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Strength of Columns under Biaxially Eccentric Loads

Résistance des colonnes soumises à des charges excentrées biaxiales

Tragfähigkeit von Stützen unter zweiaxig exzentrischer Belastung

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I Introduction

In the ultimate strength design of the columns, it is quite important to examine not only the ultimate strength but the ultimate deflection at the same time, especially, for the slender columns under biaxially eccentric loads.

Computer analysis[1] are made on: (1) the failure envelope of column cross-section and (2) the column deflection under biaxially eccentric load. The strength and the deflection at ultimate stage can be predicted by combining these two methods. This analysis,differently from those ever proposed, has characteristics of dealing with the change of the location of the eccentricity under biaxially eccentric load.

Based on the analysis and results of tests, the simple design procedure for slender column is proposed.

II Test of the columns under biaxially eccentric loads

Totally sixty-two short and slender columns are made and tested,[1] and the test programs for slender column are shown in Table 1.

All of the slender columns have the same rectangular cross-section of 6x9 cm and the same initial eccentricity of 3 cm. Columns of series A and B were reinforced with four and six dia. 6 mm bars, respectively. The column lengths of each group in two series were 60, 120 and 180 cm, respectively. Initial directions of eccentricities in a group were I, III and V or I, II, III, IV and V as shown in the remark in Table 1.

Fig. 1 shows the comparison of the measured deflections with the computed ones. The stress-strain relationship of concrete as assumed in the calculation is also shown in Fig. 1.

Fig.2 clearly shows the change of eccentricities with increasing load from the initial to the final locations, the final eccentricities being not located along the inclination of initial direction. These figures show that the analysis coincide well with the results of tests.

The torsional deformation due to biaxially eccentric load is not considered here, but it should be studied as an important problem in future.

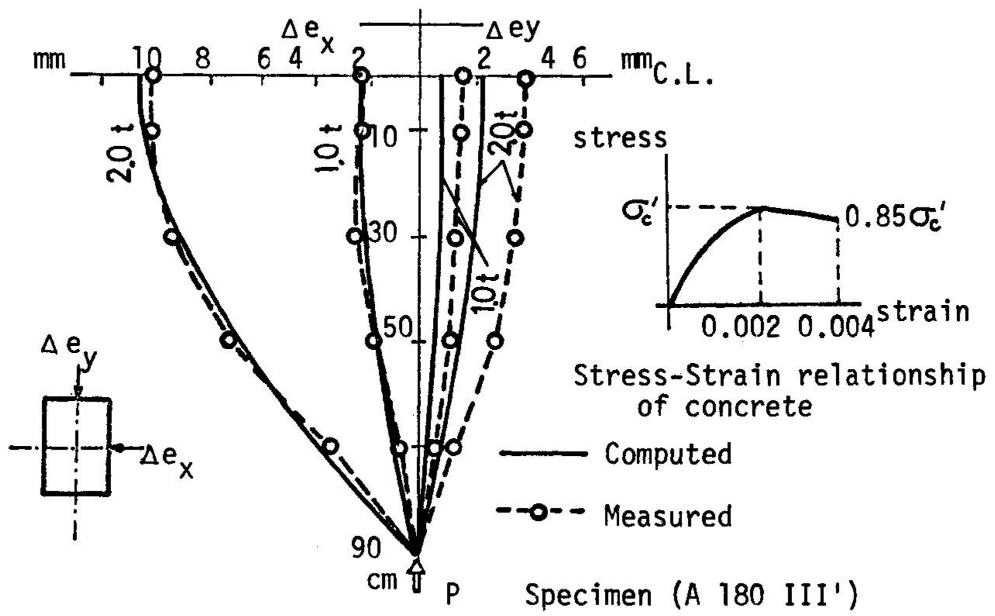


Fig. 1 Comparison of the measured deflections with the computed ones

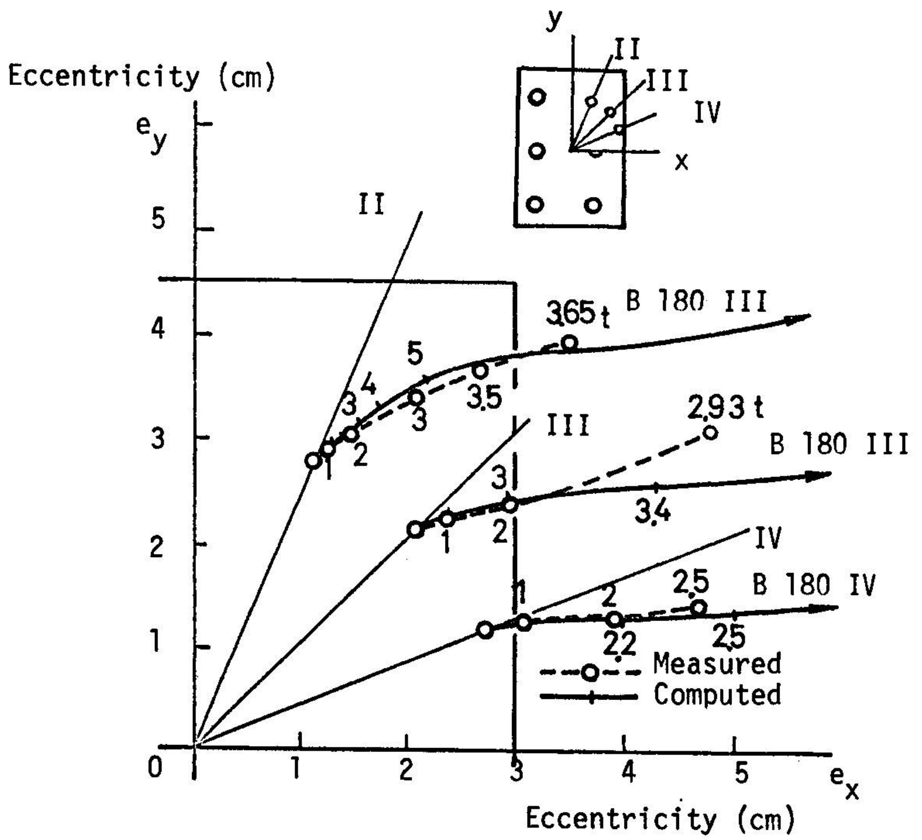


Fig. 2 Load-eccentricity relationships in the mid-height section of column

III Simple design solutions for the ultimate deflection and strength under biaxially eccentric loads

(1) Ultimate deflection and strength

A slender column when subjected to biaxially eccentric load deflects as already shown in Fig. 1 or 2.

Fig. 3 is also one of the measurements showing how the initial eccentricity of load on the critical section actually varies with increasing load for each of five slender columns subjected to uniaxially or biaxially eccentric load. The initial (e_{ro}) and final eccentricities ($e_{rp}+\Delta e_r$) projected on the e_x-e_y plane is shown in Fig. 4(a). The final eccentricities are located on the inclination (β) deviated from the initial direction (α), and P-β relationships are replotted in Fig. 4(b). Radial ultimate moments based on the final eccentricity are calculated from Fig. 4(a) and 4(b), and shown in Fig. 4(c).

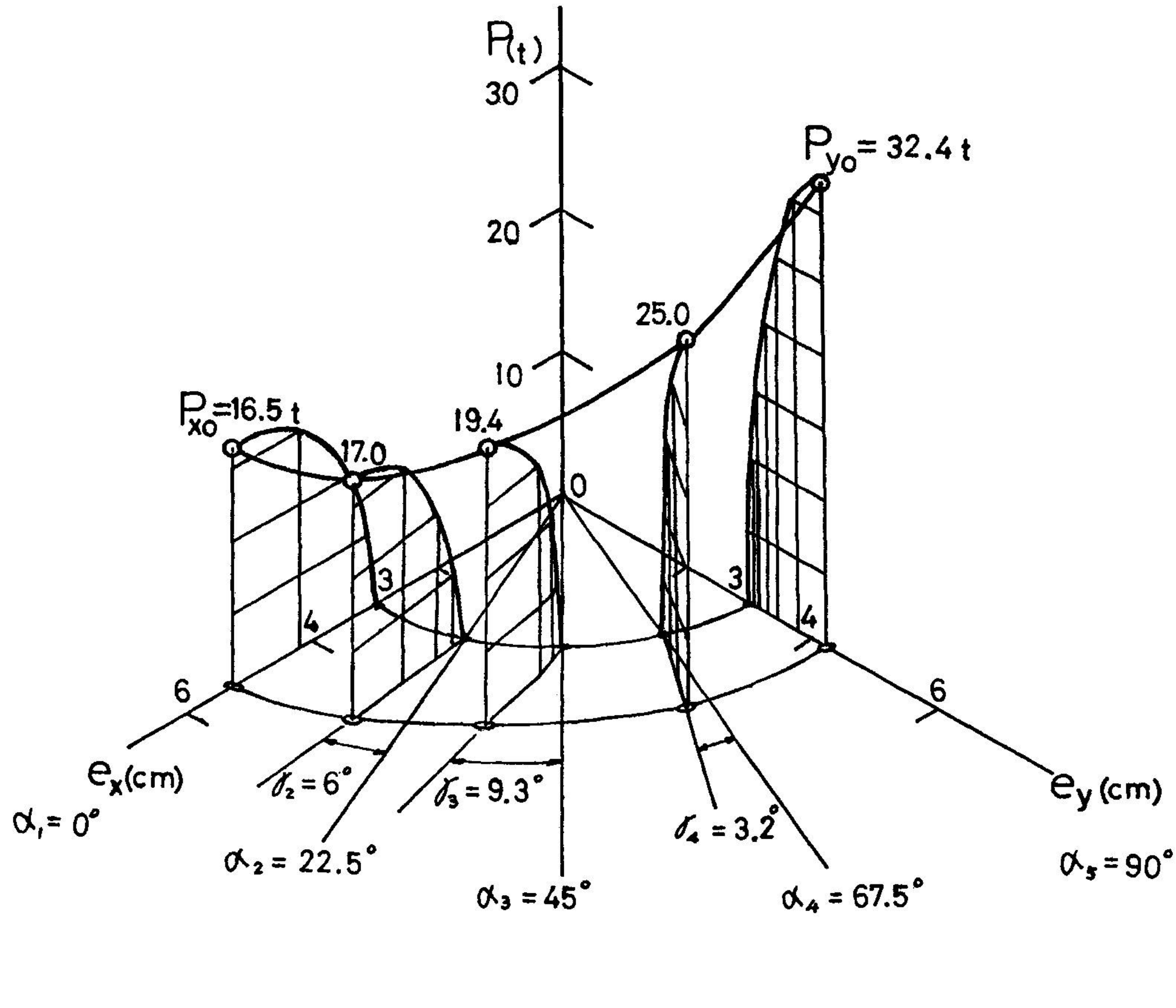


Fig. 3 Load-eccentricity relationships at the center section of biaxially loaded columns

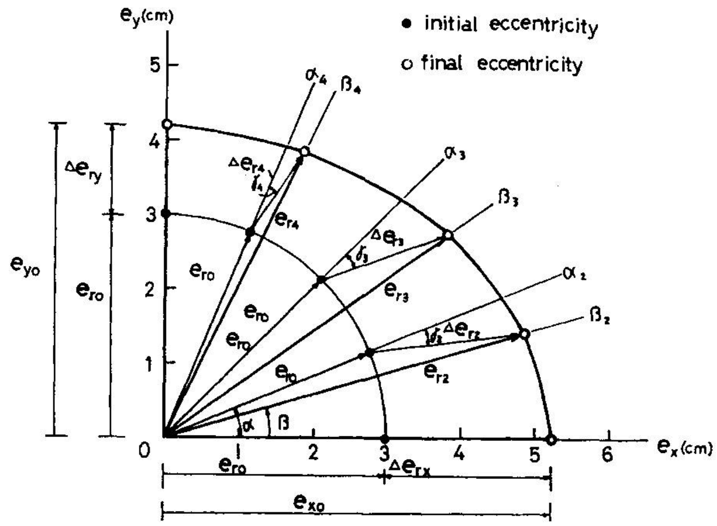


Fig. 4(a) Initial and final eccentricities projected on the e_x-e_y plane

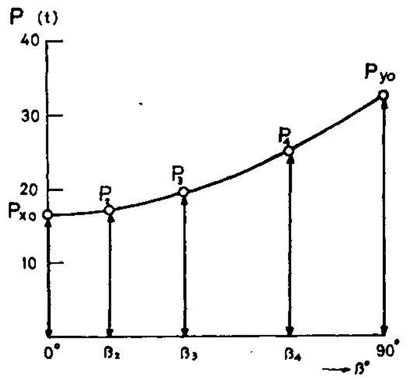


Fig. 4(b) P-β relationship

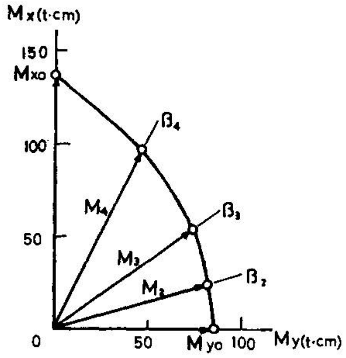


Fig. 4(c) Radial ultimate moments on final direction of eccentricity

(2) Modified additional moment method

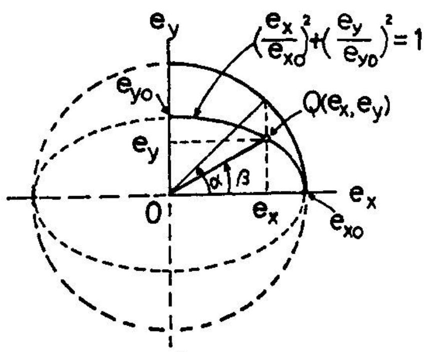


Fig. 5

Supposed radial eccentricity

Detailed calculations as illustrated in II are rather complicated, so the simple design procedure is proposed here.

It may be deduced that the radial eccentricity $Q(e_x, e_y)$ at the final stage can be approximated by the following equation, as shown in Fig. 5;

$$\left(\frac{e_x}{e_{xo}}\right)^2 + \left(\frac{e_y}{e_{yo}}\right)^2 = 1 \qquad (1)$$

The eccentricity e_x, or e_y is given by

$$e_x = e_{xo} \cdot \cos\alpha$$
$$e_y = e_{yo} \cdot \sin\alpha$$

where:
α and β in Fig. 5 are the inclinations of the lines joining the initial and final loading points (eccentricites) to the centroid of the column cross-section, respectively, and
e_{xo} and e_{yo} are the final values of uniaxial eccentricity along x and y axis, respectively.

Actual radial eccentricity at the ultimate stage is given by eq. (2):

$$e_r = \sqrt{e_x^2 + e_y^2} \quad (2)$$

and the relation between α and β is given as follows:

$$\beta = \tan^{-1}\left(\frac{e_{yo}}{e_{xo}} \cdot \tan\alpha\right) \quad (3)$$

Next, it may also be assumed that the outer limits of the ultimate moment corresponding to Fig. 5 is approximately given by the expression as shown in Fig. 6.

$$M_x = M_{xo}\sqrt{1 - \frac{M_y}{M_{yo}}} \quad (4)$$

where M_{yo} and M_{xo} are the equivalent uniaxial moments for the ultimate moment about x- and y-axis, respectively, and M_y and M_x are the components of the actual ultimate radial moment M_r.

Equation(4) gives the following relations,

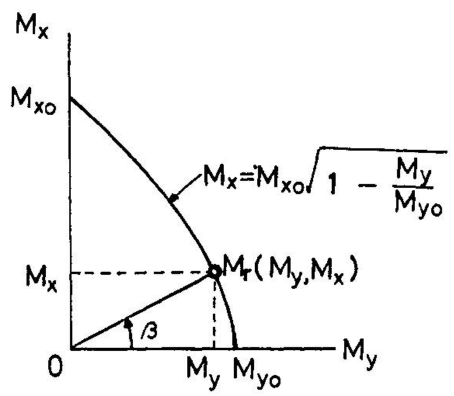


Fig. 6 Approximation for radial ultimate moment

$$M_y = M_r \cos\beta \quad (5)$$

$$M_x = M_r \sin\beta \quad (6)$$

$$M_r = \frac{M_{xo}}{A \sin\beta}\left[\sqrt{A+1} - 1\right] \quad (7)$$

$$\text{where } A = 2 \cdot \frac{M_{yo}}{M_{xo}} \cdot \tan\beta. \quad (8)$$

Thus, from equations (2) and (7), the ultimate load P_r under biaxially eccentric load can be given by

$$P = \frac{M_r}{e_r}. \quad (9)$$

(3) Comparison with tests

The detailed comparison of the proposed simple design procedures with the results of tests on actual five columns are shown, for example, in Fig. 7(a), (b) and (c), concerning the ultimate deflection (e_r), the ultimate load (P_r) and the ultimate moment (M_r), respectively.

For general use, when the uniaxial ultimate deflection and load calculated by the additional moment method introduced by Prof. Macgregor in the Preliminary Report Theme II (B.S. or FIP-CEB recomendation) are inserted into the above equations, the authors' method agreeswell with the results of tests as shown in Table 1.

Then, the simple solution proposed here for the ultimate capacities of the slender columns subjected to biaxially eccentric load may be used for design purpose.

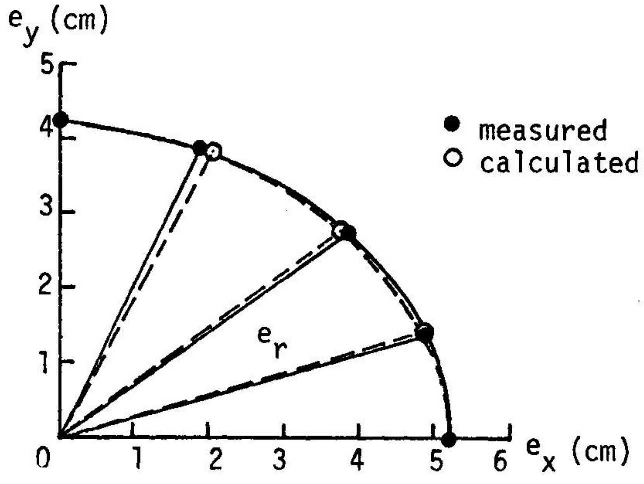


Fig. 7(a) Comparison for e_r

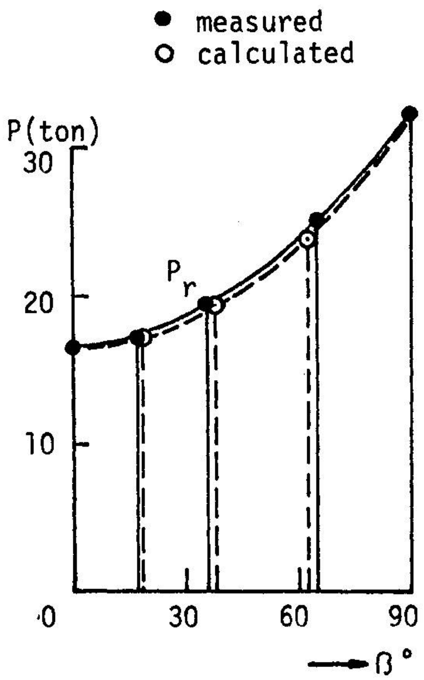


Fig. 7(b) Comparison for P_r

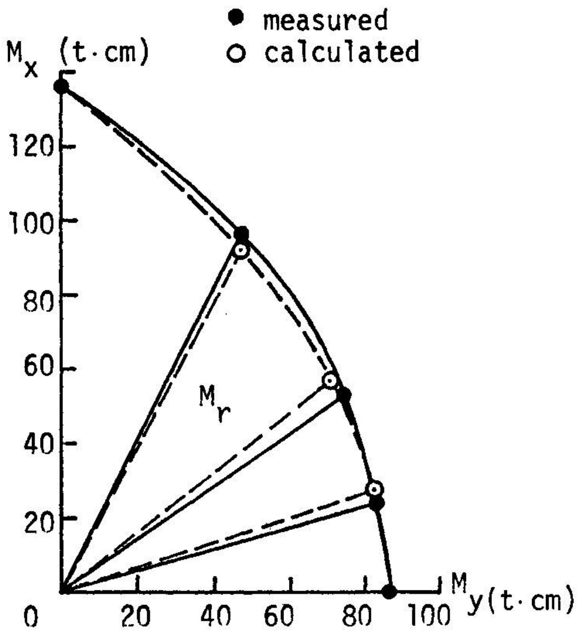
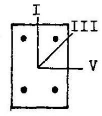
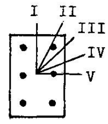


Fig. 7(c) Comparison for M_r

Table 1. Comparison between test results with calculated results

Column		l (cm)	b (cm)	h (cm)	d/h	σ_c' (kg/cm)	σ_{sy} (kg/cm)	e_{ro} (cm)	$e_{r\,test}$ (cm)	$e_{r\,cal}$ (cm)	$\frac{e_{r\,test}}{e_{r\,cal}}$	$P_{r\,test}$ (ton)	$P_{r\,cal}$ (ton)	$\frac{P_{r\,test}}{P_{r\,cal}}$	Remark
A 60	I	60	6	9	0.833	296	3430	3	3.30	3.16	1.044	7.75	8.59	0.902	
	III								3.30	3.22	1.025	5.35	4.36	1.227	
	V								3.47	3.29	1.055	3.95	3.54	1.116	
A 120	I	120	6	9	0.833	296	3430	3	4.20	3.65	1.151	7.40	7.96	0.930	
	III								4.50	3.88	1.160	3.65	3.10	1.178	
	V								4.17	4.10	1.017	2.68	2.37	1.131	
A 180	I	180	6	9	0.833	296	3430	3	5.46	4.57	1.195	6.00	7.02	0.855	III= 45°
	III								5.12	4.99	1.026	2.45	2.11	1.161	
	V								5.42	5.37	1.010	2.09	1.56	1.340	
B 60	I	60	6	9	0.833	318	3430	3	3.35	3.16	1.060	8.87	9.06	0.979	
	II								3.23	3.18	1.016	5.40	6.97	0.775	
	III								3.27	3.22	1.016	4.66	5.97	0.781	
	IV								3.50	3.27	1.070	4.40	5.51	0.799	
	V								3.59	3.28	1.095	4.69	5.39	0.870	
B 120	I	120	6	9	0.833	293	3430	3	4.34	3.66	1.186	6.20	8.02	0.773	
	II								4.48	3.73	1.201	4.33	5.52	0.784	
	III								4.50	3.89	1.157	4.16	4.30	0.967	
	IV								4.35	4.04	1.077	3.35	3.72	0.901	
	V								4.01	4.10	0.978	3.00	3.55	0.845	
B 180	I	180	6	9	0.833	318	3430	3	5.11	4.58	1.116	5.05	7.28	0.694	II = 22.5° III= 45° IV = 67.5°
	II								5.22	4.70	1.111	3.65	4.33	0.843	
	III								5.45	4.97	1.097	2.93	3.06	0.958	
	IV								6.45	5.24	1.231	2.90	2.52	1.151	
	V								6.25	5.34	1.170	2.73	2.37	1.152	
A 60	III'	60	6	9	0.833	333	3430	3	3.25	3.22	1.010	4.28	4.50	0.951	
A 120	III'	120							4.16	3.87	1.075	4.10	3.18	1.289	
A 180	III'	180							5.55	4.96	1.119	2.44	2.16	1.130	
B 60	III'	60	6	9	0.833	333	3430	3	3.38	3.22	1.050	4.75	6.05	0.785	
B 120	III'	120							4.25	3.88	1.095	4.40	4.44	0.991	
B 180	III'	180							5.55	4.96	1.119	3.00	3.09	0.971	
									Average		1.091			0.974	

IV Conclusions

A calculating method of the ultimate capacity of concrete columns with taking into consideration the column deflection under biaxially eccentric loads is proposed.

A simple design procedure is also proposed which is developed and refined so as to achieve both simplicity in use and, as far as possible, a realistic representation of actual behavior.

This method agrees well with the results of thirty model columns.

Reference

1) OKADA,K. and HIRASAWA,I.,"Ultimate Strength of Square and Rectangular Columns under Biaxially Eccentric Loads", Review of the Twenty-third General Meeting, The Cement Association of Japan, May 1969, pp356-361.

SUMMARY

This paper deals with the design method for reinforced concrete slender columns subjected to biaxially eccentric loads. Firstly, the analysis of strength and deflection of slender column at ultimate stage is made using computer, the results are compared with the results of tests.

Next, taking into account the analysis, a simple design solution is proposed. This method agrees with tests and shows applicable for general use.

RESUME

Ce rapport présente une méthode de dimensionnement pour les colonnes en béton armé élancées soumises à des charges avec excentricité biaxiale. On procède d'abord au calcul de la résistance et de la déformation ultimes des colonnes élancées en utilisant l'ordinateur; les résultats sont comparés avec ceux des essais.

On propose ensuite, en se basant sur les calculs, une méthode simple de dimensionnement. Cette méthode concorde bien avec les résultats expérimentaux; elle est applicable dans la pratique.

ZUSAMMENFASSUNG

Der Beitrag behandelt die Bemessung von schlanken Stahlbetonstützen unter zweiaxig exzentrischer Belastung. Zunächst werden die Berechnung der Traglast und der zugehörigen Verformungen schlanker Stützen mittels Computer durchgeführt und die Ergebnisse mit Versuchsresultaten verglichen.

Als nächstes wird, unter Berücksichtigung der genauen Berechnung, eine einfache Bemessungsmethode vorgeschlagen. Diese Methode führt zu Ergebnissen, die gut mit Versuchen übereinstimmen und zeigt sich als allgemein anwendbar.