Simplified calculation method for flexural and shear strength and deformation of reinforced concrete columns under constant axial loads

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Simplified Calculation Method for Flexural and Shear Strength and Deformation of Reinforced Concrete Columns under Constant Axial Loads

Une méthode de calcul simplifié de la résistance et de la déformation à la flexion et au cisaillement des colonnes en béton armé soumises à une charge axiale constante

Vereinfachte Methode für die Berechnung der Biege- und Schubfestigkeit sowie der Verformung von Stahlbetonstützen unter konstanter Normalkraft

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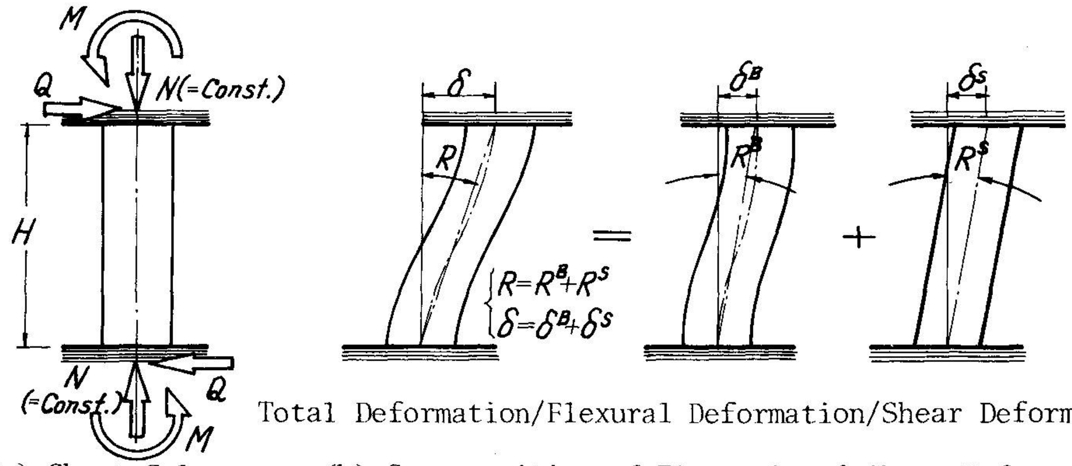
1. INTRODUCTION

The importance of the shear resistance and shear deformation of reinforced concrete restrained short columns for the aseismic design of reinforced concrete structures was already discussed and emphasized by the author [1].

In order to establish a design method of reinforced concrete columns which play an important role as one of earthquake-resisting elements [5], it is necessary to develope a simplified calculation method for the lateral sway behaviors of them under the combined action of bending moment, shear force and constant axial load. In this paper, a simplified calculation method, which is based on the superposition of flexural and shear deformations of reinforced concrete columns(see Fig.1), is presented and verified by experimental results.



Total Deformation/Flexural Deformation/Shear Deformation

(a) Short Column under Combined Forces (b) Superposition of Flexural and Shear Deformation

Fig.1 Short Column with Double Curvature

2. FLEXURAL STRENGTH AND DEFORMATION

2-1. Assumptions

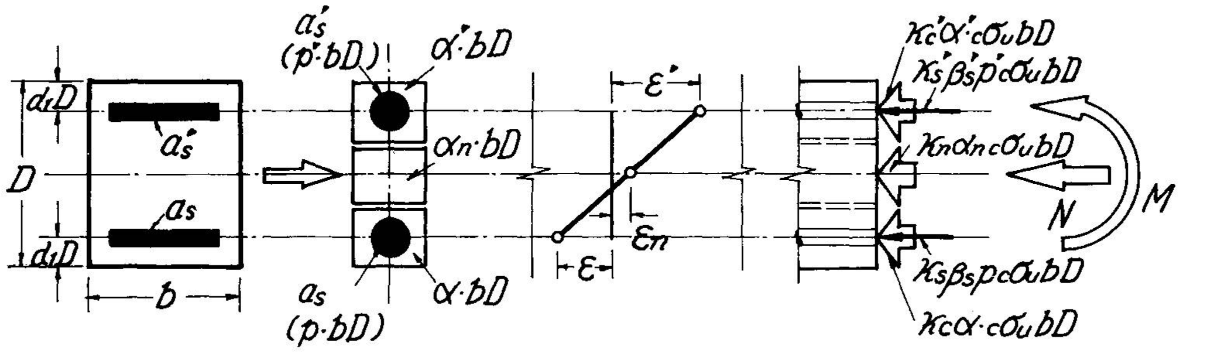
In order to calculate the flexural component of the strength and deformation of reinforced concrete columns under the combined action of bending moment, shear force and constant axial load, the "CRITICAL STRAIN POINT METHOD" of the authors [2,3] was applied, which is based on the following assumptions and idealizations.

(1) A reinforced concrete cross section may be assumed to be composed of three lumped points of concrete and two of longitudinal reinforcing steel such as shown in Fig.2(a).

(2) The strain and stress of this section are defined only on the position of these points such as shown in Fig.2(b)(c), where the strain distribution is remained linear after bending.

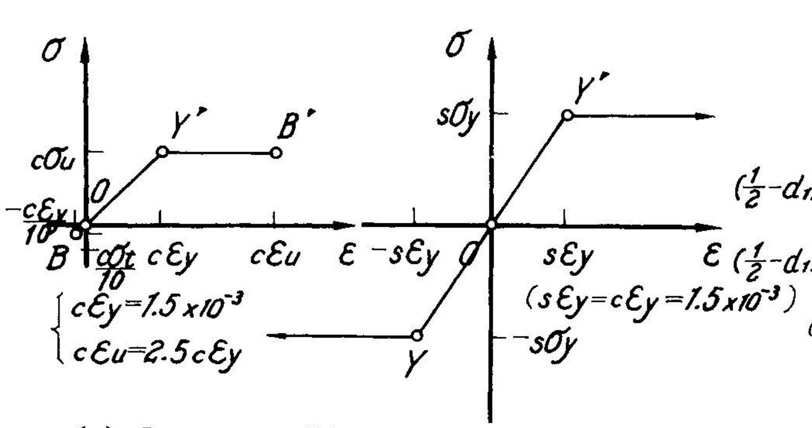
(3) The normal stress-strain relationships of concrete and reinforcing steel are assumed as shown in Fig.3(a)(b). As for concrete, B, Y' and B' show the points of tensile fracture, compressive yielding and compressive fracture, respectively. Y and Y' in the stress-strain relationship of reinforcing steel show tensile and compressive yielding points.

The concept of "CRITICAL STRAIN POINT" is illustrated on the two-dimensional plane such as shown in Fig.4. The simbols C and S in this figure correspond to the concrete and reinforcing steel on the tension side, and the dash (') and the suffix (n) show the ones at the compressive and the centroidal positions. The meaning of B,B',Y and Y' is already explained in Fig.3(a)(b).



(a) Idealized Cross Section (b) Strain Distribution (c) Stress Distribution

Fig.2 Cross Section, Strain and Stress



(a) Concrete (b) Reinforcing Steel

Fig.3 Stress - Strain Relationships

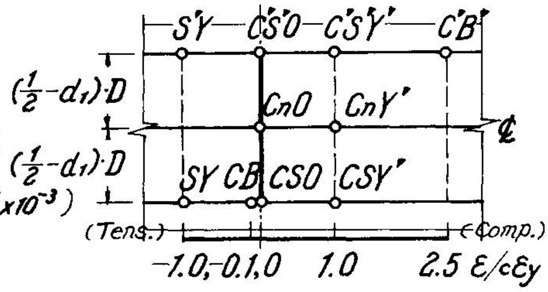


Fig.4 Critical Strain Points

2-2. Bending Moment - Axial Force - Curvature Relationships

From the stress and strain distributions of an idealized reinforced concrete cross section and the stress-strain relationships of concrete and reinforcing steel, the bending moment (m), axial force (n) and curvature (ϕ) are expressed by the following equations:

$$m = M/(1/2-d_1)\,{}_c\sigma_u bD^2 = \kappa'_c\alpha' - \kappa_c\alpha + \kappa'_s\beta'_s p' - \kappa_s\beta_s p \ , \qquad (1)$$

$$n = N/{}_c\sigma_u bD = \kappa'_c\alpha' + \kappa_{cn}\alpha_n + \kappa_c\alpha + \kappa'_s\beta'_s p' + \kappa_s\beta_s p \ , \qquad (2)$$

$$\phi = \Phi/(1-2d_1)D\,{}_c\varepsilon_y = \varepsilon'/{}_c\varepsilon_y - \varepsilon/{}_c\varepsilon_y \ , \qquad (3)$$

where κ is the stress level ratio of existing stress to yielding stress, $\beta_s p(=\beta'_s p' = {}_s\sigma_y/{}_c\sigma_u \cdot a_s/bD)$ is the reinforcement index, and dash ('), suffixes (n),(s),(c) indicate the same such as mentioned above.

When a linear strain distribution rotates, passing through a certain "CRITICAL STRAIN POINT" as the rotation center, m, n, and ϕ are able to be calculated easily and illustrated in the three-dimentional space with the Cartesian coordinates corresponding to m, n and ϕ.

As an example, Fig.5 shows the m-n-ϕ relationships of reinforced concrete cross section in which the names of lines represent those of the "CRITICAL STRAIN POINT". The dotted lines CB shows the occurrence of tensile cracking of the tensile concrete point, which is computed under the condition of the existence of the tensile resistance of concrete. When axial load is constant, bending moment (m) - curvature (ϕ) relationship is able to be easily illustrated such as shown in Fig.6.

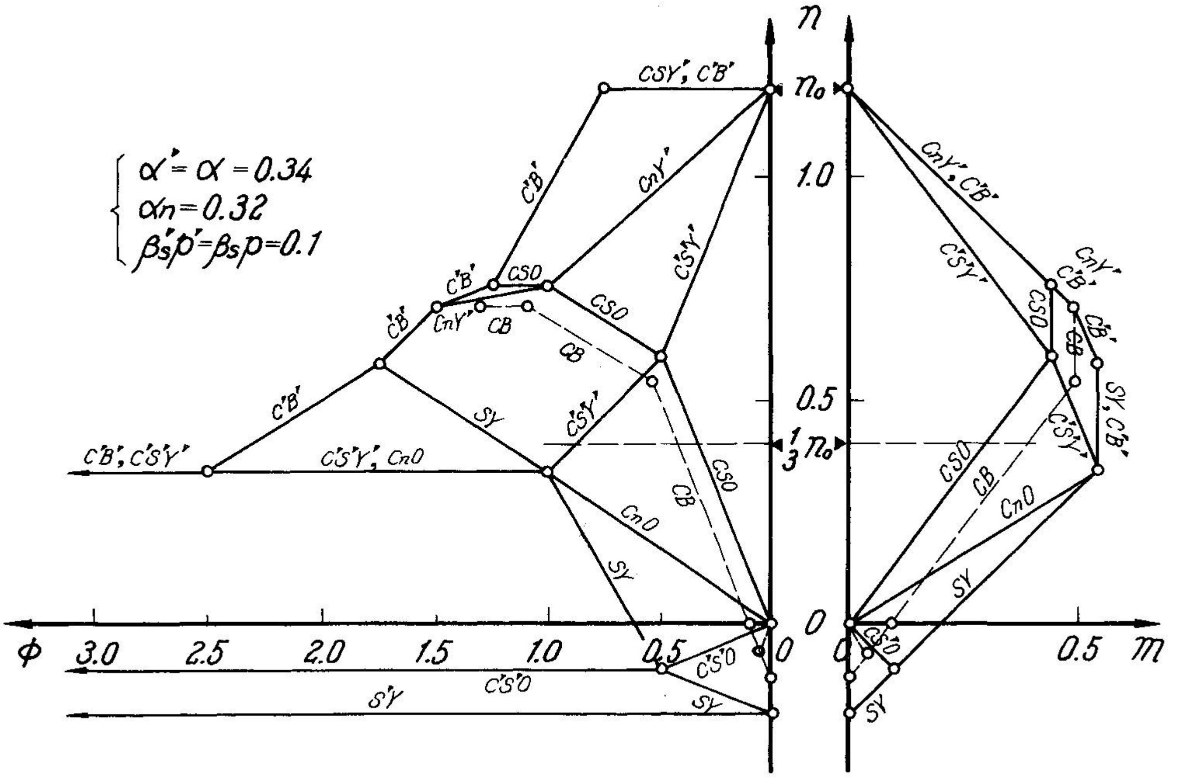


Fig.5 Bending Moment(m) - Axial Force(n) - Curvature(ϕ) Relationships

As an example, it is shown in Fig.6 under the condition of the axial load level ratio (X=1/3) which indicates the ratio of constant axial load (N) to the maximum axial strength (N_0) of columns. The "CRITICAL STRAIN POINTS" in this figure shows the points through which a linear strain distribution passes.

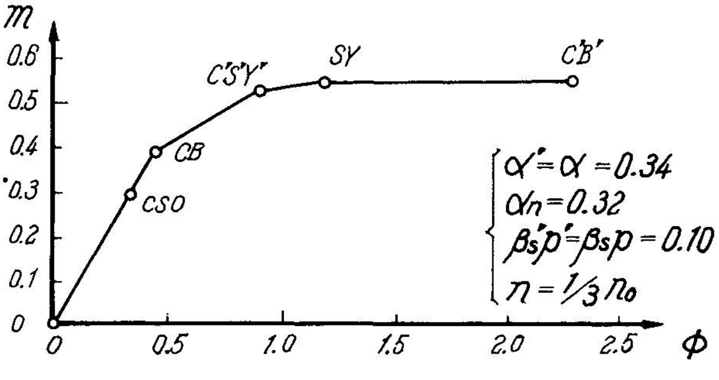


Fig.6 Bending Moment(m) - Curvature(φ) Relationship

2-3. Shear Force - Flexural Lateral Displacement Angle Relationships

When a moment - curvature relationship of a reinforced concrete cross section is given, and if the P-δ effect is able to be neglected in the case of relatively short columns, a shear force (Q) - flexural lateral displacement angle (R^B) relationship may be calculated by integrating the curvature distribution given along the longitudinal axis or by means of Mohr's theory.

Using the "CRITICAL STRAIN POINT METHOD", even in Q - R^B relationships, the physical meanings, namely, the stress and strain distribution states at the end cross section of columns are able to be shown clearly.

The plastic displacement angle increment ΔR^B may be computed by the follwing equation:

$$\Delta R^B = L_p \Delta \Phi \ , \qquad (4)$$

where L_p is the longitudinal length of plastic hinge region at the end of columns assumed to be D in this report and $\Delta\Phi$ is the plastic curvature increment in M - Φ relationship, which is based on the assumption that the lateral displacement angle of columns in the plastic range occurs only due to the rotation of the plastic hinge.

3. SHEAR STRENGTH AND DEFORMATION

3-1. Assumptions

In order to calculate approximately the shear component of the strength and deformation of reinforced concrete columns under the combined action of bending moment, shear force and constant axial load, the following assumptions are applied:

(1) Considering the condition of double-curvature deformation, the inflexion point of columns may be regarded as the critical section that determines the shear strength and deformation of short columns.

(2) The critical section mentioned above is assumed to have the area, $\frac{7}{8}(1-d_1)bD$ and the shear stress and strain distributions are assumed to be uniform over the area.

(3) The shear stress - strain relationship of concrete is assumed as shown in Fig.7, where the two dotted lines show the extreme cases, in one of which concrete shows sufficient shear ductility (with web reinforcement ratio, $\eta \geqq 1\%$), and in the other of which poor shear ductility ($\eta \fallingdotseq 0\%$). In this paper, the calculations are concerning only with the latter case.

(4) The shear yielding stress τ_y is determined from the fracture criterion of concrete under combined shear and normal stresses which is shown in Fig.8 [4]. As the abscissa of Fig.8 the constant axial load level ratio X of columns is adopt-

ed here, so τ_y is expressed by the following equation:

$$\tau_y/{}_c\sigma_u = \sqrt{-0.10X^2 + 0.09X + 0.01}\ . \tag{5}$$

3-2. Shear Force - Shear Lateral Displacement Angle Relationships

Based on the assumptions mentioned above, shear force (Q) - shear lateral displacement angle (R^S) relationships of reinforced concrete columns may be easily computed, and of course, they are analogous to the shear stress-strain relationship (see Fig.7). Finally, Q-R^S relationship until shear explosion is able to be expressed by the following equation:

$$R^S = [Q/(7/8)(1-d_1)bD]/(\tau_y/{}_c\gamma_y)\ . \tag{6}$$

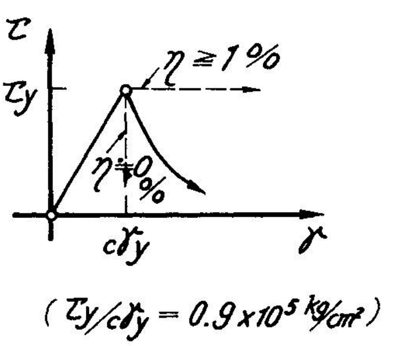


Fig.7 Shear Stress - Strain Relationships of Concrete

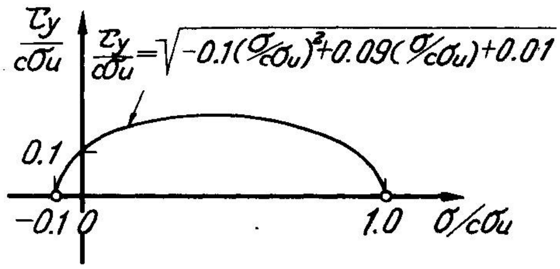
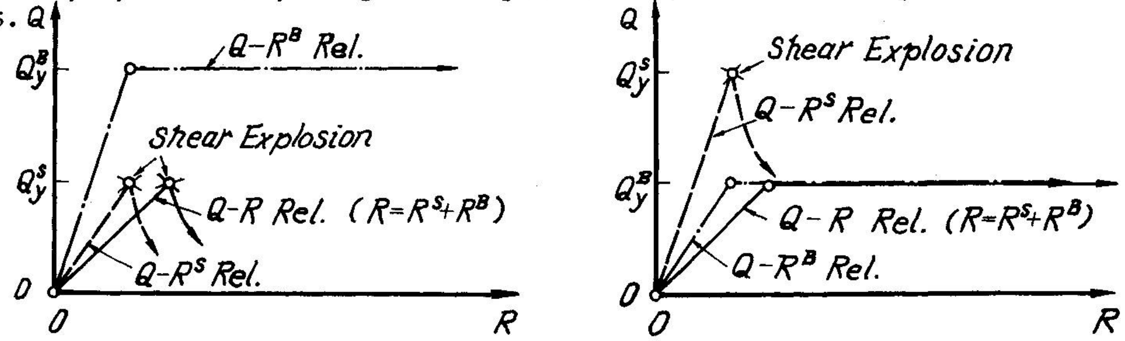


Fig.8 Fracture Criterion of Concrete

4. TOTAL STRENGTH AND DEFORMATION

4-1. Shear Force - Total Lateral Displacement Angle Relationships

By means of the superposition of the flexural and shear deformation components on the same shear force level, shear force (Q) - total lateral displacement angle ($R = R^B + R^S$) relationships of reinforced concrete columns under combined bending moment, shear force and constant axial load are able to be calculated. The processes are shown in Fig.9(a)(b) schematically. Fig.9(a) shows the shear explosion type and Fig.9(b) flexural fracture type. The fracture type is determined by the relative quantity relationship between Q_y^B (yielding shear force of Q-R^B relation) and Q_y^S (explosing shear force of Q-R^S relation), so mainly by the story height to depth ratio (H/D or shear span ratio) of columns.



(a) Shear Explosion Type (b) Flexural Fracture Type

Fig.9 Graphical Procedure of Superposition of Flexural and Shear Deformation

4-2. Comparison between Caluculated and Tested Results

The calculated Q-R relationships of reinforced concrete columns are compared with tested results. Test specimens with a 16_{cm}X16_{cm} cross section are reinforced with the same tensile and compressive longitudinal reinforcement ratios of p=p'=1.0%, and without web reinforcement. The axial load level ratios (X) are 1,2,3 and 4. A test specimen and the loading system are shown in Fig. 10. The more detailed descriptions on these tests were reported in the reference [1]. The experimental results are shown by solid lines in Fig.11(a)(b)(c)(d) in the case of H/D=1,2,3 and 4, where TC, SC and SE(X) show the points of the occurrence of tensile crack on the tensile concrete fiber, shear crack in the shear span and shear explosion, respectively. The dotted lines show the computed results corresponding to the experimental results. The symbols in parentheses on them mean the "CRITICAL STRAIN POINT", and the special symbol "SE" shows the point of shear explosion.

The coincidence between computed and experimental results is good enough except the case that X=0 and H/D=1,2, where the dowel action of longitudinal reinforcing steel seems to have considerable effects on the shear resistance.

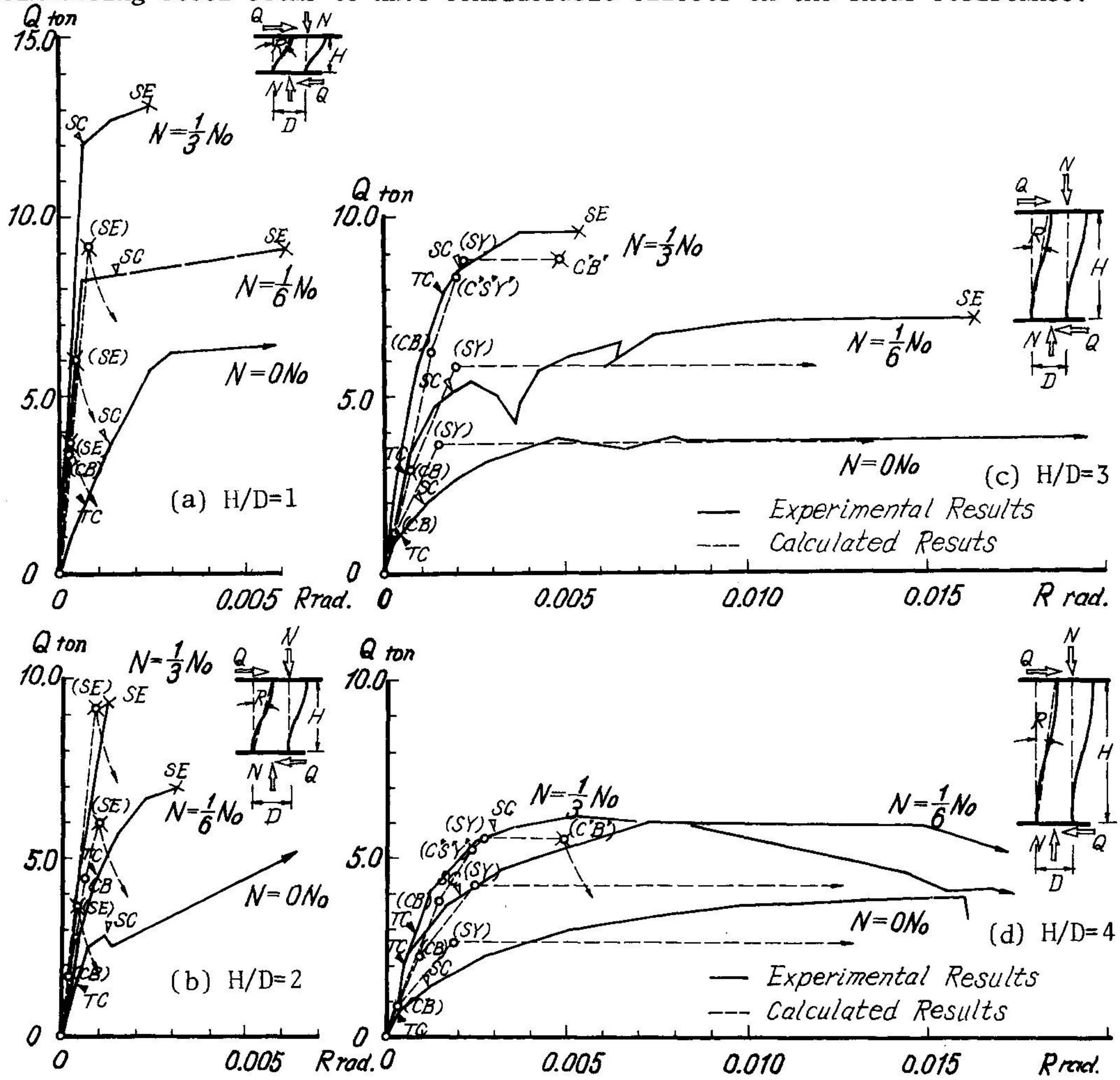


Fig. 11 Shear Force (Q) - Lateral Displacement Angle (R) Relationships

5. DISCUSSION

5-1. Maximum M-N-Q Interaction

Judging from the good coincidence between computed and experimental results, the simplified assumptions and calculation methods proposed here may be reasonable. Based on these idealizations, the interaction of the maximum M,N and Q is able to be illustrated as shown in Fig.12. A section parallel to the M-N plane of this figure shows a M-N interaction which is analogous to the Fig.5, and a section parallel to the Q-N plane shows a Q-N interaction analogous to the Fig.8. A section parallel to the M-Q plane shows a M-Q interaction in case of constant axial load, which is shown in Fig.13. In reality, however, M-Q interaction may be pseudo-elliptic such as shown by dotted line. The more precise analysis is now under consideration in the authors' laboratory.

5-2. Critical Hight to Depth Ratio

When Q_y^S is equal to Q_y^B in Fig.9, H/D means the critical height to depth ratio ($(H/D)_{cr}$ or the critical shear span ratio) that distinguishes the fracture modes of reinforced concrete columns into the shear explosion and the flexural fracture [6]. It is given by the slope of the line OC in Fig.13, too.

Finally, $(H/D)_{cr}$ is given as follows:

$$\text{if} \quad \frac{\alpha}{1+2\beta_s p} \leq X \leq \frac{\alpha+(3/4)\alpha_n}{1+2\beta_s p}, \quad (H/D)_{cr} = \frac{2(\alpha+2\beta_s p)(1/2-d_1)}{(7/8)(1-d_1)\sqrt{-0.10X^2+0.09X+0.01}}, \quad (7\text{-}1)$$

$$\text{if} \quad 0 \leq X \leq \frac{\alpha}{1+2\beta_s p}, \quad (H/D)_{cr} = \frac{2[X+2(1+X)\beta_s p](1/2-d_1)}{(7/8)(1-d_1)\sqrt{-0.10X^2+0.09X+0.01}}. \quad (7\text{-}2)$$

As an example, $(H/D)_{cr}$ is shown in H/D-$\beta_s p$ coodinates in Fig.14 with X as paramete. In the upper region than $(H/D)_{cr}$ lines, reinforced concrete columns show the flexural fracture, and in the lower region they show the shear explosion. If $X \geq [\alpha+(\frac{3}{4})\alpha_n]/(1+2\beta_s p)$, columns show compressive flexural fracture mode, then such a case may be omitted. The condition that X=1/3, and $\beta_s p \geq 0.25$ in Fig.14 belongs to such a case.

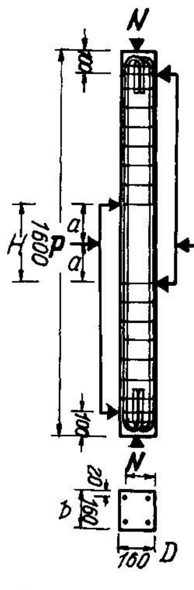


Fig.10

Test Specimen and

Loading System

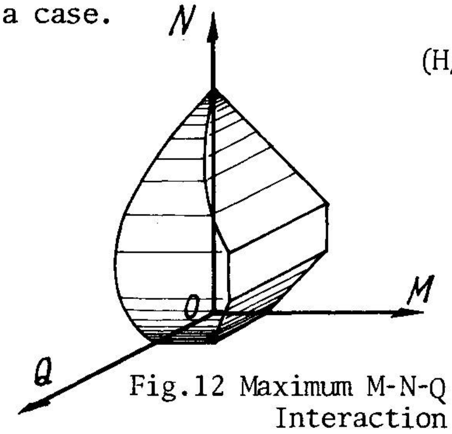


Fig.12 Maximum M-N-Q Interaction

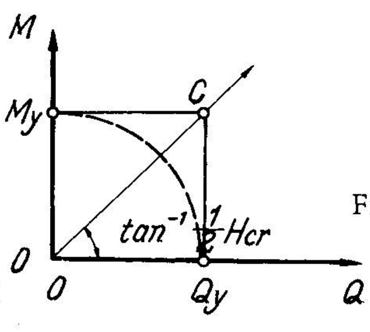


Fig.13 Maximum M-Q Interaction

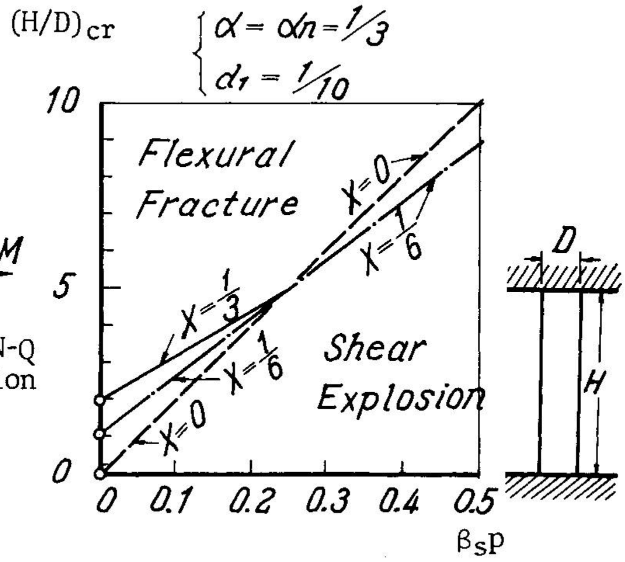


Fig.14 Critical Height to Depth Ratio $(H/D)_{cr}$- $\beta_s p$ Relationships

6. CONCLUSION

A simplified method of the calculation for the strength and deformation of reinforced concrete columns under the combined action of bending moment, shear force and constant axial load is presented. It is based on the superposition of the flexural and shear deformations (see Fig.1). The former is calculated by means of the "CRITICAL STRAIN POINT METHOD" of the authors (see Figs.2,3,4,5, 6), the latter is derived from the combined normal and shear stress characteristics at the cross section of the inflection point of reinforced concrete columns (see Fig. 7,8). The reasonable coincidence between the computed values and experimental results (see Fig.11) show the fact that these simplified calculation procedures are good enough to predict the lateral sway behaviors of reinforced concrete short columns.

7. REFERENCES

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SUMMARY

Based on the superposition of shear and flexural deformations, a simplified calculation method for the lateral sway deflections of reinforced concrete short columns under the combined action of bending moment, shear force and constant axial load is presented. The coincidence between the computed values (dotted) and the experimental results (solid lines in Fig. 11) is reasonable.

RESUME

En se basant sur la superposition des déformations à la flexion et au cisaillement, on présente une méthode de calcul simplifiée des déformations latérales des colonnes courtes en béton armé soumises à l'action combinée d'un moment de flexion, d'un effort tranchant, et d'un effort axial constant. La correspondance entre les valeurs calculées (pointillés) et les résultats expérimentaux (courbes pleines, Fig. 11) est raisonnable.

ZUSAMMENFASSUNG

Gestützt auf die Ueberlagerung von Schub- und Biegeverformungen wird eine einfache Berechnungsmethode für die seitliche Auslenkung kurzer Stahlbetonstützen unter der kombinierten Wirkung von Biegemoment, Querkraft und konstanter Normalkraft vorgelegt. Die Uebereinstimmung zwischen den berechneten Werten (gepunktete Linie) und experimentellen Ergebnissen (ausgezogene Linien in Fig. 11) ist vernünftig.