A simplified computer oriented method of ultimate strength check of a prestressed concrete column under axial load and biaxial bending

IV

A Simplified Computer Oriented Method of Ultimate Strength Check of a Prestressed Concrete Column under Axial Load and Biaxial Bending

Une méthode calcul simple par ordinateur pour le calcul de la charge ultime de grandes colonnes précontraintes en béton armé sous charge axiale et flexion biaxiale

Eine vereinfachte computerorientierte Methode zur Berechnung der Bruchlast grosser vorgespannter Stützen unter axialer Last und zweiaxiger Biegung

Subrata DALAL
Senior Structural Engineer
Trudeau, Gascon, Lalancette and Co.
Montreal, Canada

Paul ZIA
Professor
North Carolina State University
Raleigh, N.C., U.S.A.

Zenon A. ZIELINSKI
Professor of Engineering
Sir George Williams University, Montréal
Quebec, Canada

1) INTRODUCTION

Prestressing a concrete column appears to be a questionable idea if the eccentricity of the load is very small. But when the eccentricities become quite big the idea of prestressing the column becomes attractive to keep the size of the column or the amount of mild steel or both within certain limits. Prestressed concrete column under axial load and bending has been treated by several authors (1,2,3,6). Reference 4 describes a procedure to design prestressed concrete columns under uniaxial bending. Al-Rawi has developed interaction surfaces for axial load, bending moment and torsion for circular prestressed concrete columns (5).

This paper discusses a method of calculating the ultimate strength of prestressed concrete columns under biaxial bending and of course, axial load. The sections of the columns can be of any shape. The method described is suitable for computerized calculation. For our purpose, we shall investigate a hollow trapezoidal section. A similar case can be found in the design of the Olympic Stadium, Montreal. It might be said that a trapezoidal section may not be most efficient in resisting biaxial bending. But, in many practical structures, column sections might be dictated by the architect's idea.

2) DEFINITION OF THE PROBLEM

The section of the column assumed is shown in Fig. 1 with the prestressing and non-prestressing reinforcements defined, as shown in Fig. 2.

The following assumptions are made.

(i) Strains in concrete and reinforcing steel shall be proportional to the distance from the neutral axis.
(ii) The maximum strain in concrete at ultimate failure, $\varepsilon_{cu} = 0.003$.
(iii) Stress strain relationships for prestressing steel as shown in Fig. 3. The diagram can be idealized as made of 3 straight lines. The maximum strain in prestressing steel at ultimate failure $\varepsilon_{su} = 0.04$.
(iv) Stress in non-prestressed reinforcement proportional to modulus of elasticity of steel, E_s, with continuous maximum value equal to yield strength.
(v) A rectangular stress block for concrete at failure.
(vi) The column is a short column, with no buckling effect.
(vii) Bonded prestressing is used.
(viii) Though the interaction diagram between axial load and bending moment for prestressed concrete sections does not have a clearly defined balanced point, one will be assumed.
(ix) The following values of capacity reduction factors (ϕ) shall be used:
When compression governs – $\phi = 0.70$
For pure flexure – $\phi = 0.90$ with interpolated values for intermediate cases.

The typical interaction diagram between bending moment and axial load for a prestressed concrete column is shown in Fig. 4(a). This is idealized in Fig. 4(b). The interaction diagrams can be drawn if P_u, M_{uz}, M_{uy}, P_{bz}, M_{bz}, P_{by} and M_{bz} can be computed. Once the interaction diagram can be drawn, M_{uza} and M_{uya} values can be obtained from the value of P_{ud} as shown in Fig. 4(b).

Then the ultimate strength will be acceptable if

$$(M_{uzd}/M_{uza})^{1.5} + (M_{uyd}/M_{uya})^{1.5} \leq 1.0$$

3) ESTABLISHING INTERACTION DIAGRAM

As mentioned before, to establish the interaction diagrams one has to compute the values of P_u, M_{uz}, M_{uy}, P_{bz}, M_{bz}, P_{by} and M_{bz}.

(i) Value of P_u

The value of P_u for tied column as given in Ref. 4 is:

$$P_u = 0.7 \times 0.85\ [0.85\ f'_c \cdot A_c + A_{so} \cdot f_y - A_s \cdot E_s$$

$$\{\varepsilon_{se} - (\varepsilon_{cu} - f_{ce}/E_c)\}] \qquad (1)$$

The equation shows that the load capacity of the column is composed of load carried by the concrete as well as the non-prestressed vertical reinforcement minus the residual tension in the prestressing steel as given by the last term of the equation.

(ii) Value of + M_{uz}

To calculate M_{uz} we have to define the balanced condition. For the balanced condition under pure flexure the strain diagram, actual and assumed stress diagrams are shown in Fig. 5.

From the strain diagram we get

$$Y_b = [\varepsilon_{cu}/(\varepsilon_{cu} + \varepsilon_{su} - \varepsilon_{se})]\ \ d \qquad (2)$$

and we define

$$a_b = K_1\ Y_b \qquad (3)$$

where K_1 is given by

$$K_1 = 0.85 - .05\ (\frac{f'_c}{1000} - 4) \qquad (4)$$

All the variables of the above equations being given for a particular problem, Y_b can be calculated.

Now, in Fig. 6, the cross-section is given with strain and force diagrams. It is seen that if the depth of neutral axis is known all unknowns can be computed.

At this point,we can find out the depth of neutral axis by trial and error. A suggested method, suitable for computer adoption is given below.

Step 1

Assume

$$a = a_b \times 0.1$$

Step 2

Calculate the corresponding strains in the reinforcements as follows:

$$S_{sd} = \varepsilon_{cu}/(a/K_1) = K_1 \times \varepsilon_{cu}/a$$

$$\varepsilon_{102} = -\varepsilon_{cu} + S_{sd} \times Y_{01}$$

$$\varepsilon_{304} = -\varepsilon_{cu} + S_{sd} \times Y_{03}$$

$$\varepsilon_{12} = -\varepsilon_{cu} + S_{sd} \times Y_{101}$$

$$\varepsilon_{1314} = -\varepsilon_{cu} + S_{sd} \times Y_{013}$$

$$\varepsilon_{1516} = -\varepsilon_{cu} + S_{sd} \times Y_{015}$$

$$\varepsilon_{34} = -\varepsilon_{cu} + S_{sd} \times Y_{103}$$

(NOTE: If prestressing steel strain ε_{34} is greater than $\varepsilon_{su} - \varepsilon_{se}$ go back to Step 1 and start with the next value.)

Step 3

Calculate the corresponding stresses in prestressed and non-prestressed reinforcements.

For non-prestressed reinforcements

$$\text{Stress} = \varepsilon \times E_s \leq f_Y$$

For prestressed reinforcements, the stresses are to be obtained from Fig. 3 for the corresponding strains from the previous step plus ε_{se}.

Step 4

Calculate the corresponding forces in the reinforcements by multiplying by respective areas.

Step 5

Calculate the forces in concrete.

If $a \leq t_p$, $C_c = - Z_p \times a \times 0.85\ f'_c$

If $a > t_p$, $C_c = C_{cf} + C_{cw}$

where

$$C_{cf} = - Z_p \times t_p \times 0.85 \times f'_c$$

and $C_{cw} = -(a-t_p) \times 2 \times t_w \times 0.85 \times f'_c$

So, $C = C_c + C_{102} + C_{304} + C_{12}$

and $T = T_{1314} + T_{1516} + T_{34}$

Then $CTR = C/T$

Step 6

If CTR < 1, go to Step 1, increase the value of a and go through all the steps until CTR $\geq$ 1.

By going through the above six steps, the value of depth of neutral axis and hence of all internal forces developed are calculated. Then we can say the value of M_{uz} is given by:

If $a \leq t_p$

$$M_{uz} = 0.9\,[C_{102} \times Y_{01} + C_{304} \times Y_{03} + C_{12} \times Y_{101} + C_c \times 0.5 \times a + T_{1314} \times Y_{013} + T_{1516} \times Y_{015} + T_{34} \times Y_{103}] \tag{5}$$

and if $a > t_p$

$$M_{uz} = 0.9\,[C_{102} \times Y_{01} + C_{304} \times Y_{03} + C_{12} \times Y_{101} + + C_{cf} \times t_p \times 0.5 + C_{cw} \times (a+t_p) \times 0.5 + T_{1314} \times Y_{013} + T_{1516} \times Y_{015} + T_{34} \times Y_{103}] \tag{6}$$

(iii) Values of P_{bz} and M_{bz} $(+V_e)$

In computing the values of + M_{uz} we had to find the values, through an iterative process of successive increasement of a value until internal forces balance. Now we can ask the computer to either pick up or jump to the value when $a = a_b$ and get the value of all internal forces developed, namely C_{102}, C_{304}, C_{12}, T_{1314}, T_{1516}, T_{34} and C_c.

Then

$$P_{bz(+V_e)} = 0.7[C_{102} + C_{304} + C_{12} + T_{1314} + T_{1516} + T_{34} + C_c] \tag{7}$$

and, if $a \leq t_p$

$$M_{bz(+V_e)} = 0.7[C_{102} \times Y_{01} + C_{304} \times Y_{03} + C_{12} \times Y_{101} + C_c \times 0.5 \times a + T_{1314} \times Y_{013} + T_{1516} \times Y_{015} + T_{34} \times Y_{103}] \tag{8}$$

If $a > t_p$

$$M_{bz}(+V_e) = 0.7[C_{102} \times Y_{01} + C_{304} \times Y_{03} + C_{12} \times Y_{101} + C_{cf} \times t_p \times 0.5 + C_{cw} \times (a+t_p) \times 0.5 + T_{1314} \times Y_{013} + T_{1516} \times Y_{015} + T_{34} \times Y_{103}] \quad (9)$$

(iv) Values of $+M_{uy}$, P_b and M_{by}–

These values can be computed on the basis of the same principle as described before. Only the difference with the equations will be longer because of unsymmetric nature of the section.

4) ULTIMATE STRENGTH CHECK

Once the critical values are found out, the interaction diagrams can be drawn for the given section. Then the M_{uza} and M_{uya} values can be computed by getting the P_{ud} value.

M_{uza} Value

If $P_{ud} \leq P_{bz}$

$$M_{uza} = M_{uz} + (M_{bz} - M_{uz}) \times P_{ud}/P_{bz} \quad (10)$$

and if $P_{ud} > P_{bz}$

$$M_{uza} = M_{bz} - M_{bz} \times (P_{ud} - P_{bz})/(P_u - P_{bz}) \quad (11)$$

Similarly, M_{uya} values can be computed from the other interaction diagram.

Then, check whether

$$\left(\frac{M_{uzd}}{M_{uza}}\right)^{1.5} + \left(\frac{M_{uyd}}{M_{uya}}\right)^{1.5} \leq 1.0$$

If so, then the section is acceptable.

5) CONCLUSION

(a) A method is outlined to check the ultimate strength of a prestressed concrete column under axial load and biaxial bending. The method is suitable for computer adoption. With this method any shape with any kind of distribution of prestressed and non-prestressed reinforcements can be handled.

(b) The section of the column used for example, is a hollow trapezoidal box section. It is assumed as a tied column. A fifteen percentage reduction in strength is used for tied columns.

(c) It is assumed as a short column. The effect of instability is not considered. Reference No. 4 may be followed for that purpose.

6) NOTATIONS

a_b = depth of equivalent rectangular stress block kl.d (in).

A_s = $\Sigma(A_{s1} + A_{s2} + A_{s3} + A_{s4})$ - total prestressing reinforcing area (in^2).

A_{so} = $\Sigma(A_{s01} + A_{s02} \ldots A_{s016}))$- total non-prestressing reinforcing area(in^2).

A_{s1} to A_{s4} = prestressing reinforcing area (in^2).

A_{s01} to A_{s016} = non-prestressing reinforcing area (in^2).

d = effective depth of column section (in).

D = overall depth of column section (in).

E_c = modulus of elasticity of concrete (psi).

E_s = modulus of elasticity of steel (psi).

f'_c = 28 day compressive cylinder strength of concrete (psi).

f_{pu} = ultimate strength of prestressing steel (psi).

f_y = yield strength of non-prestressed reinforcement (psi).

M_{bz} or M_{by} = ultimate flexure capacity of the section at the balanced point with bending about z or y-axis respectively.

M_{uz} or M_{uy} = ultimate capacity of the section in pure bending about z or y-axis respectively.

M_{uza} or M_{uya} = ultimate M_z or M_y allowable corresponding P_{ud}.

M_{uzd} or M_{uyd} = ultimate M_z or M_y developed corresponding P_{ud}.

P_{bz} or P_{by} = ultimate load on the section at balanced point with bending about z or y-axis respectively.

P_u = ultimate axial load capacity of the section

P_{ud} = ultimate axial load developed

t_G = thickness of wider flange (in).

t_p = thickness of smaller flange (in).

t_w = thickness of web (in).

y_b = depth of neutral axis from extreme fibre (in).

y_g = distance between extreme fibre of wider flange to centroidal axis (in).

y_p = distance between extreme fibre of smaller flange to centroidal axis (in).

Y_{01} to Y_{015} = distance of non-prestressing reinforcing from extreme fibre of smaller flange (in).

Y_{101} to Y_{103} = distance of prestressing reinforcing from extreme fibre of smaller flange (in).

Z_{01} to Z_{016} = distance of non-prestressing reinforcing from Y-Y axis (in).

Z_{101} to Z_{104} = distance of prestressing reinforcing from Y-Y axis (in).

ε_{cu} = strain of concrete at ultimate failure.

ε_{se} = initial strain of prestressing steel corresponding to effective force.

ε_{su} = prestressing steel strain at ultimate failure

ϕ = capacity reduction factor

7) REFERENCES

1. Zia, P., and Moreadith, F.L., Ultimate Load Capacity of Prestressed Concrete Columns, Journal of the American Concrete Institute, Vol. 63, No. 7, (July 1966),pp.767-788.

2. Zia, P., and Guillerno, E.C., Combined Bending and Axial Load in Prestressed Concrete Columns, Journal of Prestressed Concrete Institute, Vol. 12, No. 3, (June 1967), pp.52-59.

3. Anderson, R.A. and Moustafa, S.E., Ultimate Strength of Prestressed Concrete Piles and Columns, ACI Journal, (August 1970), pp.620-635.

4. PCI Committee on Prestressed Concrete Columns, Tentative Recommendations for the Design of Prestressed Concrete Columns, PCI Journal, (October 1968), pp. 12-21.

5. Al-Rawi, G.A., Circular Prestressed Concrete Columns Subject to Concentric Forces, Bending Moment and Torsion, Ph.D. Dissertation, The Univ. of Oklahoma, (1971), (University Microfilms,Ann Arbor, Mich.).

6. Zielinski, Z.A., Research and Design Method for Prestressed Columns - Research Report, North Carolina State University, Raleigh, (1966).

7. Zielinski, Z.A., Prefabrykowane dzwigary sprezone (Textbook - Prestressed Precast Girders 300 pages), Arkady, Warsaw, (1962) - Second Edition.

SUMMARY

This paper presents a computerized method of analysis of the ultimate strength of large size prestressed concrete columns of tubular section under different axial bending and torsional loadings.

Such a column may appear in the design practice of bridges, towers or stadiums. The analysis is carried out on examples of large "column-consoles" of Olympic Stadium in Montreal.

The safety of the columns is being analyzed, using interaction surfaces describing the ultimate strength under multi-axial different moments and force action.

RESUME

La contribution présente une méthode de calcul simple par ordinateur pour le contrôle de la charge ultime de grandes colonnes précontraintes en béton armé de section tubulaire sous différents cas de charge de flexion et de torsion.

De telles colonnes se trouvent, en pratique, dans les ponts, tours et stades. L'analyse traite des exemples de grandes "colonnes-consoles" du stade Olympique de Montréal.

La sécurité des colonnes est déterminée par l'emploi de surfaces d'interaction décrivant la résistance ultime sous différents moments et forces.

ZUSAMMENFASSUNG

Der Beitrag zeigt eine computerorientierte Methode zur Berechnung der Bruchlast grosser vorgespannter rohrförmiger Stahlbetonstützen unter verschiedenen Kombinationen von Normalkraft, Biegung und Torsion.

Solche Stützen finden sich in der Praxis bei Brücken, Türmen (Masten) und Stadien. Die Berechnung wird an Beispielen grosser Säulenkonsolen des Olympia-Stadion von Montreal gezeigt.

Die Sicherheit der Säulen wird dabei mit Hilfe von Interaktionsflächen untersucht, welche die Beziehungen zwischen den verschiedenen Bruchschnittkräften wiedergeben.

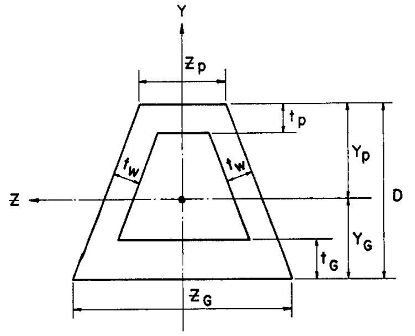


FIG. 1 DETAILS OF COLUMN CROSS-SECTION

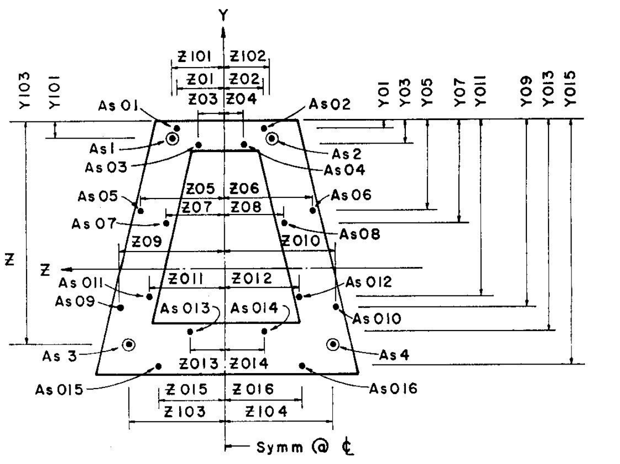


FIG. 2 REINFORCEMENT (PRESTRESSING & NON-PRESTRESSING) DETAILS

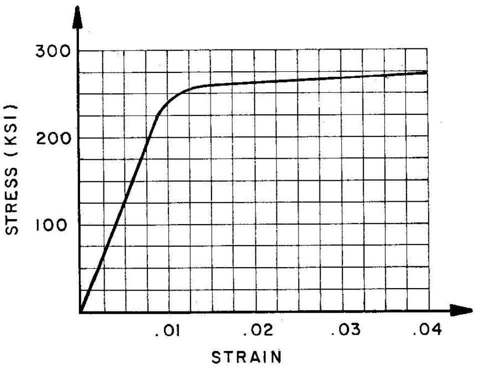


FIG. 3 (STRESS-STRAIN RELATIONSHIP FOR PRESTRESSING STEEL)

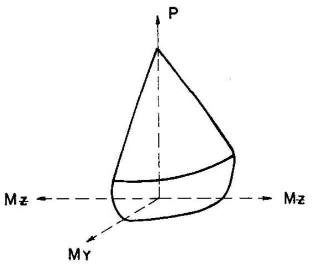


FIG. 4(a)

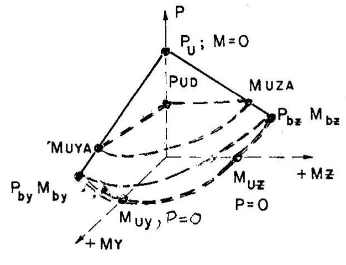


FIG. 4(b)

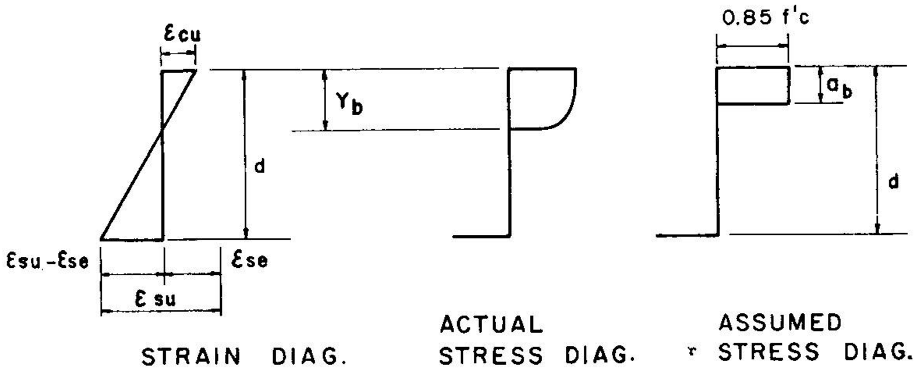
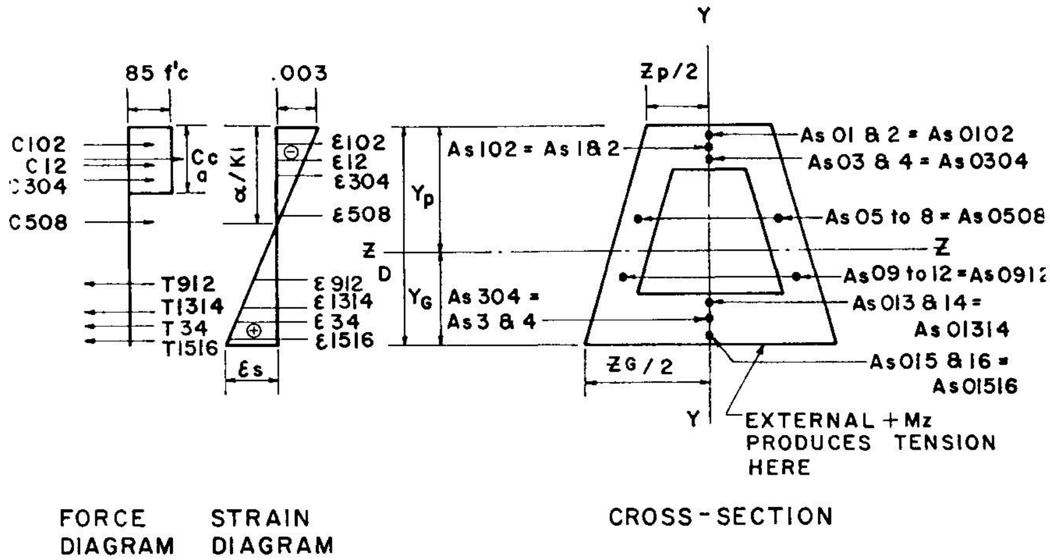


FIG. 5 STRESS & STRAIN DIAGRAMS AT FAILURE UNDER PURE FLEXURE AT BALANCED CONDITION



(For practicle purposes ignore As05 to As012)

FIG.6 VALUE OF +Muz

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