On the extreme compressive strain of concrete for calculating the ultimate strength of reinforced concrete section

On the Extreme Compressive Strain of Concrete for Calculating the Ultimate Strength of Reinforced Concrete Section

Raccourcissement ultime du béton et calcul de la sollicitation ultime d'une section en béton armé

Über den Grenzwert der Druckstauchung von Beton bei der Berechnung der Traglast von Stahlbetonquerschnitten

Hiroshi MUGURUMA
Prof., Doctor of Engineering
Shinzo TANAKA
Graduate Student
Kyoto University
Kyoto, Japan

1. Introduction

The failure of reinforced concrete section subjected to the flexural moment, axial load or combined flexural and axial loads is generally caused by crushing of concrete at the compression zone. For the calculation of ultimate strength it is necessary to assume the extreme compressive fiber strain of concrete, ε_{cu}, induced in the section at the ultimate strength. Usually, the values of ε_{cu} empirically or semi-empirically determined are adopted in the calculation. [1] For instance, ε_{cu} = 0.15 - 0.2% is assumed for pure axial loading failure and ε_{cu} = 0.25-0.35% for flexural failure under the action of pure flexural load or combined flexural and axial loads.

However, the extreme compressive fiber strain of concrete, ε_{cu}, at the ultimate strength is not always defined clearly from the theoretical view-point. The value of ε_{cu} seems to be affected by many factors such as the combination of applied axial and flexural forces, the characteristics of stress-strain curve of concrete, especially the falling branch of it after compressive strength, and the percentage of reinforcements etc. In this paper, numerical estimations are made on the value of ε_{cu} for ultimate strength calculation by using the typical stress-strain curves of concrete, and the effects of these factors upon the value of ε_{cu} are discussed.

2. Definition of concrete fiber strain, ε_{cu}, at ultimate strength

The ultimate loading capacity of reinforced concrete section subjected to combined axial and bending forces can be represented by the ultimate axial load-moment interaction curve. This can be calculated from the equilibrium of forces acting on the section under the consideration of stress and strain compatibility, where the compressive fiber strain of concrete, ε_{cu}, at ultimate strength should be assumed. While the collapse of reinforced concrete section is generally caused by crushing of concrete in compression zone, the applied load shows its' maximum value before the crushing of concrete. Fig. 1 shows such phenomenon by load-deflection curve of typical member. Thus, generally the maximum of applied

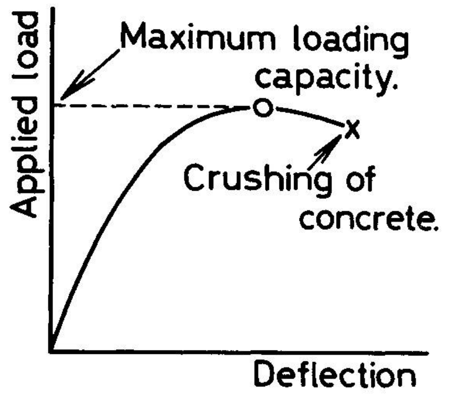

Fig. 1 Load-deflection curve of reinforced concrete member

load recorded until the collapse of section takes place by crushing of concrete is so defined as the ultimate loading capacity or the ultimate strength,[2),3)] And also the corresponding compressive fiber strain of concrete, ε_{cu}, is defined as the strain which should be used in the calculation of ultimate strength.

3. Method of numerical estimation of ε_{cu}

The purpose of this study is to obtain numerically the compressive fiber strain of concrete, ε_{cu}, at maximum loading capacity of reinforced concrete column section subjected to various combinations of applied axial load and flexural moment. To explain the procedure of numerical estimation, the rectangular column section shown in Fig. 2(a) is considered. The stress and strain distributions overall the section are shown in Fig. 2(b) and 2(c), respectively, in general form. The stress distribution shown in Fig. 2(c) is obtained in correspondence to the strain distribution in Fig. 2(b) by using the stress-strain relations of component materials. The compressive fiber strain of concrete, ε_c, and the distance of neutral axis, x, from compressive fiber of section are determined so as to satisfy the equilibrium equations for given combined loads, $P(\varepsilon_c)$ and $M(\varepsilon_c)$. That is, the applied combined loads, $P(\varepsilon_c)$ and $M(\varepsilon_c)$, are generally expressed as the function of ε_c and x. However, considering that loads, $P(\varepsilon_c)$ and $M(\varepsilon_c)$, are so applied as the eccentricity,

$$e = M(\varepsilon_c)/P(\varepsilon_c) \qquad (1),$$

becomes constant without regard to their magnitude, the distance of neutral axis, x, can also be expressed as a function of ε_c. Thus, in other words, a set of combined loads, $P(\varepsilon_c)$ and $M(\varepsilon_c)$, can be obtained numerically for a given value of ε_c under the consideration of constant eccentricity, e.

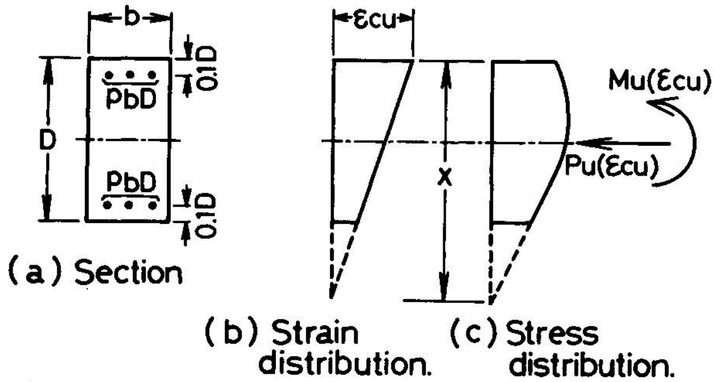

Fig. 2 Stress and strain distributions overall the rectangular column section.

Fig. 3 shows ε_c - $P(\varepsilon_c)$ or $M(\varepsilon_c)$ relation for various values of eccentricity, $e = e_1$, e_2, e_3, ······ , where the concrete fiber strain, ε_{cu}, corresponding to the peak value of $P(\varepsilon_c)$ coincides with that of $M(\varepsilon_c)$ because of the linear relation between $P(\varepsilon_c)$ and $M(\varepsilon_c)$. From the definition of ultimate strength described in 2 the peak values, $Pu(\varepsilon_{cu})$ and $Mu(\varepsilon_{cu})$, in ε_c - $P(\varepsilon_c)$ and ε_c - $M(\varepsilon_c)$ curves are defined as the maximum loading capacity for a given loading condition, $e = M(\varepsilon_c)/P(\varepsilon_c)$, and the corresponding compressive fiber strain, $\varepsilon_c = \varepsilon_{cu}$, is the strain to be used in the theoretical calculation of ultimate strength. In this paper using ε_c - $P(\varepsilon_c)$ or ε_c - $M(\varepsilon_c)$ relation as shown in Fig. 3, the ultimate strengths, $Pu(\varepsilon_{cu})$

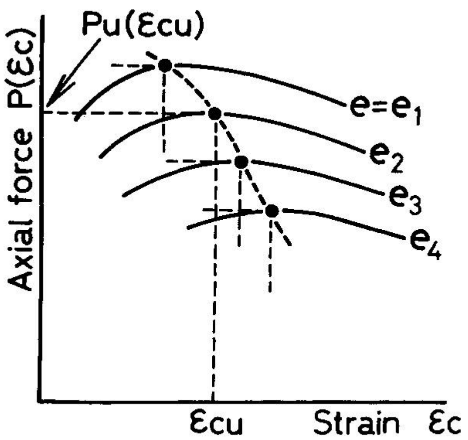

Fig.3 ε_c-$P(\varepsilon_c)$ or $M(\varepsilon_c)$relation and $Mu(\varepsilon_{cu})$, as well as corresponding compressive fiber strain of concrete, ε_{cu}, is estimated graphically for rectangular column section shown in Fig. 2(a).

Moreover, the ultimate strengths, $Pu(\varepsilon_{cu})$ and $Mu(\varepsilon_{cu})$, and corresponding concrete fiber strain, ε_{cu}, can be obtained as the values satisfying following mathematical conditions.[3)]

$$\frac{dP(\varepsilon c)}{d\varepsilon_c} = 0 \quad \text{or} \quad \frac{dM(\varepsilon c)}{d\varepsilon_c} = 0 \qquad (2).$$

Of course, a set of values, ε_{cu}, $P_u(\varepsilon_{cu})$ and $M_u(\varepsilon_{cu})$, obtained from first equation in Eq. (2) satisfys the second condition in Eq. (2) simultaneously.

4. Stress-strain curves of component materials for numerical calculation

For numerical calculation of compressive fiber strain of concrete, ε_{cu}, at the ultimate strength of reinforced concrete column section, the stress-strain curves of component materials should be assumed. To simplifying the numerical calculation, the ideal elastic-plastic relation shown in Fig. 4 is assumed for reinforcement, where the elastic modulus of $E_s = 2{,}100{,}000$ kg/cm^2 is taken without respect to the yield strength of reinforcement.

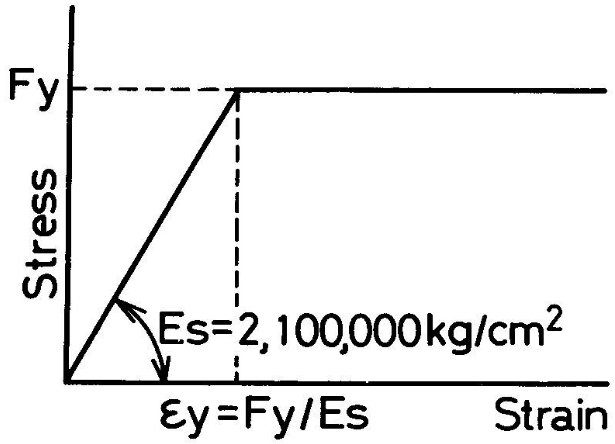

Fig. 4 Stress-strain curve of reinforcement

Fig. 5 shows three different types of typical stress-strain curve of plain concrete obtained from compressive tests on cylinder specimens of various kinds of concrete.[4),5)] The ordinate in Fig. 5 is expressed as the ratio of applied stress to the compressive strength. Curve I represents the stress-strain relation for ordinary aggregate concrete having 28-day compressive strength up to 350 kg/cm^2. Curve II is that of light-weight aggregate structural concrete. The strain at the peak stress (that is, at the compressive strength) as well as the negative inclination of strain softening region in Curve II is greater than that in Curve I. Curve III in Fig. 5 is identical one modified from Curve I for investigating the effect of the negative inclination of strain softening region upon the compressive fiber strain of concrete, ε_{cu}, at the ultimate strength. For convenience of the numerical calculation of ε_{cu}, the area surrounded by each stress-strain curve shown in Fig. 5 and its' center of gravity were calculated in correspondence to an arbitrary strain. The results are also shown in Fig. 5 in the coefficients, k_1k_3 and k_2, versus compressive strain, ε_c, relations, where k_1k_3 and k_2 denote the ratio of average stress of stress-strain curve until an arbitrary strain, ε_c, to the compressive strength, F_c, and the ratio of the location of center of gravity of corresponding area from an arbitrary strain, ε_c, to the strain ε_c, respectively. That is, coefficients, k_1k_3 and k_2, can be considered as the generalized stress block

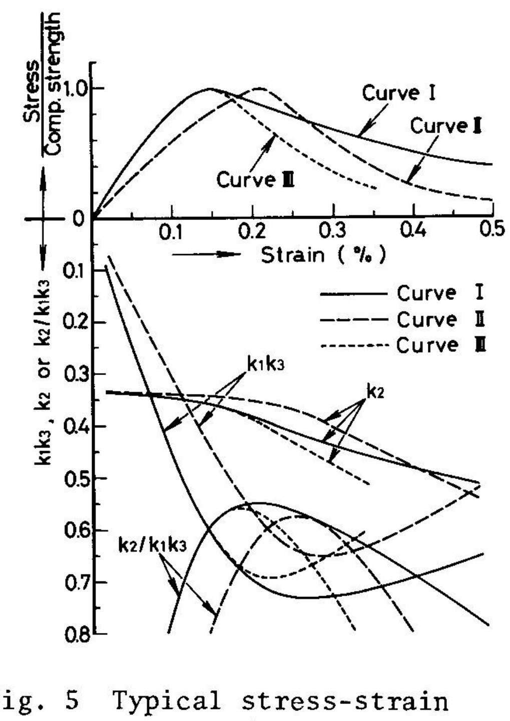

Fig. 5 Typical stress-strain curves of concrete and corresponding values of k_1k_3, k_2 and k_2/k_1k_3

coefficients of concrete in flexural compression zone of reinforced concrete section.

5. Effects of the eccentricity of applied axial load and the percentage of reinforcements upon ε_{cu}

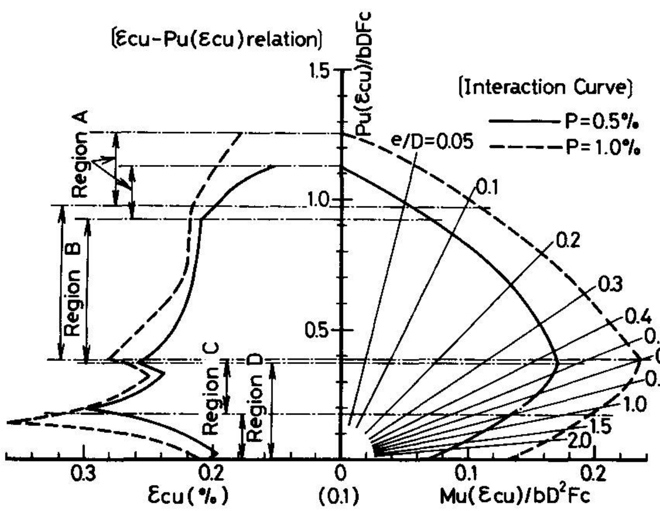

Fig. 6 ε_{cu} - $P_u(\varepsilon_{cu})$ - $M_u(\varepsilon_{cu})$ relations for rectangular column section (Curve I concrete stress-strain relation and F_y =4000 kg/cm^2 are used in calculation)

The numerical calculations of compressive fiber strain, ε_{cu}, at ultimate strength were performed on the percentage of reinforcements, p =0.5% and p = 1%, of rectangular column section shown in Fig. 2. The stress-strain relation of concrete as well as the yield strength of reinforcement adopted in calculation are Curve I in Fig. 5 and F_y=4000 kg/cm^2, respectively. The results are shown by ε_{cu}-$P_u(\varepsilon_{cu})$ relation with $P_u(\varepsilon_{cu})$ - $M_u(\varepsilon_{cu})$ interaction curve as in Fig. 6.

ε_{cu} - $P_u(\varepsilon_{cu})$ relations shown in Fig. 6 can be divided into following four regions.

Region A : Range that applied combined loads reach at their muximum without yielding of whole reinforcements.

Region B : Range that applied combined loads reach at their maximum after yielding of reinforcements in compression zone, where the stress in reinforcements in tensile zone still remains in elastic range.

Region C : Range that applied combined loads reach at their maximum after yielding of whole reinforcements.

Region D : Range that applied combined loads reach at their maximum after yielding of reinforcements in tensile zone, where the stress in reinforcements in compression zone still remains in elastic range.

In ε_{cu} - $P_u(\varepsilon_{cu})$ relation for the section of p = 0.5%, Region C disappears and is expressed border line between B and D.

It can be seen from Fig. 6 that the compressive fiber strain ε_{cu} is much influenced by the combination of applied axial and flexural loads. In Fig. 6 the pure axial loading column section shows the minimum value of ε_{cu}. These are listed in Table 1, which are a little larger than the strain, ε_{cu} = 0.15%, corresponding to the peak stress in concrete stress-strain curve. In Region A and B in Fig. 6, the compressive fiber strain ε_{cu} increases gradually with increase of the relative degree of applied moment $M(\varepsilon_c)$ to applied axial force $P(\varepsilon_c)$, that is, with increase of the eccentricity $e = M(\varepsilon_c)/P(\varepsilon_c)$. On the lower bound of Region B, the strain ε_{cu} reaches at a peak value, where the corresponding ultimate flexural moment $M_u(\varepsilon_{cu})$ becomes maximum in the interaction curve. In Table 1, the values of ε_{cu} on the lower bound of Region B are also listed. In Region C and D, considerable increase of ε_{cu} is recognized after a little decrease near the upper bound of Region C or D, and the maximum value of ε_{cu} is obtained at the middle portion of these range. After that, the value of ε_{cu} decreases rapidly to the value of about 0.21%, which corresponds to the value for pure flexure. The maximum values of ε_{cu} are also summarized in Table 1.

From such observation, it can be stated that for the exact calculation

Table 1 The values of ε_{cu} for critical point in ε_{cu} - $P_u(\varepsilon_{cu})$ relation (Curve I concrete stress-strain relation and $F_y = 4000$ kg/cm^2 are used in the calculation)

Percentage of reinforcement	P (%)	0.5	1
For pure axial loading	ε_{cu} (%)	0.152	0.180
	$P_u(\varepsilon_{cu})/bDF_c$	1.125	1.250
	$M_u(\ _{cu})/bD^2F_c$	0	0
On the lower bound of Region A	ε_{cu} (%)	0.211	0.217
	$P_u(\varepsilon_{cu})/bDF_c$	0.920	0.970
	$M_u(\varepsilon_{cu})/bD^2F_c$	0.070	0.107
On the lower bound of Region B	ε_{cu} (%)	0.267	0.281
	$P_u(\varepsilon_{cu})/bDF_c$	0.370	0.390
	$M_u(\varepsilon_{cu})/bD^2F_c$	0.171	0.233
At the peak value in ε_{cu} - $P_u(\varepsilon_{cu})$ relation	ε_{cu} (%)	0.300	0.360
	$P_u(\varepsilon_{cu})/bDF_c$	0.195	0.140
	$M_u(\varepsilon_{cu})/bD^2F_c$	0.139	0.185
For pure flexure	ε_{cu} (%)	0.204	0.210
	$P_u(\varepsilon_{cu})/bDF_c$	0	0
	$M_u(\varepsilon_{cu})/bD^2F_c$	0.068	0.132

* F_c denotes the compressive strength of concrete.

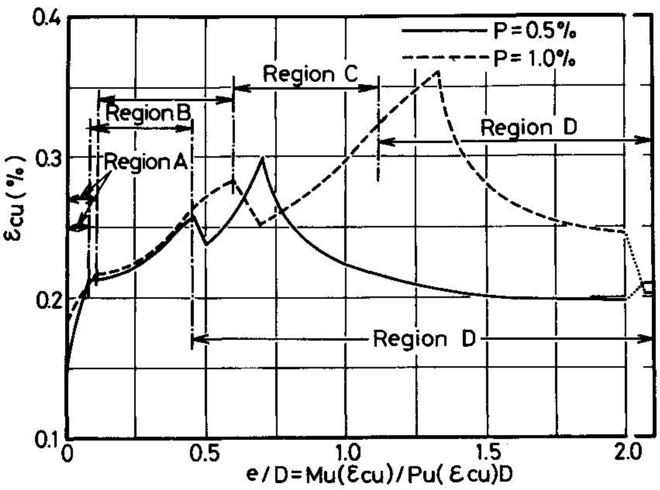

Fig. 7 e - ε_{cu} relations for rectangular column section of p = 0.5% and 1% (Curve I concrete stress-strain relation and F_y = 4000 kg/cm^2 are used in calculation)

of maximum loading capacity, especially that of corresponding deflection of reinforced concrete member the change of compressive fiber strain, ε_{cu}, as shown in Fig. 6 may be considered in correspondence to the combination of applied axial and flexural loads.

The effects of the percentage of reinforcement, p, upon the compressive fiber strain, ε_{cu}, can be observed from e - ε_{cu} relations shown in Fig. 7, which is rewritten from the results of Fig. 6. It seems from Fig. 7 that the effects of p is so small as negligible in Region A and B, while much difference of ε_{cu} between p = 0.5% and p = 1% is observed in Region C and D.

6. Effects of the shape of stress-strain curve of concrete upon the compressive fiber strain ε_{cu}

Using three different types of stress-strain curve of plain concrete shown in Fig. 5, the compressive fiber strain, ε_{cu}, at ultimate strength was calculated on the rectangular column section having p = 0.5%. The yield strength of reinforcement was assumed as 4000 kg/cm^2 in the calculation. The results are illustrated in Fig. 8. While three interaction curves are very closed with each other, much difference can be seen in three results on ε_{cu} - $P_u(\varepsilon_{cu})$ relation. Comparison between the results for concrete stress-strain Curve I and II shows that the larger compressive strain at the peak stress in stress-strain curve of

Table 2 Effects of the type of concrete stress-strain curve upon ε_{cu} for pure axial loading section of p = 5% (F_y = 4000 kg/cm2)

Type of stress-strain curve of concrete used in calculation	Ultimate axial load $P_u(\varepsilon_{cu})/bDF_c$	The value of ε_{cu} in %
Curve I	1.125	0.152
Curve II	1.160	0.208
Curve III	1.125	1.151

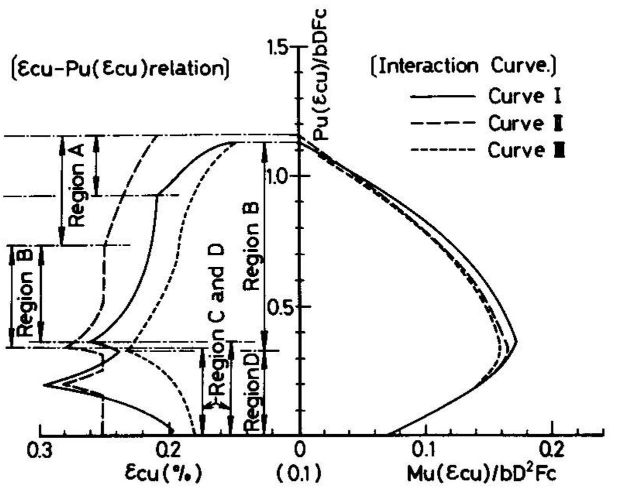

Fig. 8 Effects of the type of concrete stress-strain curve upon the compressive fiber strain, ε_{cu}, at ultimate strength (p = 0.5%, F_y = 4000 kg/cm2)

concrete results in the larger values of ε_{cu} within Region A and B in ε_{cu} - $P_u(\varepsilon_{cu})$ relation. The difference between the compressive fiber strains for Curve I and II becomes maximum in the case of pure axial loading. As a reference, the values of ε_{cu} obtained for pure axial loading are summarized in Table 2. On the contrary it appears from the comparison of the results for concrete stress-strain Curve III with that for Curve I that the increase of negative inclination in strain softening region of concrete stress-strain relation reduces the compressive fiber strain at ultimate strength of reinforced concrete section, while no obvious effect is recognized on the section subjected to pure axial force.

7. Effects of the yield strength of reinforcement, F_y, upon ε_{cu}

Fig. 9 shows the results of numerical calculation on the ultimate strengths, $P_u(\varepsilon_{cu})$ and $M_u(\varepsilon_{cu})$, and corresponding compressive fiber strain, ε_{cu}, for rectangular column section of p = 0.5% having various yield strengths of reinforcement. The stress-strain relation of concrete used in calculation is Curve I in Fig. 5.

Comparison between the results for F_y = 4000 kg/cm^2 and F_y = 5000 kg/cm^2 shows that Region A in ε_{cu} - $P_u(\varepsilon_{cu})$ relation becomes larger with increase of the yield strength of reinforcement, which results in the considerable increase of compressive fiber strain, ε_{cu}, in Region B. In Fig. 9, the strain ε_{cu} in Region B for F_y = 5000 kg/cm^2 is about 0.05% larger than that for F_y = 4000 kg/cm^2.

The use of reinforcement having smaller yield strength provides the decrease of Region A in ε_{cu} - $P_u(\varepsilon_{cu})$ curve. In Fig. 9, Region A disappears in ε_{cu} - $P_u(\varepsilon_{cu})$ relation for F_y = 2400 kg/cm^2 because the yield strain of reinforcement is smaller than the strain corresponding to the peak stress in stress-strain curve of concrete. In such case, the strain ε_{cu} for pure axial loading just coincides with the strain at peak stress in concrete stress-strain relation.

8. Conclusions

Based on the general concept that the ultimate loading capacity of reinforced concrete section is defined as the loads satisfying the equation $dP(\varepsilon_c)/d\varepsilon_c = 0$ or $dM(\varepsilon_c)/d\varepsilon_c = 0$, corresponding compressive fiber strain of concrete, ε_{cu}, was calculated numerically on the rectangular column section and the effects of the combination of applied axial and flexural loads, the percentage of reinforcement, the shape of stress-strain curve of concrete and the yield strength of reinforce-

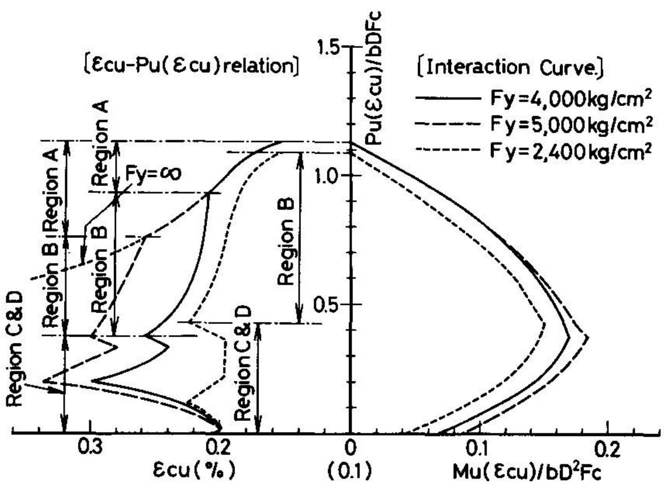

Fig. 9 Effects of the yield strength of reinforcement upon ε_{cu} - $P_u(\varepsilon_{cu})$ -$M_u(\varepsilon_{cu})$ relations of rectangular column section of p = 0.5% (Curve I concrete stress-strain relation is used in the calculation)

ment, etc. upon the compressive fiber strain were investigated. The following conclusions are obtained from the results.

(1) The compressive fiber strain ε_{cu} varies considerably with the relative intensity of applied flexural moment to axial load. ε_{cu} - $P_u(\varepsilon_{cu})$ relation obtained in this paper can be divided into four regions in corresponding to the yielding of reinforcement. The value of ε_{cu} becomes minimum in the case of pure axial loading and the increase of eccentricity results in the gradual increase of ε_{cu}. Also it decreases with increase of eccentricity after reaching the maximum value near pure flexure.

(2) No obvious effects of the percentage of reinforcement upon the value of ε_{cu} are recognized within Region A and B of ε_{cu} - $P_u(\varepsilon_{cu})$ curve, where relative intensity of applied axial load is larger in comparison with applied flexural moment.

(3) The larger strain at peak stress in concrete stress-strain curve increases the value of ε_{cu} within Region A and B of ε_{cu} - $P_u(\varepsilon_{cu})$ curve, especially the maximum increase of ε_{cu} is obtained in case of pure axial force. Also, the increase of negative slope of strain softening region in concrete stress-strain curve reduces the value of ε_{cu} considerably. These factors have no obvious effects on the ultimate axial load and flexural moment interaction curve of section.

(4) The use of reinforcements having larger yield strength results in considerable increase of ε_{cu} in region B of ε_{cu} - $P_u(\varepsilon_{cu})$ curve.

(5) The effects of the factors described in this paper upon the value of ε_{cu} may be necessary to consider for the exact estimation of ultimate strength as well as corresponding deformation of reinforced concrete member.

References

1) E. Hognestad, N. W. Hanson & D. McHenry: Concrete Stress Distribution in Ultimate Strength Design, J. of A.C.I., Vol. 52, No. 6, pp. 455~479, Dec. 1955.

2) S. Morita & N. Adachi: The Stress-Strain Behaviour of Concrete in the Compression Zone of Flexural Members, Proc. of the International Conference on Mechanical Behavior of Materials, Vol. IV, pp. 162~171, 1972.

3) R. F. Warner: Physical-Mathematical Models and Theoretical Considerations, Introductory Report on IABSE Symposium on Design and Safety of Reinforced Concrete Compression Members, pp. 1~21, April 1973.

4) F. Watanabe: Complete Stress-Strain Curve for Concrete in Concentral Compression, Proc. of the International Conference on Mechanical Behavior of Materials, Vol. IV, pp. 153~161, 1972.

5) H. Muguruma & S. Tanaka: Mechanical Properties of High Strength Concrete, Annual Report of Japan Cement Association, XXVII, 1973, (in Japanese).

SUMMARY

Numerical estimations were made on the value of extreme compressive fiber strain of concrete at ultimate strength of reinforced concrete column section subjected to various combinations of axial and flexural loads. The results showed that the extreme compressive fiber strain is much affected by many factors such as the combinations of axial and flexural loads, the caracteristics of stress-strain relation of concrete, the yiald strength of reinforcement and the percentage of reinforcements, etc.

RESUME

On a estimé la valeur de la tension de compression extrême dans le béton d'une colonne en béton armé soumise à diverses combinaisons de flexions et d'efforts axiaux. Les résultats montrent que l'allongement spécifique dans le béton varie considérablement en fonction de facteurs tels que les combinaisons de flexions et d'efforts axiaux, les caractéristiques de la courbe tension-déformation du béton, la limite d'élasticité des armatures, le pourcentage d'armature, etc.

ZUSAMMENFASSUNG

Zahlenmässige Schätzungen für den Wert der grössten Druckstauchung von Beton bei der Bruchbeanspruchung von Stahlbeton-Stützenquerschnitten unter Einwirkung verschiedener Kombinationen von Normalkraft und Biegemomenten wurden vorgenommen. Die Ergebnisse zeigten, dass die grösste Druckstauchung durch eine Reihe von Faktoren, wie die Kombination von Normalkraft und Biegemoment, den Verlauf des Spannungs-Dehnungs-Diagramms von Beton, die Streckgrenze der Bewehrung, den Bewehrungsgehalt usw. beeinflusst wird.