Predictable properties of material under incremental cyclic loading

Autor(en):   **Kato, Ben / Akiyama, Hiroshi / Yamanouchi, Yasuyuki**

Predictable Properties of Material under Incremental Cyclic Loading

Propriétés prévisibles du matériau soumis à des charges cycliques
non-constantes

Vorhersagbare Materialeigenschaften unter zunehmender zyklischer
Belastung

Ben KATO	Hiroshi AKIYAMA	Yasuyuki YAMANOUCHI
Professor	Associate Professor	Graduate Student
	University of Tokyo, Japan	

1.INTRODUCTION

This paper deals with mechanical properties of materials for structural
steels which are subjected to irregularly fluctuating external forces such as
seismic forces and wind storms. The elastic response of the structure to such a
loading is of harmonic oscillations. However the inelastic response may be
characterized by non-steady development of plastic deformations and the number
of cycles which produces large plastic strain seems to be smaller than the num-
ber discussed in the problem of fatigue. Thus the effect of accumulation of
plastic strain into a critical state would be more important than the effect of
fatigue as a collapse criterion of the material.

It is well-known that the inelastic stress-strain relation($\sigma - \varepsilon$ relation)
of materials for structural use is mathematically expressed in exponential
Ramberg-Osgood function and Masing's assumption can be applied to hysteretic
relations in cyclic loading.(1),(2) Fig.1 shows the most popular relation
adopted by many researchers for structural analysis.(3) According to Masing's
assumption, the shape of the hysteretic branch is twice as large as that of the
monotonic $\sigma - \varepsilon$ relation. Fig.2 shows a typical stress-strain relation of usual
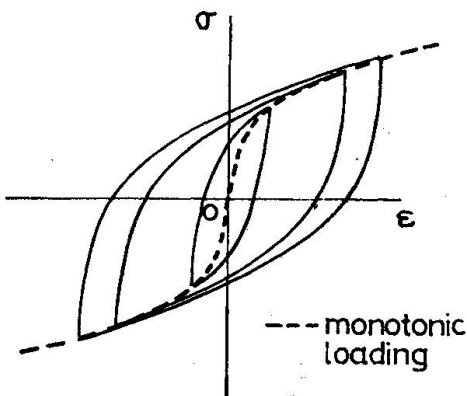carbon steels, which is characterized by yielding plateau in initial loading,
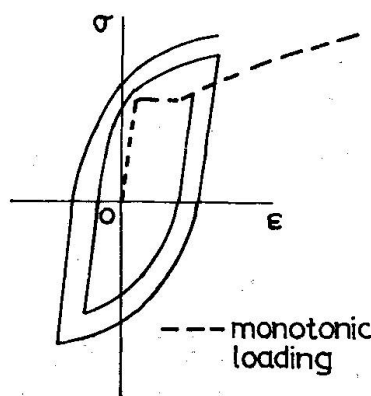

Fig.1. Ramberg-Osgood
Function

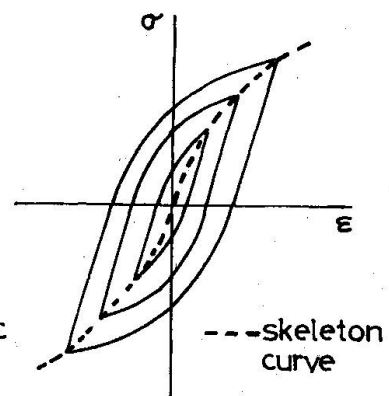Fig.2. Realistic $\sigma - \varepsilon$
Relation of Steel

Fig.3. Halford's
Expression

strain-hardening and Bauschinger effect. It is obvious that the hysteretic pattern in Fig.1 does not cover the realistic stress-strain relation of steels which allows remarkable enlargement of hysteresis loops in the direction of σ-axis due to strain-hardening.

Halford suggested a definite expression for prediction of hysteretic relation under steady-state cyclic loading as shown in Fig.3.(4) The effect of strain-hardening is well introduced in his expressions. However his pattern does not present any informations about transient paths from one stationary loop to the other.

Moreover the criterion for the fracture of the materials under incremental cyclic loading is still left unknown.

From this point of view, to extract predictable nature of the material subjected to arbitrarily changing strain history, series of material tests were carried out.

2.TESTING PROGRAM

Fig.4 shows series of test specimens. Test-series-A involves two kinds of materials and eleven specimens. In this series of testing, tensile and compressive loads were gradually increased in every loading cycles. In test-series-B two kinds of materials and 29 specimens were used. In this series of testing, loading history involves stationary loops and incremental paths. In test-series-C, also two kinds of materials and 30 specimens were used. Prior to the testing preliminary test was conducted aiming to obtain fatigue strength of materials under cyclic loading with constant stress amplitude in positive and negative direction. In this series of testing, first, stationary cyclic loading in which substantial amount of fatigue life is exhausted is applied and subsequently monotonic tensile loading is applied up to the fracture of the material. Thus the reserve of ductility after the exhaustion of fatigue life is sought after. Detailed descriptions as for the results of the tests are reported elsewhere. (5),(6),(7)
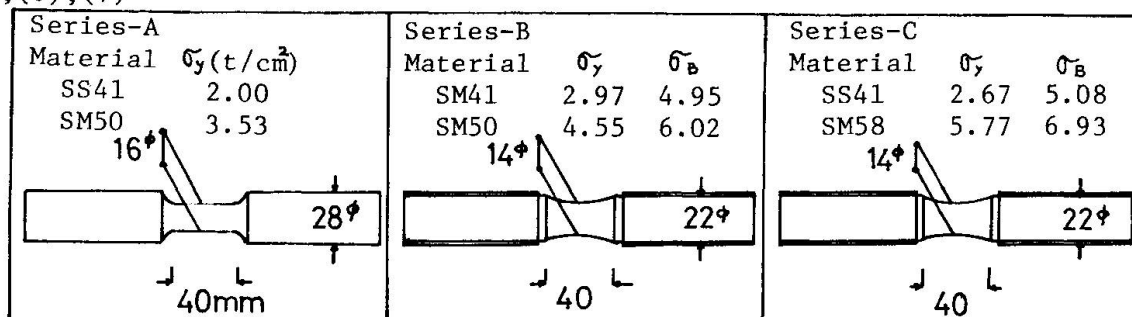

Series-A Material	$\sigma_y(t/cm^2)$		Series-B Material	σ_y	σ_B	Series-C Material	σ_y	σ_B
SS41	2.00		SM41	2.97	4.95	SS41	2.67	5.08
SM50	3.53		SM50	4.55	6.02	SM58	5.77	6.93

Fig.4. Test Specimens (σ_y=yielding stress, σ_B=breaking stress)

3. RESOLUTION OF $\sigma-\varepsilon$ RELATION INTO THREE COMPONENTS

Conventional Ramberg-Osgood function expresses continuously Bauschinger effect and strain-hardening. However these phenomena seem to be independent each other. Based on the results of test-series-A, realistic features are observed. Fig.5 shows a result of testing. In Fig.6, $\sigma-\varepsilon$ relation in each sign of loading is extended in one sequence. Bold line shows loading path which exceeds the stress level attained by the preceding loading in the same stress domain. Connecting these segments independently, two monotonic curves are obtained and compared with the relation obtained under monotonic loading in Fig.7. The initial loading path in compression loading is considerably softened by Bauschinger effect. Disregarding this deviation, the next remarkable fact can be confirmed.

A skeleton curve can be found independently in each loading domain and these curves coincide with the relation under monotonic loading.

This fact can be verified extensively even in the case that loading path includes large numbers of stationary loops(test-series-B,C). For the practical purpose to simplify the structural analysis, Bauschinger effect may be neglected in many cases. Advanced treatment of the skeleton curve is discussed extensively
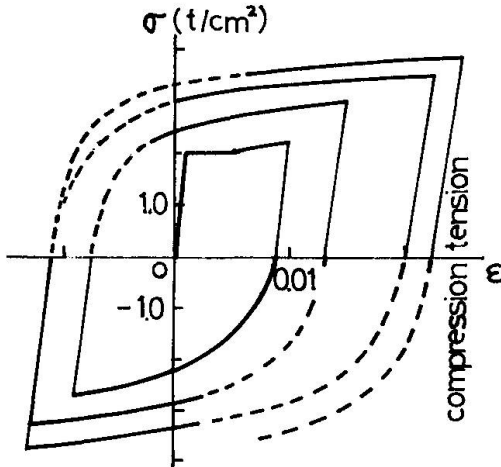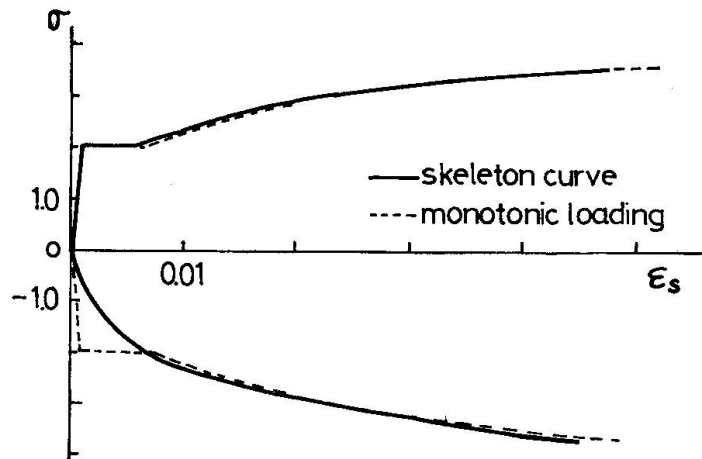
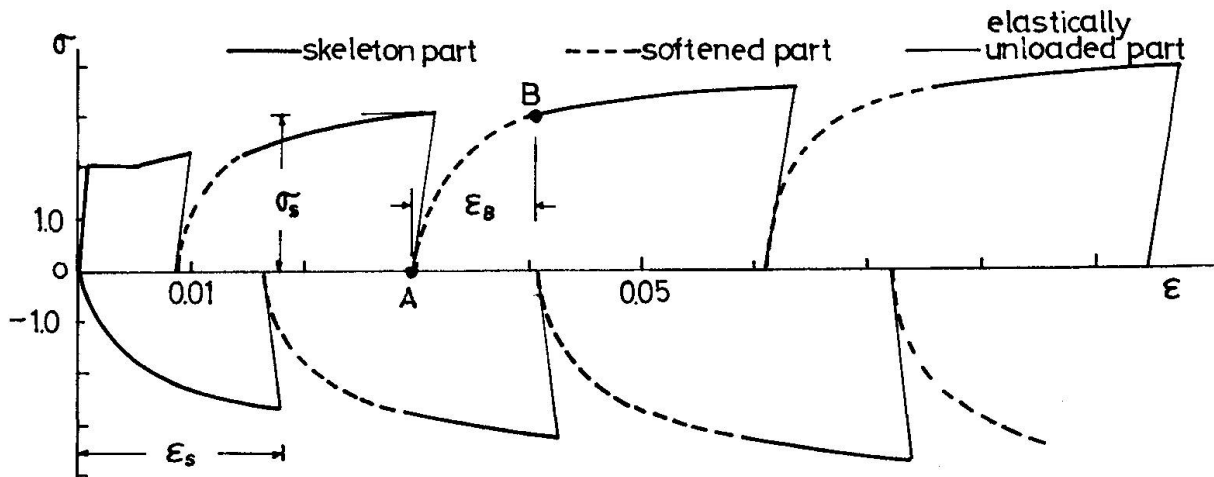Fig.5. A Result of Test-
        Series-A

Fig.7. Skeleton Curves

Fig.6. Stretched Figures of the Hysteretic Curve

elsewhere of this symposium.(8)

The stress-strain relation under cyclic loading can be resolved into the
skeleton part and the rest. The rest of σ-ε relation is composed of the elastic-
ally unloaded part and the softened part due to Bauschinger effect. Results of
test-series-A show that rigidity of the unloaded part preserves almost initial
rigidity of elasticity and the onset of the softened part lies on the zero
stress level. Therefore, problem is how to estimate the softened part illustr-
ated in Fig.6 which connects from point A where   stress is zero to point B
where stress reaches σ_s which is the maximum stress previously attained in the
same stress domain. Nondimensional expression for the softened part is obtained
by taking σ/σ_s as   the ordinate and $\varepsilon/\varepsilon_B$ as the abscissa where ε_B is termed as
Bauschinger strain. Fig.8 shows the results of test-series-A. Bold line shows
the bound of the experimental plots. Obviously the shape of the softened part is
hyperbolic and expressed as

$$(y+a)(x-a-1)+a(a+1)=0. \tag{1}$$

Slope of the shape is given as

$$\frac{dy}{dx} = \frac{-(y+a)}{(x-a-1)} \tag{2}$$

Initial slope at x=0 is given by the next equation.

$$y = (\frac{\sigma}{\sigma_s})/(\frac{\varepsilon}{\varepsilon_B}) = \frac{\sigma}{\varepsilon} \frac{\varepsilon_B}{\sigma_s} = \frac{E}{E_B} \tag{3}$$

where E=Young's modulus, $E_B= \sigma_s/\varepsilon_B$.
Therefore a is determined as

$$a = \frac{E}{(E_B-E)} \tag{4}$$

Empirically E_B is judged to be highly affected by the inelastic deformation under the opposite sign of loading. Fig.9 shows the experimental plots of E_B versus the maximum strain on the skeleton curve attained under the opposite sign of loading, $_0\varepsilon_s$. Although these data scatter widely, it is distinct that E_B decreases as $_0\varepsilon_s$ increases. The lower bound is simply expressed under the range of $_0\varepsilon_s$ below o.1 as

$$E_B = \frac{-log_{10}10_0\varepsilon_s}{6} \times E. \tag{5}$$

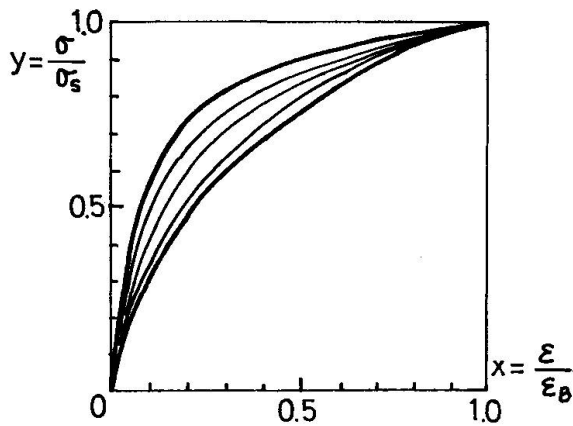Fundamental features of the softened part may be estimated by Eq(1) and Eq(5).
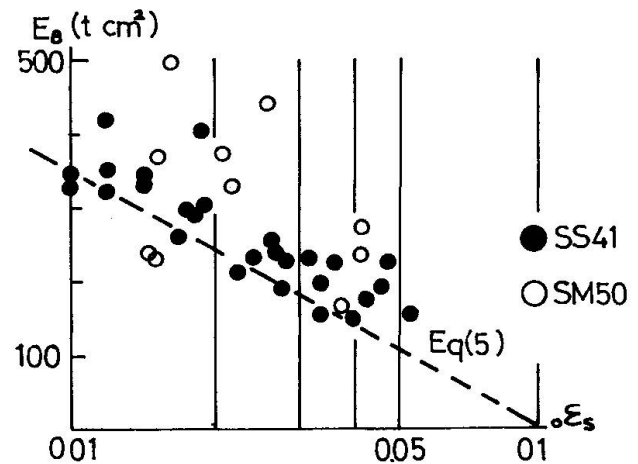

Fig.8. Shape of Softened Part

Fig.9. Secant Modulus of Softened Part

4.APPLICABILITY OF ANALYTICAL EXPRESSION

4-1. Prediction of Stationary Loops

Eq(5) was derived from the results of testing under incremental loading. If Eq(5) is extensible to such a loading as shown in Fig.10, Eq(5) may also gives the scale of the converged stationary loops. Test-series-B involves some available data suitable for comparison. Fig.11 shows the compared aspects. The prediction is almost agree with the test data. In Fig.11 broken lines show the limit of convergence above which hysteresis loops do not converge under a constant stress amplitude. It can be seen that the limit of convergence of stationary loops nearly corresponds to the limit of application of the formula.

4-2. Criterion for Ductile Fracture of Materials

Under monotonic tensile loading ductile fracture occurs immediately after the maximum stress σ_B is attained. Stress-strain relation under arbitrarily fluctuating loading is divided into skeleton part and the rest. The accumulated skeleton part in each sign of loading coincides with the monotonic σ-ε relation. Therefore the criterion for ductile fracture may be expressed as

$$\sum \Delta\varepsilon_s = \varepsilon_u. \tag{6}$$

where $\Delta\varepsilon_s$= strain on the skeleton part under tensile loading,
ε_u= strain at the ultimate state under monotonic loading.
Stationary hysteresis loops under constant stress amplitude do not make any contribution to accumulation of strains on skeleton part. Thus it can be deduced that fatigue strength and ductile fracture are independent each other.
Fig.12 shows the loading history of test-series-C. First, considerable amount of hysteresis loops under constant amplitude are exerted, and next, tensile loading
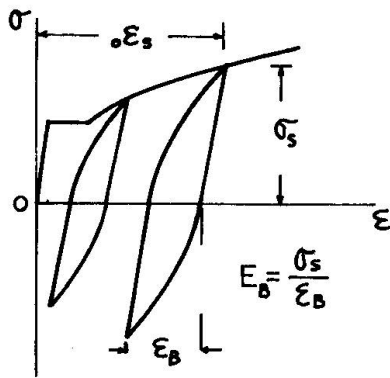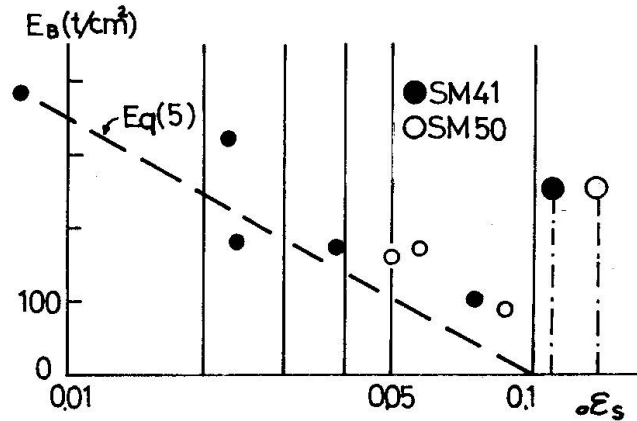
Fig.10. Loading History

Fig.11. Secant Modulus of Stationary Loops

is applied until the ultimate state is attained. Conventionally ultimate state was assumed to reach when the stress goes down to 98% of the maximum stress. Applied stress amplitude in cyclic loading ranges between $0.6\tilde{\sigma}_B$ and $0.9\tilde{\sigma}_B$ and the number of cycles N is varied from 10% to 90% of fatigue life N_f. Fig.13 shows the reserve of ductility and strength after a certain amount of fatigue life is exhausted. In Fig.13 $\Sigma\Delta\varepsilon_s$ denotes the summation of the strain on the skeleton parts under tensile loading and $\tilde{\sigma}_u$ means the maximum stress attained in each specimen. From the figure it can be seen that the reserve of strength and ductility is scarecely affected by the presence of large number of stationary loops.

5. CONCLUSION

Based on series of material tests, it was concluded that the basic feature of stress-strain relation of the steel under cyclic loading is predictable. Principal findings are followings.

Fig.12. Loading History

1. Stress-strain relation of the steel subjected to arbitrarily changing loading history was found to be composed of three parts: skeleton part, elastically unloaded part and softened part by Bauschinger effect.
2. Skeleton part coincides with $\sigma-\varepsilon$ relation under monotonic loading.
3. Softened part begins at zero stress level and terminates at the maximum stress point previously attained in the same sign of loading and is expressed by Eq(1) and Eq(5).
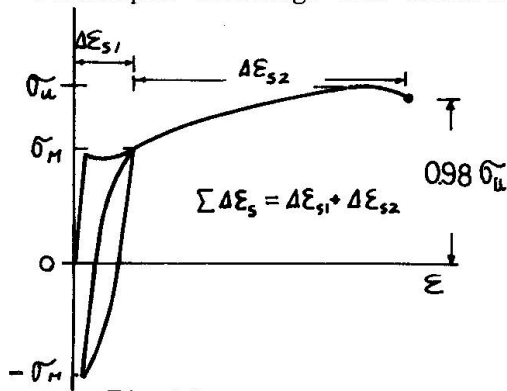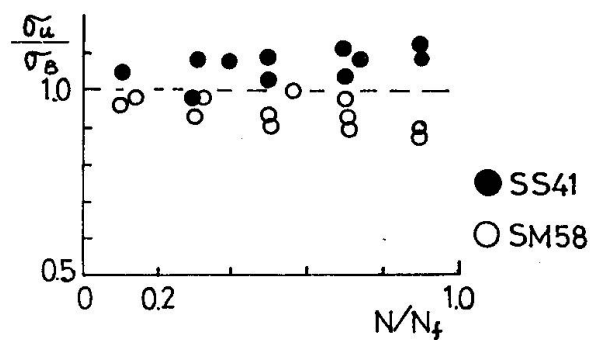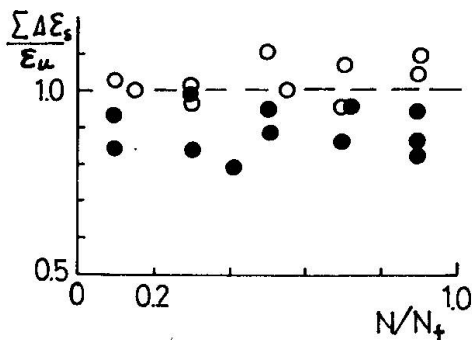

Fig.13. Reserve of Ductility and Strength

4. Scale of stationary loops involved in the loading history can be successfully predicted by Eq(5).

5. Fatigue and ductile fracture are essentially independent phenomena and the criterion for ductile fracture of the steel is given by Eq(6).

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SUMMARY

Systematic expression for the stress-strain relation of the steel under arbitrarily fluctuating loading history was sought after. Stress-strain relation of the steel was found to be composed of three parts: skeleton part, elastically unloaded part and softened part due to Bauschinger effect. The softened part can be analytically expressed and only to know the stress-strain relation under monotonic loading is required to predict the stress-strain relation of the steel subjected to complex cyclic loading.

RESUME

Dans ce travail on a recherché une expression systématique pour la relation tension-déformation de l'acier soumis arbitrairement à des fluctuations de charges. On a trouvé que la relation tension-déformation de l'acier se compose de trois parties: la partie du squelette, la partie élastique non-chargée et la partie calmée due à l'effet Bauschinger. La partie calmée peut être exprimée analytiquement et la connaissance de la relation tension-déformation sous charges monotones permet de prédire la relation tension-déformation de l'acier soumis à des charges cycliques complexes.

ZUSAMMENFASSUNG

Es wurden systematische Ausdrücke für die Spannungs-Dehnungs-Beziehung von Stahl unter beliebig ändernder Last gesucht. Die Beziehung setzt sich aus drei Teilen zusammen: Skelett-Teil, Teil der elastischen Entlastung, Teil der Enthärtung infolge Bauschinger-Effekt. Der Teil der Enthärtung kann analytisch ausgedrückt werden; zur Voraussage der Spannungs-Dehnungs-Beziehung von Stahl unter einer komplexen zyklischen Belastung ist nur die Kenntnis der Spannungs-Dehnung-Beziehung unter einseitiger Belastung notwendig.