Theme V: Rules for structural design: safety concepts

Objekttyp: **Group**

Zeitschrift: **IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen**

Band (Jahr): **13 (1973)**

PDF erstellt am: **11.07.2024**

A Probability Model for Low-Cyclic Fatigue

Un modèle de probabilité pour la fatigue à basse fréquence

Ein Wahrscheinlichkeitsmodell für nieder-zyklische Ermüdung

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Due to the random properties of material, the relationship ($\mathcal{S}$, $\mathcal{N}$) describing the dependence between the ultimate number of cycles, $\mathcal{N}$, and the ultimate load-effect, $\mathcal{S}$, is also random. Its statistical treatment makes no difficulties for middle and high ranges of load repetitions (Fig.1 b,c), however for the low-cyclic fatigue loading no method was available until now. On the other hand, a good knowledge of the ($\mathcal{S}$, $\mathcal{N}$)-relationship is necessary whenever reliabilistic considerations are to be applied to the low--cyclic problems.

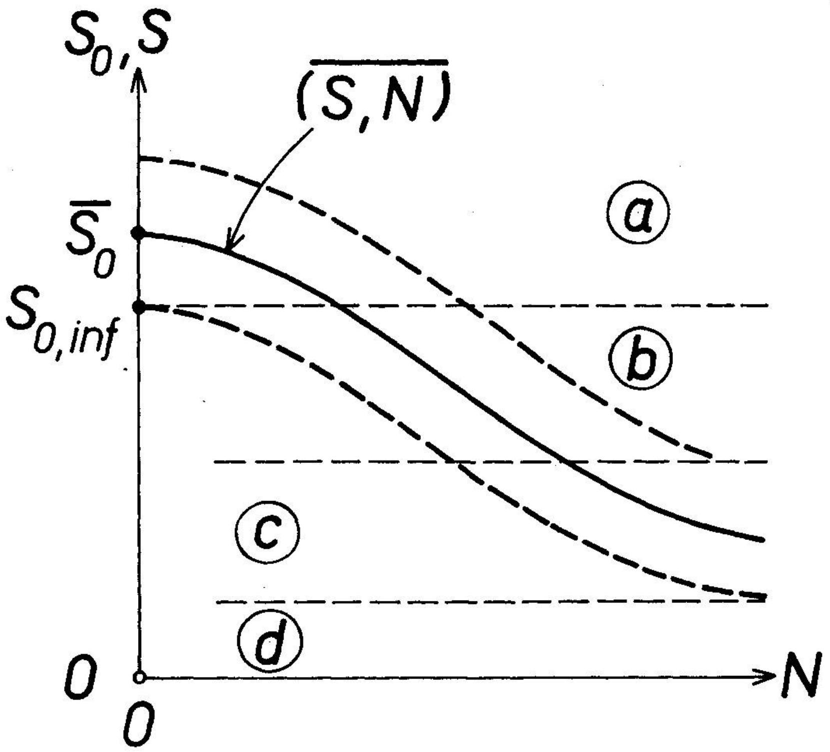

Fig.1. Fatigue ranges: (a) low-cyclic,
(b) middle-cyclic,
(c) high-cyclic,
(d) no fatigue range

The problem of fatigue is essentially two-dimensional since in the / $\mathcal{S}$, $\mathcal{N}$ / space two random variables are dealt with. According to the theory of interaction diagrams /1/ the quantiles of the ($\mathcal{S}$, N)-relationship might be found by investigating $\mathcal{S}$ under given N . Due to the nature of the phenomenon such a procedure is of course impossible and, consequently, testing of N at preliminarily selected levels of the ultimate load-effect, $\mathcal{S}$, is the only way to reach the results.

Whereas for middle and high cyclic loading only one random variable, N , enters the solution, two random variables appear in the range of low-cyclic loading. Referring to Fig.1a the low--cyclic fatigue range may be defined probabilistically as that range where under the first loading ($N = 0$) a certain probability of failure already exists. Evidently, testing a sample of specimens, if the level of fatigue loading, $\mathcal{S}_f$, is chosen so that

$$\mathcal{S}_f > \mathcal{S}_{0,inf}$$

(where $\mathcal{S}_{0,inf}$ is the lowest possible ultimate load-effect, $\mathcal{S}_0$, under single loading), certain amount of specimens fails before the $\mathcal{S}_f$ level is reached at all. Consequently, no repeated loading is possible for the failed specimens whereas the surviving specimens can be subjected to various number of load repetitions.

The ultimate number of cycles, N , is defined for a specimen as the number attached to the last completed loading cycle. It is known that fractions of cycles cannot be measured (which is particularly due to the non-linearity of the stress-strain relationship in the low-cyclic fatigue domain); the ultimate number of cycles, N , is integer. Thus, the random variable N is discrete, defined by probability mass function $p(N)$, Fig.2. On the other hand, the random variable $\mathcal{S}_0$ is obviously continuous defined by probability density function φ ($\mathcal{S}_0$).

Sumarizing these observations: the random behaviour of the ultimate load-effect at the low-cyclic loading must be modelled by a statistical distribution which consists of a continuous and a discrete portion. The random variable changes its character: in the first portion it is the ultimate load-effect (strength of material, ultimate strain, ultimate moment, etc. - according to the type of the problem), in the discrete portion it is the ulti-

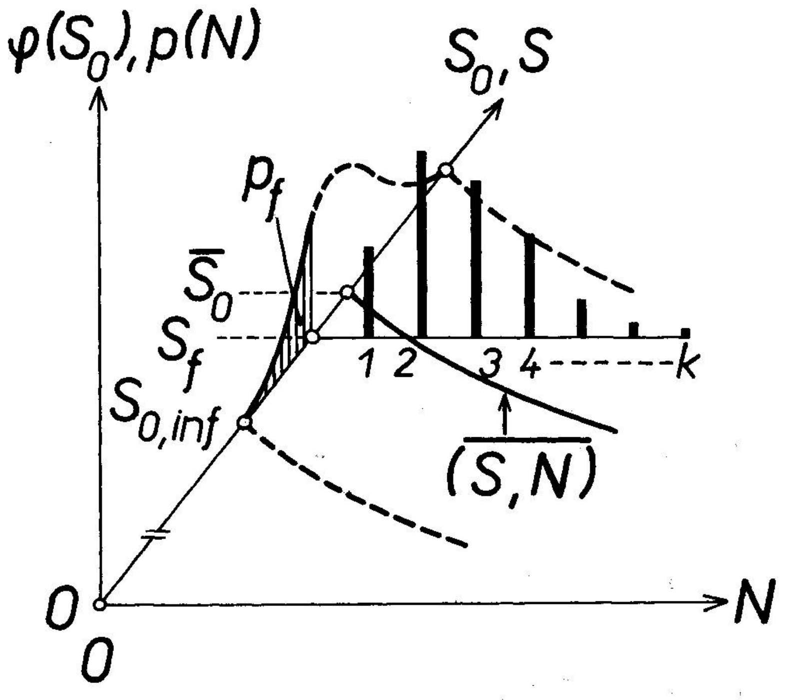

Fig.2. The broken probability distribution of the low-cyclic fatigue phenomenon

mate number of cycles, N. It is evident that both portions are mutually complementar, forming together a complete probability distribution. Let it be called "broken distribution".

The usual aim of a statistical treatment is to establish descriptors of the distribution such as the mean, variance, etc., and various quantiles, particularly the median, 0.05 quantile (characteristic strength) etc. To make this possible also with a phenomenon described by the broken distribution, the following simple procedure proves to be feasible:

Both variables, S_0 and N, are transformed by dividing individual realisations through the respective value at the break-point of the distribution, i.e. by $S_0 = S_f$ and $N = 1$, respectively. Obviously, in the continuous portion it is

$$0 \leqq S_0 / S_f \leqq 1,$$

in the discrete portion (since $N/1 = N$)

$$1 \leqq N \leqq k$$

where k is the highest observed (or possible) value of N.

The n-th general moment of the distribution is

$$\mu_n = \int_0^1 \left(\frac{S_0}{S_f}\right)^n \varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} N^n p(N),$$

hence the mean is

$$\mu_1 = \int_0^1 \frac{S_0}{S_f}\, \varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} N\, p(N)$$

and the n-th central moment

$$\nu_n = \int_0^1 \left(\frac{S_0}{S_f} - \mu_1\right) \varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} (N - \mu_1)^n\, p(N) .$$

The variance or other moment parameters can be found by usual procedure.

The above formulae can be easily adapted to formulae giving sample moments.

If quantiles for specified probabilities p are to be determined, the continuous portion of the broken distribution is used for $p \leqq p_f$, and the discrete portion for $p > p_f$; here it is

$$p_f = p\,[S_0 \leqq S_f] .$$

The type of probability distribution describing the continuous portion of the broken distribution follows from the distribution of the ultimate load-effect under single loading; e.g. normal, log-normal or any other suitable distribution may be selected. Nothing can be said at present however, about the distribution type of the ultimate number of cycles, N . In some solutions, it may prove practical to substitute the discrete distribution of N by a continuous function. Further research is here necessary.

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SUMMARY

For the statistical treatment of the low-cyclic fatigue problem a broken probability distribution can be used. It consists of a continuous portion referred to the ultimate number of cycles. Moments and other descriptors of the probability distribution can be obtained by means of transforming both variables.

RESUME

Pour l'étude statistique du problème de la fatigue à basse fréquence on peut utiliser une répartition probabiliste non continue. Ceci consiste en une partie continue relative à la charge ultime et une partie discrète relative au nombre de cycles à la rupture. Les moments et autres valeurs décrivant la répartition probabiliste peuvent être obtenus au moyen d'une transformation des deux variables.

ZUSAMMENFASSUNG

Zur statistischen Behandlung zyklischer Ermüdungsprobleme mit kleiner Amplitude kann eine diskrete Wahrscheinlichkeitsverteilung benutzt werden. Sie besteht aus einem durchgehenden Teil, der sich auf den Bruchlasteffekt bezieht, und einem diskreten Teil, der sich auf die maximale Anzahl der Zyklen bezieht. Momente und andere Indikatoren der Wahrscheinlichkeitsverteilung lassen sich durch Transformieren der beiden Variablen bestimmen.

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A Design Method using Weighted Fractiles as Design Values

Une méthode de dimensionnement utilisant des valeurs pondérées

Eine Bemessungsmethode unter Benutzung gewichteter Fraktile

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INTRODUCTION

The theoretical basis of the design method by which weighted fractiles are used as design values was presented in [4] and it is thus unnecessary to study its derivation here in detail. The main extension by the present contribution compared with the former is the application of the method to load-effects which are variable in time. This study is included in the latter part of the paper.

There are anyhow good reasons for presenting the main principles of this design method generally first.

DESCRIPTION OF THE METHOD

We study a general case of a design problem in which the condition for failure can be described by the corresponding inequality:

$$g(X_1,...,X_n) \leq 0 \qquad ... (1)$$

where $g(\cdot)$ = the corresponding limit state function, obtainable from the handbooks of statics; $X_1,...,X_n$ = the various quantities such as properties of materials, dimensions, loads or load-effects. All these quantities,invariable in time are random variables and we assume that their distributions are given in the standards.

As a design criterion we write

$$p_f = P(g(X_1,...,X_n) \leq \;) \leq p_{fa} \qquad ... (2)$$

where p_f = failure probability, which simply means that the failure probability should be p_{fa} at most. The p_{fa} - values should be given in standards and depend on the type of structure.

As will be shown later, the method can also be applied to a case when the distributions are unknown, and we define the level of reliability not by p_{fa}, but by β, defined by

$$m_Z - \beta \cdot \sigma_Z = 0 \qquad ... (3)$$

where

$$Z = g(X_1,...,X_n) \qquad ... (4)$$

and m_Z, σ_Z are the mean value and standard deviation of Z.

With distributions differing from the normal distribution, this simplification leads to considerable errors, in the aimed reliability.

We write the design equation in a deterministic form

$$g(x_1^*,\ldots,x_n^*) = 0 \qquad \ldots (5)$$

with such design values that (2) is valid. As was shown in [4] there are several combinations of values $x_1^*,\ldots,x_n^*$ which satisfy the equation (5). We choose the following values for use in (5):

$$x_i^* = m_i - \beta_i \cdot \alpha_i \cdot \sigma_i \qquad \ldots (6)$$

where m_i and σ_i are the mean and st.d. of X_i and the parameters α_i and β_i are defined as follows:

The parameters β_i describe the desired level of reliability, according to the form of the corresponding distribution:

$$F_i(m_i - \beta_i^- \cdot \sigma_i) = p_{fa} \qquad \ldots (7a)$$

$$1-F_i(m_i + \beta_i^+ \cdot \sigma_i) = p_{fa} \qquad \ldots (7b)$$

The parameters α_i describe the significance of the corresponding quantity and are defined as follows:

$$\alpha_i = a_i \,/\, \left(\sum_{j=1}^{n} a_j^2\right)^{1/2} \qquad \ldots (8)$$

where

$$a_j = \beta_j \cdot \sigma_j \cdot \partial g/\partial x_j \qquad \ldots (9)$$

From (9) it can be noted that the significance of the different quantities is thus proportional to the distance of the p_{fa} - fractile from the mean and to the derivative $\partial g/\partial x_j$. In the design, the m_i - values are chosen so that using the design values defined by (6), (5) is valid. Choice of the design values in this way is the main principle of the design method presented.

In cases in which the distributions of the different quantities are unknown the method can easily be applied distribution-free using the following relations:

$$x_i^* = m_i - \beta \cdot \alpha_i \cdot \sigma_i \qquad \ldots (10)$$

where α_i is defined by (8) but

$$a_i = \frac{\partial g}{\partial x_i} \cdot \sigma_i \qquad \ldots (11)$$

In this case the level of the reliability is not given by p_{fa} but by β, in the way shown in (3).

CHARACTERISTICS OF THE METHOD

Before studying the application of the method in cases which also include variable quantities, it is necessary to study some special features of this method.

Firstly, using (5) and (6) as design equations, it is not necessary to know the values of α_i exactly, while

$$\left.\frac{\partial p_f}{\partial a_i}\right|_{\vec{x} = \vec{x}^*} = 0 \qquad \text{by } i = 1,\ldots,n \qquad \ldots (12)$$

and therefore small variations of $\partial g/\partial x_i$, β_i and σ_i in definition of the parameters α_i do not influence the results.

It is only significant to note that always

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \qquad \ldots (13)$$

With (8) and (13), the α-values can be estimated so that they correspond approximately to the significance of the various quantities. A more exact method is division of function g(•) into subfunctions.

If we assume that Y_i i = 1,...,m are the subfunctions of g(•), according to (6), we obtain

$$y_i^* = m_i - \beta_i \cdot \alpha_i \cdot \sigma_i \qquad \ldots (14)$$

This is, however, only a p_{fai} - fractile of the quantity Y_i and we can treat the quantities X_{ij} included in it in a way analogous to the way Y_i was treated. We then obtain

$$x_{ij}^* = m_{ij} - \beta_{j/i} \cdot \alpha_{j/i} \cdot \sigma_{ij} \qquad \ldots (15)$$

where

$$F_{ij}(m_{ij} - \beta_{j/i} \cdot \sigma_{ij}) = p_{fai} \qquad \ldots (16a)$$

$$1 - F_{ij}(m_{ij} + \beta_{j/i} \cdot \sigma_{ij}) = p_{fai} \qquad \ldots (16b)$$

$$\alpha_{j/i} = a_{j/i} / (\sum_{k=1}^{n} a_{k/i}^2)^{1/2} \qquad \ldots (17)$$

where

$$a_{k/i} = \frac{\partial y_i}{\partial x_{ik}} \cdot \beta_{k/i} \cdot \sigma_{ik} \qquad \ldots (18)$$

Should Y_i and X_{ij} have approximately the same distribution, we have

$$\beta_{j/i} = \beta_i \cdot \alpha_i = \beta_{ij} \cdot \alpha_i \qquad \ldots (19)$$

where β_{ij} is defined as in (7). In this way we obtain the design values

$$x_{ij}^* = m_{ij} - \beta_{ij} \cdot \alpha_i \cdot \alpha_{j/i} \cdot \sigma_{ij} \qquad \ldots (20)$$

where β_{ij} is defined as in (7), $\alpha_{j/i}$ as in (17) and α_i by (8) where

$$a_j = \frac{\partial g}{\partial y_i} \cdot \beta_j \cdot \sigma_j \qquad \text{... (21)}$$

It should be noted that the definition of α_i - values is approximate but because of (12) the small errors are not significant.

APPLICATION TO VARIABLE LOADS

All the quantities in the design criterion (1) are assumed to have distributions which remain invariable with time. We will now study a case in which some of these quantities are variable in time.

We first make a limitation concerning the type of these quantities, the variable load-effects. We assume that they all have the same dimension i.e. they are all e.g. moments or normal forces. Secondly, we assume that all the variable quantities have, to use the terminology of Ferry Borges [3] , the same duration of elementary interval and the same number of independent repetitions. These are both considerable simplifications of the general case treated earlier in [1] .

We assume now that the number of independent repetitions, r, is given, as well as the types of the momentary distributions of the variable loads as fundamental information in standards. To be on the safe side, it seems better to assume that the number r is rather small and to define the momentary distribution as the distribution of the maximum-value in the corresponding, arbitrary elementary interval.

The definition of the design values will now be made stepwise, so that the invariable quantities are first combined with the extreme-value of the combination of the different, momentary variable load -effects.

The momentary distribution of the combination can be trivially defined from the distributions of the various, usually additional quantities. After that we only need define approximately the mean and standard deviation of the distribution of the extreme-value.

We combine this approximate distribution with the distributions of the invariable quantities, and obtain with (8) the α-parameters for the different invariable quantities and for the extreme-value of the combination of the variable quantities.

The design values of the invariable quantities can then be defined with (6). We further study the approximate design value of the extreme value of the sum of the variable quantities:

$$y^* = m_y - \beta_y \cdot \alpha_y \cdot \sigma_y \qquad \text{... (21)}$$

In (21) all the parameters are approximate, but using (7) we know that

$$1 - F_y(m_y + \beta_y \cdot \sigma_y) = p_{fa} \qquad \text{... (22)}$$

According to the form of the distribution we may now solve a new probability:

$$1-F_y(m_y+\beta_y\cdot\alpha_y\cdot\sigma_y) = p_{fay} > p_{fa} \qquad \ldots (23)$$

It should be noted that in many cases the relation p_{fay}: p_{fa} is independent of the values m_y and σ_y, e.g. with the extreme type 1 distribution. Because of (12) and (13) the slight errors in α are not significant. For these reasons we can note that in spite of the approximate way of defining α_y, we have a rather reliable value, p_{fay} as a basis for the determination of the design values of the variable quantities.

The second step is to study the momentary combination of the different variable loads. The p_{fay} - fractile with the extreme distribution corresponds to

$$1-(1-p_{fay})^{1/r} \approx p_{fay}/r \text{ - fractile} \qquad \ldots (24)$$

with the momentary distribution. We denote this with

$$p_{fay}/r = p_{fam} \qquad \ldots (25)$$

The design values of the different variable quantities may now be defined as earlier:

$$x_i^* = m_i-\beta_i\cdot\alpha_i\cdot\sigma_i \qquad \ldots (26)$$

where m_i and σ_i are the mean value and the standard deviation of the different momentary distributions. The other parameters are defined as follows:

$$1-F(m_i+\beta_i\cdot\sigma_i) = p_{fam} \qquad \ldots (27)$$

$$\alpha_i = a_i/(\sum_{j=1}^{m} a_j^2)^{1/2} \qquad \ldots (28)$$

where $a_j = \frac{\partial y}{\partial x_j}\cdot\beta_j\cdot\sigma_j \qquad \ldots (29)$

in which y simply means the sum $x_1+,\ldots,+x_m$, and m is the number of the variable loads.

EXAMPLE

We now study a simple function $g(\cdot)$ with resistance R, invariable load-effect S_g, and two variable load-effects S_{p1} and S_{p2}:

$$g(\cdot) = R-S_g-S_{p1}-S_{p2} \leq 0 \qquad \ldots (30)$$

We take as fundamental information, which is supposed to be given in the standards, the following values:

$$p_{fa} = 10^{-6} \quad ; \quad r = 100$$

Further, we assume that the variation-coefficients and the types of distributions are given in table 1. The dependence p_{fa}/β is given in fig. 1.

		Type of distr.	m_i	V_i
R	X_1	Log.normal	6,0	0,10
S_g	X_2	Normal	1,0	0,05
S_{p1}	X_3	Extreme I	1,0	0,15
S_{p2}	X_4	Extreme I	0,5	0,10

Table 1

	m_i	β_i	σ_i	$\beta_i \cdot \sigma_i$	α_i	x_i^*
X_1	5,9	3,8	0,59	2,24	0,81	4,08
X_2	1,0	4,7	0,05	0,24	0,08	1,02
X_3+X_4	2,1	10,4	0.16	1,67	0,59	
X_3	1,0	9,7	0,15	1,46	0,95	2,39
X_4	0,5	9,7	0,05	0,49	0,32	0,66

Table 2 $\Phi = 2,36$

	m_i	β_i	σ_i	$\beta_i \cdot \sigma_i$	α_i	x_i^*
X_1	5,0	3,8	0,50	1,90	0,75	3,57
X_2	1,0	4,7	0,05	0,24	0,10	1,02
X_3	1,0	10,4	0,15	1,56	0,62	1,97
X_4	0,5	10,4	0,05	0,52	0,21	0,61

Table 3 $\Phi = 2,00$

	m_i	β_i	σ_i	$\beta_i \cdot \sigma_i$	α_i	x_i^*
X_1	6,0	3,8	0,60	2,28	0,82	4,13
X_2	1,0	4,7	0,05	0,24	0,08	1,02
X_3	1,52	10,4	0,15	1,56	0,54	2,36
X_4	0,67	10,4	0,05	0,52	0,18	0,76

Table 4 $\Phi = 2,40$

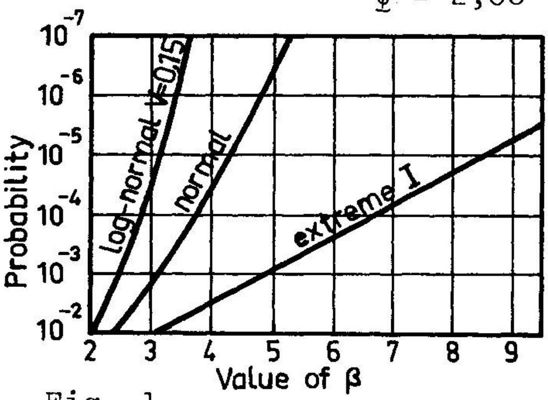

Fig. 1.

We first obtain as mean and standard deviation of $(S_{p1}+S_{p2})_e$:

$$m = 2,1 \quad ; \quad \sigma = 0,16$$

where index e indicates the extreme. According to (25) and (27) we obtain

$$p_{fam} = 4 \cdot 10^{-6} \quad ; \quad \beta_3 = \beta_4 = 9,7$$

and the results with (26) are given in table 2.

After this, the same example is calculated in two different ways. Firstly, by studying only the combination of the momentary load-effects. This is a kind of lower-bound solution which approaches the "exact" solution with small values or r. The results of this calculation are given in table 3. Secondly, we study the combination of the distributions of the extreme values, which contrarily to the former case is an upper bound solution and is approached by the "exact" solution in cases in which the significance of the variable loads is concentrated in one of them. In our example, it can be seen that the significance of quantity X_3 is superior in relation to X_4 and therefore the results in table 2 and table 4 are rather close to each other.

The differences between the cases may be described by the fictive central safety-factors Φ corresponding to the mean values obtained in the different cases.

CONCLUSIONS

The present design method implies standardized distributions for the most usual quantities needed in the design of structures. This will anyhow be a requirement in future irrespective of the type of design method chosen. Statistical research on the types of distributions of the various quantities is therefore necessary and useful.

The method has several applications, some of which are more exact but complicated and the others which are simplier. These will be presented on a larger scale in another paper to be published later.

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SUMMARY

A statistical design method is presented which is characterized by using weighted fractiles as design values. This paper gives special attention to the application in which quantities invariable in time are combined with quantities variable in time, primarly load-effects. An example is also presented to illustrate the method.

RESUME

On présente une méthode statistique de dimensionnement utilisant des valeurs pondérées en-dessous d'un certain seuil de la courbe de fréquence (courbe de Gauss) comme grandeurs de dimensionnement. Ce travail traite tout spécialement le cas de grandeurs invariables dans le temps combinées avec des grandeurs variables, en premier lieu les influences de la charge. Un exemple numérique est présenté pour illustrer la méthode.

ZUSAMMENFASSUNG

Es wird eine statistische Bemessungsmethode gezeigt, die durch gewichtete Fraktilen als Bemessungsgrössen charakterisiert ist. Die Abhandlung beachtet speziell jene Anwendung, bei der die zeitunabhängigen Grössen mit zeitabhängigen Grössen kombiniert werden, hauptsächlich bei Belastungseffekten. Ein Beispiel veranschaulicht die Methode.

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Ordnungsstatistik in der Zuverlässigkeitsbeurteilung von Tragwerken unter wiederholter Belastung

Order Statistics in Reliability Assessment of Structures under Repeated Loadings

"Statistique d'ordre" pour l'appréciation de la fiabilité des charpentes soumises à des charges répétées

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1. EINFÜHRUNG

Das Problem der Zuverlässigkeitsbeurteilung von ermüdungsempfindlichen Tragwerken wird vielfach durch die Anwendung derselben Konzepte behandelt, die für die einmalige Aufbringung einer Last (Bruchlast) ausreichend sind.

Die Zuverlässigkeit unter Ermüdungsbelastung kann durch diese Methoden nicht befriedigend gelöst werden und zwar aus dem einfachen Grund, da die Verteilung von Ermüdungslebensdauer unter variablen (und sogar unter konstanten) Belastungsamplituden nicht mit der Verteilung der einer gewissen Lebensdauer zugeordneten Festigkeit, welche tatsächlich die Verteilung der "Residualfestigkeit" ist, in Beziehung gebracht werden kann.[1]

Daher muß die Zuverlässigkeit bei Ermüdungsbeanspruchung mit der Verteilung der Ermüdungslebensdauer, mit der Vorhersage der Ordnungsstatistik, bekannt als die "Zeit bis zum ersten Versagen"[2] und dem mit dieser Vorhersage verbundenen Risiko in Zusammenhang gebracht werden.

Die Zuverlässigkeitsanalyse, die auf dem Begriff der "Zeit bis zum ersten Versagen" aufbaut, ist besonders relevant bei Tragwerken, für welche die Vermeidung eines einzigen katastrophalen Versagens als primäres Bemessungskriterium gilt. Das Versagenskriterium kann aber hier auch allgemein als der Beginn einer Rißbildung oder die Erreichung eines bestimmten Zerstörungsniveaus festgelegt werden.

Das vorhin erwähnte Verfahren, das schon seit einiger Zeit zur Flugtragwerksbemessung verwendet wird, ist aber auch im Bauingenieurwesen von großer Wichtigkeit, da das Auswechseln von ermüdungsgeschädigten Konstruktionselementen z.B. bei Brücken oder Hochhäusern zum Teil mit viel größeren Schwierigkeiten und Kosten verbunden ist als bei Flugtragwerken.

Das Konzept der Zeit bis zum ersten Versagen hat mehrere praktische Vorteile. Diese sind u.a. die Möglichkeit der tatsächlichen Überprüfung während des Betriebes, die starke Beeinflussung der Ergebnisse durch verschiedene Verteilungen der Lebensdauer, die automatisch die Unterschiede der verschiedenen Versagensmechanismen reflektieren, und die Abhängigkeit von der Anzahl der betrachteten Konstruktionsdetails.

2. WAHRSCHEINLICHKEITSVERTEILUNG DER ERMÜDUNGSLEBENSDAUER

Die Anzahl der Material- und Lebensdauerversuche von Tragwerken, die technisch und wirtschaftlich durchführbar sind, schließt eine Ableitung einer Verteilungsfunktion der Ermüdungslebensdauer aus Beobachtungen mit Methoden der statistischen Inferenz aus.

Es ist daher notwendig, aufgrund physikalischer Überlegungen probabilistische Modelle auszuwählen, die den physikalischen Phänomenen entsprechen. Erst dadurch wird eine Extrapolation von einer kleinen Anzahl von Versuchsergebnissen auf die für die Zuverlässigkeitsanalyse wichtigen niederen Bereiche vertretbar.

Die physikalische Annahme der mit zunehmender Anzahl von Lastaufbringungen zunehmenden Rißbildung kann in die probabilistische Form einer mit zunehmender Zeit t ansteigenden "Risikorate"[3]

$$Z(t) = \frac{f(t)}{1 - F(t)} \qquad (1)$$

gebracht werden.

In Gl.(1) ist f(t) die Wahrscheinlichkeitsdichte und F(t) die Summenverteilung.

Die Risikofunktionen verschiedener Verteilungsfunktionen haben einen sehr unterschiedlichen Charakter. Während die Funktion z.B. für eine Gammaverteilung asymptotisch zu einem konstanten Wert ansteigt, nimmt die Risikofunktion der Lognormalverteilung nach anfänglicher Zunahme wieder ab und konvergiert mit zunehmender Zeit langsam zu Null. Die zwei-Parameter-Weibullverteilung (asymptotische Verteilungsfunktion der kleinsten Werte),

$$F(y) = 1 - \exp\left[-\left(\frac{y}{\beta}\right)^{\alpha}\right] \text{ für unbekannte } \alpha,\ \beta > 0, \qquad (2)$$

hingegen weist eine mit zunehmender Zeit monoton ansteigende Risikofunktion auf und entspricht daher den physikalischen Gegebenheiten am besten, da sich mit zunehmender Zeit, die eine zunehmende Rißbildung hervorruft, das Versagensrisiko erhöht.

Daher wurde den weiteren Ausführungen die Weibullverteilung als Modell der Ermüdungslebensdauer zugrunde gelegt.

3. FORMULIERUNG DES ORDNUNGSSTATISTISCHEN PROBLEMS

Aufgrund eines umfassenden experimentellen und analytischen Programms wurde in Ref.4 festgestellt, daß für eine große Mannigfaltigkeit von Tragwerken oder Tragwerksdetails und Belastungsarten die Streuungen der Logarithmen der Zeiten bis zum Versagen nahezu konstant sind. Dies bedeutet, speziell auf Gl.(2) bezogen, daß der streuungsbestimmende Formparameter α der Weibullverteilung einen konstanten Wert für eine Reihe von verschiedenen Bedingungen hat. Dieser unbekannte Wert der Weibullform α wurde durch eine Versuchsreihe von mehr als tausend anwendbaren Stichproben geschätzt und seine direkte Abhängigkeit von der Art des Materials festgestellt (für Aluminium $\alpha = 4$). Wenn dieser Formparameter genau festliegt, kann er für die weiteren Berechnungen als konstanter Wert angenommen werden.

Die zentrale Tendenz der Grundgesamtheit oder Größenparameter β in Gl.(2) wird vorteilhaft durch Ermüdungsversuche an Modellen in Originalgröße bestimmt. Von den verschiedenen für die statistische Schätzung der Parameter der Weibullverteilung anwendbaren Methoden wird von Mann[5] die "Maximum Likelihood" Schätzung als die beste rekommendiert. Die Punktschätzung der charakteristischen Weibull-Lebensdauer β mit bekanntem α ist:

$$\hat{\beta} = \left[\frac{1}{n_f}\left(\sum_{i=1}^{n_f} Y_i^{\alpha}\right)\right]^{1/\alpha} \qquad (3)$$

hierin bedeuten n_f die Anzahl der Versagensbeobachtungen in einer Versuchsstichprobe und Y_i die Variable der Zeit bis zum Versagen.

Gl.(3) ergibt eine Schätzung mit minimaler Varianz und ohne bias mit bekannter Stichprobenverteilung. In diesem Fall kann auch, wenn gewünscht, der niedere Grenzwert der Intervallschätzung berechnet werden. Die Stichprobenverteilung der Weibullschätzung $\hat{\beta}$ wird dann in Form eines Vertrauensniveaus, das auch die Stichprobengröße in Betracht zieht, angegeben.

Wenn nun die Zuverlässigkeit R eines aus einer Grundgesamtheit willkürlich gewählten Tragwerks oder Tragwerkdetails als ein geeignetes Maß für die Sicherheit oder reparaturfreie Periode beurteilt wird, würde die Angabe eines akzeptierbaren Wertes R genügen, um eine "sichere Lebensdauer" zu gewährleisten. Als "sichere Lebensdauer" wird hier jene Lebensdauer betrachtet, bei welcher die Wahrscheinlichkeit des anfänglichen Auftretens von Ermüdungsrissen in einem Detail genügend klein ist.

In vielen oder möglicherweise allen Fällen ist die willkürliche Festlegung eines annehmbaren Wertes R eine nicht ausreichende Grundlage für die Bestätigung einer sicheren Lebensdauer für eine Anzahl von Details oder Tragwerken.

Im Hinblick auf den sicheren oder reparaturfreien Betrieb von Tragwerken oder Details erwartet man für die ganze oder nahezu ganze Anzahl von Elementen eine akzeptierbare Zuverlässigkeit.

In Bezug zur sprichtwörtlichen Kette, die nicht stärker als ihr schwächstes Glied sein kann, ist es nur eine Frage der Definition, die Zuverlässigkeit der Mehrheit der Elemente mit der Zuverlässigkeit des schwächsten der (angenommenen) unabhängigen nominell identischen Elemente gleichzusetzen. Es ist daher notwendig, die Verteilung der Ermüdungslebensdauer dieses schwächsten Gliedes zu bestimmen und aus dieser dann eine sichere Lebensdauer abzuleiten. Diese neue Zufallsvariable ist ein Beispiel einer Ordnungsstatistik.

Glücklicherweise ist bei Unabhängigkeit der Versuche die genaue Berechnung der ordnungsstatistischen Verteilung von irgendeiner angenommenen, geschätzten oder bekannten Verteilung der Grundgesamtheit relativ einfach.

Aus grundlegenden ordnungsstatistischen Betrachtungen erhält man

$$1 - F_{Y_1}(y_1) = [1 - F_Y(y)]^N \qquad (4)$$

wobei F_{Y_1} die Summenverteilung der Zeit bis zum ersten Versagen, F_Y die Summenverteilung der Grundgesamtheit und N die Anzahl der betrachteten Details oder Tragwerke bedeuten.

Speziell für die Weibullverteilung (Gl.2) ist

$$1 - F_{Y_1}(y_1) = \exp\left[-\frac{y_1}{\beta_1}\right]^{\alpha} \qquad (5)$$

hierin bedeutet $\beta_1 = \beta N^{-\frac{1}{\alpha}}$. Dies zeigt, daß für diese Verteilung die Funktion $(1 - F_{Y_1}(y_1))$ selben Extremaltyps ist wie $(1 - F_Y(y))$ und zwar mit identischem Formparameter α, aber mit einem um

($N^{-\frac{1}{\alpha}}$) reduzierten charakteristischen Wert ß.
Gl.(5) ergibt nach y_1 aufgelöst:

$$Y_1 = \beta_1 \{\ell n(\frac{1}{1-F_{Y_1}(Y_1)})\}^{1/\alpha} \tag{6}$$

Setzt man nun für $1 - F_{Y_1}(y_1) = R$ in die obige Gleichung, so erhält man:

$$Y_R = \beta_1 \{\ell n(\frac{1}{R})\}^{1/\alpha} \tag{7}$$

wobei Y_R gleich der sicheren Lebensdauer für eine bestimmte Zuverlässigkeit R ist. Gl.(7) kann auch als "umgekehrte Zuverlässigkeitsfunktion" bezeichnet werden.

4. DER STREUUNGSFAKTOR

Die im Kapitel 3 entwickelten Formulierungen finden in der Bemessung ermüdungsbeanspruchter Tragwerke oder Details in Form eines sogenannten "Streuungsfaktor" ihren Niederschlag[6]. Dieser ist das Verhältnis zwischen getesteter oder berechneter Durchschnittslebensdauer oder charakteristischer Lebensdauer und der Bemessungs- oder sicheren Lebensdauer.

Der für die Anzahl der ermüdungsbeanspruchten Details und spezifizierte Zuverlässigkeit Rechnung tragende Streuungsfaktor ist

$$S_R = \frac{\hat{\beta}}{Y_R} \tag{8}$$

wie auch in Bild 1 gezeigt wird.

5. ERGEBNISSE UND SCHLUSSFOLGERUNGEN

Die zur Durchführung der Analyse notwendigen experimentellen Daten wurden aus Ref.4 entnommen. Bild 2 zeigt jene Einflüsse an, welche die Anzahl der Details, das Zuverlässigkeitsniveau und der Formparameter der Weibullverteilung auf den Streuungsfaktor ausüben. Hierbei wird die starke Abhängigkeit des Streuungsfaktors von der Anzahl der Details, die im Betrieb der Ermüdungsbeanspruchung ausgesetzt sind, ersichtlich. Zusätzlich sei hier bemerkt, daß dieses Verhältnis logarithmisch linearer Natur ist. Die Ergebnisse zeigen weiterhin deutlich die Möglichkeit und den Vorteil einer direkten Einflußnahme auf das Risikoniveau bei der Bemessung ermüdungsbeanspruchter Tragwerke.

Das zur Bestimmung des Formparameters α in Ref.4 für Aluminium durchgeführte Versuchsprogramm (α = 4.o) sollte auch auf konventionelle Baustähle ausgedehnt werden.

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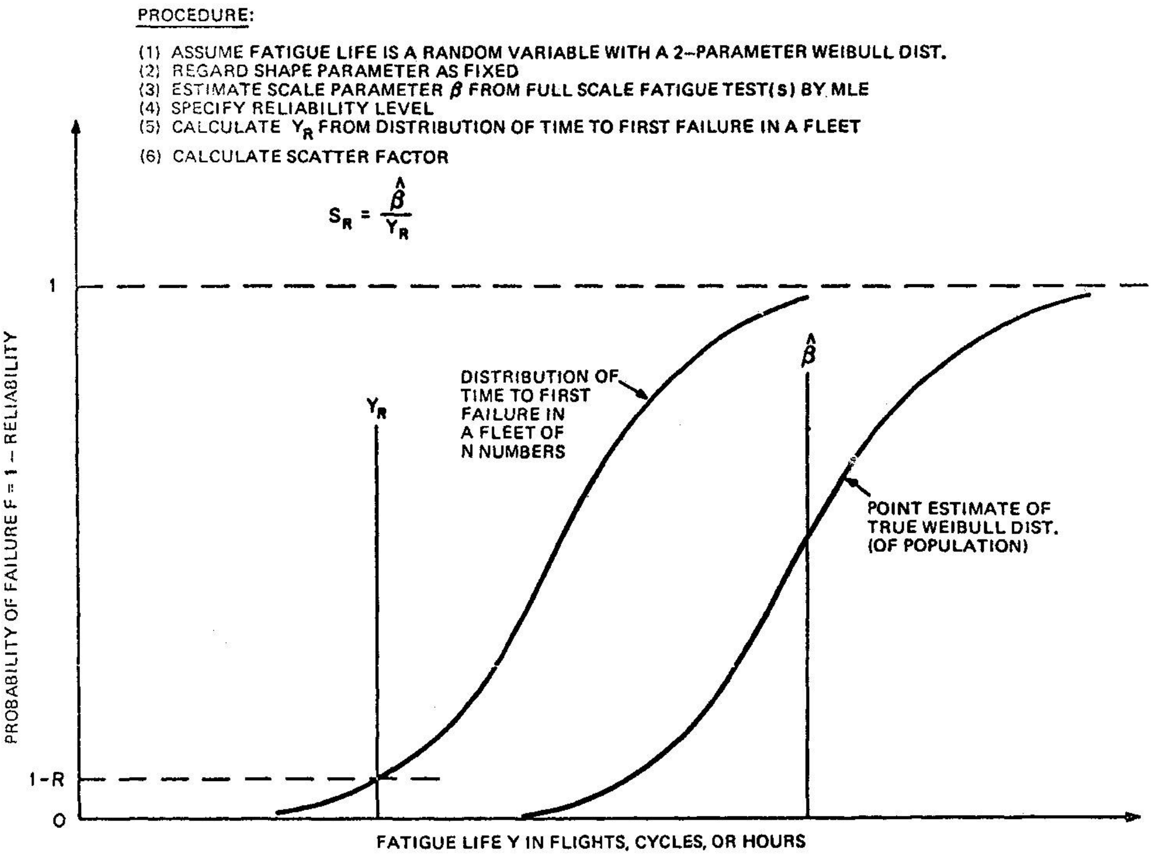

FIGURE 1 SCHEMATIC REPRESENTATION OF RELIABILITY PLAN

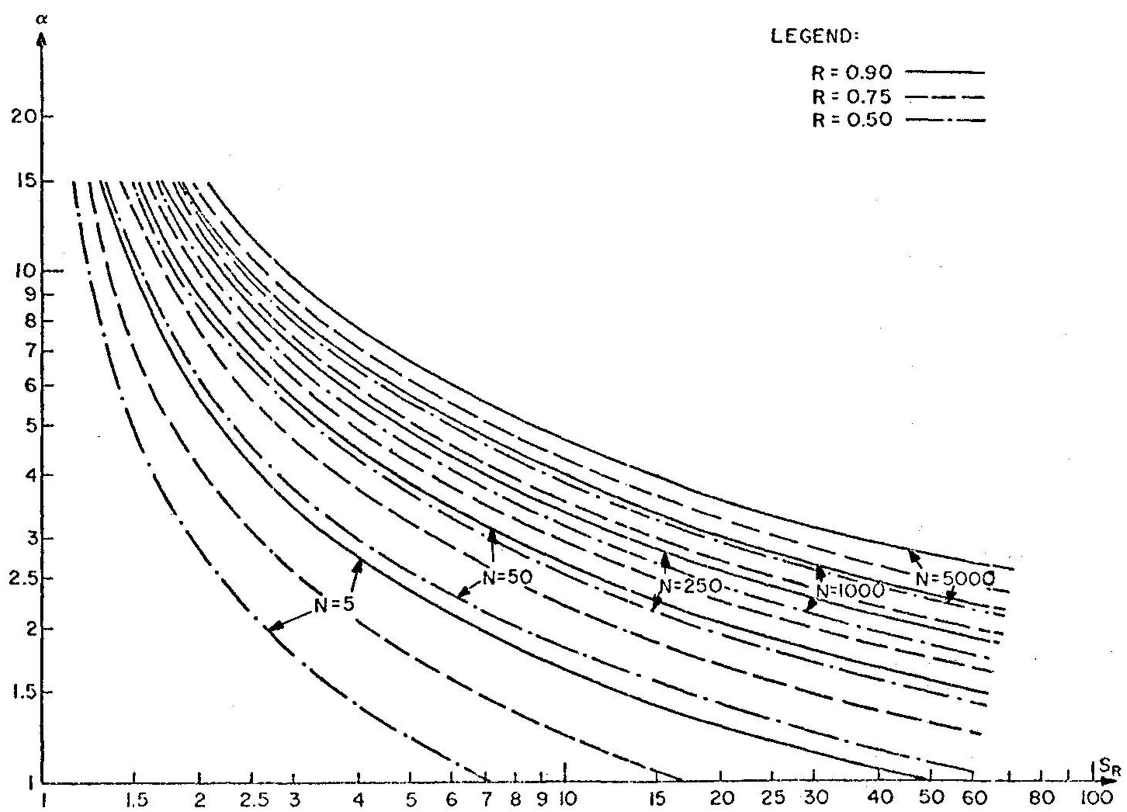

FIGURE 2 SHAPE PARAMETER VERSUS SCATTER FACTOR

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ZUSAMMENFASSUNG

Eine ordnungsstatistische Methode der Risikobeurteilung von Tragwerken, die wiederholten Belastungen ausgesetzt sind, wird vorgeschlagen. Die zwei-Parameter Weibullverteilung (asymptotische Verteilung der kleinsten Werte) wird als probabilistisches Modell der Ermüdungslebensdauer verwendet. Der Begriff der "Zeit bis zum ersten Versagen" wird für die Berechnung der sicheren Lebensdauer angewandt. Das Verhältnis dieser beiden Variablen wird als "Streuungsfaktor" bezeichnet und in der tatsächlichen Bemessung verwendet. Ein numerisches Beispiel zeigt das gegenseitige Verhältnis zwischen dem Formparameter der Weibullverteilung, der Anzahl der einer Ermüdungsbeanspruchung ausgesetzten Details und der Zuverlässigkeit auf.

SUMMARY

An orderstatistical approach in the risk assessment of structures which are subjected to repeated loadings is proposed. The two-parameter Weibull distribution (asymptotic distribution of the smallest values) is utilized to model fatigue life. The concept of the time to first failure is applied to calculate the certifiable life. The ratio of these two variables is then termed as "scatterfactor", to be used in actual design. A numerical example shows the interrelationship between the shape parameter of the Weibull distribution, the fleetsize and the reliability.

RESUME

On propose dans ce travail une méthode de statistiques d'ordre pour juger les risques auxquels est soumise une structure sous l'action de charges répétées. La répartition à deux paramètres de Weibull (répartition asymptotique des plus petites valeurs) est utilisée comme modèle probabiliste de la résistance à la fatigue. Le concepte de "temps jusqu'à la première fissure" est employé pour le calcul de la durée de vie la plus sûre de la structure. Le rapport de ces deux variables est appelé coefficient de dispertion et est utilisé pour le dimensionnement réel. Un exemple numérique montre le rapport réciproque entre le paramètre de forme de la répartition de Weibull, le nombre d'éléments soumis à un effort de fatigue et la sûreté de la méthode.

V

Representation for Dynamic Loading

Représentation pour les sollicitations dynamiques

Darstellung für dynamische Beanspruchung

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1. Introduction

It is well known that structural behaviour can be qualitatively different for loads with different space distribution or time history. This fact is obvious especially for high stress levels, characterised by non-linear behaviour, and for the nature of limit-states. Failure types like those due to fatigue, to buckling, to strong shocks, etc., illustrate this statement. It could be presumed that for loading patterns that are only slightly different structural behaviour should be in almost cases no more strongly different (a well known exception: behaviour of two identical columns, one of them subjected to pure compression, the second to compression and small lateral forces). A challenge is raised: how to define a general tool for measuring a difference or a "distance" between two loading patterns and between corresponding behaviour modes of a structure? The question is of special interest for dynamic loads, that are basically multi-parameter loads. The fact that several parameters have to be considered in any case of dynamic loading makes this discussion, the main object of which is the use of the above mentioned measuring tool and its implications for design philosophy, especially appropriate for dynamic loading.

2. The space of loadings associated with a structure

The representation of material or structural characteristics or behaviour is often related to quantities like stresses at a point, internal forces at a member section, etc. The loadings will be chosen as basic parameters for this purpose throughout the paper, because they permit to represent characteristics and phenomena which could not be related to previous parameters (stresses, internal forces), especially for high loading levels (e.g. in case of plastic behaviour).

The set [S] of loadings S possibly acting on a structure will be imagined. Two elements of this set, S' and S", can differ by the system of points of application, by the direction and intensity of some forces or of some imposed displacements, by the time history. Two loadings, S' and S", given, their sum, S=S'+S", will be defined by a loading consisting of all the forces of S' and of all the forces of S". The product of a loading S with a scalar number q, q.S, will be a loading having forces q times greater than S. Two loadings, S' and S", given, a linear combination of them, q'S'+q"S", will be also a loading. A general property is put to evidence:

L_1: The set of loadings possibly acting on a structure is a linear space.

A new general property can be also put to evidence:

L_2: Two loadings, S' and S", given, a scalar product of them, (S'.S"), can be defined by the relation

$$(S'.S'') = \frac{1}{T_o}\int_o^{T_o}\left[\int_V \sum_{i,j}^{1,n} s'_{ij}(x_k,t).e''_{ij}(x_k,t)\, dV(x_k)\right] dt \qquad (1)$$

where T_o represents the length of a time interval that covers the life of the structure dealt with, while $s'_{ij}(x_k,t)$ and $e''_{ij}(x_k,t)$ represent stresses and strains (linearly determined) corresponding to the loadings S' and S".

The set of loadings, [S], that satisfies the properties L_1 and L_2, is a Hilbert space.

The norm of a loading, $\|S\|$, will be given by the relation

$$\|S\| = \sqrt{(S.S)} \qquad (2)$$

and the distance between two loadings, d(S'.S"), will be given by the relation

$$d(S',S'') = \|S' - S''\| \qquad (3)$$

A small distance d(S',S") means that the two loadings are practically not different from the view point of space distribution and time history. n linearly independent loadings, S_i (i=1...n) once given, they can define a base of an n-dimensional Euclidean sub-space, $[S]_n$, of [S]. Any loading of this sub-space can be expressed as a linear combination,

$$S = \sum_i^{1,n} q_i S_i \qquad (4)$$

An arbitrary loading S given, a best approximation of it by elements of the sub-space $[S]_n$ could be defined by the relation

$$\|S - \sum_i^{1,n} q_i S_i\| = \min. \qquad (5)$$

which permits a determination of the coordinates q_i.

The space [S] permits a representation not only of loadings, but also of structural characteristics. If structural behaviour corresponding to a loading (or a point) S is considered, sets of loadings for which structural behaviour is similar are represented by domains B_1 associated with certain states of stress. Boundaries R_1 of such domains are corresponding to limit-states. As and example, the states of stress corresponding to an elastic-plastic frame are represented in fig. 1 Structural behaviour for a given load corresponds to the domain to which the point belongs.

A random loading is represented by a random point S. In case of the use of an n-dimensional sub-space $[S]_n$, the distribution can be represented by the probability density $g(q_i)$. Random structural characteristics are represented by the fact that the location of domains B_1 is random, i.e. a fixed point S can belong randomly to one or another of the domains B_1. The probabilities $F_1(s)$ of not exceeding the limit-state R_1 for a loading S=s represent structural characteristics from a stochastic view point (they can be dealt with

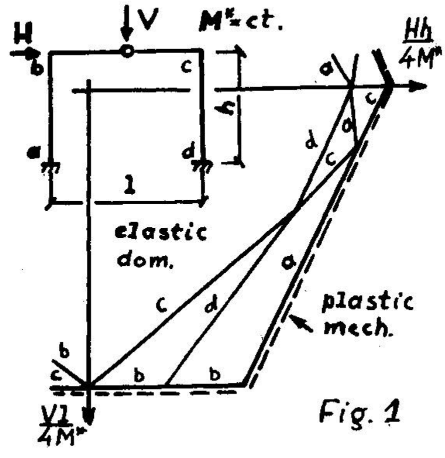

Fig. 1

as conditional probabilities: probabilities of not exceeding R_1 <u>if</u> S=s).

The probabilities H_1, of survival without exceeding the limit-states R_1, have to be determined by means of formulae of total probabilities,

$$H_1 = \int F_1(s)\ g(s)\ dV(s) \tag{6}$$

(dV(s): element of volume in the space dealt with).

An elementary structural event is the following: structural characteristics are represented by a system of given (fixed) domains B_1 and boundaries R_1, while the loading process is represented by a given point S=s. The structural performance depends essentially on the position of the point S=s with respect to the system of domains B_1 and boundaries R_1. If both kinds of factors are random, their randomness is different. In case of a one-parameter loading, S, and of one single limit-state, R, the randomness has to be represented on two different axes in the plane. In the general case of multi-parameter loadings, randomness of loadings and of structural characteristics should be represented simultaneously in the cartesian product of the space [S] with itself, [S]*[S]. Neglecting this aspect could lead to errors in evaluating structural safety, like in case of using semi-probabilistic techniques.

3. Idealisations

Loadings depend practically in any case on several parameters which can vary randomly. This is especially the case of dynamic loadings. On the other hand, the number of parameters on which loadings acting on a structure depend is theoretically infinite in any case. Safety analysis can be done, even in research activity, only for a moderate number of parameters (say a few parameters of space distribution and a few parameters of time history). The possibilities of practical analysis in design activity are still poorer. This gap between reality and practical possibility of analysis raises an obvious need for idealisation, but for an idealisation made on a consistent basis.

The problem of idealisation could be kept in view like in the following illustrative example. Imagine one wants to analyse a body (or a figure) located in a three-dimensional space. The tool for analysis does not permit more than one single two-dimensional analysis, i.e. projection of the figure on a plane is a primary step, which is to be followed by the analysis. The problem to be solved before performing the two-dimensional analysis is that of an optimum choice of the plane on which the initial problem will be projected. This plane has to be as significant as possible (for example, in case one wants to project an axi-symmetrical ellipsoid the plane has to be parallel to the rotation axis, to provide maximum of information). To have a more realistic image of the problem, the idealisation could be illustrated by another example: how to determine an m-dimensional sub-space that is sufficiently significant for a problem formulated in an n-dimensional space ($m < n$, or even $m \ll n$), and leads to a convenient amount of work in analysis?

The problem of permissible idealisations is sharply raised also in case of testing a structure up to failure (especially for dynamic tests). Failure is actually obtained, in any test, for a well defined loading pattern (space distribution and time history), while research activity should analyse the behaviour up to failure for any possible pattern So one obtains, instead of a whole boundary R_1, only a point of it, located on a radius corresponding to the loading pattern. A test in itself is, from this view point, a kind of poor sampling.

A general tool for evaluating the degree of accuracy of idealisations is the comparison of loading patterns which are the most significant for structural behaviour or for the risk of damage, with their projection on the sub-space adopted for analysis, by means of distances determined according to relations (3) or (5).

4. Design values

The problem of deffining and determining design values for practice has been formulated first in the frame of the semi-probabilistic approach, that represents at present the most advanced tool of wide use in design. More recent and consistent developments related to the adoption of an approximate probabilistic approach are not yet widely used in practice. It seems therefore reasonable, at this time moment, to refer the problem of design values rather to the semi-probabilistic approach. An important feature of the approach recommended for practice is the (implicit) assumption of a single-parameter loading. The (implicit) assumption of a single-limit-state structural behaviour is perhaps less significant. Although the evaluation of the risk of failure is different for the semi-probabilistic and for the approximate probabilistic approaches, some problems raised for the first one could be kept in mind also for the latter one.

In case of a single-parameter loading (this parameter is an intensity-type one) one can define a characteristic value Q_k and a design value Q for the distribution of this parameter. The analysis of structural behaviour under the given (static) loading scheme leads to defining a certain limit-state which is reached for a certain (limit) value of the loading parameter. The randomness of structural characteristics leads to a certain statistical distribution of this limit value and permits to define a characteristic value R_k and a design value R for the parameter dealt with. The semi-probabilistic approach requires that Q does not exceed R.

Imagine now a multi-parameter loading, represented in an n-dimensional space $[S]_n$. The direct generalisation of characteristic value or of design value is in this case a characteristic boundary, or a design boundary, respectively. A representation of such boundaries, for loading and resistance, is given in fig. 2 for a two-dimensional case. The sense of boundaries for R is quite clear. A point of such a boundary is obtained if, for a given direction, the loading intensity corresponding to a limit-state is determined. It is not the same for loadings, because the number of boundaries corresponding to a given global probability of being exceeded is infinite (in fig. 2 there are represented two possible characteristic boundaries, Q'_k and Q''_k).

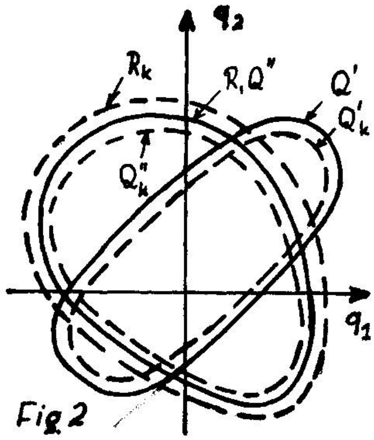

Fig 2

A first problem is the following: how to define a characteristic or a design boundary for loadings? The most natural answer could be: choosing it to be homothetical to the characteristic boundary R_k. But is this answer whenever possible the right one?

Assuming now an accurate definition (leding for example to boundaries Q' and Q'_k of fig. 2) has been adopted, a new problem is raised: how to replace a given boundary by a more elementary one (for example a polyhedral one) in order to make practical computations possible? Should one circumscribe to an elliptic domain a rectangle or an octogone? Should one look for another idea?

* * *

Some problems raised in this discussion might seem too theoretical and sophisticated. Nevertheless they cannot be avoided if a more consistent system of design rules is to be developed. The challenge for an improved approach of multi-parameter loadings is highly actual and this is true especially for dynamic loads, about which safety requirements of design codes say so little.

SUMMARY

The discussion is concerned with problems raised by multi-parameter loadings to which any type of dynamic loading belongs. Some general properties of the set of loadings that can act on a structure are dealt with. Problems of approximation and permissible idealisation are discussed on this basis. Difficulties raised by the definition of design values for multi-parameter loadings are then emphasized.

RESUME

La discussion est consacrée aux problèmes posés par les charges dépendant de plusieurs paramètres, dont les charges dynamiques font part. On présente quelques propriétés générales de l'ensemble des charges qui puissent agir sur une structure. On discute ensuite des problèmes d'approximation et de schématisation admissibles. On met en évidence des difficultés générées par la tentative de définir des valeurs de calcul pour les charges dépendant de plusieurs paramètres.

ZUSAMMENFASSUNG

Der Beitrag befasst sich mit Fragen der Mehr-Parameter Belastungen, zu denen die dynamischen Belastungen immer gehören. Es werden einige allgemeine Eigenschaften der Summe der auf ein Bauwerk möglicherweise wirkenden Belastungen dargelegt, und auf dieser Basis Fragen der Näherungen und der zulässigen Vereinfachungen erörtert. Ferner werden die Schwierigkeiten dargelegt, die sich beim Versuch ergeben, die Berechnungswerte für Mehr-Parameterbelastungen zu definieren.

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V

Design Criteria for Structures Subject to Repeated Limited Strains

Critères de dimensionnement pour les structures soumises à des déformations répétées et limitées

Bemessungskriterien für Tragwerke, die wiederholten begrenzten Verformungen ausgesetzt sind

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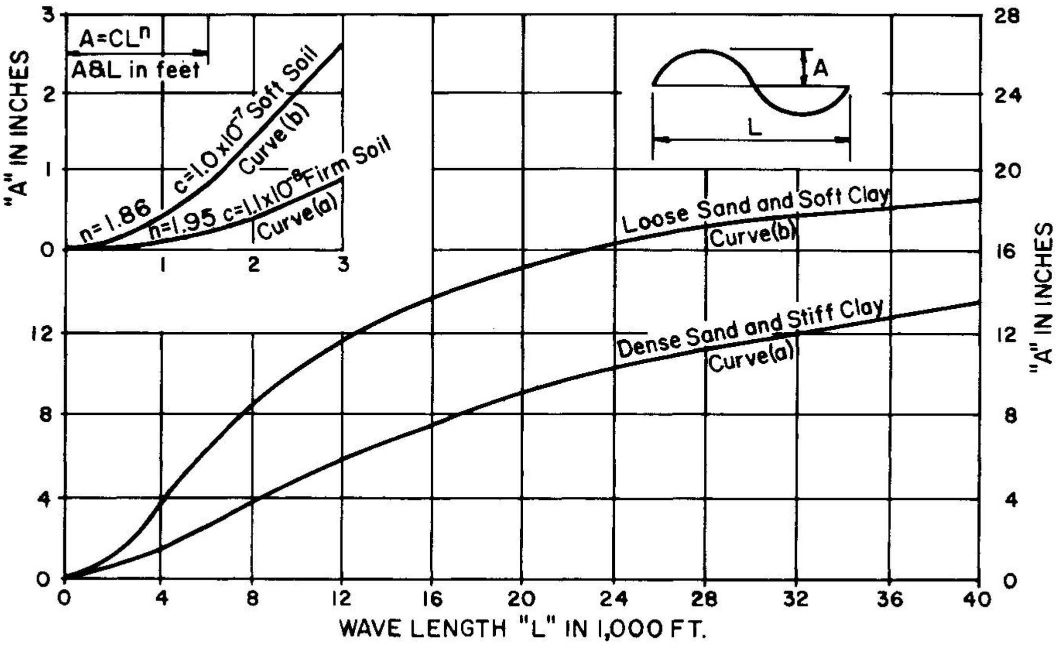
Most investigations of periodic structural phenomena have concentrated on repetition of a defined load or force function, and have attempted to deduce or observe the resulting deformations. This approach is appropriate for transient gravity loadings, such as live loads of transport vehicles.

However, for many important problems, the imposed periodic phenomenon is a displacement or strain, and what is sought is the resulting structural reaction. The largest classification of such problems is those relating to earthquakes, whose effects have been too often described in terms of inertial reaction forces, and too infrequently in terms of motions and deformations, which are the primary seismic phenomena.

In the case of buried structures, such as tunnels, subways, and underground chambers, it is obvious that the seismic deformations imposed on the structure must be the same as those occurring within the surrounding earth. The design process then consists of defining the seismic deformations of the earth, and checking the ductility of the structure against the imposed repeated strains. In this case, the ultimate deformability is of only academic interest – it is necessary and sufficient merely to determine that the structure has sufficient deformability.

As an example of the application of this principle, the earthquake design criteria for subways for the San Francisco Bay Area Rapid Transit System (the BART project) may be cited (Ref. 1). The basic definition of the seismic deformations was provided by Dr. George Housner of the California Institute of Technology, in the form of a spectrum of wave lengths vs. amplitude (Fig. 1), based on many years of observations of California earthquakes.



BART DESIGN EARTHQUAKE SPECTRUM

Fig. I

From this basic spectrum, a graph of curvature of the seismic ground waves may be constructed, and from this the elastic strains imposed by the earthquake on any buried structure may be determined. For the conditions of the BART project, the maximum unit strain resulting from ground curvature was:

$$\xi = 5.2 \frac{A}{L}$$

where ξ = unit strain, inch/inch
A = amplitude of seismic wave, feet
L = length of seismic wave, feet

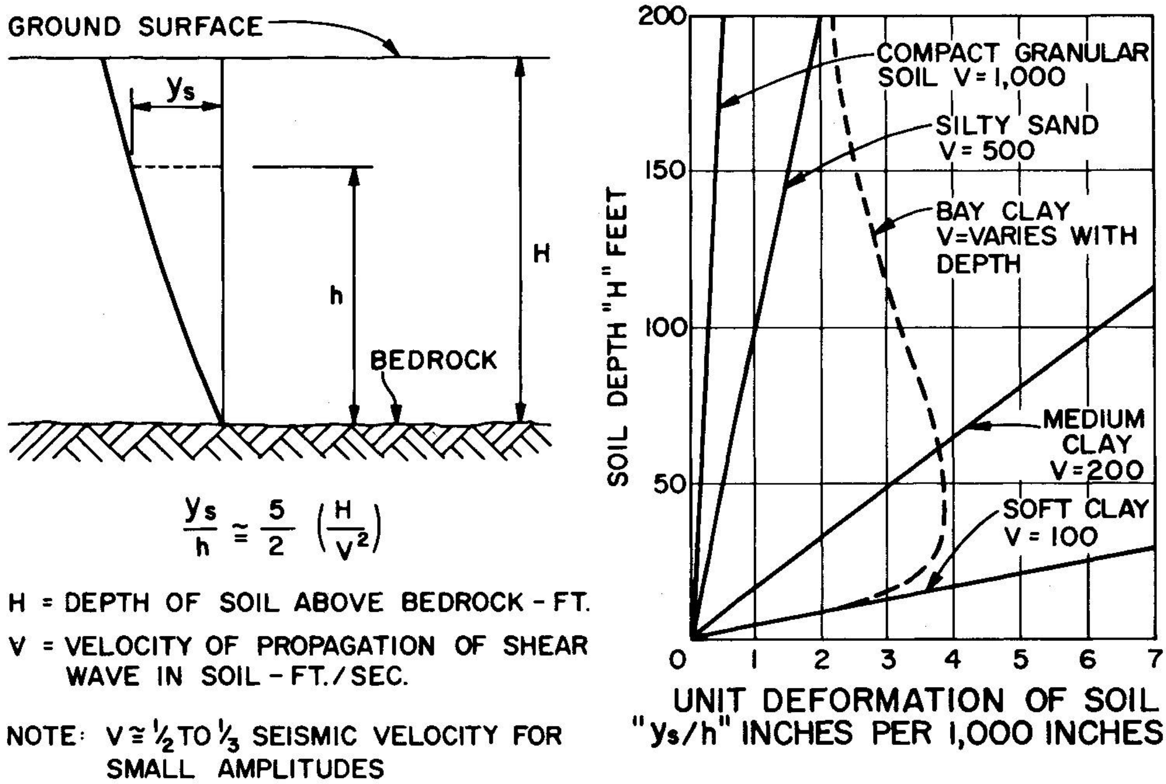
For a typical subway line structure, the maximum unit strain in the concrete walls was 52×10^{-6} in/in, which is well below the strain limit for flexural rupture (see Fig. 2).

	LINE STRUCTURE		STATION BOX	
TYPICAL OVERALL WIDTH, W, FEET CRITICAL WAVE LENGTH, L=6W, FEET	35 210		70 420	
SOIL TYPE	DENSE SAND	SOFT CLAY	DENSE SAND	SOFT CLAY
AMPLITUDE–INCHES –FEET	0.0044 0.00037	0.025 0.0021	0.017 0.0014	0.090 0.0075
RADIUS OF CURVATURE, $R = \frac{L^2/4\pi^2 A}{5280}$, MILES	580	101	610	114
UNIT STRAIN INDUCED BY OBLIQUE WAVE, FOR $\psi = 32°$, $\varepsilon = 5.2 \frac{A}{L}$, MILLIONTHS INCH/INCH	9	52	17	93
UNIT STRESS INDUCED, PSI FOR E = 4,000,000 PSI	36	208	68	372

EXAMPLES OF STRAINS DUE TO CURVATURE DISTORTION

Fig. 2

In addition to strain from direct curvature, a buried structure is also subject to shearing strains resulting from the lag of motion in the overburden behind that in the basement rock. The intensity of this shearing strain is determined by the depth of the overburden, its dynamic rigidity (measured by its seismic velocity), and the characteristics of the vibration of the rock. These may be reduced to a graph in the form of Fig. 3, again representing conditions on the BART project.



SHEARING DISTORTION OF GROUND
BART DESIGN EARTHQUAKE

Fig. 3

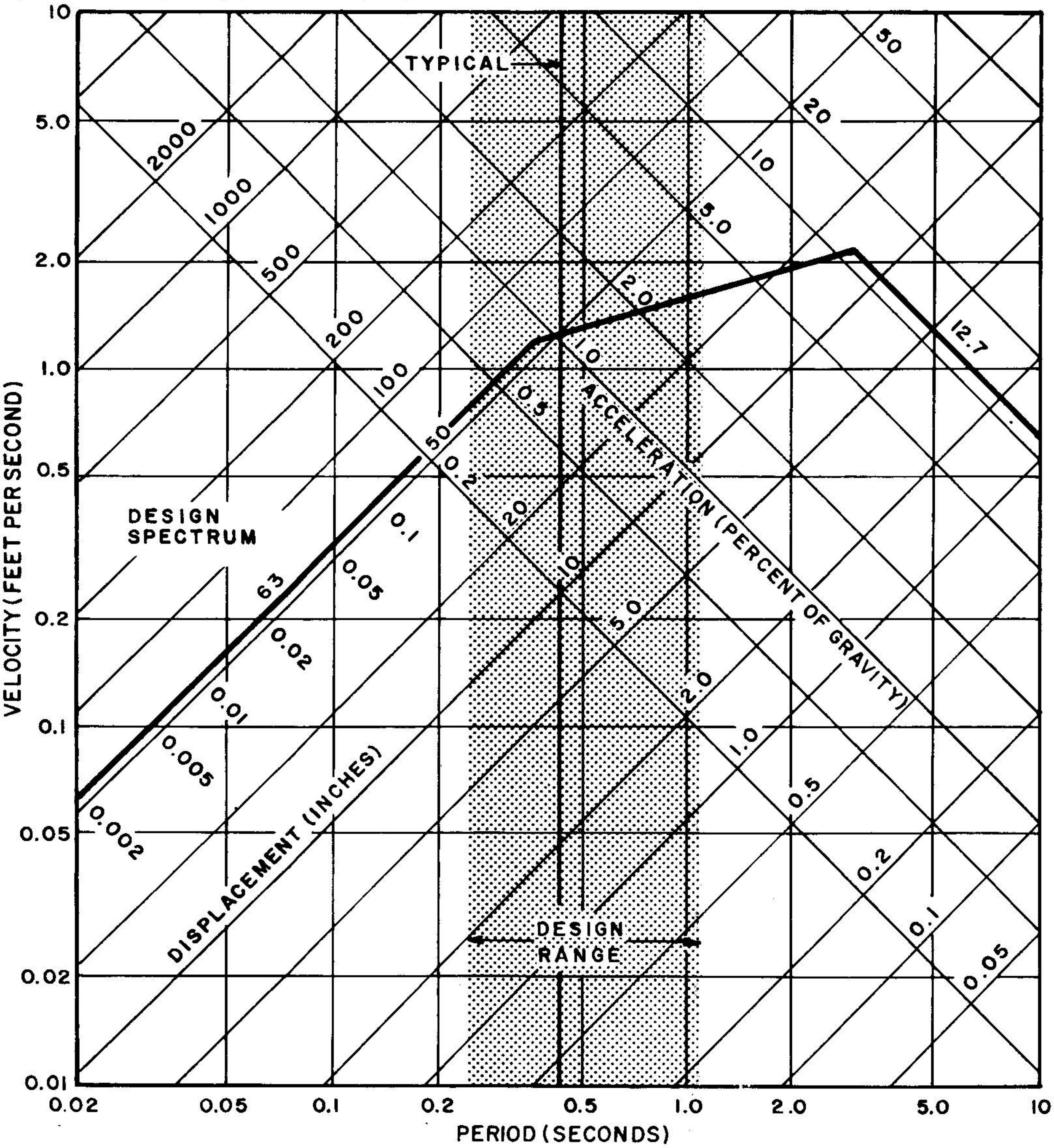
From such a graph, the shearing distortions imposed on a buried structure may be determined, and the design criterion is that the structure must retain its capacity to carry static loads and earth pressures, while subject to the repeated shearing distortions so defined. For the BART subway, this turned out to be no problem except in a few special cases in extremely soft soils. Further details are given in Ref. 1.

It is less obvious that similar principles apply to the aseismic design of many aboveground structures. The response of an elastic structure to earthquake excitation of its foundation is a function of its natural frequency of vibration. By limiting this natural frequency, the designer may control the amplitude of the vibratory response of the structure. The periodic strains imposed on the elements of the structure may thus be limited, and the design may proceed in a manner analogous to that for buried structures.

This principle may also be illustrated by reference to the BART project, particularly to the special seismic design criteria for the aerial structures, which consist of single-column reinforced concrete pier shafts supporting concrete box girder superstructure spans.

The basic ground deformation spectrum for soft alluvium (Fig. 1) may be replotted in the form of Fig. 4, a four-dimensional plot of acceleration, velocity, displacement (or amplitude), and natural period of vibration as devised by Dr. N. M. Newmark of the University of Illinois. This graph represents the response of a single-degree-of-freedom oscillator to the vibrations of the BART design earthquake. Since the single-column pier supporting a concentrated superstructure mass may be closely represented by an inverted pendulum (a single-degree-of-freedom oscillator), the amplitude of its response may be controlled by limiting its natural frequency of vibration.

A high frequency of vibration (or low period) was ensured by providing stiff pier shafts. The specific criteria adopted for the BART aerial structure piers were that in response to the design earthquake the reinforcing steel should not be stretched to more than twice the yield strain, and the compressive strain in the concrete should not exceed 0.0038 in/in (Ref. 2). These criteria determined the allowable lateral deformation (or amplitude of vibration) for any combination of pier height and width. Figure 4 then indicates the acceleration imposed by the design earthquake, and the limiting pier shaft frequency (or stiffness) required.



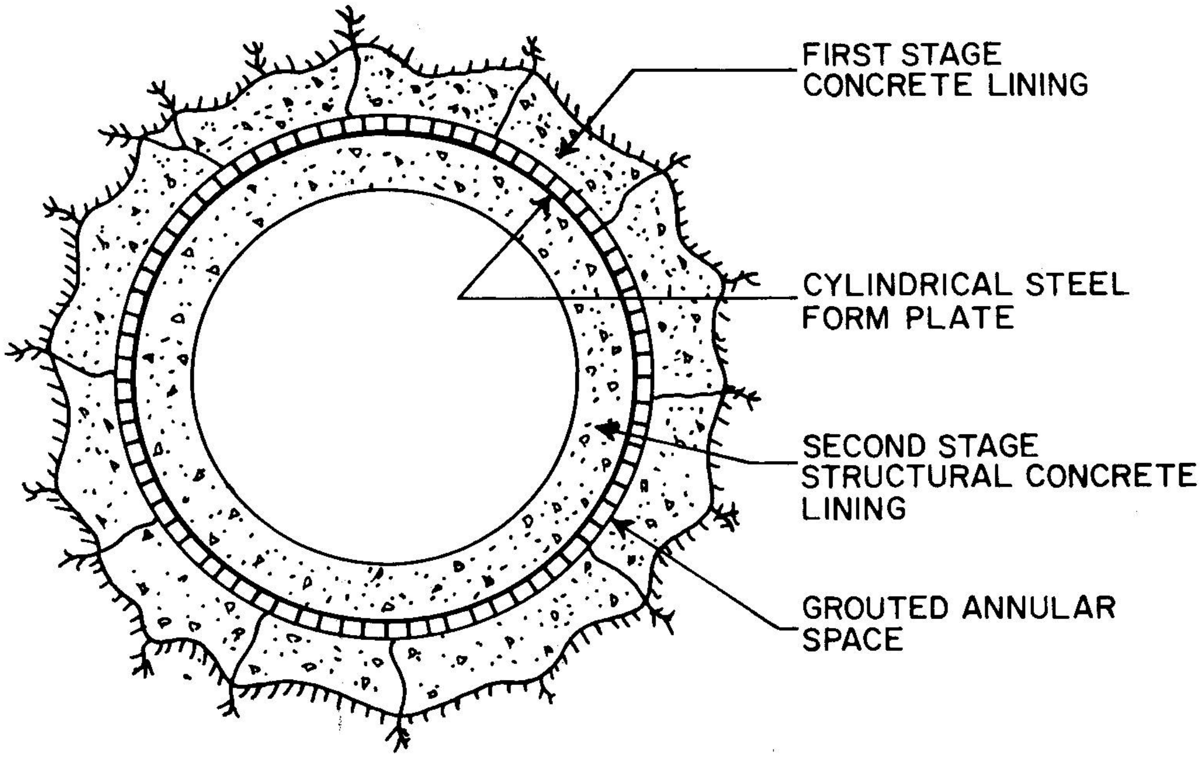
BART AERIAL STRUCTURE RESPONSE SPECTRUM

Fig. 4

A different structural problem involving repeated, limited strains occurs in hydroelectric projects, where it is necessary to design steel-lined penstocks encased in rock tunnels. The water pressure, which may be subject to wide and frequent fluctuations, is contained by the combined action of the thin steel lining and the surrounding rock. Considerable economic benefits could be realized by reducing the thickness of the steel lining, and allowing it to undergo repeated inelastic strains, which would be limited by the elastic reaction of the rock.

Unfortunately, a lack of practical experience with large-scale steel structures repeatedly strained into the inelastic range has inhibited designers from venturing into this unexplored territory, and current practice is to limit the strain in the steel liner under normal operating conditions to the elastic range. The confinement provided by the rock is recognized by increasing the steel working stress to about 3/4 of the yield strength (Ref. 3).

An interesting solution to this problem was devised by the designers of the Dez Dam in Iran (Ref. 4). The project includes steel-lined buried penstocks operating under a head of approximately 200 meters. The penstock tunnels were lined in two stages (see Fig. 5). First, the rough excavation was lined with a moderate-strength concrete to produce a smooth cylindrical cavity. Within this, a thin cylindrical steel form plate was erected, leaving an annular opening between it and the outer concrete lining. A high-strength concrete interior lining was then placed and cured. Finally, high pressure grout was introduced into the annular opening, cracking the outer lining to force grout into the surrounding rock, and prestressing (or pre-straining) the inner structural lining. The principle here is to introduce an initial compressive strain equal to or greater than the tensile strain anticipated under working conditions.



DEZ DAM PRESSURE TUNNEL
SCHEMATIC CROSS SECTION
Fig. 5

The principles of designing for limited strains are relatively easily set forth. To establish quantitative design criteria, it will be necessary to accumulate performance history records on actual projects where these principles have been used. All designers having experience in this field can contribute to the profession by publicizing any case histories of which they have knowledge.

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SUMMARY

In designing structures to resist earthquakes, the concept of a design criterion of controlled strains is proposed. For buried structures, the structure deformation must be the same as the ground deformation, and sufficient ductility to conform to this deformation is required. For aboveground structures, the strain or amplitude of vibration may be controlled by limiting the structure's natural frequency or stiffness.

For buried penstocks, the strain of the steel liner is limited by the confinement of the surrounding rock. Pre-compressing the liner may counteract tensile strains produced by working loads.

RESUME

Pour le dimensionnement de structures devant résister aux tremblements de terre on présente le concept d'un critère de dimensionnement basé sur les déformations contrôlées. Pour les structures enterrées la déformation de la structure doit être la même que celle du sol, et une certaine déformabilité est nécessaire pour permettre cette déformation. Pour les structures en surface, la déformation ou l'amplitude des vibrations peuvent être contrôlées en limitant la fréquence propre de la structure ou la rigidité.

Pour les vannes enterrées, l'élongation du manchon en acier est limitée par la roche environnante. En utilisant un manchon précontraint on peut diminuer les élongations produites par les charges actives.

ZUSAMMENFASSUNG

Zur Bemessung erdbebensicherer Tragwerke wird der Begriff eines Bemessungskriteriums, gestützt auf kontrollierte Dehnungen, vorgeschlagen. Für unterirdische Bauwerke müssen die Deformationen gleich denen des Bodens und eine genügende Duktilität für diese Deformationen vorhanden sein. Für oberirdische Bauten lässt sich die Dehnung oder die Schwingungsamplitude kontrollieren, indem die Eigenfrequenz des Bauwerkes oder seine Steifigkeit in gewissen Grenzen gehalten wird.

Für unterirdische Turbineneinläufe ist die Dehnung der Stahl-Auskleidung infolge der Einengung durch den umgebenden Fels begrenzt. Die Vorspannung der Auskleidung kann den durch die Gebrauchslast hervorgerufenen Zugdehnungen entgegenwirken.

V

The Resistance of Prestressed Concrete to Dynamic Loading. Its Fatigue Resistance; Miner's Hypothesis

La résistance du béton précontraint à la charge dynamique. Sa résistance de fatigue; hypothèse de Miner

Die Festigkeit von vorgespanntem Beton gegenüber dynamischer Belastung. Seine Ermüdungsfestigkeit; Hypothese von Miner

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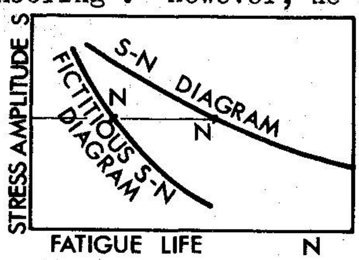
1. Different Kinds of Dynamic Loading

There are three kinds of dynamic loading: impact, vibration and fatigue which are mostly interconnected. With impact the structure should be very flexible to dissipate energy. In the case of vibration some rigidity is essential and it is important that the natural frequency of the member differs from that of the vibration in order to avoid resonance. Obviously fatigue occurs at the same time. Ferry Borges[1] showed in Fig.9 the Wöhler diagram(known as S-N curve) and in Fig.10 the Goodman diagram (which is based on constant load range). The latter is usually of little importance because of the varying load spectrum, and Miner's hypothesis for the cumulative loading is often considered. Ferry Borges has clearly stated that "Basic safety concepts differ with the various branches of engineering". However, he mentioned in eq.12 that instead of the value "1" only "0.3" may apply, which is based on experience with aircraft. In this case random loading with heavy impact takes place and, instead of the S-N curve applying to ordinary fatigue, a fictitious curve (Fig.1) ought to be considered, as shown by Freudenthal and Heller[2], referred to in (1), with smaller values.

FIG. 1.



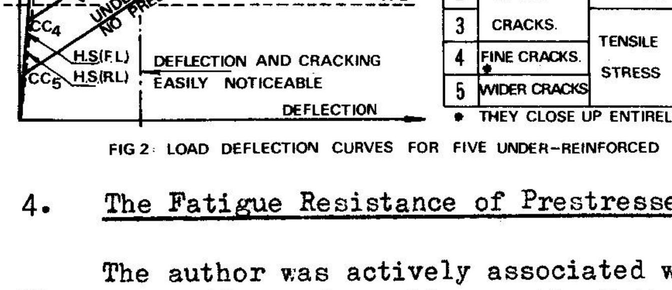
2. The different load spectra, occurring in Civil Engineering

Generally the impact effect is taken into account in the magnitude of the loading, as e.g. with bridges, and the cumulative effect of the load spectrum during the expected life can be assessed by means of Miner's hypothesis. With road bridges there are many millions of relatively light loadings and a limited amount of heavy loadings[3]. Wind loading is based on the wind speed and the dynamic pressure, dependent on various factors. The cumulative effect due to frequency and magnitude of the wind gusts during the expected life must be considered. Based on the data available, it is a question of calculated risk to assume the suitable safe return period of maximum wind speed during the expected lifetime of a structure. It may also be necessary to consider excitations due to vortex oscillations. Special hurricanes or tornadoes are usually ignored, since their locations, maximum speed and duration are unknown. The design of a structure subjected to wind can be based on the fatigue resistance of the members, provided the structure is sufficiently stiff to avoid excessive vibration. It is important to investigate aerodynamically the behaviour of a model in a wind tunnel for various wind speeds up to the maximum and to examine the possibility of excessive vibration due to vortex shedding, as the frequency due to a possible excess loading should differ from the natural frequency to avoid resonance. This might not be possible if the structure is too light, but it might be achieved by the provision of suitable dampers when the amplitude of the vibration caused by wind excitation can be reduced to a satisfactory amount by means of energy dissipation. Examples are mentioned under heading 6.

With impact, great flexibility is required and in earthquake districts impact forces acting in opposite direction must be considered with a possibility of dissipating energy, the magnitude of the forces being usually not known. Sufficient amount of displacement must be allowed, which may be reduced by dampers. The cumulative effect in fatigue may be similar to the random loading in aircraft engineering with a reduced Miner's value.

3. The behaviour of prestressed concrete

The behaviour of prestressed concrete varies to a great extent, dependent on the relative magnitude of the prestressing force. The Author has shown in 1950[4] that the load deflection curves, shown diagrammatically, comprise 3 stages (except for type 1): elastic rigid, elastic flexible and plastic range, starting at PD (permanent deformation). With ordinary reinforced concrete, the permanent deformation occurs earlier. Different types of under-reinforced beams of the same section may be obtained (Fig.2). This was based on the distinction between full and partial prestressing with relatively large and reduced prestressing forces. Fig.2 also shows deflection curves. It was assumed that the ultimate loads of the 5 types, of gradually increasing displacements, are slightly reduced with decreasing prestressing forces. However, it is now known that they are almost equal. A structure between type 1 (at which cracking and failure may occur simultaneously) and type 2 is most suitable for a structure which must be stiff with minimum deformation (e.g. a turbo foundation to resist heavy vibration). Fig.2 has been incorporated in the Appendix of the "First Report on Prestressed Concrete" of the Institution of Structural Engineers, London 1951[5]. Successful impact tests on flexible masts were reported by the author in 1956[5].



C.C. : COMMENCEMENT OF CRACKING.
F. : FAILURE.
M.O.R.: MODULUS OF RUPTURE.
P.C. : PRESTRESSED CONCRETE.
P.D. : PERMANENT DEFLECTION.
W.L. : WORKING LOAD.
H.S. : HOMOGENEOUS SECTION
(F.L): FIRST LOADING
(R.L): AFTER REPEATED LOADING.

No.	CONDITION AT W.L.	BOTTOM FIBRE STRESS AT W.L HOMOGENEOUS SECTION.		COMMENCEMENT OF CRACKING	KIND OF FAILURE	RANGE BETWEEN NOTICEABLE DEFLECTION & FAILURE
1	NO VISIBLE CRACKS.	COMPRESSIVE STRESS.	CONSIDERABLE.	AT FAILURE	SUDDEN BRITTLE	NONE
2			APPROACHING ZERO	MUCH ABOVE W.L	GRADUAL YIELDING	RATHER SMALL.
3		TENSILE STRESS	BELOW M.O.R.	ABOVE W.L		LARGE
4	FINE CRACKS.*		ABOVE M.O.R.	BELOW W.L		LARGER
5	WIDER CRACKS		MUCH ABOVE M.O.R.	MUCH BELOW W.L		VERY LARGE

* THEY CLOSE UP ENTIRELY ON REMOVAL OF THE LOAD

FIG 2: LOAD DEFLECTION CURVES FOR FIVE UNDER-REINFORCED P.C. BEAMS OF THE SAME SECTION.

4. The Fatigue Resistance of Prestressed Concrete

The author was actively associated with extensive research from 1951-1969. First the effect of cracking on the fatigue resistance was studied. At tests in Liège 1951, it was ascertained that the static failure load was not reduced by 3 million previous load cycles when cracks had opened and closed 3 million times[6]

This is illustrated in Fig.3 in which the 3 loading ranges with increasing upper load ranges are seen. Although there is a difference in stiffness between the static load deflection curves before and after each million of load cycles, as seen in (a), this difference is insignificant compared with the general behaviour up to failure, as seen in (b). Fig.4 shows another example of a test result of 1954[7]. First a static loading was carried out until cracks became visible and static loadings were repeated after 583,000 and 747,000 cycles. The permanent sets were very small, but the displacements substantial. Thus the displacements capable of dissipating energy were mainly of elastic nature which is of great advantage at vibrations and earthquake resistance. It should be noted that in both examples tensioned and non-tensioned prestressing wires were provided. In another test almost 9 million load cycles were successfully applied with gradual increasing load range up to 68% of the failure load[8]. In all these cases only the cumulative effect of fatigue was investigated in association with British Railways[9].

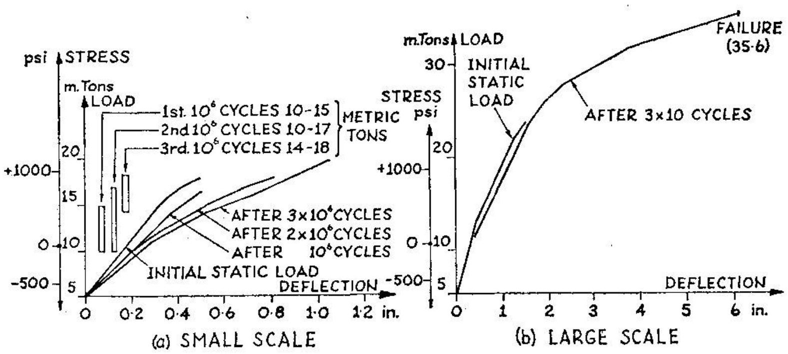


Fig. 3 ——Load-deflection Diagram: Fatigue Test, Composite Slab, Liège, 1951.

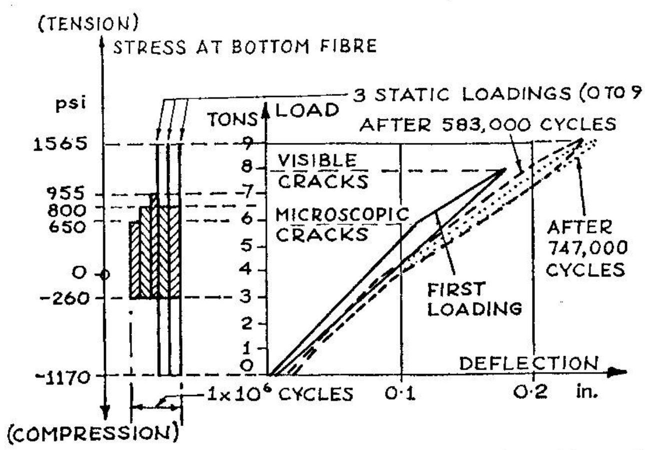


Fig. 4.——Load-deflection Diagram: Fatigue Tests, 1954.

The author was associated with further research at the DUKE University, USA 1965-69. It was the purpose to obtain S-N curves of the fatigue resistance of various prestressed concrete beams, subjected to constant load ranges[10]. A relatively very high fatigue resistance was obtained for constant load ranges between 30% and 70 to 90% of the static failure load, based on the assumption that 30% corresponds to the dead load and the load factor of safety is 1.5 for the dead load and 2.5 for the live load, as was required for bridges, corresponding to a service load of 52% of the static failure load. Also in this case tensioned and non-tensioned tendons (but in this case strands) were provided. Failure commenced due to brittle fracture of some wires of the tensioned strands, as long as the upper limit did not exceed 85% of the static failure load. With a higher upper limit crushing of the concrete occurred. After the brittle failure of some wires, the beams were capable of taking up an appreciable number of further load cycles before final failure.

Further subject of research was to compare the results obtained from constant load ranges with corresponding stress ranges of the steel itself (tested in the air) and to investigate the applicability of Miner's hypothesis. Some of the test results were presented in 1972[12]. Fig.5 shows the basis of comparison, as derived by Hu. The investigations by Warner and Hulsbos[13], Tide and Van Horn[14] and Hilmes and Ekberg[15] agree very well and these results can be presented by a simplified S-N curve, shown in Fig.6. It is seen that for a small stress range of 10% (between 40 and 50% of the strength) 10 million load cycles can be

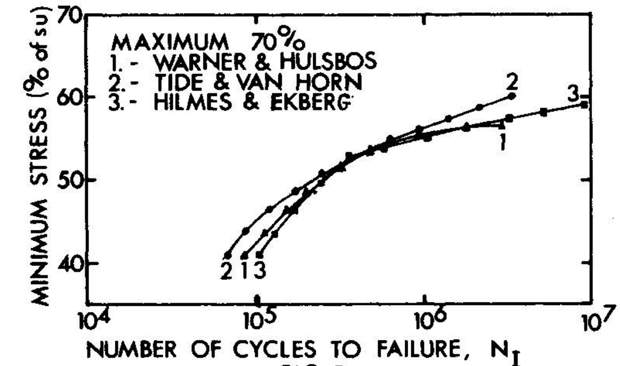


FIG. 5.

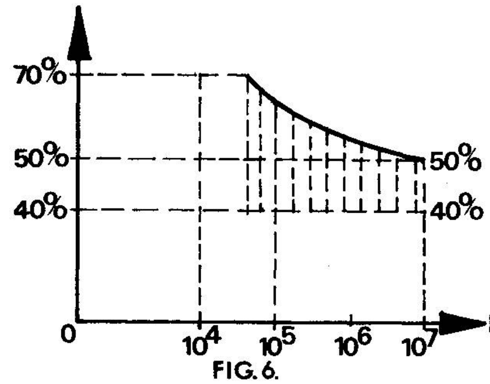


FIG. 6.

expected. However, a corresponding beam test for a constant load range between 30 and 52% of the static failure load withstood only about 2.5 million cycles before failure. Some of the test beams showed results which were relatively much higher than those corresponding to the appropriate stress ranges in the steel, whereas others were much lower. This was apparently the consequence of unsatisfactory bond, which became evident from the crack distribution (great spacing and upper forking)[12]. The test specimens were produced in a prestressing plant without constant supervision, whereas all the previous specimens had been carefully produced at the Laboratory of the University. From a number of investigations at which strain measurements were made at intermediate static loadings it was ascertained that a gradual loss of the effective prestress had taken place with increasing number of cycles. This loss was greater at the beams with bad bond; at a certain state a rapid reduction in the initial tensioning stress took place, as indicated in Fig.7. This is a very important result, since it shows that with bad bond the effective prestress decreases, thus in spite of a constant load range increasing stress ranges have to be considered.

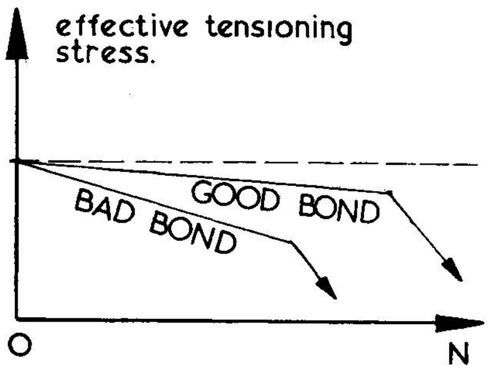


Fig.7 Reduction in Tension stress during Fatigue.

Venuti[14] carried out tests which he wanted to base on statistical considerations. He tested e.g. 18 beams to 50% of the static failure load in which case there ought not to have occurred any failure after 5 million cycles, because the stress range in question corresponded to about 10 million cycles. However only 11 beams survived the loading after 5 million cycles and with two of the 7 beams which failed earlier, compression failure took place. At another group of beams the load was increased to 60% of the failure load. In this case one beam sustained 3 million cycles, whereas the remaining beams failed after much shorter fatigue life, the smallest number being only 53,000 cycles (also with compressive failure). These results can be explained similarly to some of the latest tests at Duke University as being caused by insufficient bond, quite different from the excellent behaviour of the previous beam tests[11].

It would obviously be quite unsatisfactory to have to consider the possibility of such uncertainty, since a prestressed concrete beam with pretensioned tendons must have sufficient bond. Otherwise a completely unsuitable material would be obtained which would be worse than reinforced concrete with usually well anchored steel. However, it may be stated that small test specimens require a more careful vibration than larger members and that generally prestressed concrete members are well produced. Nevertheless the manufacture of prestressed concrete members should be well supervised.

5. Miner's Hypothesis

The simple formula by Miner can obviously not be taken as a strict rule but only as a hypothesis. From Fig.1 it is seen that the S-N curve on which it is based needs a revision in the case of heavy impact combined with random loading, as with aircraft loading. However, this does not seem to apply to structural members used in civil engineering, exposed to wind pressure or bridge loading; provided that the effect of impact is duly taken into account in the loading. Obviously, if the bond is insufficient and the effective prestress is gradually reduced owing to loss of bond (Fig.7), it cannot be expected that Miner's value remains valid, since even during constant load ranges the stress range gradually would increase.

The tests[12] have indicated that with satisfactory bond the known test results about the fatigue resistance of the steel in the air can be used as a basis for assessing the fatigue resistance of a prestressed member subjected to the corresponding stress ranges. It has even become evident that with good bond, the fatigue resistance of the prestressing steel in a prestressed concrete member is better than that of the steel in the air. This depends on the design. With good crack distribution it appears understandable that the resistance of the prestressing steel embedded in the concrete is higher than that of the steel in the air. It is hereby assumed that failures at testing of prestressing steel with fractures at the anchorage are excluded, where Miner's rule is investigated. Thus it may be stated that the original value for Miner's hypothesis seems generally to be applicable to fatigue of prestressed concrete members in civil engineering. However, a fictitious S-N curve might have to be considered according to Fig.1, or the Miner value may have to be reduced considerably (say to 0.3) where heavy impact occurs in connection with random oscillations as is the case with aircraft. This might also be applicable to earthquakes.

6. Some design notes (based on well-manufactured prestressed concrete)

Bridges: Present design criteria are rather safe. This would allow substantial increase in maximum loads or revision of future design loading (i.e. increase of live load), if based on a combined limit state of serviceability and collapse (i.e. avoiding fatigue failure and excessive deformation).

Structures subjected to wind: Because of the possibility of adjusting its rigidity prestressed concrete is very suitable. Model tests in wind tunnels and artificial dampers are preferably provided as discussed in section 4. Three examples (with two of which the author was associated) are described in the paper by Bobrowski and Cramer[17]) under Theme II.

Where earthquake resistant constructions are required: It is obviously impossible to deal with these problems in a few lines. Research has clearly shown that suitably designed prestressed concrete is capable of large displacement and dissipating energy due to impact forces with little permanent set. In columns it would be necessary to provide tendons at opposite sides and to allow sufficient displacement.

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(It seems to be surprising that the author's most telling research presented some twenty years ago (e.g. see Figures 3 and 4) has apparently been completely ignored in the basic reports of the Symposium).

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SUMMARY

Resistance of prestressed concrete to dynamic loading is discussed. Great resilience against impact is important, but stiffness is essential for heavy vibrations. Miner's hypothesis seems satisfactory at repeated loading with limited impact. At random loading with heavy impact, as with aircraft and possibly also for earthquakes, Miner's value ought most likely to be reduced.

RESUME

On discute dans ce travail la résistance du béton précontraint soumis à des charges dynamiques. Une grande résiliance contre les chocs est importante, mais la rigidité est essentielle pour les vibrations à basse fréquence. L'hypothèse de Miner semble satisfaisante en cas de charge répétée et de chocs limités. En cas de charge stochastique accompagnée de chocs importants, tel que les chocs produits par les avions et probablement aussi pour les tremblements de terre, la valeur de Miner devrait très probablement être réduite.

ZUSAMMENFASSUNG

Es wird der Widerstand von vorgespanntem Beton gegen dynamische Belastung diskutiert. Grosse Elastizität gegen Stoss ist wichtig, doch ist die Steifigkeit ausschlaggebend für starke Vibrationen. Die Hypothese von Miner scheint bei wiederholter Belastung mit begrenzter Stärke der Stösse zu genügen. Bei beliebiger Belastung mit starken Stössen, wie bei Flugzeugen oder eventuell Erdbeben, sollte der Wert von Miner sehr wahrscheinlich reduziert werden.

V

Dynamic Wind Loads and Cladding Design

Surcharge dynamique due au vent et calcul du parement

Dynamische Windbelastung und Bemessung von Gebäudeverkleidungen

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Windstorm damage to cladding (exterior surfaces) of buildings often results in considerable economic loss and sometimes personal injury. The same care should therefore be given to cladding design and research as is given to the structure. This discussion looks at two aspects of the design of cladding to resist wind loads: (a) how behaviour under dynamic wind loads affects resistance of both metal (ductile) and glass (brittle) panels; (b) what risks of cladding failure are implicit in North American design rules. The problem of determining actual wind loads on cladding, the greatest unknown in the design problem, is not discussed.

Design of Cladding to Resist Wind Loads

Because turbulent wind is a dynamic loading, recent design approaches for structures are based on a dynamic component that takes into account wind turbulence, size of building (averaging effect on turbulence), and dynamic amplification. A similar approach is suitable for cladding design (Fig. 1)

$$\frac{R}{FS} \geq w_o = \bar{w}\, C_g = \bar{w}\,(1 + g.I_T) \qquad (1)$$

where R is the design resistance, FS the design safety factor, w_o the design wind pressure, $\bar{w}$ the mean wind pressure equal to the mean velocity pressure $\bar{q}$ times the shape factor C_p, C_g the gust factor, I_T the rms (root mean square) intensity of wind pressure turbulence and g a peak factor relating peak wind pressure to rms intensity of turbulence. For a stationary Gaussian loading, the expected peak factor $\bar{g}$ is given in Table I, based on the theory given in Ref. (1) and assuming there are 0.3 peaks per second, a figure obtained from full-scale pressure measurements[2].

When I_T is zero the loading is static and the resistance R corresponds to that for a statically loaded panel. Since the effect of dynamic loads on resistance is related only to the turbulent component of the wind, it is appropriate to consider this and other structural effects, such as dynamic amplification, by a modification of the peak factor g in Eq. (1), i.e. by multiplying g by a "gust material" factor, k_m,

$$C_g = 1 + k_m . g . I_T \qquad (2)$$

The "gust material" factor, k_m, depends both on the material and structural behaviour and on the nature of dynamic loading; it will be shown that for cladding this factor is nearly equal to 1.

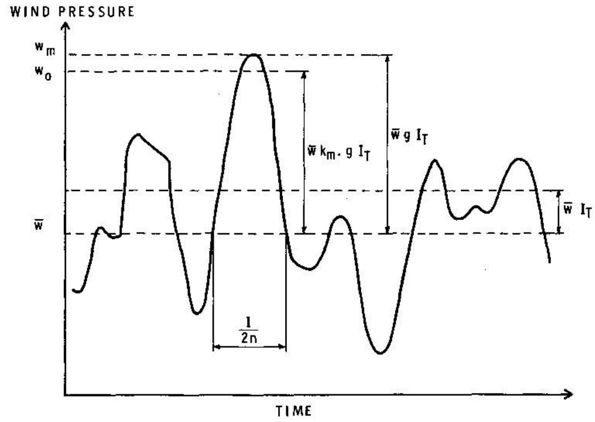

Figure 1: Wind Pressure Representation for Cladding Design

The natural frequency of metal and glass panels varies from about 5 to 50 Hz. In most cases this frequency is considerably higher than significant wind turbulence frequencies.[3] Thus during elastic deformations a panel can be represented as a statically loaded structure and dynamic amplification can be neglected. Only for unusual, high-frequency wind turbulence created locally by building corners[4] or by surface irregularities, such as mullions, is dynamic magnification likely to become significant.

Metal Cladding

Metal panels fail by yielding, by buckling, or by fracture of the connections. Two aspects of metal behaviour are discussed in this paper: rate effect, i.e. change of yield strength with rate of loading, and the effect of plastic deformation. Metal fatigue is neglected in this study.

TABLE I - PEAK FACTOR AND GUST FACTORS FOR GLASS

Duration of Load	Expected Peak Factor for Loading $\bar{g}$	Glass (Fig. 3)	
		$k_m . g$	Minimum, C_g
one hr.	3.9	4.0	2.0
10 min.	3.4	3.2	1.7

According to information on rate effect for steel,[5] a ten fold increase in rate of loading increases the yield stress, σ_y, by approximately 2 ksi. Since the standard ASTM testing rate [6] corresponds approximately to 10 ksi per second, the rate effect can be approximated by

$$\sigma_y - \sigma_o = 2 \log \left(\frac{\dot{\sigma}}{10}\right) \qquad (3)$$

where σ_o is the yield stress according to the standard test.

Plastic deformation can also absorb an exceptional temporary overload; although the material may yield there may not be sufficient permanent deformation to require replacement of the panel. Vickery looked into this problem for a steel building structure and found that the mean wind speed necessary to produce severe damage is about 20 per cent greater than that necessary to just produce yielding.[7]

Consider a simply-supported panel, undergoing plastic bending at midspan. The panel is subjected to a half sinusoidal cycle of wind pressure with maximum amplitude $w_m - \bar{w}$ and forcing frequency n Hz (Fig. 1). The amount of plastic deformation, δ_p, during an excursion of wind pressure beyond yield pressure w_y can be calculated numerically from the equation of motion. Rate effect is taken into account in accordance with Eq. (3) by expressing w_y in terms of w_o, corresponding to the standard test yield resistance.

The following damage criteria are assumed: permanent deflection: $\frac{\delta_p}{L} = \frac{1}{50}$, and section failure: $\frac{\delta_p}{\delta_y} = 5$, whichever occurs first. The first criterion corresponds to the need for panel replacement and the second to reaching of maximum bending resistance. In corrugated panels, local buckling occurs before very large plastic deformation.

Calculations were carried out for 3 types of cladding spanning 10 ft: (1) sheet metal (weight 1 psf, n_o = 20 Hz); (2) normal roof decking or wall cladding (weight 8 psf, n_o = 10 Hz); (3) reinforced concrete or masonry (weight 50 psf, n_o = 20 Hz). In the calculations it was assumed that w_y = 50 psf and $\bar{w}/w_y = 0.5$.

The results, given in Fig. 2 in terms of k_m, show that for wind frequencies less than about 1 Hz, very little can be gained by taking into account plastic deformation and rate effect. Recalculation for normal cladding, Case (2), assuming strain hardening factors (ratio of inelastic stiffness to elastic stiffness) of 0.01 and 0.1 indicate that strain hardening is no more significant for dynamic resistance than it is for static resistance.

Thus design wind loads for metal cladding failing primarily by yield can be determined by assuming that the panel is a static structure which fails when the wind pressure exceeds the standard plastic resistance. Some extra resistance is available for rare local high-frequency wind turbulence.

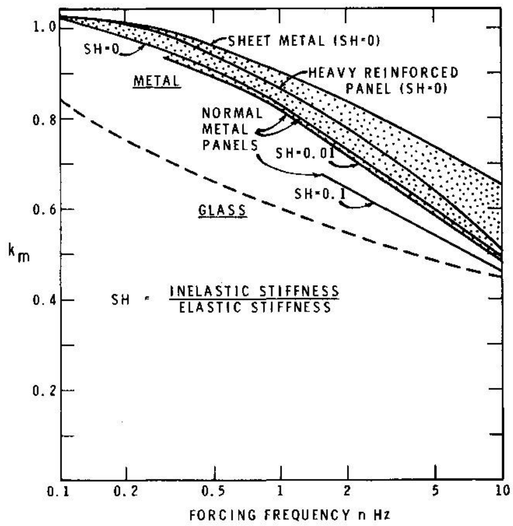

Figure 2: Gust Material Factor k_m for One Cycle of Loading

Windows

Glass is a brittle material in which failure starts at an invisible surface or edge flaw. Its strength is dependent on window size (in accordance with the theory of weakest flaws), rate (or duration) of loading, and, to a lesser extent, on temperature and relative humidity. Rate of loading, the only effect of interest to this study, can be approximated by the following criterion:[8]

$$\int_o^{t_f} w^{12}\, dt = C_M \qquad (4)$$

where w is the pressure and t_f the time at failure. On the basis of manufacturers' loading tests[9] - a gradually applied pressure in which failure occurs at about one minute, the constant C_M is determined from Eq. (4) to be $4.62\ w_o^{12}$, where w_o is the failure pressure from the standard test. Based on one peak of sine wave loading (Figure 1), Fig. 2 shows that glass is considerably more sensitive to rate effect than metal, although for a load of very short duration (high n) metal gains on glass because of its ductility.

Because of this sensitivity to rate effect, one peak of loading does not provide useful information for glass. Figure 3 shows the results of applying the damage criterion to a sustained random wind pressure:

$$\int_0^T w^{12}\, dt = \bar{w}^{12} \int_0^T [1 + I_T\, x(t)]^{12}\, dt = C_M \qquad (5)$$

where x is assumed to be a stationary Gaussian random process with a mean of zero and a standard deviation of one. The damage will be different for each random process of duration T but the expected damage can be determined from Eq. (5) by expanding into powers of x and replacing $\int_0^T x^n\, dt$ by its expected value based on the normal distribution, $1.3 \ldots (n-1)T$. From Eqs. (1) and (3), w_o can be eliminated and the results can be expressed in terms of $k_m \cdot g$ or C_g.

Figure 3 shows the results as a function of rms intensity of turbulence, I_T, for 2 load durations: 10 minutes and 1 hour. The results indicate constant values of $k_m \cdot g$ for high intensity of turbulence and constant values of C_g for low intensity of turbulence; these values are given in Table I. The former applies to most windows near the ground and the latter to windows in tall unobstructed buildings. The values of $k_m \cdot g$ compare closely to the expected peak factors $\bar{g}$, which do not consider dynamic structural effect. The results of this study therefore indicate that, except for windows subject to small turbulence, the value of k_m for glass can also be taken as 1.

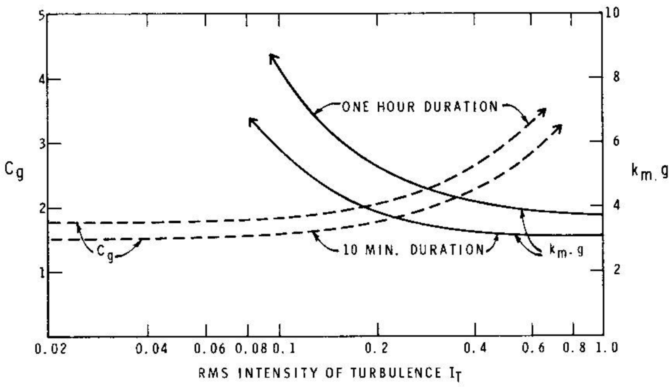

Figure 3: Gust Factors For Glass - Stationary Gaussian Loading

Risk of Failure

The consequences of cladding failure due to wind are not as serious as for structural collapse. North American building codes allow for this, either by a one-third increase in allowable stress for metal cladding,[10] or by a reduced return period for design wind load.[11]

Annual failure risks implied by North American design rules are compared in Table II based on the following assumptions: (1) metal yield strength is distributed normally with a mean value 1.15 times the specified resistance (guaranteed minimum) and a coefficient of variation 0.10; (2) glass window strength is distributed normally with a coefficient of variation 0.25; (3) maximum annual hourly wind loads follow the extreme value Type 1 distribution with a coefficient of variation 0.30.

Annual failure risks of 0.01 to 0.3 per cent in Table II appear to be considerably higher than indicated by actual damage, probably due to conservative design assumptions. Comparative figures in Table II, however, indicate that

TABLE II - ANNUAL FAILURE RISKS FOR CLADDING

Code	Design Wind Code		Metal		Glass	
	Return Period, Years	Gust Factor, C_g	Safety Factor	Annual Failure Risk, %	Safety Factor	Annual Failure Risk, %
NBC[a] 1970	10	2.5	1.67	0.007	2.5	0.17
NBC[a] 1965	30	2.0	1.25	0.26	2.5	0.21
ANSI[b] 1972	50	2.0	1.25	0.13	2.5	0.16

a National Building Code of Canada - Sections 4.1, 4.6 and 4.7

b American National Standards A58.1 - 1972. See also Ref. (10)

recent Canadian design rules for metal cladding are too conservative; a safety factor of 1.25 would give an implied failure rate of 0.16 per cent per year. Recent window failures in tall buildings[12] indicate that falling glass from a broken window initiates failure of a number of other windows (a kind of progressive collapse). For this and psychological reasons it is recommended that design failure risks be reduced for tall buildings.

Conclusions

A study of metal for rate effect and ductility, and of glass for rate effect, shows that for turbulent wind loads, both types of cladding can be considered as statically loaded structures in which failure occurs when wind pressure exceeds the structural capacity as determined by standard tests. The factor k_m in Eq. (2), which takes into account dynamic behaviour under turbulent wind loads, can be generally taken as 1. For glass, minimum values of the gust factor, C_g, in Table I are recommended to avoid failure of windows subject to steady winds.

Annual failure risks for cladding indicate that recent Canadian rules for metal cladding are too conservative. Existing safety factors for windows in tall buildings appear to be too small.

The area of greatest uncertainty in the design of cladding to resist wind loads is in the wind loading itself, in particular the intensity of turbulence, I_T, and shape factor, C_p. This information should be obtained from full-scale measurements and boundary-layer wind-tunnel tests.

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SUMMARY

A study is made of the behaviour of ductile (metal) and brittle (glass) panels under dynamic wind loading. The results show that the effect of such factors as dynamic amplification, rate of loading and ductility can generally be neglected. Failure risks implicit in North American codes are compared; some changes in design rules are suggested, including an increase in safety factor for glass windows in tall buildings.

RESUME

On a fait une étude sur le comportement des panneaux ductiles (métal) et cassants (verre) dans des conditions de chargement dynamique dû au vent. D'après les résultats obtenus, l'effet de facteurs comme l'amplification dynamique, la vitesse de chargement et la ductilité est généralement négligeable. Les risques de rupture que sous-entendent les codes nord-américains sont mis en regard; quelques modifications dans les règles de calcul sont suggérées, y compris une augmentation du facteur de sécurité des fenêtres vitrées dans les bâtiments très élevés.

ZUSAMMENFASSUNG

Das Verhalten von zähen (Metall) und spröden (Glas) Materialien in Gebäudeverkleidungen unter dynamischer Windbelastung wird untersucht. Die Ergebnisse deuten an, dass der Effekt von Faktoren wie dynamisches Aufschaukeln, Belastungsgeschwindigkeit und Zähigkeit im allgemeinen vernachlässigt werden kann. Die in nordamerikanischen Normen enthaltenen Bruchwahrscheinlichkeiten werden verglichen. Gewisse Verbesserungen in den Berechnungsmethoden werden vorgeschlagen, u. a. eine Erhöhung des Sicherheitsfaktors für Glas in hohen Gebäuden.

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V

Plastic Shakedown of Structures with Stochastic Local Strengths

Ruine des structures en domaine plastique avec résistance locale empirique

Plastisches Versagen von Tragwerken mit stochastischen lokalen Festigkeiten

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1. INTRODUCTION

A procedure for evaluating bounds on the plastic collapse probability of redundant, ductile structures with stochastic local strengths has been presented elsewhere [1] and brought to the direct knowledge of I.A.B.S.E. members by a contribution to the Free Discussion at the Amsterdam Congress [2]. The original form of the procedure allowed the determination of bounds to the reliability of structures subjected to load systems measured by a single parameter with known probability density law; these bounds could be made closer by successive approximations.

Later, the procedure has been extended to multi-parameter loading [3], and shown to be just a particular case of a more general formulation applicable to all structural problems whose solution is given by a maximum or minimum condition [4].

In this and a companion paper [5], the procedure is further extended and applied to the shakedown/incremental collapse problem of plastic structures. In fact, it is well known in classical limit analysis (see e.g. Refs. 6, 7) that a ductile structure can be led to gradual collapse ("incremental collapse") by repeated loads of smaller intensity than the corresponding non-repeated loads, even if the lowering of the yield limit with plastic cycles of stress is (unsafely) neglected. This effect, which is quantitatively significant in a number of cases of practical engineering importance, appears not to have been examined before for structures made up of elements with random strengths; also Professor Ferry-Borges' Introductory Report for this Symposium [8] considers explicitly effects of the "fatigue" type only.

The theorems which allow the introduction of this effect into probabilistic limit analysis are presented in detail in the quoted Ref. [5], and are briefly summarized below. This paper is more specifically devoted to a numerical example.

2. BOUNDS ON THE PROBABILITY OF SHAKEDOWN

For simplicity's sake, only structures with one significant strength component (bending moment M) will be considered.

Let $m_{e\ max}$ and $m_{e\ min}$ be the elastic diagrams of maximum and minimum bending moment M^e_{max} and, respectively, M^e_{min}, calculated allowing all the admissible loadings to act in turn on the structure. If m_o' and m_o'' are the diagrams of positive and negative limit moments M_o' and, respectively, M_o'', the static theorem states that shakedown takes place if at least one diagram m^* of self-equilibrated moments exists such that the inequality:

$$M_o'' \leq M^e_{max} + M^* \leq M_o'; \quad M_o'' \leq M^e_{min} + M^* \leq M_o' \tag{2.1}$$

holds in every section of the structure.

On the contrary, by the kinematic theorem, shakedown cannot take place if a rigid-plastic mechanism exists such that:

$$\Sigma_i\, M^e_{i\ max}\, \Delta\dot{\phi}_i + \sum_j\, M^e_{j\ min}\, \Delta\dot{\phi}_j > \sum_i M'_{oi}\, \Delta\dot{\phi}_i + \sum_j M''_{oj}\, \Delta\dot{\phi}_j \tag{2.2}$$

i and j being the dummy indices which distinguish sections undergoing positive and, respectively, negative plastic rotations. Therefore, if m_o' and m_o'' are random functions and if a number of self-equilibrated diagrams m_1^*, m_2^* ..... m_n^* is considered, denoting by E_1 the event that (2.1) holds at least for one m_i^*, and by S_d the actual probability of shakedown, one gets:

$$S_{d\psi} = \text{Prob}\,\{E_1\} \leq S_d \tag{2.3}$$

Similarly, consider a number of mechanisms k_1, ... k_r and denote by E_2 the event that (2.2) does not hold for any k_i. Then:

$$S_{d\gamma} = \text{Prob}\,\{E_2\} \geq S_d \tag{2.4}$$

From (2.3) and (2.4), bounds on the probability of incremental collapse $P_d = 1 - S_d$ follow obviously:

$$P_{d\psi} = 1 - S_{d\psi} \geq P_d \geq 1 - S_{d\gamma} = P_{d\gamma} \tag{2.5}$$

The two events (2.1) and (2.2) are mutually exclusive and exhaustive. Therefore, bounds (2.3) and (2.4) are sharp, in the sense that the difference $S_{d\gamma}-S_{d\psi}$ approaches zero when all self-equilibrated diagrams and all possible mechanisms are considered for the definition of events E_1 and E_2.

3. NUMERICAL EXAMPLE

Consider the simple frame in Fig. 1, already studied in [1,2,3], of nominally constant section throughout, subjected to loading dependent on two parameters, W_1 and W_2.

Assume that only the eleven critical sections shown in Fig. 1 are significant, and that in each critical section $|M''_{oi}| = M'_{oi} = \tilde{M}_{oi}$. Assume further that the $\tilde{M}_{oi}$ (i = 1, ..., 11) constitute a set of independent random variables distributed according to the Gauss probability law P_{oi} (M_{oi}) with mean value $\bar{M}_{oi} = \bar{M}_o = 10$ tm and standard deviation $\sigma_{oi} = \sigma_o = 1$ tm in every section. The set of admissible loading is defined by the following inequalities:

$$0 \leq W_1 \leq W_{1m} \quad ; \quad 0 \leq W_2 \leq W_{2m} \tag{3.1}$$

where W_{1m} and W_{2m} are the "extreme load-parameters" on which the probability of incremental collapse depends. The results of the analysis are presented mainly as level curves, respectively $C_{\psi P}$ and $C_{\gamma P}$, of the bounding incremental collapse probabilities $P_{d\psi}$ and $P_{d\gamma}$, drawn in the plane of the load-parameters W_{1m}, W_{2m}.

To obtain the curves $P_{d\psi}$ $(W_{1m}, W_{2m}) = P$ (curves $C_{\psi P}$), consider three diagrams $\bar{m}_1^*$, $\bar{m}_2^*$, $\bar{m}_3^*$ which, on the expected (or average) structure [1,2,3], can be associated with the three failure mechanisms shown in Fig. 2. Fig. 3 shows such diagrams for a particular value of $\alpha = W_{2m}/W_{1m}$. Each of these diagrams is also associated with extreme loads which lead the expected structure to incremental collapse according to one of the quoted mechanisms. Therefore, if the actual loading $(W_{1m}, \alpha W_{1m})$ has to be multiplied by a factor $\gamma_{\alpha i}$ in order to cause collapse of the expected structure according to mechanism k_i, three self-equilibrated diagrams are associated to each pair $(W_{1m}, \alpha W_{1m})$ by setting

$$m_i^* = \frac{\bar{m}_i^*}{\gamma_{\alpha i}} \quad (i = 1,2,3) \tag{3.2}$$

Now, put:

$$M_{ij} = \max \{|M^e_{i\ max} + M_i^*|, |M^e_{i\ min} + M_i^*|\} \tag{3.3}$$

$$\left.\begin{aligned} P_r &= \Pi_j \left[1 - P_{oj} \ (M_{rj})\right] \\ P_{rs} &= \Pi_j \left[1 - P_{oj} \ (\max \{M_{rj}, M_{sj}\})\right] \\ P_{123} &= \Pi_j \left[1 - P_{oj} \ (\max \{M_{1j}, M_{2j}, M_{3j}\})\right] \end{aligned}\right\} \begin{aligned} &j = 1,.....,11 \\ &r,s = 1,2,3 \\ &s > r \end{aligned} \tag{3.4}$$

From (3.3) (3.4) it is possible to obtain the expression for $P_{d\psi}$:

$$1-P_{d\psi} \ (W_{1m}, W_{2m}) = (1-P_1) + (1-P_2) + (1-P_3) - P_{12} - P_{23} - P_{13} + P_{123} \tag{3.5}$$

Function $P_{d\psi}$ $(W_{1m}, \alpha W_{1m})$ is plotted in Fig. 4 in a semi-logarithmic diagram versus the load-parameter W_{1m}, for a number of values of the ratio α, and for $P_{d\psi} \leq 0.1$. Intersecting these curves by horizontal lines it is possible to obtain points of the level curves $C_{\psi P}$. These curves are shown in Fig. 5, for $P = 10^{-5}$, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 0.5.

As for level curves $P_{d\gamma}$ $(W_{1m}, W_{2m}) = P$ (curves $C_{P\gamma}$), it is necessary to associate to the three collapse mechanisms so far considered also the ones that lead to localized fracture the expected structure, paying attention to the critical sections 1,4,6,8,11. Furthermore, it is convenient, in order to keep the calculations simple, to consider separately each failure mechanism, and to calculate only the individual failure probabilities. By this simplification, it turns out that curves $C_{\gamma P}$ are composed by a number of straight lines. In order to illustrate the procedure, consider, for instance, mechanism k_2 (Fig. 2). The equation of virtual work for this mechanism is:

$$3 \ W_{2m} + 2.1816 \ W_{1m} = \tilde{M}_{o1} + 2 \ \tilde{M}_{o6} + 2 \ \tilde{M}_{o8} + \tilde{M}_{o11} \tag{3.6}$$

The second member of (3.6) is a Gaussian random variable $\tilde{D}_a$ with mean value $\bar{D}_a = 6 \ \bar{M}_o$ and variance $\sigma_D^2 = 10\sigma_o^2$. In order to keep the failure probability lesser than P, it is necessary that the first member of (3.6) remains lesser than $\bar{D}_a + Z_P \cdot \sigma_D$, Z_P being the value of the standard normal variable $\tilde{Z}$ which is not exceeded with probability P. For P = 0.1, the equation of a side of the polygonal $C_{\gamma P}$ is given by:

$$3W_{2m} + 2.1816 \ W_{1m} = 6 \ \bar{M}_o - 1.28\sqrt{10} \ \sigma_o \tag{3.7}$$

Similarly, the equation for the alternate plasticity mechanism relevant, for instance, to section 1 is:

$$M^e_{1max} - M^e_{1min} = 0.1818\ W_{1m} + 0.8864\ W_{2m} = 2\ \tilde{M}_{o1} \qquad (3.8)$$

For P = 0.1, the equation of another side of the polygonal $C_{\gamma P}$ would be:

$$0.0909\ W_{1m} + 0.4432\ W_{2m} = \bar{M}_o - 1.28\ \sigma_o \qquad (3.9)$$

Note now that the same equation (3.9) is obtained when section 11 is considered as critical, and rupture occurs if eq. (3.8) holds in section 1 or in section 11. Therefore, if Q is the probability that eq. (3.8) holds in only one section, the compound probability will be

$$2Q - Q^2 = P \qquad (3.10)$$

if

$$Q = 1 - \sqrt{1 - P} \qquad (3.11)$$

Replacing Z_P by Z_Q, it is possible to obtain directly the overall probability of rupture relevant to both mechanisms: for P = 0.1, eq. (3.11) yields Q = 0.05, Z_Q = 1.65, and the equation (3.9) becomes:

$$0.0909\ W_{1m} + 0.4432\ W_{2m} = \bar{M}_o - 1.65\ \sigma_o \qquad (3.12)$$

Curves $C_{\gamma p}$ are shown in Fig. 6 for P = 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 0.5. It is soon noted that only the first three mechanisms yield significant bounds on the probability of incremental collapse. This is clearly due to the definition (3.1) assigned to the set of admissible loading; however, even under these conditions, Fig. 7 shows a contraction of the shakedown domains with respect to the corresponding domains relevant to monotonic loading.

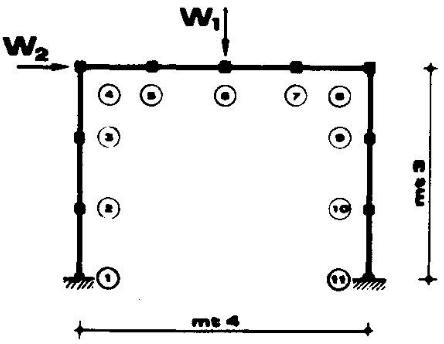

Fig. 1: Example Frame

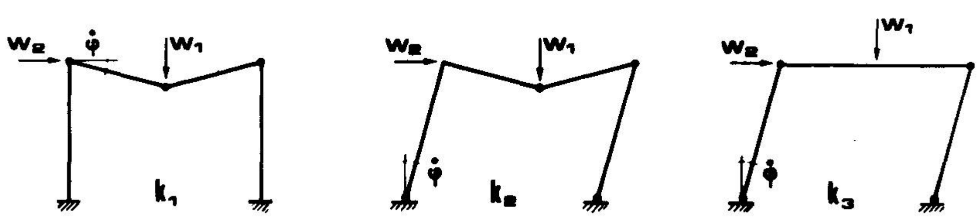

Fig. 2: Basic mechanisms

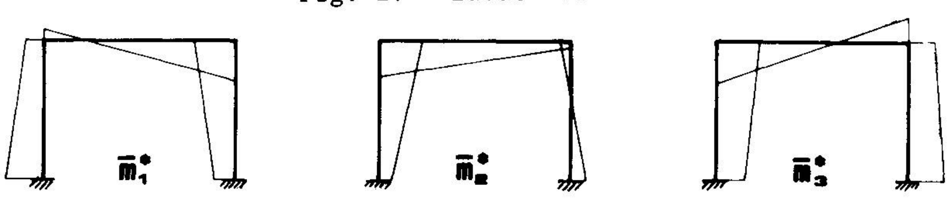

Fig. 3: Self-equilibrated diagrams of bending moment

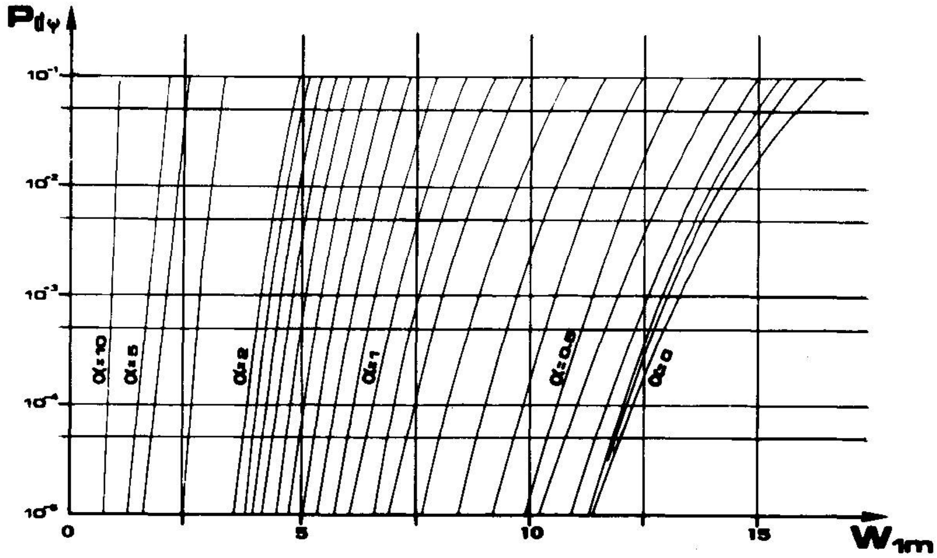

Fig. 4: Upper bound probability $P_{d\psi}$ vs. load parameter W_{1m} ($\alpha = W_{2m}/W_{1m}$)

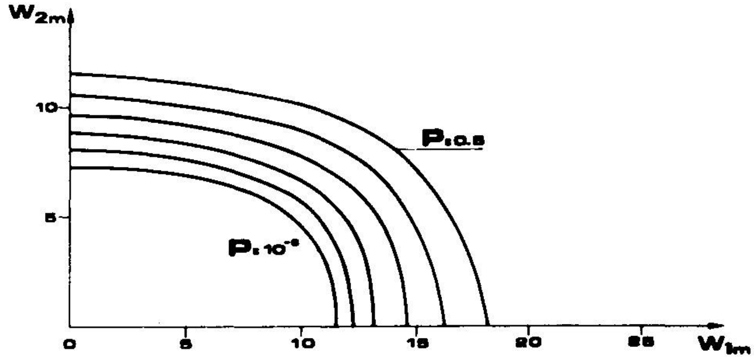

Fig. 5: Level curves $C_{\psi P}$ of the upper bound probability $P_{d\psi}$

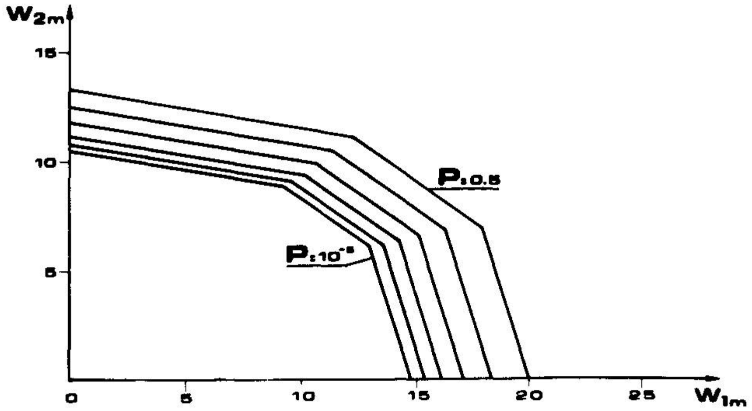

Fig. 6: Level curves $C_{\gamma P}$ of the lower bound probability $P_{d\gamma}$

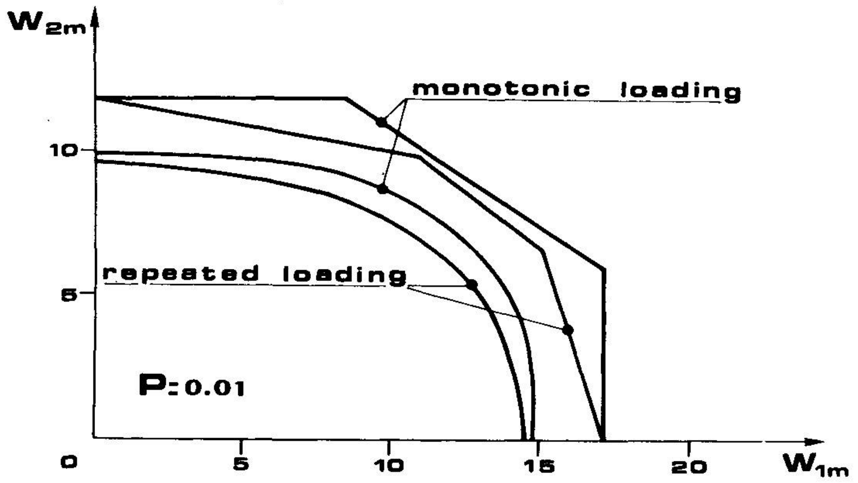

Fig. 7: Comparison between monotonic and repeated loading

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ACKNOWLEDGEMENT:

This research has been supported by the Italian National Research Council (C.N.R.).

SUMMARY

A method for bounding the shakedown probability of an elastic-perfectly plastic structure with stochastic local yield condition, subjected to repeated cycles of loading is presented. The procedure is then illustrated by a numerical example with reference to a framed structure.

RESUME

On présente dans ce travail une méthode pour estimer la probabilité de ruine d'une structure élastique parfaitement plastique avec condition de fluage local stochastique, soumise à des cycles de charge répétés. Le procédé est ensuite illustré par un exemple numérique traitant un portique.

ZUSAMMENFASSUNG

In dieser Arbeit wird eine Methode zur Begrenzung der Shakedown-Wahrscheinlichkeit eines elastisch-idealplastischen Tragwerkes mit stochastischen lokalen Fliessbedingungen unter wiederholten Belastungszyklen vorgelegt. Das Vorgehen wird sodann anhand eines numerischen Beispiels, bezogen auf ein Rahmentragwerk, veranschaulicht.

V

The Analysis and Design of Steel Structures Subject of Variable Repeated Loading

Analyse et dimensionnement des structures en acier soumises à des charges variables répétées

Analyse und Bemessung von Stahltragwerken unter variabler wiederholter Belastung

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1. Introduction

When a framed structure has to resist well-defined repeated loads, the frequency being such as to eliminate considerations of fatigue, two approaches to design are possible. The structure may be designed using elastic methods so that there is a specified factor of safety against yield or, alternatively, the plastic methods of design may be used whereby the designer chooses an appropriate load factor against collapse.

When the nature of the repeated loading is such that two or more combinations of peak loads may occur in approximately cyclic order, plastic design can become inappropriate due to the danger of either alternating plasticity or incremental collapse taking place at load levels below the plastic collapse load. In such cases, both steel and reinforced concrete structures may be rationally and economically designed on the basis of the shakedown load. This can be seen as a hybrid elastic-plastic approach whereby the structure has a specified reserve of strength against the onset of either alternating plasticity or incremental collapse.

The main objections to such an approach are on the grounds of analytical difficulty. In this paper, a suitable family of computer-orientated techniques for both analysis and design are described. These are not significantly more expensive to use than the usual approaches to automatic plastic analysis and design (which they embrace as a special case). The basis of these techniques is an unusual formulation of the equilibrium equations for plane, rigid-jointed frames.

2. Equilibrium Equations for Plane Frames

These equations have two forms and it is convenient to consider the general case first. It is assumed that the shape of the structure has been determined and that the member sizes are required. The computation is commenced by making an arbitrary assumption regarding the member cross sections (eg all members identical). Then hinges are inserted, one for each degree of statical indeterminacy in such a way that they do not constitute a mechanism.

The complete stiffness equations for the modified structure can be written down with the terms corresponding to the inserted hinges partitioned from the terms corresponding to the original structure.

$$\left[\begin{array}{c|c} S_{11} & S_{12} \\ \hline S_{21} & S_{22} \end{array}\right] \left[\begin{array}{c} D \\ \hline \theta_H \end{array}\right] = \left[\begin{array}{c} L \\ \hline M_H \end{array}\right] \qquad \ldots\ldots \quad (1)$$

Thus, S_{11} is stiffness matrix of the original structure, and the other submatrices contain additional terms associated with the extra hinges. L and D are matrices containing the vectors of applied load and corresponding displacements and M_H and θ_H are matrices containing the moments at and rotations of the hinges. We are only concerned with the terms above the partition line and these can be considered in two alternative ways, as follows.

(1) For the original structure, loaded by several alternative load combinations each of which has a corresponding column in the load matrix L, the rotations θ_H are zero so that the displacements D_L due to L are given by

$$D_L = S_{11}^{-1} L \qquad \ldots\ldots \quad (2)$$

(2) For the unloaded structure, consider a unit rotation at the location of each inserted hinge in turn. Then θ_H becomes a unit matrix and L is null so that the corresponding displacements D_θ are given by

$$D_\theta = S_{11}^{-1} \left[-S_{12} \right] \qquad \ldots\ldots \quad (3)$$

It is advantageous to combine (2) and (3) so that both can be solved in one operation.

$$\left[\begin{array}{c|c} D_L & D_\theta \end{array}\right] = S_{11}^{-1} \left[\begin{array}{c|c} L & -S_{12} \end{array}\right] \qquad \ldots\ldots \quad (4)$$

The result on the left hand side of (4) consists of a series of displacement vectors which can be considered one at a time and the distributions of bending moment calculated for the 'c' critical sections of the structure. Thus are obtained:-

(1) From vectors D_L, bending moments $\mathcal{M}_i$ that are in equilibrium with the applied loads L. (These are in fact correct elastic bending moments for an assumed structure).

(2) From vectors D_θ, distributions of residual bending moment K_{ij} that are in equilibrium with zero applied load and are mutually independent. The number of these distributions is equal to the degree or redundancy 'r' of the structure.

Combining these results gives generalised equilibrium equations for the structure in which x_j is an arbitrary multiplier for the jth distribution of residual bending moment.

$$M_i = \mathcal{M}_i + \sum_{j=1}^{r} K_{ij} x_j \quad (i = 1, 2, \ldots\ldots c) \qquad \ldots\ldots \quad (5)$$

The cost of these equations is a single elastic analysis of the structure with a possibly large number of right hand sides. The advantage is complete generality regarding structural shape and loading and a convenient form for the techniques that follow.

3. Analysis for the Shakedown Load - A Problem Orientated Technique

The most general form of the equilibrium equations has been derived with design problems in mind. For the analysis of a given structure, equations (1) are the correct stiffness equations and it is convenient to insert a hinge at each critical section in order to obtain the particular form of the equilibrium equations.

$$M_i = \mathcal{M}_i + \sum_{j=1}^{c} K_{ij}\,\theta_j \quad (i = 1, 2, \ldots\ldots c) \qquad \ldots\ldots \quad (6)$$

In (6), θ_j are actual plastic hinge rotations which are initially zero and $\mathcal{M}_i$ are actual elastic bending moments from which can be extracted the maximum and minimum values at each critical section after considering all possible combinations of loading.

The analysis for the shakedown load involves following the formation of plastic hinges as the load level increases while maintaining a distribution of residual bending moment that satisfies the shakedown theorem(1). Let first yield take place at critical section 'ℓ' with load factor λ_1 and bending moments typified by M_1. Then for further loading, the bending moment at section ℓ must remain at the full plastic moment 'M_p' and one or other of (7) must apply.

$$\lambda \mathcal{M}^{max} + K_{\ell\ell}\,\theta_\ell = +M_p$$

$$\text{or} \quad \lambda \mathcal{M}^{min} + K_{\ell\ell}\,\theta_\ell = -M_p \qquad \ldots\ldots \quad (7)$$

At an arbitrary load factor $\lambda_1 + d\lambda$, θ can be evaluated from (7) and bending moments typified by M_1' calculated. A linear prediction can then be made for the load factor λ_2 at which the next hinge forms.

$$\lambda_2 = \lambda_1 + d\lambda \frac{(\pm M_p - M_1)}{(M_1' - M_1)} \qquad \ldots\ldots \quad (8)$$

The smallest λ_2 obtained when the maximum and minimum elastic bending moments are considered at each critical section in turn locates the next plastic hinge at (say) section 'm'. Equations (9) now govern further loading.

$$\lambda \mathcal{M}_\ell^{\substack{max\\min}} + K_{\ell\ell}\,\theta_\ell + K_{\ell m}\,\theta_m = \pm M_p$$

$$\lambda \mathcal{M}_m^{\substack{max\\min}} + K_{m\ell}\,\theta_\ell + K_{mm}\,\theta_m = \pm M_p \qquad \ldots\ldots \quad (9)$$

The plastic hinge rotations θ_ℓ and θ_m can be calculated at an arbitrary load factor greater than λ_2, a linear prediction made for the load factor at which the next plastic hinge forms, and the process continued. The formation of successive plastic hinges can be followed until the shakedown load is reached when an incremental collapse mechanism exists and the matrix of participating influence coefficients for residual bending moment becomes singular.

Further consideration of this technique, including a simple yet accurate allowance for frame instability, is given elsewhere(2).

4. Analysis for the Shakedown Load - A Linear Programming Technique

Here, the equilibrium equations are also obtained from an analysis of the actual structure so that the maximum and minimum elastic bending moments are available but we use the general form of the equations. The static formulation of the problem then arises directly from the shakedown theorem, thus:-

Maximise λ

Subject to
$$\lambda \mathcal{M}_i^{max} + \sum_{j=1}^{r} K_{ij} x_j - \sum_{j=1}^{r} K_{ij} x'_j \leqslant M_{pi}$$
$$-\lambda \mathcal{M}_i^{min} - \sum_{j=1}^{r} K_{ij} x_j + \sum_{j=1}^{r} K_{ij} x'_j \geqslant M_{pi} \qquad (i = 1, 2, \ldots\ldots c) \qquad \ldots\ldots (10)$$

This is a linear programming problem which can be readily solved using the dual simplex algorithm. The alternative kinematic formulation of the problem can be obtained as the linear programming dual of (10) and given physical meaning[3] but the above statement has simpler constraints and for that reason is preferred.

Although the linear programming approach leads to a viable method of analysis, the problem-orientated approach is considerably more efficient. Indeed, the method described in section 3 is believed to be the most efficient method available for the calculation of the failure loads of plane frames with frame instability whether or not repeated loading is involved.

5. Automatic Design for Minimum Weight at the Shakedown Load

Here, the linear programming approach becomes more appropriate but a new difficulty presents itself. The statement of the problem depends on a knowledge of the maximum and minimum elastic bending moments which, in turn, depend on the relative member stiffnesses which are initially unknown. An iterative technique is required whereby assumed member stiffnesses are successively improved and this is found to converge rapidly.

The statement of the problem in its static form involves the usual minimisation of a linear weight function which is the sum of the products of length 'L_k' and full plastic moment 'M_{pk}' taken over the 'n' groups of members of identical full plastic moment. Application of the shakedown theorem results in a static yield condition similar to that obtained in section 4 so that the full statement is:-

Minimise
$$\sum_{k=1}^{n} L_k M_{pk}$$

Subject to
$$M_{pk} - \sum_{j=1}^{r} K_{ij} x_j + \sum_{j=1}^{r} K_{ij} x'_j \geqslant \mathcal{M}_i^{max} \qquad (i = 1, 2, \ldots\ldots c)$$
$$M_{pk} + \sum_{j=1}^{r} K_{ij} x_j - \sum_{j=1}^{r} K_{ij} x'_j \geqslant \mathcal{M}_i^{min} \qquad \ldots\ldots (11)$$

This too is a linear programming problem in a form readily solved by the use of the dual Simplex algorithm. A similar statement to this, and its kinematic dual, have been discussed in connection with plastic design and it has been shown that a simple serviceability constraint can be readily incorporated[3].

6. Approximate Minimum Weight Design - A Problem Orientated Approach

This technique results in designs that are almost invariably within 1 or 2% of the minimum weight but at a greatly reduced cost in computer time. The method is best described with reference to an example.

Fig 1 shows a frame which represents a simple design problem in which two unknown full plastic moments are to be chosen, M_{p1} for the columns and M_{p2} for the beam. The vertical load can vary between zero and 1.0. Table 1 summarises the design process. Each of the seven critical sections has two columns, the numbers corresponding with Fig 1. The first number refers to the maximum elastic bending moment, the second to the minimum and these moments are evaluated in arbitrary units for a frame of arbitrary stiffness in row 1. Three residual bending moment distributions corresponding to unit rotations of hinges at sections 2, 3 and 4 are shown in rows 2, 3 and 4. The merit of any particular design is judged by the usual linear weight function, in this case $Z = 200M_{p1} + 100M_{p2}$, which appears in the right hand column.

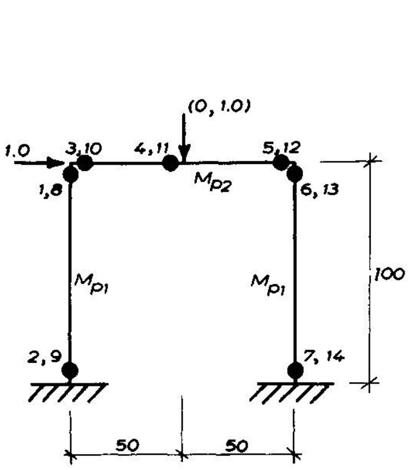

Fig 1. Simple Design Problem

The design process involves successively reducing the value of the weight function while satisfying the condition of static yield; equilibrium being ensured by the use of the generalised equilibrium equations (5).

The numerically largest bending moments present in the columns and the beams represent a feasible initial design. Thus at the commencement, M_{p1} = 32.74 units (section 14) and M_{p2} = 29.76 units (section 5) and Z = 9542 units. The moments governing the current design at each stage are marked with asterisks.

	Plastic Moment	M_{p1}		M_{p2}			M_{p1}		M_{p1}		M_{p2}			M_{p1}		Z
	Section	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
1		-13.10	-24.41	21.43	0	29.76*	-21.43	-28.57	-21.43	-28.57	13.10	-16.67	21.43	29.76	-32.74*	9524
2	m_2	952	20950	952	3333	7619	-7619	-12380	-952	20950	952	3333	7619	-7619	-12380	
3	m_3	-10950	952	10950	-6667	-2381	2381	7619	-10950	952	10950	-6667	-2381	2381	7619	
4	m_4	6667	3333	-6667	6667	6667	-6667	-3333	6667	3333	-6667	6667	6667	-6667	-3333	
5	$.000568\ m_3$	-6.22	+0.54	+6.22	-3.79	-1.35	+1.35	+4.33	-6.22	+0.54	+6.22	-3.79	-1.35	+1.35	+4.33	
6		-19.32	-23.87	27.65	-3.79	28.41*	-20.07	-24.24	-27.65	-28.03	19.32	-20.46	20.07	-28.41*	-28.41*	8522
7	$m_2' = m_2 + .909\ m_3$	-10910	21820	10910	-2728	5454	-5454	-5454	-10910	21820	10910	-2728	5454	-5454	-5454	
8	$m_4' = m_4 - .636\ m_3$	13640	2727	-13640	10910	8182	-8182	-8182	13640	2727	-13640	10910	8182	-8182	-8182	
9	$-.0000344\ m_4'$	-0.47	-0.09	+0.47	-0.38	-0.28	+0.28	+0.28	-0.47	-0.09	+0.47	-0.38	-0.28	+0.28	+0.28	
10		-19.79	-23.96	28.12	-4.17	28.13*	-19.79	-23.96	-28.13*	-28.13*	19.79	-20.83	19.79	-28.13*	28.13*	8438
11	$m_2'' = m_2' - 2.50\ m_4'$	-45000	15000	45000	-30000	-15000	15000	15000	-45000	15000	45000	-30000	-15000	15000	15000	
12	$-(0.52 \times 10^{-7}) m_2''$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
13		-19.79	-23.96	28.12*	-4.17	28.12*	-19.79	-23.96	28.12	28.12*	19.79	-20.83	19.79	-28.12*	-28.12*	8437

Table 1. Shakedown Design of Simple Portal Frame

This design can be improved by either adding or subtracting a small proportion of any of the available residual bending moment distributions in rows 2, 3 and 4. The one that makes possible the greatest weight reduction is to be chosen. For example, if a small proportion of row 2 is subtracted from row 1, the moments at sections 5 and 14 both reduce and the weight reduces. A limit is reached when the increasing moment at some other section becomes equal to the falling moment governing the design at that section. When each of rows 2, 3 and 4 are considered in this way, it is found that the greatest weight reduction is obtained by adding 0.000568 x row 3 to row 1 thus equalising the moments at sections 13 and 14. This step is summarised in rows 5 and 6 in Table 1.

Before proceeding further, it is necessary to ensure that this equality of bending moments is not violated at a later stage. This can be readily achieved by combining the distributions of residual bending moment so that they too exhibit the same equality. This involves reducing by one the number of such distributions, the equations necessary to achieve this and the resulting distributions being shown in rows 7 and 8.

Rows 6 - 8 can now be seen to be a starting point for another weight reducing step, being directly analogous to rows 1 - 4. This step is summarised by rows 9 and 10. The process can be continued until each of the available residual bending moment distributions has been combined with the elastic bending moment distribution at which stage no further weight reducing steps can be made. This final design is given in row 13 of the table.

For this example the final design is in fact optimal, being identical to that obtained by linear programming. In general this will not be so but the final design will be so close to the optimum that the difference is unsignificant when translated into available sections. Here also, iteration of member stiffnesses is usuall required though with this particular example, the final design results in a frame of uniform section which coincides with the initial assumption so that no iteration is required.

This technique has been described in greater detail, with particular reference to collapse design, in reference 4, where examples of its use are given and compared with solutions obtained by linear programming.

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SUMMARY

This paper is concerned with plane steel frameworks for which the limiting load is the shakedown load. It describes techniques for both analysis and design which are all based on an unusual derivation of generalised equilibrium equations. Both linear programming formulations and problem-orientated techniques are described, the latter being considerably more efficient.

RESUME

Ce travail étudie des cadres plans en acier pour lesquels la charge limite est la charge de rupture. Il décrit des méthodes pour l'analyse et le dimensionnement basées sur une dérivation inhabituelle des équations d'équilibre généralisées. Il présente à la fois les méthodes de programmation linéaire et les techniques spécifiques adaptées au problème, les dernières étant plus efficaces.

ZUSAMMENFASSUNG

Dieser Aufsatz befasst sich mit ebenen Stahlrahmen, für die die Grenzlast die "Shakedown"-Last ist. Er beschreibt Methoden zur Berechnung und Bemessung, die sich alle auf einer ungewöhnlichen Ableitung der verallgemeinerten Gleichgewichtsgleichungen aufbauen. Es werden Formulierungen zum linearen Programmieren und problemorientierte Techniken beschrieben, wobei letztere bedeutend leistungsfähiger sind.

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Safety, Durability and Reliability of Metal Structures

Sécurité, durabilité et fiabilité des constructions métalliques

Sicherheit, Dauerhaftigkeit und Zuverlässigkeit von Metallkonstruktionen

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1. General Aspects.

The design of safe, durable and maintainable engineering structures requires the development of a general methodology as well as of specific procedures for
a) the verification of their functional and structural integrity (serviceability and safety);
b) the prediction of their expected service life under anticipated operational conditions (durability);
c) the assessment of the risk associated with such verification and predictions (reliability).

Such procedures must necessarily be integrated to involve all phases, from the planning phase of materials evaluation and selection to the final phase of reliability demonstration and the setting-up of an inspection and maintenance program. It is the lack of recognition of the necessity for integration of all phases as well as of the complexity of the interactions between inherent material properties, selected design criteria, structural details, manufacturing processes and conditions of operation that has been responsible for some of the most costly errors in materials selection and design that have been committed. Materials evaluation for the purpose of assurance of superior structural integrity and reliability is not identical with conventional evaluation of inherent material properties, but involves the comparative study of alternative systems and process technologies in which all interactions are considered and from which the basic parameters for the subsequent integrity-, durability- and risk analysis can be deduced.

The significance of any of the above aspects will vary with the type of structure, its expected operational utility and usage and the material and fabrication process used in its construction. Thus for instance the fact that the surveillance procedure for riveted railroad bridges consists exclusively of periodic visual and (possibly) sonic in-service inspections of the rivets and gusset plates in the principal connections, while the surveillance procedure for long-range aircraft involves several extensive out-of-service inspections of the complete load-carrying structure reflects the differences in structural type, operational usage and material of construction rather than a basic difference in the general reliability outlook of a particular industry. The contention that "problems of structural safety and reliability are viewed in different terms in different branches of engineering" (Introductory Report, p. 9) would be valid only if it were meant to imply that the degree of acceptance of a rational approach to problems of safety and reliability is not uniform in the various branches of engineering. Unfortunately, this acceptance is, at present, still rather low in all branches of engineering concerned with mechanical and structural design (as distinct from the design of electronic systems); the apparent differences in approach are nothing else but a reflection of the differences in the assessment of the relative significance of the above three aspects of structural analysis and design.

The Introductory Report attempts to deal with the problem of design for repeated loading in terms of the first aspect (integrity) by restating as "Generalized Formulation" the by now widely accepted probabilistic concept of safety under the single application of a critical load or load combination.[1] The conclusion is obvious that "in these general terms the problem (safety for repeated loading) apparently has no practical solution" (Introductory Report, p. 10).

Attempts have been made to extend the simple probabilistic ultimate load safety concept to design for repeated loads producing fatigue damage by a method [2] that resembles the somewhat vague concept suggested in Section 3.5 but permits the introduction of modern more recent procedures of fracture mechanics for the purpose of fatigue life prediction and of order statistical concepts for risk assessment.[3].

2. Fracture Mechanics Model.

This method is based on the combination of the equation for the unstable crack-propagation stress intensity $K_c = F.\sigma_u\sqrt{c_u}$ determining the gross ultimate load stress σ_u for a structural part with crack extension under the (repeated) stress intensity range $\Delta K = F.(\Delta\sigma)\sqrt{c}$, of the form $(dc/dN) = M(\Delta K)^r$, where F, M and r are constants that depend on geometry and material. Integration of the crack-extension equation over N stress-intensity cycles with initial defect c_i produces an expression for the gross stress σ_R at ultimate failure of the damaged structural part that has been subject to $N < N_F$ stress intensity cycles (residual strength) where $N_F \sim (K_c/K_i)^{r-2}$ denotes the number of cycles at which the initial defect c_i attains the critical value c_u (ultimate strength):

$$\sigma_R \sim \text{Const.}\sigma_u\left[1 - \frac{N}{N_F}\right]^{\frac{1}{r-2}}$$

Unfortunately this simple method is applicable only to monolithic single-load path structures containing severe (pre-existing) material or manufacturing defects (cracks, inclusions, welds) in which fatigue failure arises as the terminal condition of slow crack extension from a pre-existing defect, a condition to be avoided in any good design. Even in this case, however, the estimate of N,

N_F or σ_R is subject to considerable statistical uncertainty because of significant scatter of observed crack propagation rates, critical crack-size and other relevant parameters. The scatter range for the same structural metal of the crack-propagation rates alone is between 1:5 and 1:10, and combines with the uncertainties involved in the assessment of the severity of pre-existing defects and in the specification of the operational loading spectra. Only in the case of essentially non-statistical operational loading (railroad bridges, pressure vessels) is it at all possible to rely on the existence of an "endurance limit" of a metal like steel and to disregard fatigue in the design of structural members by keeping the maximum stress amplitude below the "endurance limit".

Even if fracture mechanics analysis is only used for the determination of appropriate inspection intervals by considering the slow propagation of the largest undetectable crack to critical size, the inherent uncertainties of recorded crack-propagation rates at constant stress range intensities, as well as the additional uncertainties arising from attempts at superposition of such rates at variable stress range intensities, in view of the severe interactions between high and low intensities mainly due to residual stress fields at the crack-roots, preclude a better than order-of-magnitude prediction of "safe" inspection intervals; such interaction may also severely reduce or destroy the "limiting levels" of stress-intensity below which crack-propagation is nonexistent. Nevertheless, the knowledge of crack-propagation rates for structural metals under conditions representative of the structure and of its operation is basic in the analysis of fatigue behavior and adequate material selection.

In the case of a redundant, multiple-load-path "damage-tolerant" structure with or without pre-existing defects this oversimplified model of the fatigue process bears no resemblance to reality and is therefore inapplicable. It completely disregards the stage of fatigue-crack initiation which, contrary to the assumption made in the fracture mechanics interpretation of fatigue, makes up a significant portion of the fatigue life, even in well-designed monolithic structures in the production of which a sufficiently high level of quality control has been imposed.

3. Fatigue Life Prediction.

The lack of confidence in analytical procedures of fatigue life prediction arises from the fact that, so far, no analytical rule of fatigue damage accumulation applied to variable stress-amplitudes and variable mean stress produces life predictions that consistently agree with life test results more closely than one order of magnitude. This is not only the result of statistical uncertainty, but of the impossibility to combine all physical aspects of fatigue damage accumulation in a structure with redundancies and complex interactions subject to a complex, neither purely stochastic nor purely deterministic load sequence, into an analytical damage accumulation rule that would reflect the trend of damage accumulation as well as the statistical uncertainties involved. The widely used linear damage accumulation rule, with simple modifications to account for "interaction" and "residual stress" effects [4] has the advantage of maximum simplicity. Identification of the loading conditions producing the major portion of the fatigue damage, for instance the ground-air-ground cycle in the transport airplanes, and isolated study of the damage accumulation under such conditions alone might produce a more reliable analytical procedure of fatigue life estimation by concentrating on the dominant damage mechanism. Even in this case, however, the analytically predicted life will represent not more than an order-of-magnitude approximation, always to be verified by full scale tests.

The fatigue performance of a structure is significantly affected but not decisively determined by the fatigue performance of the material under conditions of laboratory tests. It has been estimated that more than 90 percent of fatigue failures in structures and machine parts are primarily due to faulty design of details or to production defects; many of these failures could therefore be avoided by increased attention to design and design details, and by stricter control of production processes, without improving the fatigue performance of the material itself. On the other hand, it is unlikely that serious effects of faulty design or of inadequate surface treatment can be fully compensated by the selection of an alloy of better fatigue performance in laboratory tests.

The character of fatigue failures in various types of structures and structural parts depends on the relative significance of the different factors by which their fatigue performance is affected. Thus fatigue in axles, shafts, pins, etc. under relatively steady operating conditions, as in motors, machinery, ships and railroad equipment is dominated by the stress-concentrations associated with characteristic design-features, such as fillets, section-changes, key-ways, holes, corners, etc. As diameters increase, inhomogeneities in the metal and residual stresses arising in the forming process (cooling gradients, metallurgical transformations) become increasingly important. Fatigue in riveted structures is dominated by stress-concentrations and fretting in the connections; for structures under highly variable loading, it is significantly affected by the plastic redistribution of stresses under high load-amplitudes. Fatigue under acoustic noise is affected by the character of the noise spectrum as well as by panel geometry, by the damping of the excited modes of the structure and the intensity of the noise itself. Thus it is known that the same noise level applied to the same structure produces different fatigue lives depending on whether the energy is concentrated in a single frequency (siren noise) or distributed over a multidude of frequencies (jet-noise). Fatigue in welded structures is dominated by metallurgical changes in the weld-affected zone combined with excessive rigidity of and residual thermal stresses in the connections. Fatigue of parts or structures repaired or built-up by welding is invariably caused by the combined metallurgical, thermal and mechanical effects associated with the welding. Fatigue of complex structures under variable operating conditions, such as airframes or high-temperature service equipment, can usually not be attributed to a dominant cause, unless failure is clearly due to faulty design of details.

It is to be expected that the smaller the number of contributing factors and the better they can be controlled, the easier is design information obtainable from specimen fatigue tests, and the more reliable the design for fatigue and the prediction of fatigue life on the basis of this information. The more the fatigue performance depends on a combination of several factors, the more difficult the separate assessment of the effects of the individual factors. The more important therefore the full-scale fatigue test of the structure for the double purpose of eliminating the weakest spots by observation of the actual sequence of localized failures, and of predicting the order of magnitude of the fatigue life of the structure in which these spots have been adequately strengthened. Accelerated fatigue tests, at constant high stress amplitude, in which the sequence and character of the service failures is not duplicated and a different type of failure is produced, are useful only as means of locating points of excessive initial damage; they do not provide information for the estimate of operational life.

4. Aspects of Fatigue Design.

The dependence of fatigue failure in structures on a combination of factors the effect of each of which can only be specified with a certain margin of uncertainty makes it unrealistic to attempt to predict the fatigue life of a structure even under rather closely defined operational condition in any but a statistical manner. Fatigue design of structures is therefore a problem of "reliability"-analysis rather than of stress- or strength-analysis. Fatigue design differs significantly from ultimate-load design by the complexity and resulting vagueness of the correlation of load, stress and carrying capacity. The large number of contributing factors, the combined effects of which determine the fatigue life of a structure, necessarily produce a rather wide scatter in the fatigue life even of nominally identical structures under nominally identical operating conditions.

The approach to fatigue design of structural parts and structures must be determined by their "fatigue-sensitivity", defined as a measure, at a specified service life, of the probability of the structure to fail in fatigue rather than under a single application of the "ultimate" load.[5] As this probability will depend, among other, on operating conditions and the expected service life, the "fatigue-sensitivity" of a structure cannot be specified independently of such conditions; the same structure may be "fatigue-insensitive" under one set of conditions and "fatigue-sensitive" under another. Under conditions of low fatigue-sensitivity it will be usually unnecessary to design for a specific operational life in fatigue; fatigue design may be simply limited to elimination, by constructive means, of the most obvious sources of crack initiation, so as to avoid a possible reduction of the expected fatigue life to within range of the expected life with respect to the ultimate load. Specific design for fatigue, supplemented by extensive fatigue testing is necessary only for medium and high "fatigue-sensitivity" for which fatigue failure is the expected type of failure; tests of full scale structures are an integral part of fatigue design. The prediction of the expected operational life of the structure and the prevention of catostrophic consequences of possible failure within this period (which cannot be excluded because of the irreducible uncertainty of such prediction) is the dual purpose of such design.

Fatigue design for finite life as a problem of "reliability" must be concerned with a rational measure of reliability; an expedient measure is provided by the probability of failure-free operation during the specified service life L. Statistically this is the probability of survival as a function of service life, the survivorship-function, which is thus identical with the "reliability function" R(L). It is the advantage of this statistical measure of reliability that it permits the quantitative correlation, under simplifying assumptions, of the reliability of the structure with the reliability of its components, as well as of the reliability under fatigue conditions with the reliability under ultimate load conditions. The introduction of a quantitative measure or scale of "fatigue sensitivity" and a classification of structures or structural designs in terms of such a measure makes it possible to utilize the existing results of full-scale fatigue tests in the fatigue analysis of newly designed structures, by concentrating in this analysis on those factors by which the fatigue sensitivity of the new structure is expected to differ from that of previously designed structures of specified fatigue sensitivity, for which both testing and operating experience has been accumulated.

The procedures of reliability analysis of structures under fatigue conditions are based on the concept of "risk" of failure after N load applications, r(N), in terms of which a quantititive measure of "fatigue sensitivity can be defined. Since failure of a structure or part can be caused either by chance coincidence of an extremely rare "ultimate load" with an initial resistance sufficiently low to produce instantaneous collapse, or by fatigue under repeated load-intensities significantly lower than the ultimate load, and represented by a spectrum of operational loads, a reasonable measure of fatigue sensitivity is the ratio of the risk of fatigue failure to that of "ultimate load" failure at any load application. This ratio

$$f(N) = \frac{r_F(N)}{r_u(N)}$$

where r_F denotes the risk of fatigue failure, r_u that of "ultimate load" failure, can therefore be designated as a "coefficient of fatigue sensitivity" of the structure at a certain "age" N; since $r_F(N)$ may be assumed to increase with age by definition of the phenomenon of "fatigue", the fatigue sensitivity of a structure will also tend to increase with age. The larger f(N), the larger the probability, at any value of N, that the structure will fail in fatigue rather than by ultimate load collapse. If $N=N^*$ denotes the expected operational life of the structure, the value $f(N^*)=f^*$ characterizes the fatigue sensitivity of the structure at the end of this life rather than at any "age" N, and is therefore an important design parameter:

It is tentatively assumed that a rational classification of the fatigue sensitivity of systems or structures might be based on the following scale.

(I)	$0 < f^* < 0.1$	fatigue insensitive structures
(II)	$0.1 < f^* < 1.0$	moderately fatigue sensitive structures
(III)	$1.0 < f^* < 10$	highly fatigue sensitive structures
(IV)	$10 < f^*$	fatigue critical structures

Class (I) structures need be designed for ultimate load only; Class (II) structures should be designed for ultimate load, but with careful consideration of details with respect to fatigue performance; only the critical components and connections are fatigue-tested by accelerated procedures to eliminate fatigue-prone details. Class (III) structures should be both designed and tested for fatigue, with at least one full-scale fatigue test under a representative load spectrum, in addition to component and connection fatigue tests sufficient to estimate their usable minimum operational life. Class (IV) structures or parts, if they cannot be avoided, should be designed for fatigue alone and used only if a sufficient number of replications of full-scale fatigue test can be performed to permit a reliable statistical estimate of mean or median fatigue life and of the scatter range. However, important structures, the failure of which would have serious consequences, should be designed so as not to fall into Class (IV). This may be done either by reducing the risk of fatigue failure or by limiting the operational life, or by imposing stringent procedures of inspection for fatigue cracks, the discovery and repair of which would make the maximum inspection period the "critical" operational life, to be used in evaluating the fatigue sensitivity.

In terms of the approach to the various aspects of design structures under repeated loading outlined above and believed to reflect the objective conditions of the problem, some of the concepts with which the Introductory Report seems to be concerned are either outdated (section 3.2), of dubious validity (Section 3.3, in particular Eq. 12) or confusing if not misleading (Section 4.1) in the lack of recognition that design for "limit states" (critical loading conditions) and design for repeated loading are different but complimentary aspects of design.

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SUMMARY

The Introductory Report deals with the problem of design for repeated loading on the basis of a generalization of the probabilistic concept of safety under a single load application. The short-comings of this approach are discussed and the basic differences between design for a single limit state and design for fatigue are outlined. The concept of "fatigue sensitivity" of a structure is introduced as a key to the selection of a rational design procedure.

RESUME

Le rapport introductif traite le problème du dimensionnement en cas de charges répétées sur la base d'une généralisation du concepte probabiliste de sécurité pour une seule application de la charge. Les imperfections de cette méthode sont discutées et les différences fondamentales entre le dimensionnement pour un état limite simple et le dimensionnement à la fatigue sont soulignées. Le concept de "sensibilité à la fatigue" d'une structure est introduit comme point de repère pour le choix d'une méthode de dimensionnement rationnelle.

ZUSAMMENFASSUNG

Der einführende Teil befasst sich mit dem Problem der Bemessung für wiederholte Belastung aufgrund einer Verallgemeinerung des wahrscheinlichkeitstheoretischen Sicherheitskonzepts unter einer einzigen Lastaufbringung. Die Vereinfachungen dieser Näherung werden diskutiert und die grundlegenden Unterschiede zwischen Bemessung auf einen einzigen Grenzzustand und Bemessung auf Ermüdung hervorgehoben. Als Schlüssel zur Auswahl vernünftiger Bemessungsmethoden wird der Begriff der "Ermüdungs-Empfindlichkeit" eines Tragwerks eingeführt.

Structural Safety of Steel Highway Bridges subjected to Repeated Vehicle Loads

Sécurité des pont-routes en acier soumis à des charges de trafic répétées

Tragwerksicherheit von Stahl-Autobahnbrücken unter wiederholter Fahrzeugbelastung

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1. Introduction

Recently, several probabilistic studies on stresses at main girders of highway bridge subjected to repeated moving loads, have been published. The present study is based on a different concept from past studies, and is intended to estimate the distribution of peak stress frequency at a main girder, taking into account statistic factors on traffic flow. Numerical examples for model bridges demonstrate practicability of the proposed analysis for static and fatigue failurs.

2. Estimation of Working Stresses

It is assumed that an effect of the high frequency components of a stress wave is covered by an impact coefficient and that local minimum stresses can be disregarded. When a sequence of concentrated loads , $P_1, \cdots\cdots, P_N$, moves at a uniform speed, V , on a simple beam as shown in Fig. 1, a bending moment, $Y(t)$, at the span center, A , and at a time, t , is presented by an influence line shown in the figure, as follows:

$$Y(t) = \sum_{i=1}^{N} (P_i \cdot a_i) \qquad (1)$$

If a load P_J is at the span center as shown in Fig. 1, the occurrence of peak value will be given on the following condition:

$$\left| \sum_{i=1}^{I} P_i - \sum_{i=K}^{N} P_i \right| < P_J \qquad (2)$$

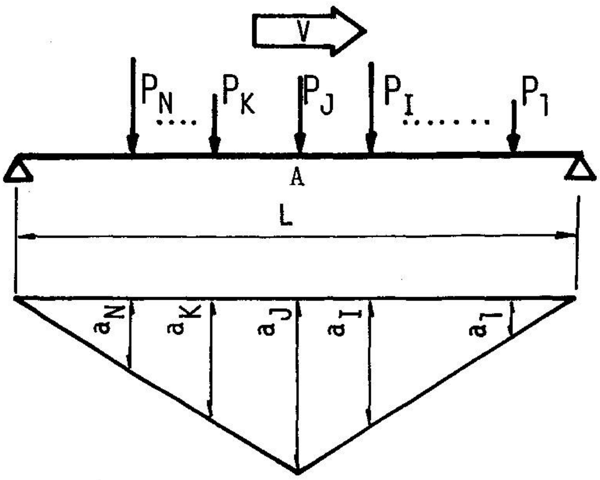

Fig. 1 A Sequence of Moving Loads and Influence Line

Then, the following mathematical amounts should be computed for distribution of peak stress: the probability density function of P_J, $P_J \cdot a_J$ and $\sum_{i=1}^{N} (P_i \cdot a_i)$; the joint probability density function of $\sum_{i=1}^{I} P_i$ and $\sum_{i=1}^{I} (P_i \cdot a_i)$, $\sum_{i=K}^{N} P_i$ and $\sum_{i=K}^{N} (P_i \cdot a_i)$, and $\left| \sum_{i=1}^{I} P_i - \sum_{i=K}^{N} P_i \right|$ and $\sum_{i=1}^{I} (P_i \cdot a_i) + \sum_{i=K}^{N} (P_i \cdot a_i)$.

3. Models of Traffic Flow and Models of Highway Bridges

(1) Traffic flow: Actual measurements of wheel loads of a vehicle have been reported, but are a few ones published on the measurements of total weight of vehicles. Fig. 2 shows one of them[1] in Japan. It is assumed that the distribution of vehicle total weight can be expressed by the superposition of the three kinds of normal distribution corresponding to each vehicle group. The combination ratio and distribution pattern illustrated in Table 1, as a standard example, are selected so that the excesses probability may have an inclination to coincide generally with, and to be somewhat larger than, the actual measurements shown in Fig. 2. The Probability, $p(i)$, that the number of moving vehicles covering the length of $L/2$ is i , is expressed by the following equation, on the assumption that the probability density function of headway follows the exponential function of $\mu=0.02i$:

$$p(i)=(\mu L/2)^i e^{-\mu L/2} / i! \quad (3)$$

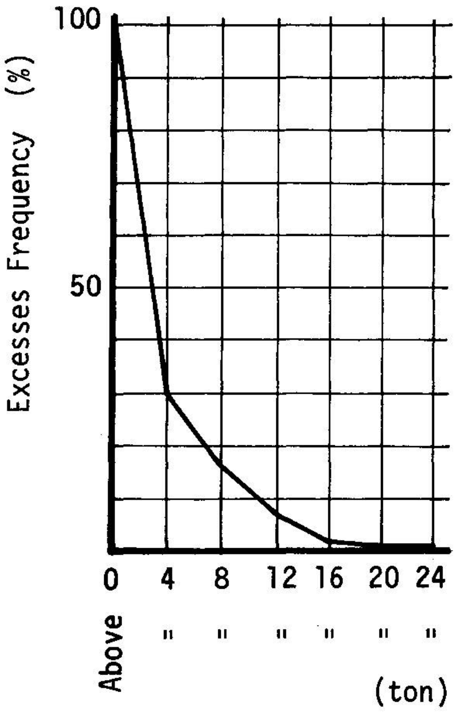

Fig. 2 Excesses Frequency of Vehicle Total Weight

Table 1. Standard Examples for Distribution of Vehicle Total Weight

Vehicle	Average Weight (tons)	Standard Deviation of Weight (tons)	Combination Ratio
Heavy Trucks	20	6.0	0.02
Light Trucks	10	4.0	0.23
Passenger Cars	2	1.2	0.75

(2) Highway bridge models illustrated for study are the standard composite girder bridges in Japan as shown in Table 2. Stress calculations are made for a lower flange at the span center as a representative member. The 1st class bridge is designed for 20-tons trucks, and the 2nd class is for 14-tons trucks. The assumption is made that simultaneously loaded vehicles are 5 numbers for 40 m span and 3 for

24 m with the occupying length of 8.0 m, and are concentrated loads independent from each another.

Table 2. Models of Highway Bridge

Notation	Class	Width (m)	Span Length (m)	Girder	Dead Load Stress (kg/cm2)
ST-1	1st	6.0	40.0	Outside	1108
ST-2	1st	6.0	40.0	Inside	1045
ST-3	1st	6.0	24.0	Outside	888
ST-4	1st	6.0	24.0	Inside	840
ST-5	2nd	6.0	24.0	Outside	1122

4. Material Properties

The static strength[2] of the steel SM50 for representative members of the model bridge is expressed by its yielding stress, the distribution pattern of which is represented by the lognormal distribution, and the mean value is 3750 kg/cm^2 and the coefficient of variation of which is 0.08. The fatigue strength[3] of the steel SM50 is expressed by the time strength, the distribution of which is of the lognormal one, the median value of which at 2x10^6 numbers of loading cycles is 1500 kg/cm^2. An inclination of the *S-N* curve for the material is deterministic and its mean value is 6.0, and the standard deviation of logarithmic fatigue life is 0.25 .

5. Calculation Results for Distribution of Peak Stress

Results of the calculations of excesses probability of the stress due to live load and impact at the model bridges ST-1∿5 are summarized in Fig. 3 . In the calculation, the excesses probability is taken one for outside girders when a vehicle passes through the span center on the lane right above the girders, and it is taken one for inside girders when a vehicle passes through the span center on either lane.

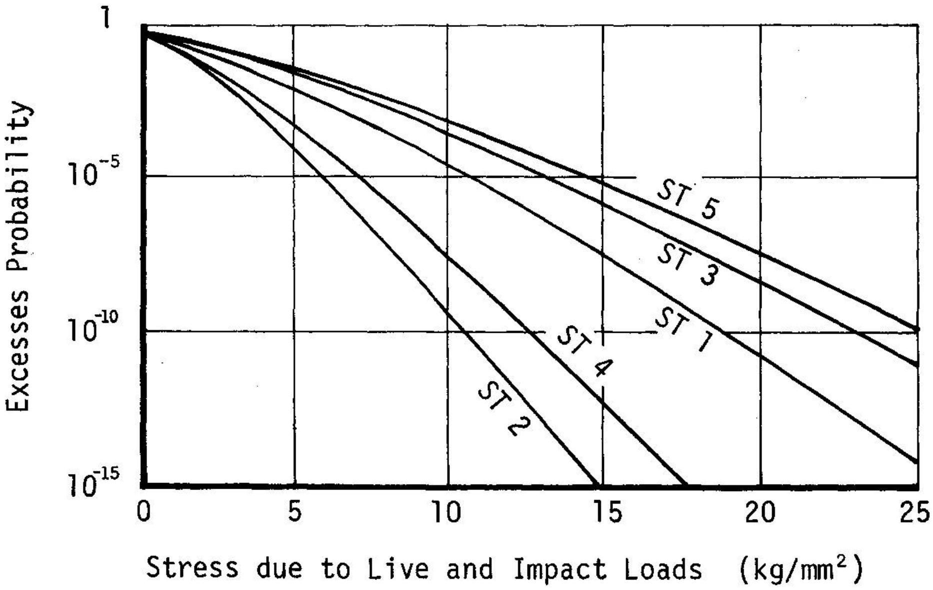

Fig. 3 Calculation Summary for Peak Stress Distribution

6. Equation for Probability of Static Failure

When loads are applied either in equal intervals or at prescribed time instants so that the life of the structure can be measured in terms of the number,

n , of load application, and if the resistance, R , is a random variable, the case considered in which the distribution functions $F_S(y)$ of applied load S , and $F_R(x)$ of R , are time invariant, implying that all the loads, S , applied in sequence belong to one and the same distribution and the material and structure suffer no deterioration. Then, the probability of failure of a structure, p_{fn} , resulting from a series of n load applications, is expressed as, with the density function, $f_R(x)$, of R :

$$p_{fn} = \int_0^{\infty} f_R(x)\{1 - (F_S(x))^n\}\,dx \,. \quad (4)$$

7. Equation for Probability of Fatigue Failure

(1) Rule of fatigue damage accumulation : The most widely known and used procedure for cumulative fatigue damage is the linear damage rule commonly referred to as the Miner rule. Also, various methods have been proposed[4)] as alternative to the linear damage rule. Since the proportion of too small stresses to the total actual stress is large, instead of the standard S-N relationship, the modified linear damage rule be applied to the present situation.

(2) Assumption for calculation: The S-N curve reaches the fatigue limit at 2×10^6 cycles of loading and it takes a horizontal line for $N > 2 \times 10^6$ numbers of cycle. Scattering of the fatigue limit will be expressed in terms of the scattering of time strength or fatigue life. The slope, θ , of a sloping portion of the S-N curve may be taken deterministic. The minimum stress due to dead load may be assumed to be deterministic and its correction may be made in the modified Goodman diagram.

(3) Equation for the probability for fatigue failure is, then , obtained as follows:

$$p_f = P_r\left(\sum_{i=1}^{n} \frac{S_i^{\theta}}{N_0 S_0^{\theta}} > 1\right) , \quad (5)$$

where S_i = a stress due to i-th loading, N_0 = a fatigue life expectancy to the stress S_0 , and θ = a slope of the S-N curve. Eq. (5) presupposes that the S-N curves are presented as straight lines on a log-log plot, and that the modified linear damage rule is valid. The number N of cycle of loading can be treated as deterministic, because, as the result of great number of repetition of measurement for traffic volume per day, the distribution of day traffic volume will be given. Since the number of loading on a highway bridge during its useful life is very large, randomness of $\sum_{i=1}^{n} S_i^{\theta}$ is not taken into account. With the above discussion, Eq. (5) will be transformed into

$$p_f = P_r\left(\frac{N}{N_0 S_0^{\theta}} \int_0^{\infty} S^{\theta} f(S)\,dS > 1\right) , \quad (6)$$

where N_0 = the number of repetition to failure due to independent loading of a stress level S_0 (probabilistic), S_0 = the standard stress level (probabilistic), S = a stress level due to live load and impact (probabilistic), $f(S)$ = the probability density function of S .

Then, the further transformation will give $$p_f = \int_0^C f_{N_0}(x)\, dx \quad , \qquad (7)$$

where f_{N_0} = the probability density function of fatigue life between zero to tension, $0 \sim S_0$, and $C = \frac{N}{S_0^\theta}\int_0^\infty S^\theta f(S)\, dS$.

8. Presentation of Calculation Results

With the numerical values for strength factor given at the Chapter 3, the probability of failure, p_f , for static failure and fatigue failure can be calculated by Eqs. (4) and (7), respectively. The calculation results of p_f are summarized in Fig. 4, with chainlines for the static failure and full lines for the fatigue failure.

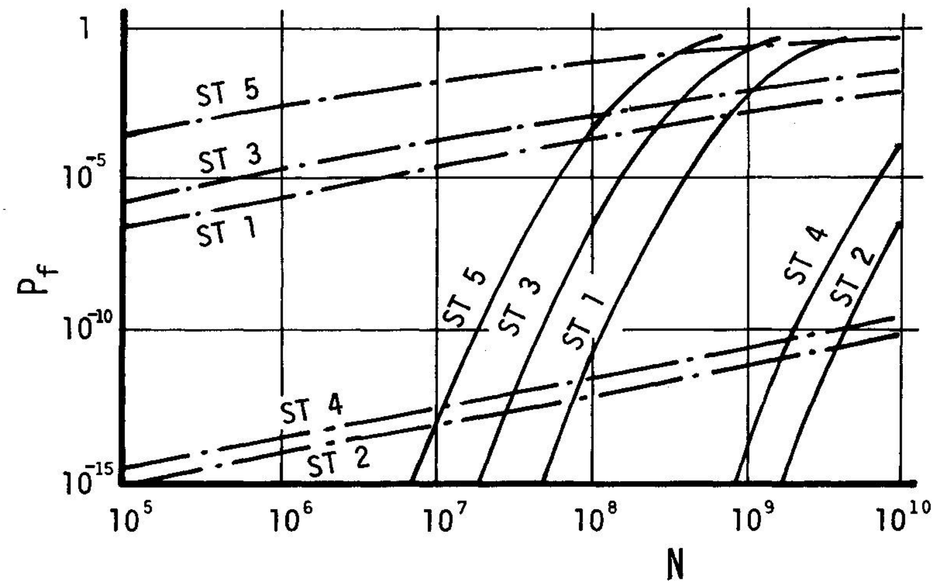

Fig. 4 Probability of Failure of Main Girders

Among the models, the case of ST-5 indicates the largest values for the both of the failures, and ST-3, ST-1, ST-4 and ST-2 show smaller values in this order, corresponding to the amount of the excesses probability of peak stress. It is known that, at the case of ST-5, the static failure is governing for the number of loading less than $6x10^8$ and the fatigue failure has a priority for the number larger than that. Such a feature can be seen also at the rest cases, but the case of ST-1 will be governed practically by the static failure.

9. Conclusions

At the present study, a method of stress estimation for the safety of main girders of a highway bridge, is proposed, and its fundamental concept is discussed and the calculation method is given. Also it is pointed out that application of the stress estimation method to actual highway bridges can reflect the actual condition fairly well. Taking into consideration the traffic flow factors such as vehicle weight distribution and vehicle headway distribution, the probability of static failure and fatigue failure of main girders at five simple-supported composite beam highway bridges, is calculated for illustration. As a result, it is known that the static failure governs the safety of illustrated girder models under the condition of the traffic volume of, for example, about 10,000 vehicles per day and of the useful life of about 50 years, rather than the fatigue failure.

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SUMMARY

In order to appraise the structural safety of a main girder of a steel highway bridge subjected to a great number of repetition of variable vehicle loads under a service condition, a probabilistic method for estimating working girder stresses is proposed, together with a basic safety concept and numerical examples. The method makes it possible to estimate the frequency distribution of peak stresses working at main girders, taking into consideration statistical factors on traffic flow, in addition to probabilistic variation of material properties.

RESUME

On présente dans ce travail une méthode probabiliste pour estimer les contraintes à la fatigue dans les poutres maîtresses, associée à un concept de base de la sécurité et à des exemples numériques pour déterminer la sécurité structurale d'une poutre maîtresse d'un pont-route en acier soumis à un grand nombre de charges variables répétées. Cette méthode permet d'estimer la distribution de la fréquence des pointes de tensions dans les poutres maîtresses, en considérant des facteurs statistiques sur le flux de trafic et une variation probable des propriétés du matériau.

ZUSAMMENFASSUNG

Um die Tragwerksicherheit eines Hauptträgers einer Autobahnbrücke aus Stahl bei vielen Wiederholungen von verschiedenen Fahrzeugbelastungen unter Betriebsbedingungen abzuschätzen, wird eine wahrscheinlichkeitstheoretische Methode zur Schätzung der Betriebsbeanspruchungen in den Trägern, zusammen mit einem grundlegenden Sicherheitskonzept und numerischen Beispielen vorgeschlagen. Die Methode ermöglicht die Abschätzung der Frequenzverteilung der Spitzenspannungen in den Hauptträgern unter Berücksichtigung statistischer Faktoren des Verkehrsflusses, zusätzlich zu der wahrscheinlichkeitsbedingten Variation der Materialeigenschaften.

V

On the Safety of Steel Members against Buckling under Random, Reversible Loads

De la sécurité des barres métalliques soumises au flambement et à des charges alternées aléatoires

Zur Frage der Sicherheit von Stahlstäben unter zufälliger Wechselbeanspruchung

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1. INTRODUCTION

Design rules for compressed steel members are nowadays based on extensive experimental and numerical evidence on the ultimate buckling strength of real, industrially fabricated bars upon virgin loading [1]. If full or partial stress reversal may occur in the member, that is not the only possible ultimate state. As Klöppel and Klee have shown for steel St 37 [2], only 1.5 cycles at a reversed plastic strain of 1 ‰ will lower the yield plateau by 13 %. Beside this strain softening at low strain levels which affects the extreme points of the hysteretic loop, the proportional limit inside the loop is lower than in the virgin loading (Bauschinger effect). At least theoretically, we may thus expect buckling collapse at lower loads after the first plastic excursion in tension. On the other hand, tensile plastic strains may reduce the most important imperfection of the member : initial crookedness ; and sequences of alternating high loads have very small probabilities in most civil engineering structures.

2. SIMPLIFYING ASSUMPTIONS

The member considered has an idealized two point section and a sine-shaped initial crookedness (Fig. 1). Its amplitude - known to be 1/1000 or smaller in industrial bars [1] - is used as a fixed parameter.

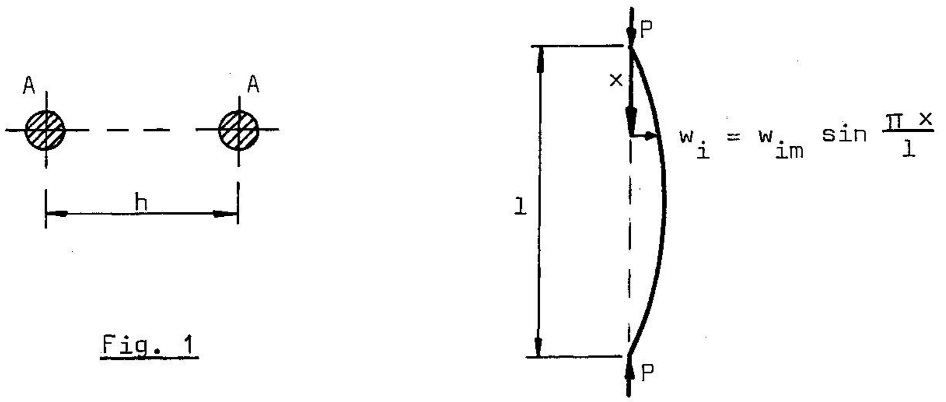

Fig. 1

A bilinear stress-strain relation (Fig. 2) is used for the flanges. Informations on the parameters γ and σ_p are obtained as follows : in the range between 1.5 and 10.5 cycles at plastic strains below 3 ‰ , the extreme point of the hysteretic loop lies on a yield plateau with $\sigma_c = 0.87\ \sigma_y$, where σ_y denotes the yield stress in the virgin state [2].

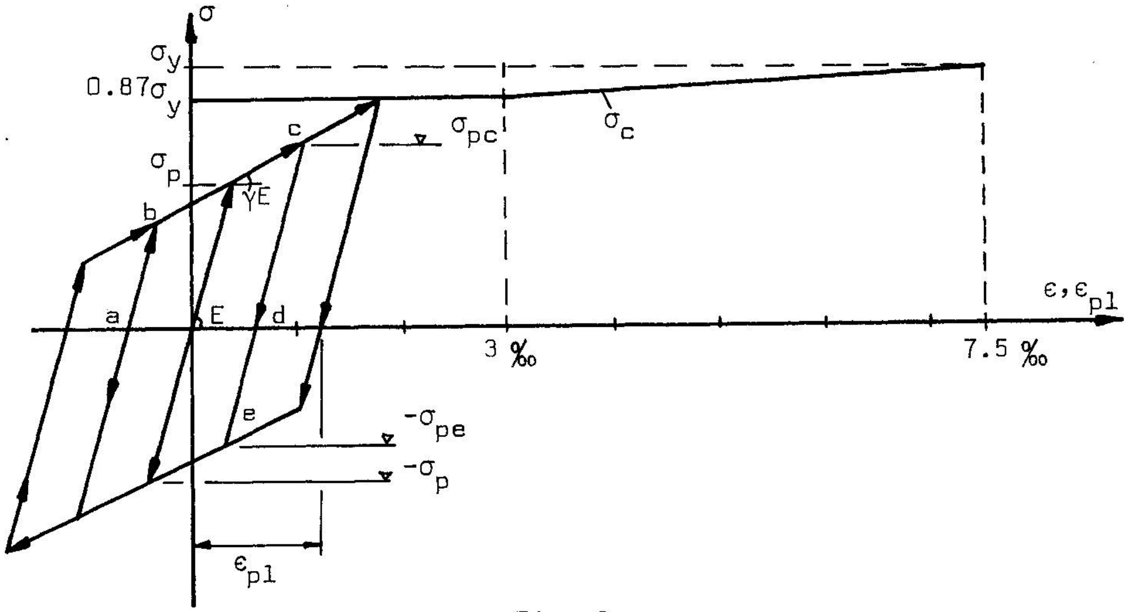

Fig. 2

Following Morrow [3], the dissipation energy spent in one cycle is

$$\Delta A = 4\ \frac{1-n'}{1+n'}\ \varepsilon_{pl}\ \sigma_c$$

where $n' \sim 0.15$, independently of the cycle number. In our simplified bilinear loop

$$\Delta A_B = 4\ \sigma_p\ \varepsilon_{pl}$$

so that $\Delta A = \Delta A_B$ leads to

$$\sigma_p = \frac{1-n'}{1+n'}\ \sigma_c \sim 0.74\ \sigma_c$$

The parameter γ is then, by a geometric argument

$$\gamma = \frac{1}{\dfrac{E\ \varepsilon_{pl}}{\sigma_c - \sigma_p} + 1}$$

Strictly speaking, it would be necessary to iterate, in each load step and at each point of the numerical solution scheme on both flanges, for the correct value of γ associated with the actual ε_{pl}. Instead, a fixed value of γ is used, based on approximate ε_{pl} before collapse. The assumed value γ is conservative if no stresses above σ_c occur before collapse.

3. NUMERICAL SOLUTION OF THE DETERMINISTIC PROBLEM

Numerical solution follows the well known Engesser-Vianello procedure for each load step. The initial crookedness for a step is the residual deflection from the previous step. If at a point on one of the two flanges stress history of the previous step was a-b-c-d, (Fig. 2), proportional limits in the bilinear law of the next step will be σ_{pc} and $-\sigma_{pe}$, at that point. It can be shown that the Engesser-Vianello procedure converges whenever the absolute value of the longitudinal force in the member is smaller than the ultimate load in compression. Thus, the procedure will not converge - and this is confirmed by numerical experience - if a tensile force of larger absolute value than the ultimate compressive force is applied. A possible solution is to use a backward version of the conventional procedure : given initial deflections $w_i(x)$ and a first guess $w_1(x)$ of incremental deflections due to P, find incremental curvature

$$\kappa_1 = - w_1''$$

by numerical differentiation and new excentricities

$$e = e\ (\kappa_1, P)$$

making use of the material properties. An improved approximation

$$w_2 = e - w_i$$

is so obtained.

4. SIMULATION

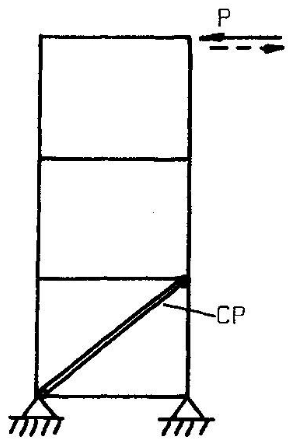

Fig. 3

A stochastic loading scheme containing the probabilities of high absolute values of P and the probability of sign changes is chosen as follows :

$|P|$ has extreme type I distribution

$$\Phi_{|P|} = \exp\left[-\exp\left(-\alpha\ (P-P_o)\right)\right]$$

The sign of P is subject to a two-state Markov chain with one-step transition probabilities:

	+	–
+	1-Q	Q
–	Q	1-Q

Q=o stands for cases with no sign changes. This is the reference situation of usual design. Q=1 indicates certain stress reversal at each step, as may be the case if P stems from some technical device with fixed operative schedule and random loading. A value of Q=0.5 may be justified in conjunction with P from wind load extrema. More complex situations, as forces from vibrations or earthquake, with a high correlation between subsequent peaks $|P|$, cannot be simulated by this simple scheme, since the values of $|P|$ are uncorrelated.

Simulation runs for a specific case and Q=0, 0.5, 1.0 are currently in production. Probabilities of failure will be esteemed from a sufficient number of randomly simulated individual histories over a lifetime of 600 load applications (monthly extrema over 50 years), and the differences in failure probability arising between the standart case Q=0 and the case Q=0.5 and Q=1.0 will be tested for statistical significance.

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SUMMARY

A numerical simulation scheme is set up to test the effect of random stress reversal on the safety against buckling of members with initial out-of-straightness.

RESUME

On établit un procédé de simulation numérique pour déterminer la probabilité de ruine de barres sujettes au flambement, soumises à des charges alternées aléatoires.

ZUSAMMENFASSUNG

Es wird ein numerisches Simulationsverfahren angegeben zur Bestimmung der Versagenswahrscheinlichkeit von Knickstäben bei zufällig auftretender Wechselbeanspruchung.