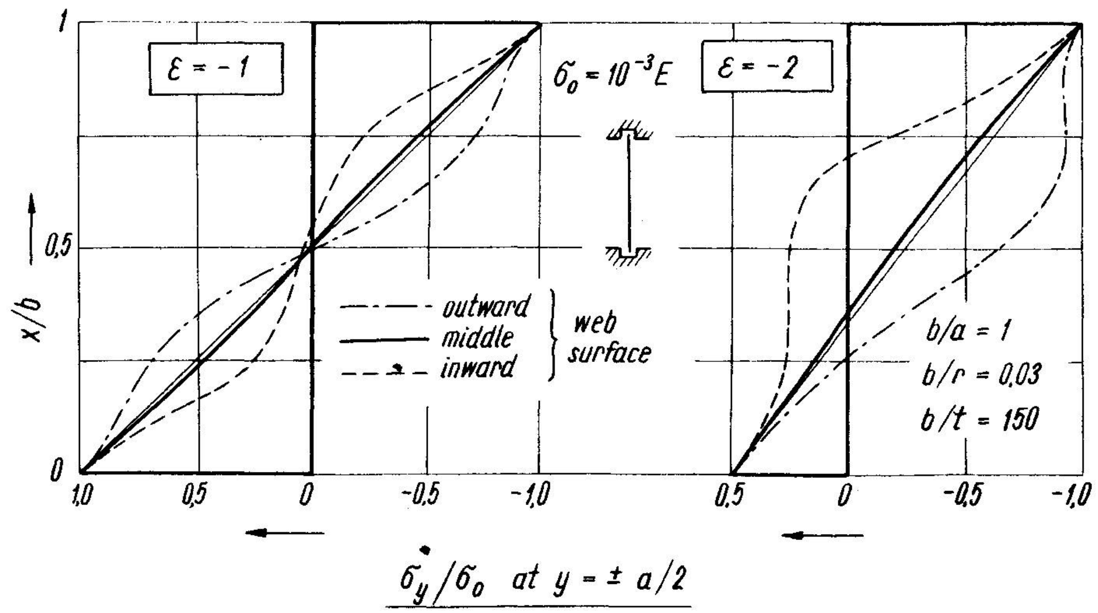

1
0,5
0
x/b
ε = -1
σ0 = 10^-3 E
ε = -2
outward
middle
inward
web surface
b/a = 1
b/r = 0,03
b/t = 150
1,0
0,5
0
-0,5
-1,0
σy/σ0 at y = ± a/2

Fig. 6

to -0,5, -1,0 and -2,0. The curves of w/t (solid lines) have been calculated for indicated nondimensional parameters b/a, b/r, b/t and the stress level $\sigma_0 = 10^{-3}E = 2100$ Kp/cm^2, σ_0 being the higher of the absolute values of edge stresses $\bar{\sigma}_{yu}$ and $\bar{\sigma}_{yl}$. Poisson's ratio equal $\nu = 0,3$ has been assumed.

As seen from inspection of diagrams on left-hand side of Fig.3, w/t-values for $\varepsilon = -2$ (solid line) fall beyond the range of validity of (second-order) theory of small deflections. Of course, they are somewhat exaggerated because for most of the panel area the final membrane forces $|n_y(x)|$ are smaller than membrane forces $|\bar{n}_y(x)|$ assumed in the third Eq.(4) - see Figs. 4 to 6 for comparison. Values obtained by first-order theory of cylindrical shells (dashed lines in Fig.3) do not convey a true picture of deformation of cylindrical web panels with relatively large radiuces of curvature. From comparison of diagrams in Fig.3, a favorable effect of fixity of web panel at the junction with curved flanges on limiting deflections is evident.

The stresses are of primary interest to designers. Longitudinal stresses σ_y in the middle web surface and on the outward and inward web surfaces are shown in Figs. 4 to 6. The curves of Fig.4 pertain to stress pattern with $\varepsilon = -0,5$. Stresses at end section of the panel are plotted for both boundary conditions considered. Diagrams of Fig.5 pertain to $\varepsilon = -1$ and panels with simply supported curved edges, while those of Fig.6 refer to panels with fixed curved edges and two stress ratios: $\varepsilon = -1$ and $\varepsilon = -2$.

Extremal values of normal deflection w with a corresponding ordinate x at which these values do occur are assembled for comparison in Tables 1 and 2, for two sets of boundary conditions considered. Several values of parameters ε, r/b, a/b and t/b, and two stress levels: $\sigma_0 = 10^{-3}E = 2100$ Kp/cm^2 and $\sigma_0 = (2/3)10^{-3}E = 1400$ Kp/cm^2 are taken into account.

Tables 1 and 2 also comprise extremal values of stress increase on either surface of web panel, in the tension and the compression zone of the panel. The stress increase above the initial value $\bar{\sigma}_y(x) = \bar{n}_y(x)/t$ at a distinct point with ordinate x is equal to

$$\Delta\sigma_y = \left(\frac{n_y}{t} \pm \frac{6m_y}{t^2}\right) - \frac{\bar{n}_y}{t} \qquad (12)$$

Table 1. Extremal values of normal displacement w and of stress increase $\Delta\sigma_y$ in cylindrical web panels simply supported along curved edges

Nondimensional parameters			Stress level	Extremal normal displacement at middle section				Extremal stress increase			
				in tension zone		in compression zone		at middle section		at end section	
r/b	a/b	t/b	σ_0 (10^{-3}E)	w/t	at x/b	w/t	at x/b	in tension zone	in compression zone	in tension zone	in compression zone
				Stress pattern ε = - 0,5							
33,3	0,5	1/100	2/3	0,016	0,25	-0,004	0,85	0,083	-0,024	0,189	-0,054
			1	0,022	0,25	-0,006	0,85	0,077	-0,023	0,184	-0,054
		1/150	2/3	0,047	0,25	-0,013	0,85	0,102	-0,034	0,259*	-0,080
			1	0,063	0,25	-0,019	0,85	0,090	-0,034	0,244*	-0,080
	1,0	1/100	1	0,130	0,30	-0,006	0,90	0,101	-0,026	0,328*	-0,049
		1/150	1	0,318	0,30	-0,018	0,90	0,091	-0,021	0,403*	-0,072
100	0,5	1/150	1	0,021	0,25	-0,006	0,85	0,032	-0,012	0,084	-0,028
				Stress pattern ε = - 1,0							
33,3	0,5	1/100	2/3	0,013	0,20	-0,015	0,80	0,071	-0,085	0,162	-0,176
			1	0,019	0,20	-0,022	0,75	0,068	-0,089	0,159	-0,180
		1/150	2/3	0,040	0,20	-0,055	0,80	0,094	-0,141	0,228	-0,276
			1	0,056	0,20	-0,092	0,80	0,087	-0,160	0,218	-0,293
	1,0	1/100	1	0,069	0,20	-0,104	0,75	0,073	-0,106	0,223	-0,258
		1/150	1	0,186	0,20	-0,445	0,75	0,082	-0,193	0,305	-0,395*
100	0,5	1/100	1	0,006	0,20	-0,008	0,80	0,024	-0,031	0,054	-0,061
		1/150	1	0,019	0,20	-0,031	0,80	0,031	-0,057	0,075	-0,102
				Stress pattern ε = - 2,0							
33,3	1,0	1/100	1	-		-0,305	0,65	0,143	-0,290	0,192	-0,492*
		1/150	1	-		-4,700	0,65	-	-2,518	-	-1,505*

and is related to stress level σ_o .

With regard to extremal values of normal stresses σ_y on web surfaces the following observations should be made. As the extremal value of $\Delta\sigma_y$ in cylindrical panels simply supported along curved edges occurs at some distance from the curved edge, the sum $|\sigma_y| = |\sigma_y + \Delta\sigma_y|$ at that point falls in most cases below σ_o. Exceptional cases are indicated by an asterik in Table 1. In those cases extremal values of the sum $|\sigma_y + \Delta\sigma_y|$ are higher than σ_o

In cylindrical panels fixed along curved edges bending moments $m_y = \nu m_x$ do occur along those edges (with the exception of corner points where $m_x = m_y = 0$) and, accordingly, the sum $|\sigma_y + \Delta\sigma_y|$ at the curved edges is higher than σ_o. This being taken into account, the extremal values of $\Delta\sigma_y$ at middle section given in Table 2 refer to field or edge points of that section, wherever the absolutely largest value does occur.

Normal stresses $\sigma_x \equiv \Delta\sigma_x$ due to bending moments m_x deserve attention. For example, σ_x at upper edge of middle section of a web panel with $\varepsilon = -1$, $r/b = 33,3$, $a/b = 1$, $t/b = 1/150$ according to Table 2 is equal to $\sigma_x = \Delta\sigma_y/\nu = -0,129\,\sigma_o/\nu = -0,43\,\sigma_o$. For $\varepsilon = -2$ it is even higher compare last line of Table 2 , but still below the approximate upper limit $\sqrt{3/(1-\nu^2)}\,\sigma_o = 1,81\,\sigma_o$ derived from a solution to a case of rotational symmetry. (Normal stresses σ_x due to bending moments m_x at the joints of web panels and flanges are of secondary importance as far as ultimate strength of the girder is concerned. However, they are significant in design of welded joints for fatique strength.)

Shear stresses $\tau_{xy} = n_{xy}/t$ resulting from transverse deflection of the cylindrical panel are, in the considered range of curvatures, very small und amount to a few percent of σ_o .

4. Results and conclusions

Calculations based on second-order small deflections theory for two stress levels, situated in the range of working and yield stresses of structural steel, do not provide full insight into behaviour of thin cylindrical webs in curved plate girders with increasing load. Displacemets and stresses in the considered range increase virtually in proportion to load (i.e. to stress level σ_o) and to girder curvature (i.e. to $1/r$). More

Table 2. Extremal values of normal displacement w and of stress increase $\Delta\sigma_y$ in cylindrical web panels fixed along curved edges with free displacement in x-direction

Nondimensional parameters			Stress level	Extremal normal displacement at middle section				Extremal stress increase					
								at middle section				at end section	
			σ_o	in tension zone		in compression zone		in tension zone		in compression zone		in tension zone	in pression zone
r/b	a/b	t/b	10^{-3}E	w/t	at x/b	w/t	at x/b	$\Delta\sigma_y/\sigma_o$	at x/b	$\Delta\sigma_y/\sigma_o$	at x/b		
						Stress pattern ε = - 0,5							
33,3	0,5	1/150	1	0,051	0,30	-0,011	0,80	0,073	0,25	-0,022	0,80	0,205	-0,053
	1	1/150	1	0,223	0,35	-0,001	0,90	0,112	0	-0,022	0,70	0,294	-0,041
100	0,5	1/150	1	0,017	0,30	-0,004	0,80	0,026	0,25	-0,008	0,80	0,070	-0,018
	1	1/150	1	0,079	0,35	-		0,039	0	-0,007	0,70	0,108	-0,015
						Stress pattern ε = - 1,0							
33,3	0,5	1/100	1	0,014	0,25	-0,018	0,75	0,051	0,25	-0,068	0,75	0,121	-0,138
		1/150	1	0,041	0,25	-0,071	0,75	0,063	0,25	-0,125	0,75	0,167	-0,225
	1,0	1/100	1	0,036	0,25	-0,048	0,70	0,065	0	-0,076	1,0	0,137	-0,153
		1/150	1	0,101	0,25	-0,197	0,70	0,088	0	-0,129	1,0	0,186	-0,227
100	0,5	1/100	1	0,004	0,25	-0,006	0,75	0,018	0,25	-0,024	0,75	0,041	-0,047
		1/150	1	0,014	0,25	-0,024	0,75	0,023	0,25	-0,044	0,75	0,057	-0,077
	1,0	1/100	1	0,012	0,25	-0,017	0,70	0,022	0	-0,026	1,0	0,047	-0,053
		1/150	1	0,034	0,25	-0,070	0,70	0,029	0	-0,045	1,0	0,071	-0,076
						Stress pattern ε = - 2,0							
33,3	1,0	1/100	1	-		-0,154	0,60	0,064	0,30	-0,145	1,0	0,093	-0,285
		1/150	1	-		-0,934	0,60	0,221	0,30	-0,457	0,65	0,242	-0,422

relevant are the following observations: with increasing distance between vertical stiffeners deflections grow very markedly; stresses do increase as well, but to a lesser degree. A reduction of web thickness results in a very prenounced increase of deflections; remarkably enough, bending stresses become higher as well.

Flexibility of vertical stiffeners in real structures would cause a further increase of deflections and bending stresses at middle sections of web panels. Only the results obtained by ordinary (first-order) theory of cylindrical shells are available for comparison. It can be be inferred from them that in moderately stiffened cylindrical panels - as is the case with plane webs designed for stability and not for ultimate strength - this increase can be as high as by one third or more

In general, displacements and additional stresses due to bending under design conditions remain within acceptable limits in the parameter range considered. As evident from Figs.4 to 6, the mean membrane stresses $\bar{\sigma}_y$ drop only slightly - as the result of transverse web deflection - from the originally assumed linear pattern. Consequently, the reduction of web-area contribution to overall section modulus of the curved girder amounts only to a few percent and is insignificant.

5. Scope of further research

A more intrinsic analysis by large deflections theory is needed to clarify the perfarmance of thin cylindrical webs under loads well in excess of working loads - in particular, when extremely thin webs are investigated. Presumably, for higher loads, still within elastic range, deformed configuration characterized by one half-wave in longitudinal direction changes into another one with more half-waves.

Web performance under an initial stress pattern which includes longitudinal stresses $\bar{\sigma}_y$ as well as shear stresses $\bar{\tau}_{xy}$ remains to be investigated.

Experimental work is necessary, i.a. to check the influence of plastic zones on ultimate strength of thin cylindrical webs.

Acknowledgement. The results presented in this paper have been obtained by J.Wachowiak in 1966 in the course of prepa-

ration of his dissertation [6], under the guidance of the senior author, at the Department of Civil Engineering, Technical University, Gdańsk.

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SUMMARY

Stresses and displacements in thin cylindrical webs of curved plate girders are analysed on the basis of second-order theory of small deflections by means of Galerkin's method. A cylindrical web panel (Fig. 2) rigidly supported along curved edges and fixed along straight edges (at vertical stiffeners) is considered. Stresses on middle web surface and on outward and inward web surfaces as well, at end and middle section of the web panel, are shown in Fig. 4 to 6. Numerical results are assembled in Tables 1 and 2.

RESUME

Les auteurs déterminent les contraintes et les déflexions de l'âme mince cylindrique des poutres courbes, en utilisant la théorie du second ordre pour les petites déformations à l'aide de la méthode de Galerkin. On considère un panneau d'âme cylindrique appuyé le long des membrures et encastré au droit des raidisseurs verticaux. Les figures 4 à 6 représentent les contraintes de la surface moyenne ainsi que des surfaces intérieure et extérieure, aux extrémités et au milieu du panneau d'âme. Les tableaux 1 et 2 contiennent des résultats numériques.

ZUSAMMENFASSUNG

Spannungen und Verformungen in dünnwandigen, kreiszylindrischen Stegen von gekrümmten Vollwandträgern werden aufgrund der Theorie II. Ordnung für kleine Verschiebungen, mit Hilfe des Galerkin'schen Verfahrens untersucht. Es wird eine Teilschale (Fig. 2), die an gekrümmten Rändern starr gestützt und an geraden Rändern (an den Vertikalsteifen) eingespannt ist, betrachtet. Spannungen in der Mittelfläche sowie an der äusseren und inneren Schalenoberfläche, im Endquerschnitt bzw. Mittelquerschnitt der Teilschale, werden in Abb. 4 bis 6 gezeigt. Zahlenresultate sind in den Tafeln 1 und 2 zusammengestellt.