Strength of laterally loaded reinforced concrete columns

Autor(en): **Chen, Wai F. / Chen, Andrie C.T.**

Objekttyp: **Article**

Zeitschrift: **IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen**

Band (Jahr): **16 (1974)**

PDF erstellt am: **11.08.2024**

Persistenter Link: https://doi.org/10.5169/seals-15745

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.
Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.
Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Ein Dienst der *ETH-Bibliothek*
ETH Zürich, Rämistrasse 101, 8092 Zürich, Schweiz, www.library.ethz.ch

http://www.e-periodica.ch

IV

Strength of Laterally Loaded Reinforced Concrete Columns

Résistance des colonnes en béton armé soumises à une charge latérale

Tragfähigkeit seitlich belasteter Stahlbetonstützen

Wai F. CHEN
Associate Professor
Department of Civil Engineering
Fritz Engineering Laboratory
Lehigh University
Bethlehem, Pennsylvania, USA

Andrie C.T. CHEN
Design Engineer
Gannett, Flemming, Corddry and
Carpenter, Incorporated
Harrisburg, Pa., USA

INTRODUCTION: Most theoretical investigations on reinforced concrete beam-columns have been limited to beam-columns with no lateral loads [Fig. 1(b)]. This investigation was made to obtain solutions of a beam-column with a concentrated lateral load applied at mid-span [Fig. 1(a)]. In the analysis, the moment-curvature-thrust relationships are first established. These relationships are then approximated by appropriate mathematical expressions. Finally, the computer programs developed in Refs. [1] and [2] are modified and utilized to obtain solutions of the problems shown in Fig. 1. Non-dimensional interaction curves relating thrust, slenderness ratio, and lateral load for the maximum load carrying capacity of the reinforced concrete beam-columns are presented. The analytically obtained results are then compared with the prediction based on the empirical ACI formula (moment magnifier method) and good agreement is observed in most cases.

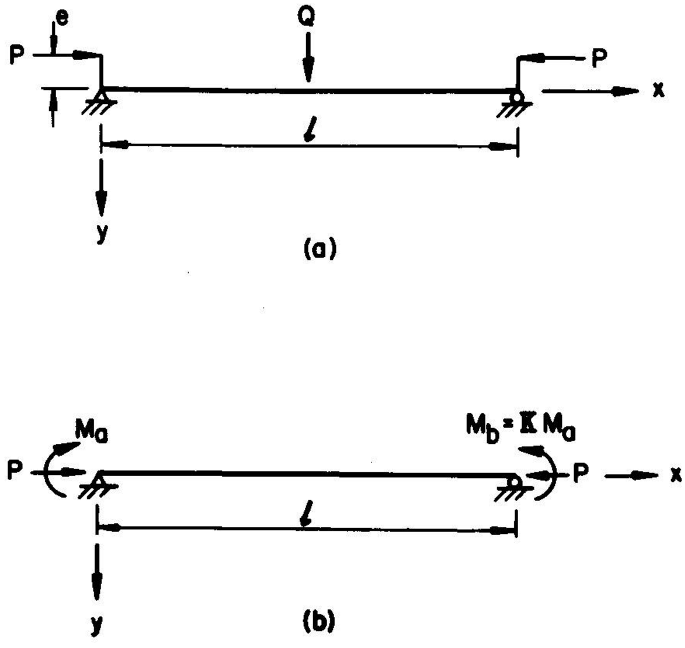
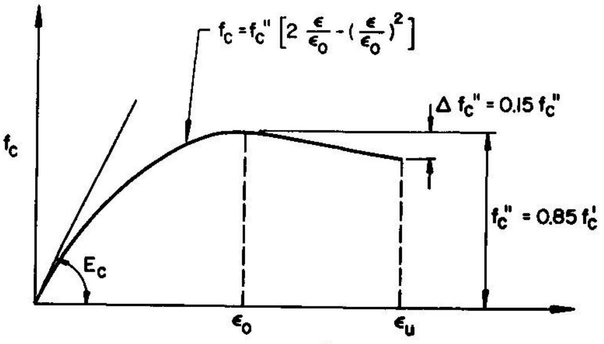
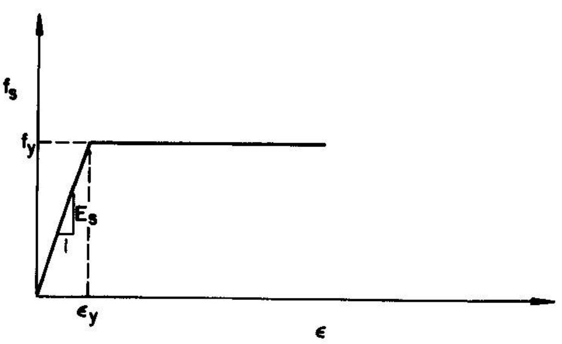


Fig. 1 Beam-Columns Studied in the Paper



(a) Concrete



(b) Steel

Fig. 2 Stress Strain Relationships used in the Paper

STATEMENT OF THE PROBLEM: It is assumed that the beam-column will fail in the plane of the loading only. The failure is either caused by excessive bending of the beam-column as a whole or by compressive crushing of the critical cross section because of the limited deformability of concrete in compression. The crushing strain ϵ_u used in the analysis is 0.004 in/in. For the case, Fig. 1(a), it is further assumed that the axial force P at the ends of the beam-column will be applied first and maintained at a constant value as the lateral load Q is continuously increased from zero to a maximum value and then allowed to drop off steadily beyond the maximum point. For the case, Fig. 1(b), proportional loading is assumed for M_a and M_b, i.e., $M_b = KM_a$.

MOMENT-CURVATURE-THRUST RELATIONSHIPS: Several widely accepted assumptions are adopted herein in obtaining the moment-curvature-thrust relationships of reinforced concrete sections. For the concrete, Hognestad's stress-strain relationship [3] in compression is used and its tensile strength is neglected, [Fig. 2(a)]. The initial modulus of elasticity for concrete is taken [3] as $E_c = 1{,}800{,}000 + 500\ f''_c$ in which $f''_c = 0.85\ f'_c$. For reinforcing steel, the stress-strain relationship is assumed to be linearly elastic-perfectly plastic, [Fig. 2(b)] and E_s is taken at 30,000,000 psi. The creep effects of both materials are neglected.

A symmetrically reinforced rectangular concrete cross section is considered herein (Fig. 3). Assuming linear strain distribution over the section, the corresponding stress distribution and thus the moment-curvature-thrust relationships of the section can be obtained. Details of the method and the computer program are described elsewhere [4].

It is convenient to introduce quantities so that the moment-curvature-thrust relationships and hence the basic differential equations may be written in a form more appropriate for computation. The nondimensional moment, curvature and thrust are

$$m = \frac{M}{M_B} \quad , \quad \varphi = \frac{\Phi}{\Phi_B} \quad , \quad p = \frac{P}{P_o} \tag{1}$$

in which M_B is the balanced moment which produces at ultimate strength, simultaneously, a strain of $\epsilon_u = 0.004$ in the extreme fiber of concrete and the strain $\epsilon_y = f_y/E_s$ at the first yield on the tension steel; Φ_B is the corresponding curvature. And P_o is defined as

$$P_o = f''_c A_c + A_s f_y \tag{2}$$

in which A_s = total steel area and A_c = concrete net area.

In general, the moment-curvature curve with a constant thrust for a reinforced concrete section can be separated into three regimes: first regime, the slope of the curve is nearly constant; second regime, the rate of change of the slope is relatively large, third regime, the rate of change of the slope is small and the moment asymptotically approached the maximum moment capacity m_{pc} of the cross section. The dimensionless maximum curvature is denoted by φ_{pc}.

Following the previous work [5,6], the actual moment-curvature-thrust relationships may be closely represented by the following three mathematical expressions:

In the first regime $(0 \le \varphi \le \varphi_1)$

$$m = a\varphi \tag{3}$$

In the second regime ($\varphi_1 \le \varphi \le \varphi_2$)

$$m = b - \frac{c}{\varphi^{1/2}} \tag{4}$$

In the third regime ($\varphi_2 \le \varphi \le \varphi_{pc}$)

$$m = m_{pc} - \frac{f}{\varphi^3} \tag{5}$$

where a, b, c, f, φ_1, φ_2, φ_{pc}, and m_{pc} may be treated as the arbitrary curve-fitting parameters. The parameters will be a function of axial force p only. These functions may be expressed as polynomials of p for a given cross section. The choice of the curve-fitting parameter functions to fit an actual m-φ-p curve is not as difficult as it would seem to be. A detailed procedure to guide the choice of these parameter functions is given in Ref. 6. The approximate equations (Eqs. 3, 4, and 5) and the actual moment-curvature curves for a section with a constant thrust are compared in Figs. 3 and 4. The actual moment-curvature curves (solid curves in the figures) are calculated for the following cross section and material properties: $d'/t = 0.1$, $A_s/bt = 0.04$, $f_y = 45$ ksi, and $f'_c = 3$ ksi where b, d' and t are defined in the inset of Fig. 3. Since the failure of a beam-column is usually controlled by the second and third regimes, it can be concluded that the approximate m-φ-p equations are sufficiently accurate for practical use.

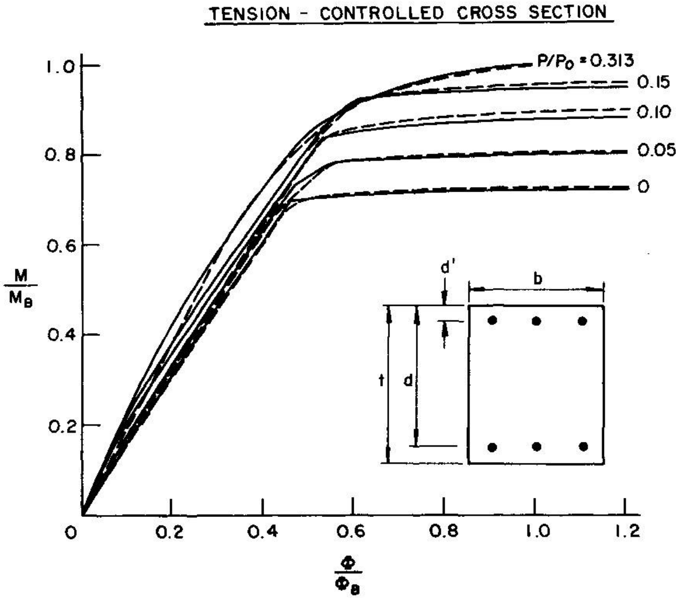


Fig. 3 Moment-Curvature-Thrust Relationships (Approximate Curves Shown Dashed)

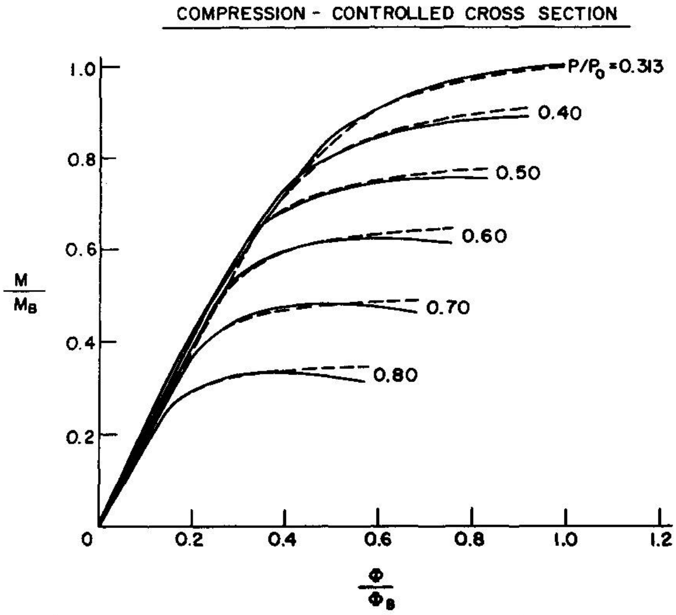


Fig. 4 Moment-Curvature-Thrust Relationships (Approximate Curves Shown Dashed)

NUMERICAL RESULTS: Using the approximate moment-curvature-thrust relationships, the basic differential equations of the beam-column are expressed in terms of curvature. The solutions to these differential equations with various boundary conditions are reported in Refs. [1] and [5].

A modified version of the program in Ref. [2] is used to solve the reinforced concrete beam-column problem as shown in the inset of Fig. 5. Computer programs are now available at Lehigh University. The results obtained directly from the program are the values of lateral load and the corresponding mid-span curvature (Fig. 5) The maximum lateral load, at which the beam-column will fail either by the instability of the beam-column or by the crushing of the mid-span cross section, is also indicated in the output of the program. By varying the beam-column length and the magnitude of the applied thrust, a series of maximum load interaction curves for combinations of axial force, P/P_o, and lateral load Q/Q_o, ($Q_o = 4\, M_B/\ell$) that can be safely supported by the reinforced concrete beam-column are obtained and plotted in Figs. 6 through 10, for the eccentricity ratios $e/t = 0.0$, 0.1, 0.3, 0.6 and 1.0, respectively.

The Column-Curvature-Curve method developed in Ref. [1] is utilized in the numerical solutions of the reinforced concrete beam-column shown in Figs. [11] and [12]. The interaction curves obtained are also compared in the figures with the results from Ref. 7, and good agreement is observed.

COMPARISON OF RESULTS WITH ACI MOMENT MAGNIFIER FORMULA: In designing reinforced concrete members carrying combined axial compression and bending moment, use is frequently made of the so-called moment magnifier method. According to the method, the influence of the second-order PΔ moment caused by the lateral deflection of a slender column on column strength is considered by multiplying the first-order moment M_o by the moment magnifier factor, δ. Formula recommended for estimating the magnification factor, δ is given in the 1971 ACI Building Code [8].

The general form of the formula which will be used in the comparison is Formula (10-5) given in Section 10 of the ACI Code. It takes the form

$$\delta = \frac{C_m}{1 - (P/P_c)} \tag{6}$$

where C_m = a coefficient depending on loading and support conditions to be taken as 1 and $P_c = \pi^2\, EI/\ell^2$ in which EI is computed by the approximate formula

$$EI = \frac{E_c\, I_g}{5} + E_s\, I_s \tag{7}$$

where I_g = moment of inertia of gross concrete section about the centroidal axis, neglecting the reinforcement, and I_s = moment of inertia of reinforcement about the centroidal axis of the member cross section.

The maximum loads determined by the ACI Formula (6) are compared in Figs. 6 to 10 with the numerical results obtained here. In general, the ACI Formula is seen to give good predictions for most cased investigated. For the case $e/t = 0$ (Fig. 6), the formula tends to underestimate the load carrying capacity for high slenderness ratio columns, especially in the high axial force range. For the case $e/t = 0.1$ (Fig. 7), the formula gives an excellent prediction for all slenderness ratios considered. For the cases $e/t = 0.3$, 0.6 and 1.0 (Figs. 8, 9 and 10), the trend reverses and the formula tends to overestimate the load carrying capacity for low slenderness ratio columns, but the difference is usually small. It may be concluded from this study that the current ACI formula is valid not only in predicting the ultimate strengths of beam-columns without transverse loads but also in estimating the ultimate strength of laterally loaded beam-columns.

CONCLUSIONS: Ultimate strength of a laterally loaded reinforced concrete beam-column has been obtained. Nondimensional interaction curves relating thrust, slenderness ratio, and lateral load for the maximum load carrying capacity of the reinforced concrete beam-column are presented. The maximum lateral loads determined by the analytical method have been compared in Figs. 6 to 10 with ACI moment magnification formula. Results indicated that ultimate strength predicted by ACI is in good agreement with calculated theoretical values.

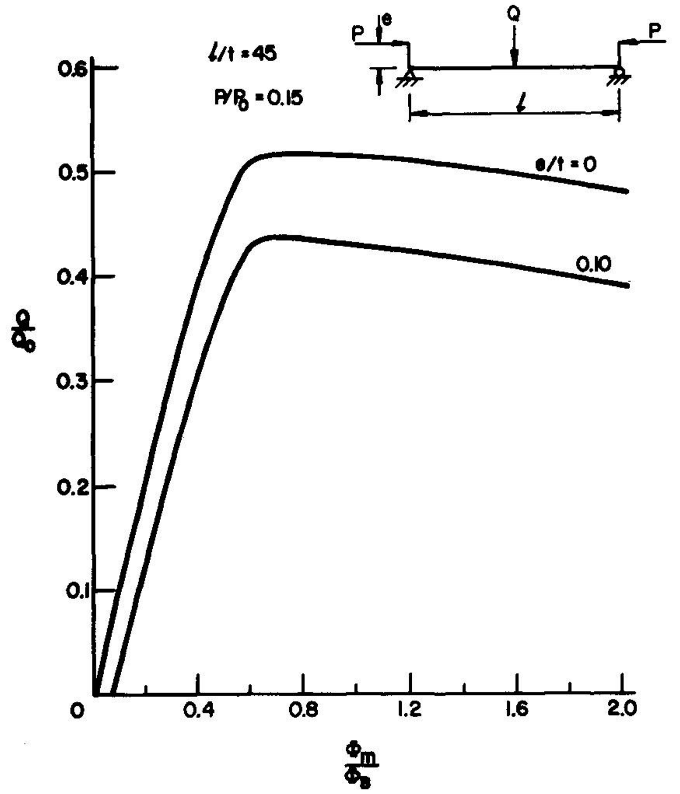


Fig. 5 Lateral Load vs. Mid-Section Curvature Curves

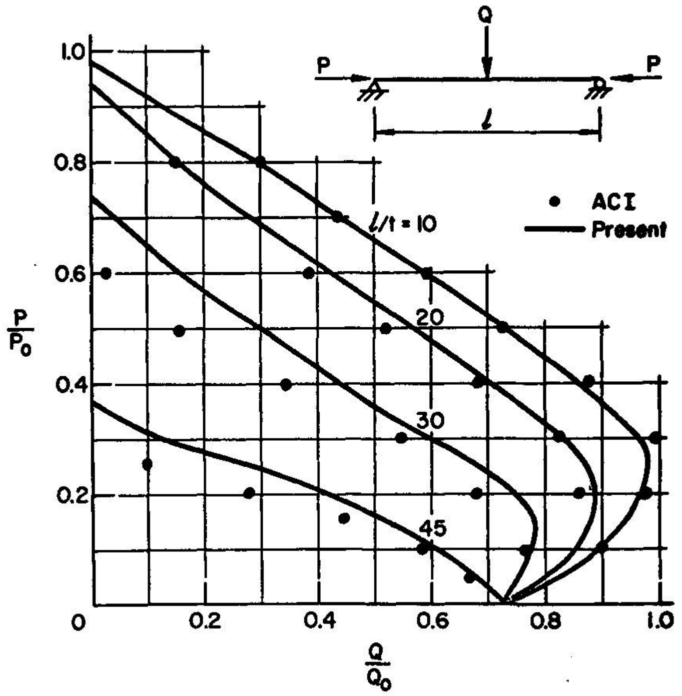


Fig. 6 Interaction Curves for Lateral Load vs. Thrust for e/t = 0

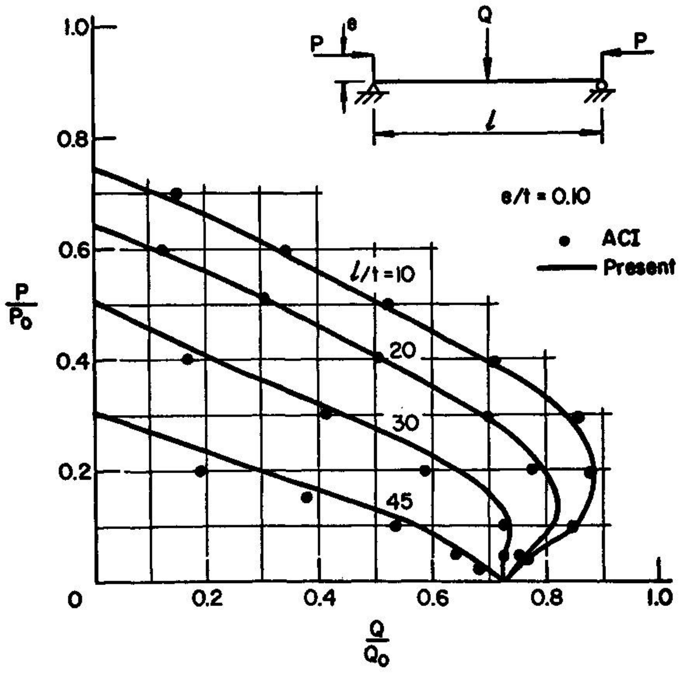


Fig. 7 Interaction Curves for Lateral Load vs. Thrust for e/t = 0.1

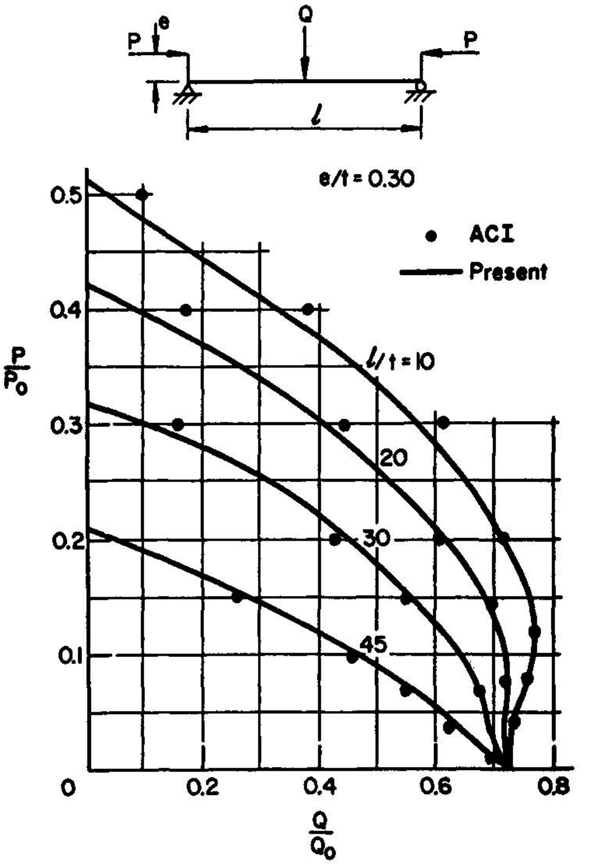


Fig. 8 Interaction Curves for Lateral Load vs. Thrust for e/t = 0.3

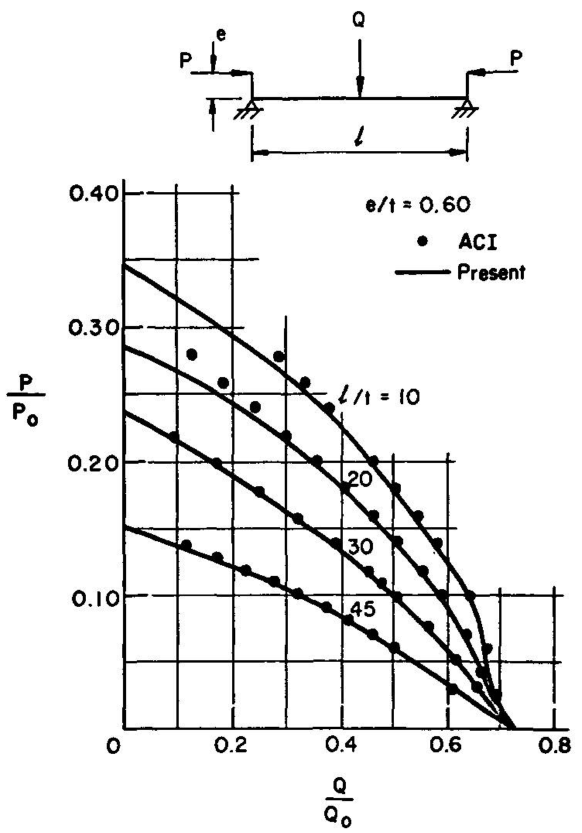


Fig. 9 Interaction Curves for Lateral Load vs. Thrust for e/t = 0.6

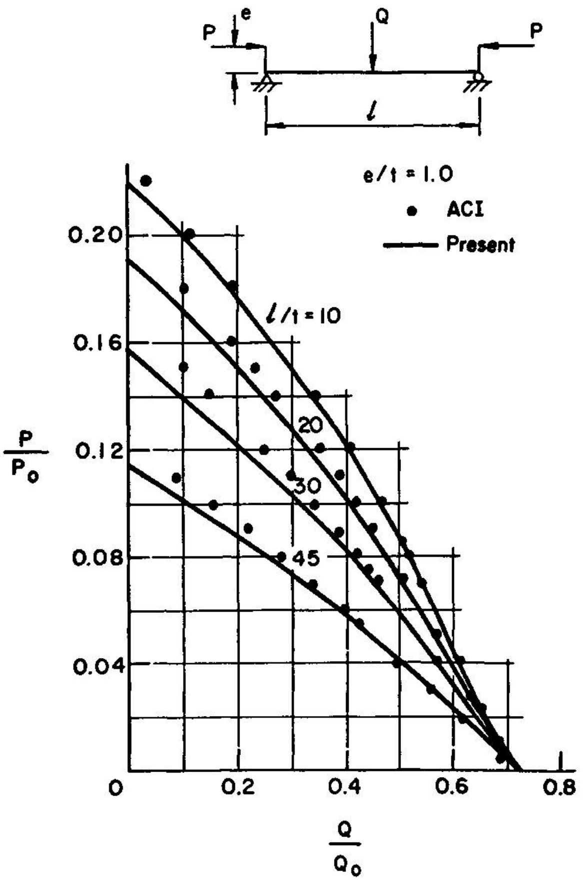


Fig. 10 Interaction Curves for Lateral Load vs. Thrust for e/t = 1.0

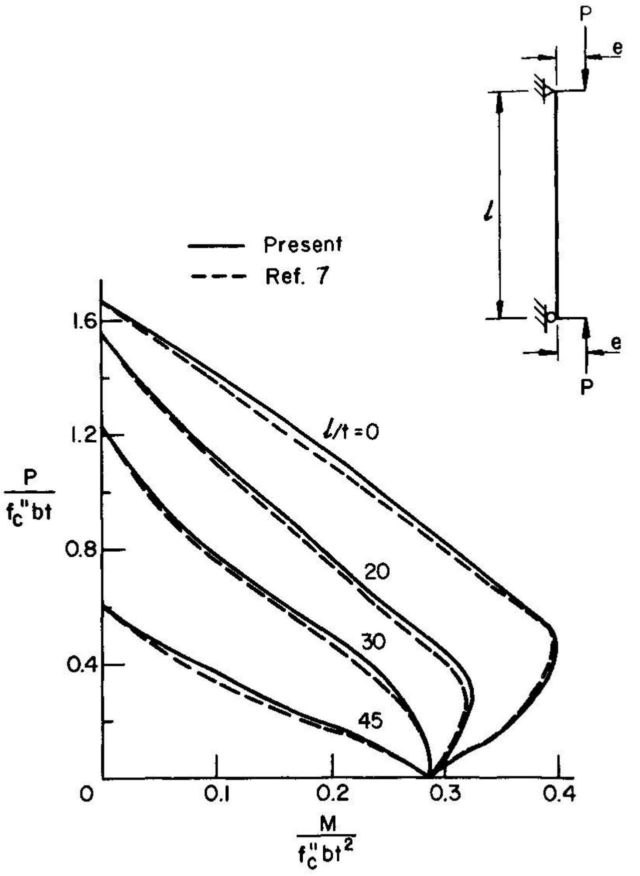


Fig. 11 Comparison of Interaction Curves --Equal End Eccentricities

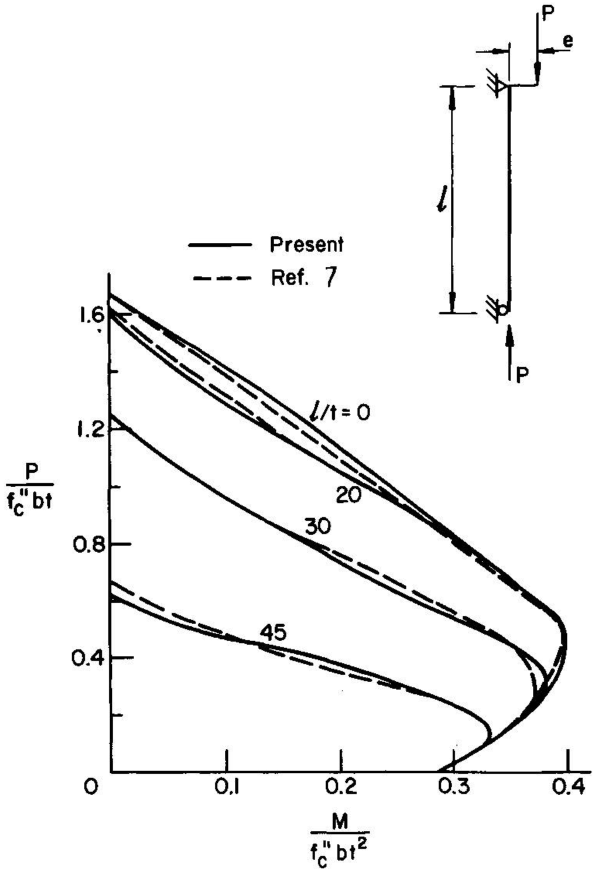


Fig. 12 Comparison of Interaction Curves--One End Eccentricity

REFERENCES:

1. Chen, W. F. and Atsuta, T., "Column Curvature Curve Method for Analysis of Beam-Columns", The Structural Engineer, The Journal of the Institute of Structural Engineers, London, England, Vol. 50, No. 6, June 1972, pp. 233-240.

2. Iyengar, S. N. and Chen, W. F., "Computer Program for an Inelastic Beam-Column Problem", Fritz Engineering Laboratory Report No. 331.7, Lehigh University, May 1970.

3. Hognestad, E. A., "A Study of Combined Bending and Axial Load in Reinforced Concrete Members", Bulletin No. 399, University of Illinois Engineering Experiment Station, Urbana, Ill., 1951.

4. Chen, A. C. T. and Chen, W. F., "Solutions to Various Reinforced Concrete Beam-Column Problems", Fritz Engineering Laboratory Report No. 370.2, September 1970, Lehigh University.

5. Chen, W. F., "General Solution of Inelastic Beam-Column Problems", Journal of the Engineering Mechanics Division, ASCE, Vol. 96, No. EM4, August 1970, pp. 421-441.

6. Chen, W. F., "Further Studies of Inelastic Beam-Column Problems", Journal of the Structural Division, ASCE, Vol. 97, No. ST2, February 1961, pp. 529-544.

7. Pfrang, E. O. and Siess, C. P., "Behavior of Restrained Reinforced Concrete Columns", Journal of the Structural Division, ASCE, Vol. 90, ST5, Proceeding Paper 4111, October 1964, pp. 113-136.

8. ACI Committee 318, "Building Code Requirements for Reinforced Concrete", (ACI 318-71), American Concrete Institute, Detroit, 1971.

SUMMARY

Analytical solutions are obtained for the maximum load carrying capacity of a reinforced concrete beam-column subject to combined axial thrust and a concentrated lateral load applied at mid-span. Numerical results are presented in the form of interaction curves relating axial thrust, lateral load and slenderness ratio. The analytically obtained results are compared with the prediction based on the empirical ACI formula and good agreement is observed in most cases.

RESUME

On obtient une solution analytique pour exprimer la charge ultime d'un système colonne-poutre en béton armé soumis à l'action conjuguée d'une force axiale et d'une charge latérale concentrée appliquée à mi-hauteur. On présente les résultats numériques sous forme de courbes d'interaction qui mettent en relation la force axiale, la charge latérale et l'élancement. On compare les résultats du calcul avec ceux obtenus par les formules empiriques ACI et on observe une bonne concordance dans la plupart des cas.

ZUSAMMENFASSUNG

Rechnerische Lösungen werden erhalten für die Tragfähigkeit von Balken-Stützen-Systemen unter der kombinierten Wirkung von Normalkraft und konzentrierter seitlicher Last in Feldmitte. Die numerischen Ergebnisse werden in Form von Interaktionsdiagrammen dargestellt, abhängig von Normalkraft, seitlicher Belastung und Schlankheit. Die rechnerisch erhaltenen Resultate werden mit den empirischen ACI-Formeln verglichen, wobei in den meisten Fällen gute Uebereinstimmung herrscht.