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The Application of Tri-axial Strength Theory to the Computation of Deformation and Limited Stress Values in Massive Concrete Structures

L'emploi de la théorie de la résistance triaxiale au calcul des valeurs des déformations et des contraintes limitées dans les structures massives en béton

Die Anwendung der Theorie der drei-axialen Festigkeit und der begrenzten Spannungswerte in massiven Betonwerken

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SUMMARY

The paper suggests a method of adjusting tri-axial strengths of concrete for long-term temperature and creep effects, and gives an example to illustrate how the stresses due to load, temperature variations and creep may be calculated in mass concrete structures.

J. B. Newman has produced a space envelope based on short-term cylinder tests at ambient temperature from which the axial strength of concrete cylinders subject to radial pressure may be determined.

P. J. E. Sullivan has studied experimentally the influence of high temperatures on uniaxial strength and rates of creep. It is important to extend these effects to triaxial conditions of load and restraint, such as exist at hot spots in pressure vessels. The paper suggests a method based on a tetrahedral frame as a model of the behaviour of concrete as a two-phase material.

RESUME

Ce rapport propose une méthode pour ajuster les résistances triaxiales du béton en fonction des effets des températures à long terme et du fluage. Il montre de quelle manière il est possible de calculer les contraintes dues à la charge, les variations de température et de fluage

en présence de structures massives en béton.

J.B. Newman a proposé une enveloppe spatiale fondée sur des essais de brève durée sur cylindres, à température ambiance, à partir desquels il est possible d' obtenir la résistance axiale de cylindres en béton, soumis à une pression radiale.

P.J.E. Sullivan a étudié expérimentalement l' influence des températures élevées sur la résistance monoaxiale et les vitesses de fluage. Il est important d' étendre ces effets à des conditions triaxiales de charge et de contenance telles qu' on les trouve sur les points chauds de un caisson. L' etude propose une méthode qui se base sur un cadre en tétraèdre prise comme modèle du comportement du béton comme matériau bi-phase.

ZUSAMMENFASSUNG

Der Artikel schlägt ein Verfahren zur Regulierung der dreiachsigen Spannungen des Betons vor, auf Grund der Wirkungen von langdauernden Temperaturen und Kriechen. Ferner bietet er ein Beispiel dafür, wie die Lastbeanspruchungen, Temperaturänderungen und Kriechen bei massiven Betonkonstruktionen berechnet werden können.

J.B. Newman hat eine räumliche Enveloppe erkannt, die sich auf kurzzeitlichen Versuchen auf Zylinder und bei Raumtemperaturen bezieht, womit es möglich ist, eine achsiale Spannung der unter radialen Druck stehenden Zylindern zu erhalten.

P.J.E. Sullivan hat den Einfluss von hohen Temperaturen auf die monoachsiale Festigkeit und auf das Kriechen experimentiert. Es ist wishtig, dass die Wirkungen unter der Bedingung einer dreiachsigen Last verbreitet werden, wie es an den warmen Stellen eines Behälters geschieht. Im Artikel wird ein Verfahren vorgeschkgen, das sich auf einem Tetraeder Rahmen als Modell des Verhaltens des Betons als zweiphasen - Material bezieht.

N O T A T I O N

PART 1

Ref. Figs. 1, 2 and 3

$\sigma_1, \sigma_2, \sigma_3$	Stress in directions 1, 2, 3.
σ_c	Ultimate short-term cylinder strength under uni - axial stress.
E_1, E_2, E_3	$\frac{\text{average stress}}{\text{average strain}}$ of a concrete element in direction 1,2,3 at any stage of load

$E_{\sqrt{2}}$ — $\frac{\text{average stress}}{\text{average strain}}$ of a concrete element in a diagonal direction through the point of contact of stones

E_c — $\frac{\text{average stress}}{\text{average strain}}$ at failure of a concrete cylinder under uni-axial short-term load

e — Strain

e_3 — Resultant strain in direction σ_3.

ν_1, ν_2 — Instantaneous "Poisson Ratio" in relation to stress in directions 1 and 2, giving secondary strain in direction 3 at any stage of load equal to

$$\frac{\text{overall normal strain of concrete element}}{\text{overall axial strain of concrete element}}.$$

ν_c — ν_1 at failure of concrete cylinder for short-term uni-axial load.

PART 2

M_o — Resultant bending moment at zero time.

k — Coefficient

p — Unit pressure

l — Span

N — Pre-stress force

H — Ring tension due to pressure

e — Eccentricity of pre-stress

M_{co} — Continuity moment at zero time

ϕ_o — Resultant rotation at zero time at a section

E — $\frac{\text{change of stress}}{\text{change of strain}}$ in an interval due to stress only

E_o — E value at zero time

D — Depth of section

f_o — Stress in any layer at any section at zero time.

y — Distance of layer from neutral axis

n_r — Depth of neutral axis at r^{th} interval

E_r — Value of E at r^{th} interval at T_r.

e_r	Total strain at r^{th} interval
e_{cr}	Creep in the r^{th} interval at T_r and f_r.
T_r	Temperature in the r^{th} interval.
ϕ_r	Change of rotation of any section in the r^{th} interval.
f_r	Stress change at any layer in any section at the r^{th} interval. $\bar{f}_r$ = resultant stress.
ϵ_T	Coefficient of thermal expansion

PART 1

The Influence of Temperature and Creep on Tri-axial Strength

The triaxial strength of concrete has been extensively studied and reported in ref. 1 for short-term loading at ambient temperature and, in refs. 2 and 3, the variation of short-term E and 7-day creep values with stress and temperature for uni-axial loading. The Influence of temperature on the uni-axial strength of concrete has been investigated and reported in ref. 4. The strength of concrete in such structures as pressure vessels is influenced by both the triaxial conditions of load, creep and temperature effects. It is, therefore, of interest to attempt to adjust tri-axial strengths derived from uni-axial values for these effects.

A designer, using advanced computer techniques, will then be able to include more realistic values of strength and deformation in the calculations.

In ref. 5 and Figs. (1), (2) and (3), concrete, as a two-phase material, is considered as approximating to spherical hard stones packed tightly together around softer mortar matrices, so that the centres of the stones of an element are positioned at the points of a double tetrahedron. The average E value of the material along a diagonal between the stones through the points where they nearly touch is high, and is only slightly influenced by the deformations of the thin coating of mortar between them. The average E value of the concrete in an axial direction, which includes the pocket of soft mortar between the stones, is low, and is considerably reduced at high stresses, and also in effect by a rise in temperature.

The load-deformation characteristics of such an element of concrete can be represented by a double tetrahedron framework of rods with tri-axial members (ref. Fig. (3)). The relative axial stiffness of the rods can be determined, so that the load-deformation characteristics of the concrete element are simulated. It is possible then (ref. 5) to show that the value of ν_c is related to $E_c / \sqrt{2}\, E_{\sqrt{2}}$ by the curve in Fig. 4. If it is assumed that $\sqrt{2}\, E_{\sqrt{2}}$ remains constant, because its value is not greatly altered by stress and temperature change, then ν_c may be related to E_c only.

Concrete under short-term uni-axial load at ambient tempera-

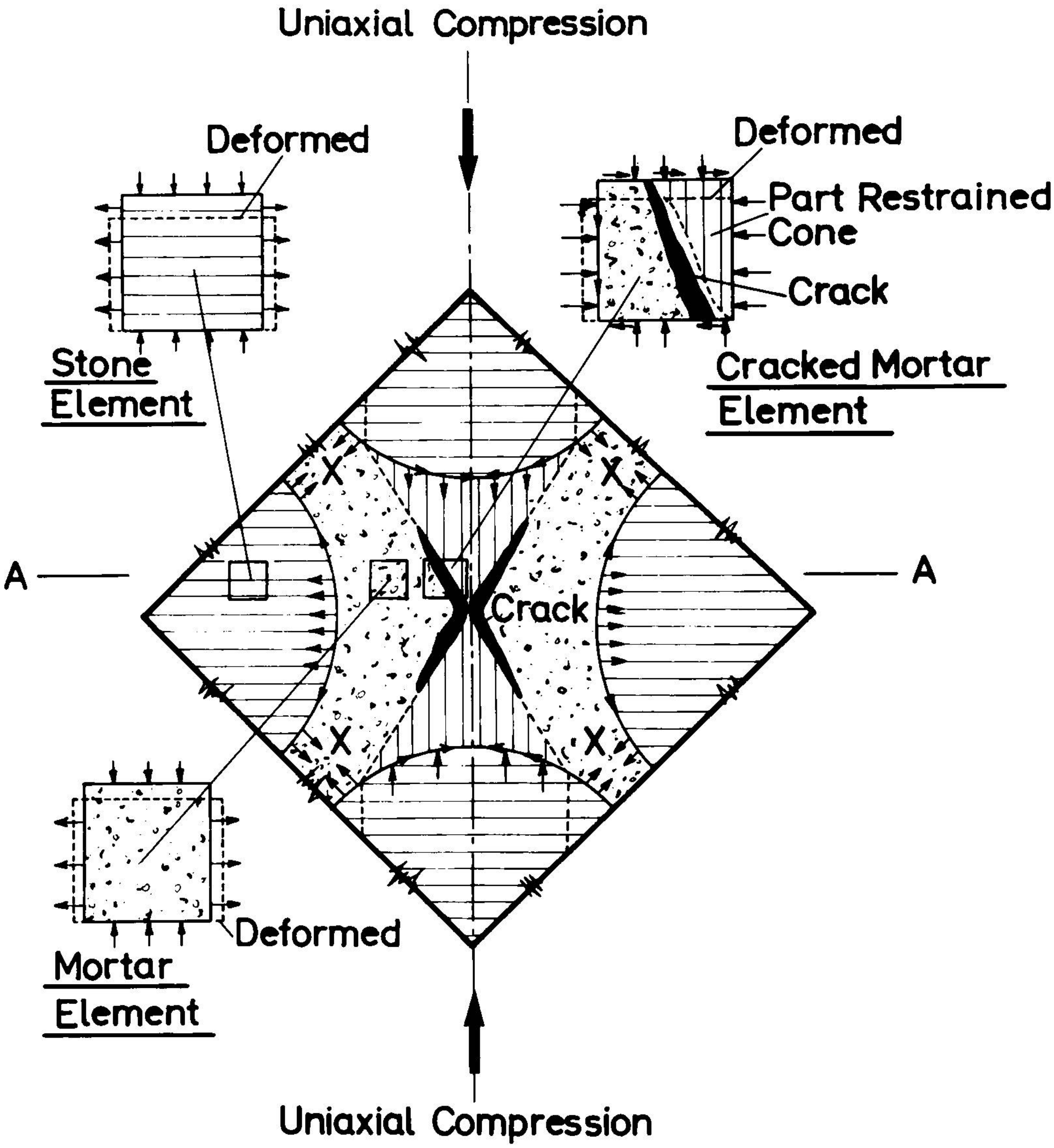

FIG. 1

Idealised concrete element - Elément idéalisé en béton - Idealisiertes Betonelement.

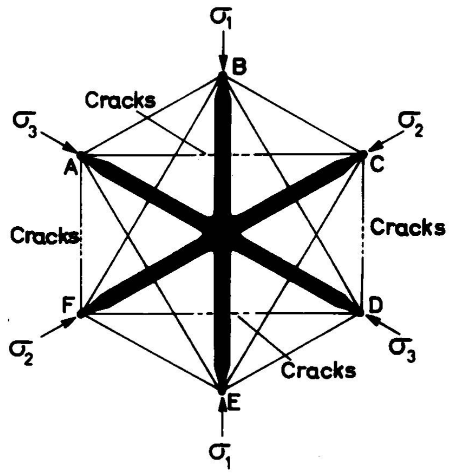

FIG. 3

Isometric of tetrahedral model - Isométriques du modèle tétrahédric - Isometrische Linien des Tetraheder- Modells.

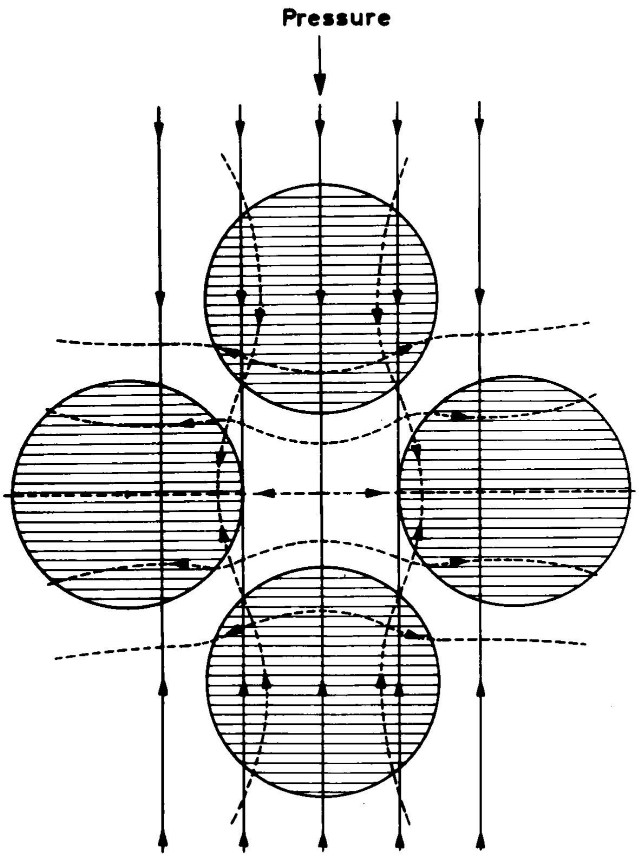

FIG. 2

Stress resultants - Résultantes des contraintes - Resultierende Spannungen

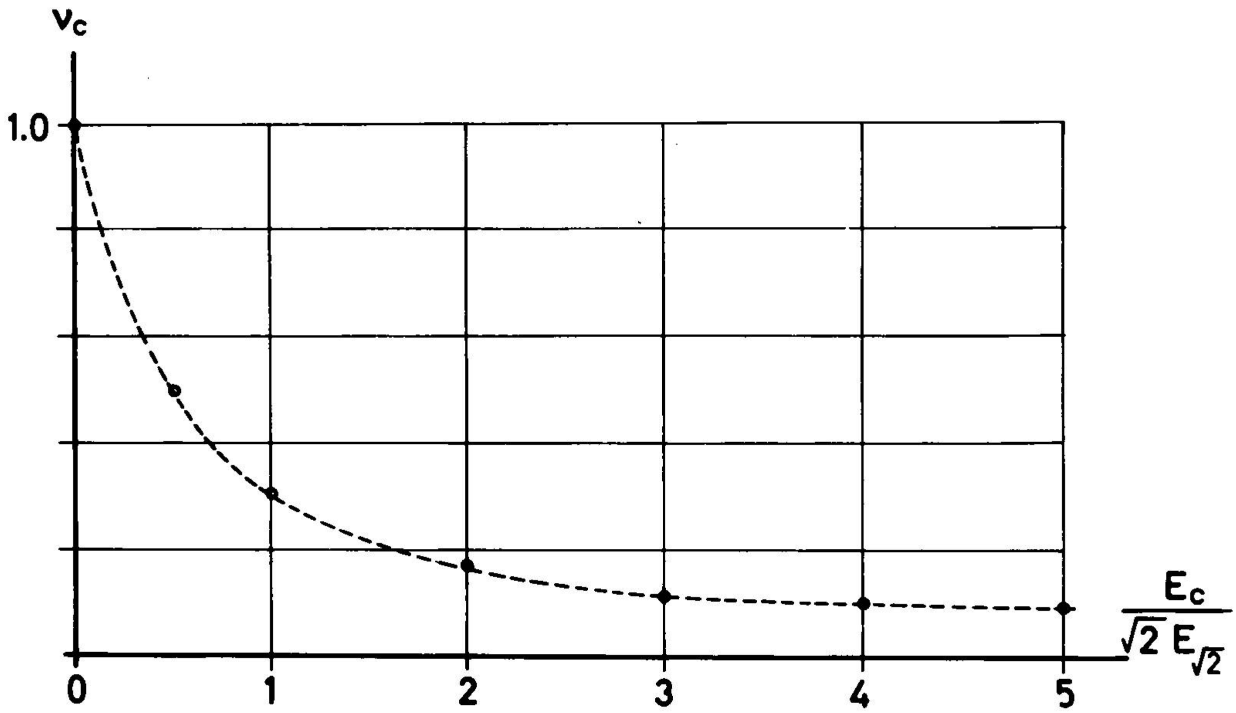

FIG. 4

Plot of ν_c against $E_c/\sqrt{2}\,E_{\sqrt{2}}$ - **Représentation** de ν_c en fonction de $E_c/\sqrt{2}\,E_{\sqrt{2}}$ - ν_c in Funktion des $E_c/\sqrt{2}\,E_{\sqrt{2}}$.

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ture fails at a compressive strain of σ_c/E_c. It can be assumed that failure is mainly due to micro-cracks extending to a condition of instability which occurs at an extension strain normal to σ_c of $(\sigma_c/E_c)\,\nu_c$. If it is assumed that, under triaxial load, instability occurs at a similar value of extension strain, this limiting strain can be adopted as a criterion of failure. Assuming the definitions of ν and E, given in the notation, the following classical equation can be established when $\sigma_1 > \sigma_2 > \sigma_3$:-

$$e_3 = \frac{\sigma_c}{E_c} \times \nu_c = \frac{\sigma_1 \nu_1}{E_1} + \frac{\sigma_2 \nu_2}{E_2} - \frac{\sigma_3}{E_3} \quad \dots\dots\dots\dots\dots\dots (1)$$

which transforms to:-

$$\frac{\sigma_1}{\sigma_c} = \frac{E_1}{\nu_1}\left(\frac{\nu_c}{E_c} + \frac{\sigma_3}{E_3\,\sigma_c} - \frac{\sigma_2\,\nu_2}{E_2\,\sigma_c}\right) \quad \dots\dots\dots\dots\dots\dots (2)$$

Assuming the following typical values $E_c = E_1 = E_2 = E_3 = 2 \times 10^6$; $\nu_c = \nu_1 = \nu_2 = 0.2$

$$\frac{\sigma_1}{\sigma_c} = 1 + 5\frac{\sigma_3}{\sigma_c} - \frac{\sigma_2}{\sigma_c} \qquad \text{.........................} \quad (3)$$

If $\sigma_2 = \sigma_3$

$$\frac{\sigma_1}{\sigma_c} = 1 + 4\frac{\sigma_2}{\sigma_c} \qquad \text{............................} \quad (4)$$

Equation (4) agrees with the relationship shown in Fig. 5, obtained by Newman (ref. 6) from a large number of tri-axial tests under a range of confining pressures. It appears, therefore, that the equations proposed for obtaining σ_1 and the diagram for ν values are reasonable, and are probably adequate for relating tri-axial strengths and deformations to uni-axial values obtained for various conditions of temperature and creep. It can be assumed that E values for **material** along a line which includes the full width of the mortar matrix are reduced

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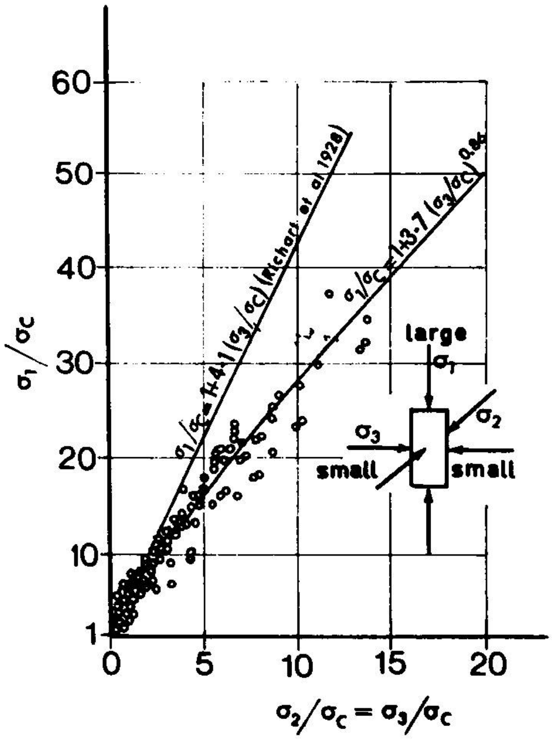

FIG. 5

$\frac{\sigma_1}{\sigma_c}$ plotted against $\frac{\sigma_2}{\sigma_c}$ (ref. 6) - $\frac{\sigma_1}{\sigma_c}$ en fonction de $\frac{\sigma_2}{\sigma_c}$ (ref. 6)

$\frac{\sigma_1}{\sigma_c}$ in Funktion des $\frac{\sigma_2}{\sigma_c}$ (ref. 6) by rises of temperature and stress, and therefore that ν values are increased. The concrete is also weakened by a rise in temperature, resulting in a reduction in tensile strength of the mortar matrix and the bond to the stones, as shown in Fig. 6.

Examples

(1) No temperature or creep:-

Assume σ_c = 6,000 p.s.i. (42 N/mm^2)
σ_2 = 1,000 P.S.I. (7 N/mm^2)
σ_3 = 300 p.s.i. (2.3 N/mm^2)

Substituting in equation (3):-

$$\frac{\sigma_1}{\sigma_c} = 1 + \frac{1500}{\sigma_c} - \frac{1000}{\sigma_c}$$

σ_1 = 6,500 p.s.i. (45 N/mm^2)

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(2) $\sigma_2 + \sigma_3$ as in (1), but temperature = 200°C, duration of creep 7 days at stress $0 \cdot 4\,\sigma_c$;

From Fig. 6 uni-axial strength = $0 \cdot 8\,\sigma_c$;
Fig. 7 strain doubles in 7 days - assume all E values are halved;
Fig. 4 since E values are halved, ν values increase 1.2 times.

Substituting in equation (2), adjusted values of E and ν :-

$$\frac{\sigma_1}{0 \cdot 8\,\sigma_c} = \frac{0 \cdot 5 \times 2}{1 \cdot 2 \times 0 \cdot 2}\left(\frac{1 \cdot 2 \times 0 \cdot 2}{0 \cdot 5 \times 2} + \frac{\sigma_3}{0 \cdot 5 \times 2\,\sigma_c} - \sigma_2\,\frac{1 \cdot 2 \times 0 \cdot 2}{0 \cdot 5 \times 2\,\sigma_c}\right) =$$

$$= 1 + 4.2\,\frac{\sigma_3}{\sigma_c} - \frac{\sigma_2}{\sigma_c}$$

If σ_c = 6,000 p.s.i. (42 N/mm^2) σ_3 = 300 p.s.i. σ_2 = 1,000 p.s.i.

σ_1 = 4,800 + 1,000 - 800 = 5,000 p.s.i. (35 N/mm^2)

which is a reduction of 23% due to temperature and creep. Equation (2) is very sensitive to changes in E values which, in actual structures, will vary for the same concrete mix. It would be wise, in the above example, to assume a possible 50% reduction in strength, but the calcu-

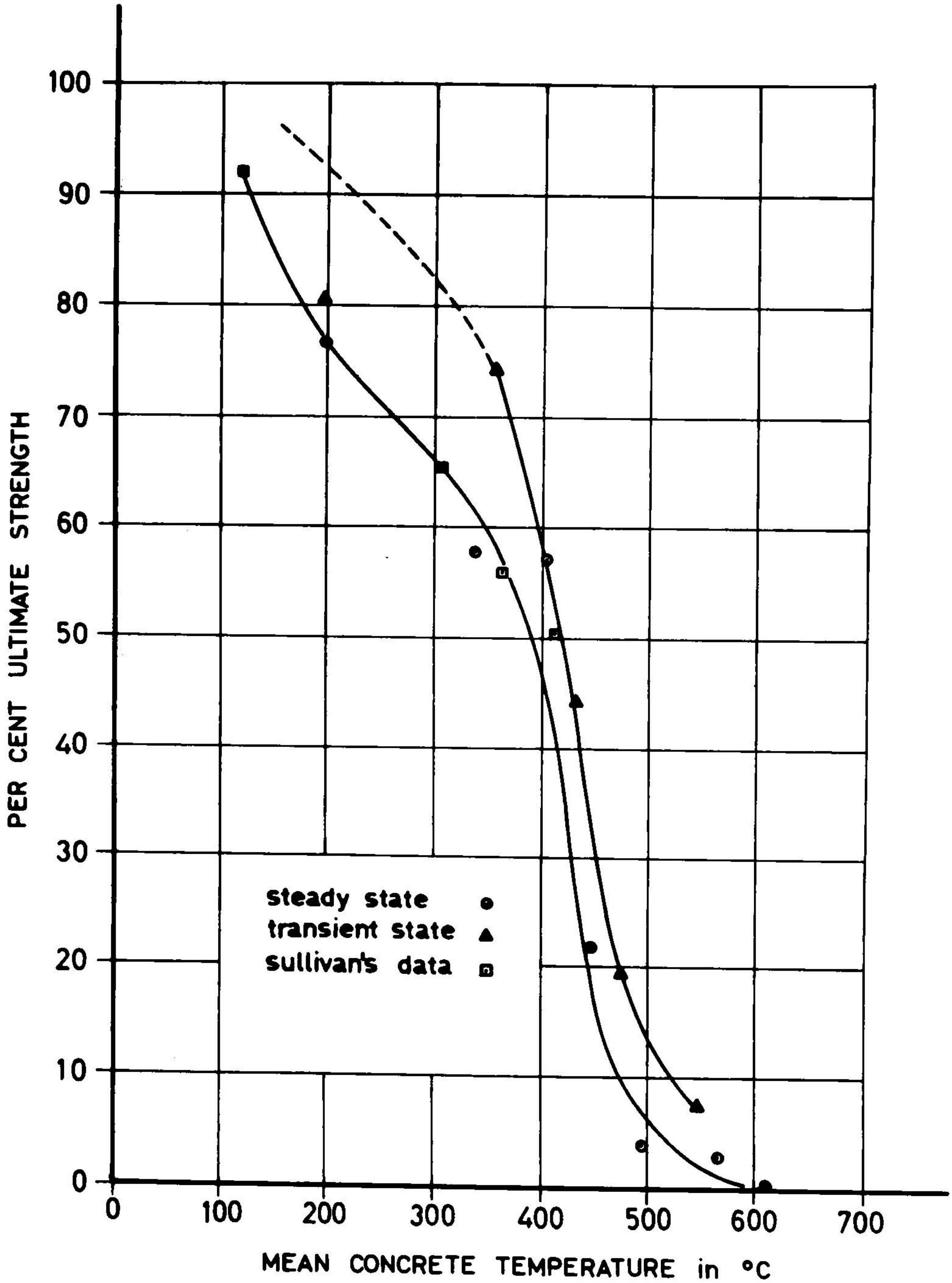

FIG. 6

Effect of mean concrete temperature on per cent ultimate strength of plain concrete specimens tested under transient (series II) and steady states (series III).

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Strength against temperature - Résistance en fonction de la températu re - Festigkeit in Funktion der Temperatur.

lated stress will also be considerably reduced by creep and the reduced values of E used in the stress calculations. The values of E_2 or E_3 will reduce less than the value of E_1 due to temperature and creep, since the values of σ_2 and σ_3 are less than σ_1. In the extreme, only the value of E_1 would be halved in equation (2), which would halve the value of σ_1. The results of Glücklich (ref. 7) indicate that the ultimate strength of unbound concrete is reduced by long-term creep.

PART 2

Example:

Calculation of σ_1 in an Octagonal Pressure Vessel

The following assumptions are made (ref. Figs. 8, 9):-

1. An octagonal strip of wall of unit depth at mid-height of a vessel is only restrained against bending by the continuity of the octagonal ring.
2. No serious error is made, if the strain distribution across the wall is assumed to be linear.
3. The distribution of temperature along each side of the octagon is indentical and symmetrical.
4. H acts concentrically at zero time.
5. The increase of diameter of the octagon produces no bending stress.

Calculations of stress and changes of stress are carried out for successive time intervals O to r. At zero time, the temperature is assumed to be uniform and ambient, so that only pressure and pre-stress stress the vessel. The stresses calculated for zero time due to pressure and pre-compression are assumed to remain constant.

The change of stress in each layer at each section of a side of the octagon for each successive time interval following zero time is due to:-

(a) Creep during the interval at T_r;

(b) Change of length due to change from T_{r-1} to T_r;

(c) Change of rotation due to restraints to a linear distribution of strain across sections and to zero total rotation along a length 1/2. The value of E for the r^{th} interval is assumed to be the value at the end of the $r-1^{th}$ interval, adjusted for T_r if greater accuracy is required. e_{cr} is the creep for the r^{th} interval at $\bar{f}_{r-1}$ and T_r.

At each interval r, values of e_c for T_r and $\bar{f}_{r-1}$, and E_r for $\bar{f}_{r-1}$ and T_r may be found from Fig. 7. If great accuracy is required, the time intervals must be short, and compatibility must be established at each interval by trial and adjustement between values of E_r and e_{cr}, and $\bar{f}_r$ and T_r.

The side of the octagonal ring is assumed to have unit thickness.

<u>Zero Time</u> (ref. Figs. 8 and 9):-

$$M_o = k\,pl^2 - N\,l - M_{co} \qquad \ldots\ldots\ldots\ldots (1)$$

$$\phi_o = \frac{12\ M_o}{E_o\ D^3} \qquad \ldots\ldots\ldots\ldots (2)$$

$$\int_o^{1/2} \phi_o\, dx = O \qquad \ldots\ldots\ldots\ldots (3)$$

$$f_o = \phi_o\, y\, E_o + \frac{N - H}{D} \qquad \ldots\ldots\ldots\ldots (4)$$

Equations (1), (2) and (3) give M_{co} M_o and ϕ_o.

<u>At r^{th} interval</u> (ref. Figs. 10 and 11).

<u>Equating total compression and tension</u>

$$\sum_o^{n_r} E_{r-1} (\phi_r\, y - e_{cr} + (T_r - T_{r-1})\ \varepsilon_T)\ \delta y\ \delta x$$

$$= \sum_o^{1 - n_r} E_{r-1} (\phi_r\, y - e_{cr} + (T_r - T_{r-1})\ \varepsilon_T)\ \delta y\ \delta x \ldots (5)$$

$$\sum_o^{1/2} \phi_r\ \delta_x = O \qquad \ldots\ldots\ldots\ldots (6)$$

$$f_r = E_{r-1} (\phi_r\ y - e_{cr} + (T_r - T_{r-1})\ \varepsilon_T) \qquad \ldots\ldots (7)$$

Equations (5) and (6) give n_r and ϕ_r.

Resultant stress at any section at r^{th} interval = $\underline{f_o + \sum_1^r f_r}$

<u>Shear</u>

The unit shear at any layer section may be calculated as an average over a length δx. The unit shear equals the difference over the length δx of the total longitudinal stress between the layer and the outside of the section divided by δx.

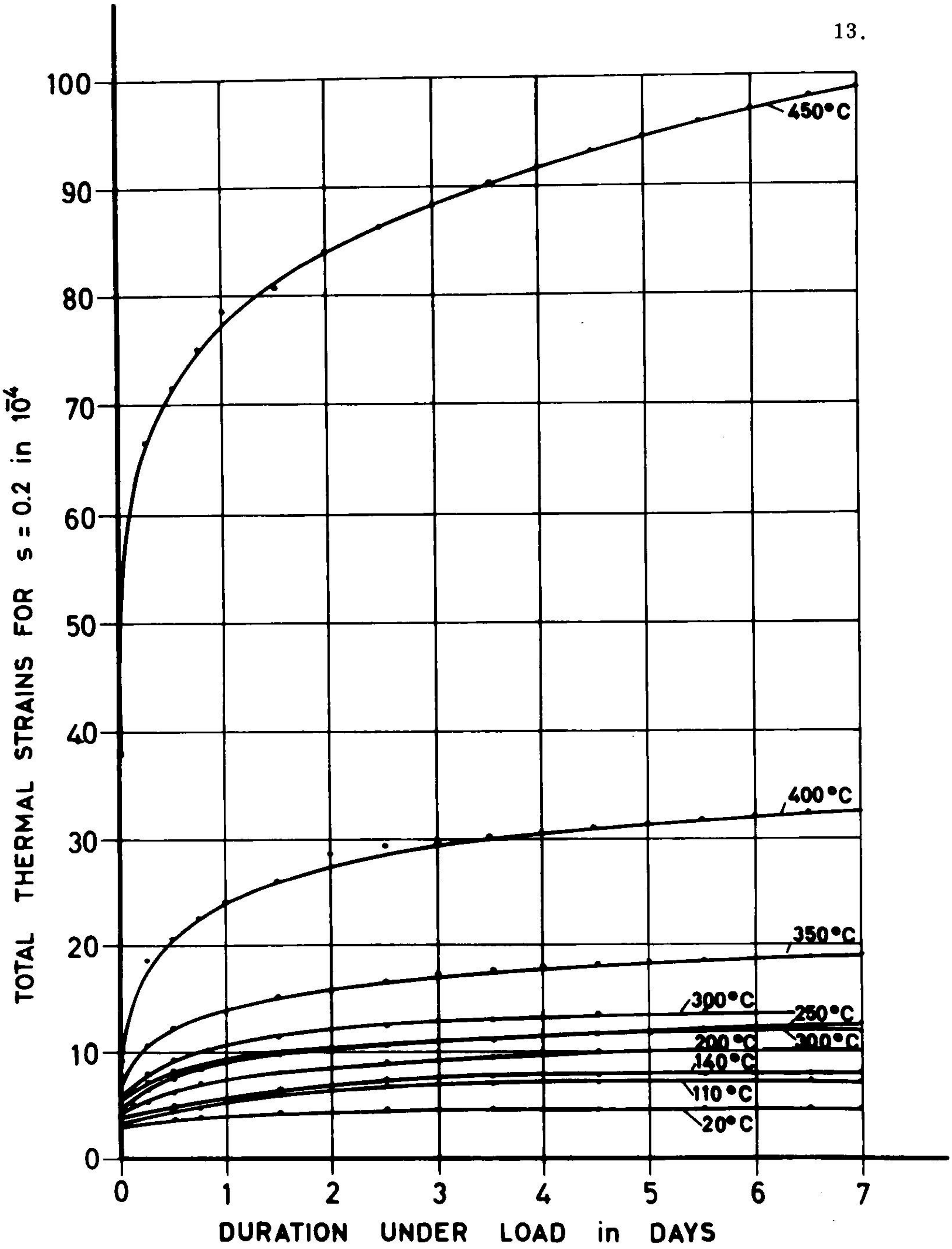

FIG. 7 A

High temperature creep for a stress / cold strength ratio of 0, 2 . Ref. 3 other stress ratios.

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Creep against temperature - Fluage en fonction de la température - Kriechen in Funktion der Temperatur.

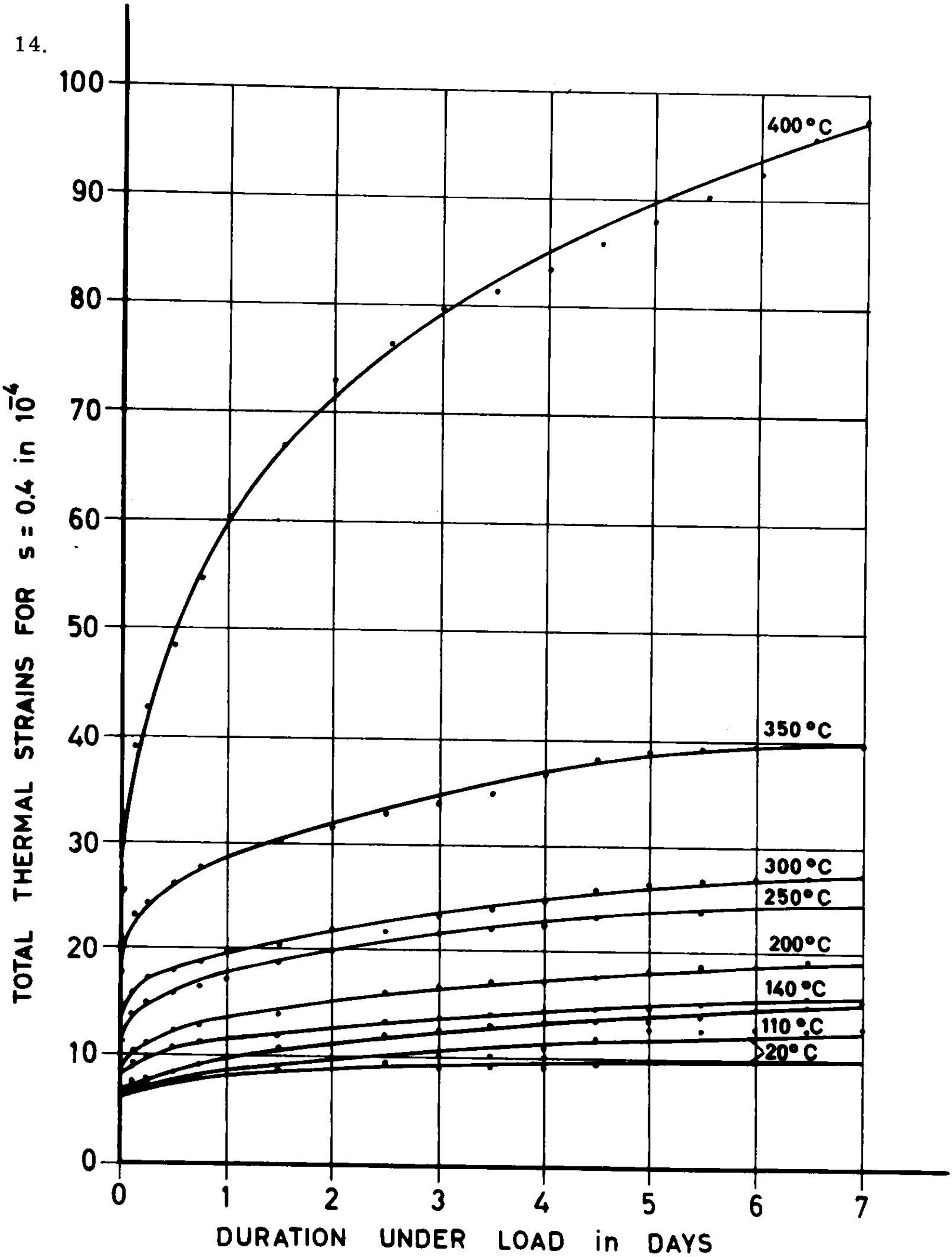

FIG. 7 B

High temperature creep for a stress / cold strength ratio of 0. 4 . Ref. 3 other stress ratios.

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Creep against temperature - Fluage en fonction de la température - Kriechen in Funktion der Temperatur.

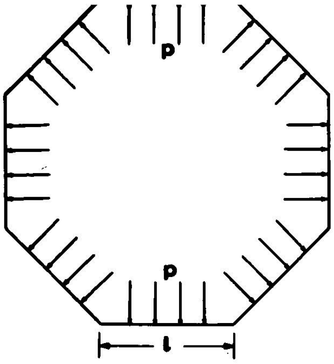

FIG. 8

Octagonal pressure vessel - Caisson octagonal en pression - Oktagonaler Druckbehalter.

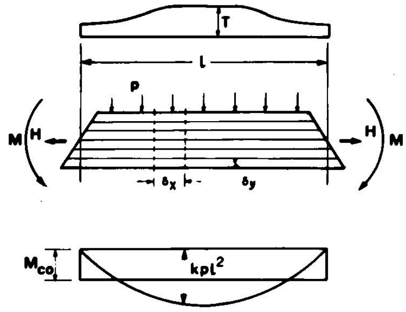

Fig. 9

Temperature and bending moment distribution - Distribution de la température et des moments fléchissants - Verteilung der Temperatur und des Biegemomentes.

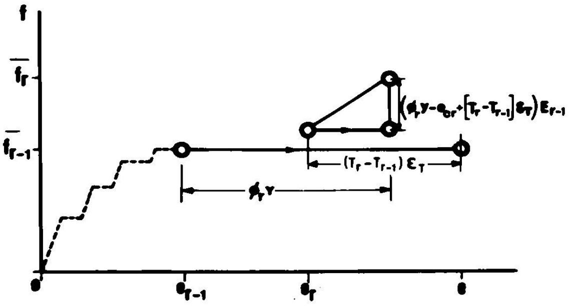

FIG. 10

Stress change at r^{th} interval - Changement de contrainte à l' interval r-ème - Spannungsänderung bei dem r-ten Intervall.

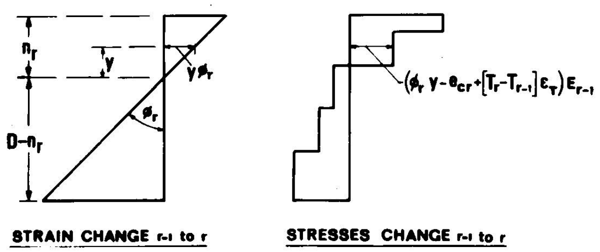

FIG. 11

Rotation at r^{th} interval - Rotation de contrainte à l' interval r-ème - Rotation bei dem r-ten Intervall.

ACKNOWLEDGEMENT

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