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THEORETICAL INVESTIGATIONS OF COLUMNS UNDER BIAXIAL LOADING

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ABSTRACT

Columns in frame structures are often loaded eccentrically with respect to both principal axis. Under load the axis displace and twist themselves. The displacement and the twisting are depending on each other. The result is a space curvature.

The present numerical investigations are based on approximative calculations. In doing so, the moment curvature is improved by an iteration method. In every step of the iteration the plastic behaviour of each part of the column is taken into consideration. The influence of the torsional rigidity is considered as well as that of the residual stresses. It was assumed a parabolic figure of the residual stresses in the flanges and the web of the cross-section. The geometrical imperfections are considered as an eccentricity of the point of application of the compressive forces.

Several numerical values of the geometrical imperfections, of the residual stresses and of the slenderness ratio has been investigated with regard to columns of H-shape. It was possible to put through the extensive calculations only by using a computer. The procedures were programmed for a computer with large storage capacity, type S 4004/45. The various influences are shown in diagrams. The load-carrying capacity of the columns are compared to each other.

1. Introduction

Many extensive papers of the Commission 8 of the European Convention for Constructional Steelwork are in hand concerning compressed hinged columns [1] .

The geometrical imperfection exists in the direction of one principal axis. Other kinds of imperfections are considered, such as residual stresses concerning several types of cross-section.

Columns in frame structures are sometimes loaded eccentrically with respect to both principal axis. Under load the axis displace and twist themselves. The displacement and the twisting are depending on each other. The result is a space curvature.

In the following paper you will see some numerical results. They shall help to decide, if there is a possibility to compute a biaxial loaded column in a similiar way as a column excentrically loaded with respect to one principal axis.

2. Theoretical basis

The present investigations are based on approximative calculations. The whole of the mathematical model is given in [2].

In the cases, which are here calculated, the axis of the beam may be straight. But it is also possible that there is an initial imperfection such as a parabolic curve.

The stress-strain relationship of the material is elastic-perfectly plastic.The effects of strain-hardening and shear on the yielding are neglected.

In the given model the potential energy is used in regard to the theory of deformation. The cross-section is bisymmetrical, see equation (1) and figure 1.

$$\Pi = \frac{1}{2}\int_{x=0}^{L} \Big\{ GJ_D\,\vartheta'^2 + EF_{ww}\,\vartheta''^2 + EF_{zz}\cdot\zeta''^2 + EF_{yy}\,\eta''^2 + N\left(\eta'^2 + \zeta'^2 + i_p^2\cdot\vartheta'^2 + 2e_z\cdot\eta''\vartheta - 2e_y\,\zeta''\vartheta\right) - 2q_z\,\zeta \Big\}\,dx \qquad (1)$$

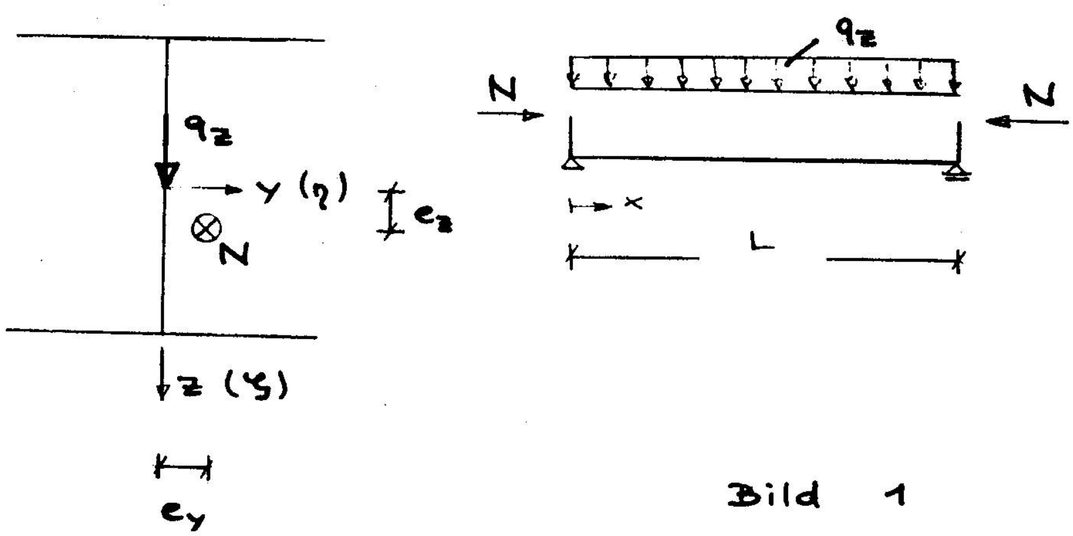

Bild 1

To explain the symbols:

GJ_D	torsional rigidity of St. Venant
EF_{ww}	warping rigidity
EF_{zz}	rigidity about Y-axis
EF_{yy}	rigidity about z-axis
i_p^2	$(F_{yy} + F_{zz})/F$
N	axial load
ϑ	twisting
F	area of cross-section

The soluction of Eq. (1) is based on assuming poynomial expressions for the displacements η and ζ and the twisting ϑ. We call these polynomial expressions "Hermitesche Interpolationspolynome", $H_j(x)$, shown in Eq. (2.)

$$\begin{aligned} \vartheta &= \sum_{i=1}^{9} a_i \sum_{j=1}^{8} H_j(x) \\ \eta &= \sum_{i=1}^{9} b_i \sum_{j=1}^{8} H_j(x) \\ \zeta &= \sum_{i=1}^{9} c_i \sum_{j=1}^{8} H_j(x) \end{aligned} \qquad (2)$$

We will find the bending moments and the twisting moment by the solution of the Eq. (1) by the method of Ritz. After that we will calculate the stresses at any point of the beam as usual. Residual stresses can be added. If we have stresses in some part of the beam, which are greater than the yield stresses, so the corresponding cross-sections have been partly plasticized.

Following that the real curvatures η'', ζ'', ϑ'' are defined by a "cross-section-iteration".
The plastification is considered by using the idealized stress-strain relation. By a further iteration we get the real displacements η, ζ, ϑ by numerical integration regarding every station along the member.

From this we find the new bending moments M_y, M_z.
With these improved bending moments the "cross-section-iteration" will be repeated. Both of the iterations are put through as long as there are no more modifications in the displacements.

The ultimate load is reached, as soon as the load-deflection curve do not increase anymore, see fig. 2

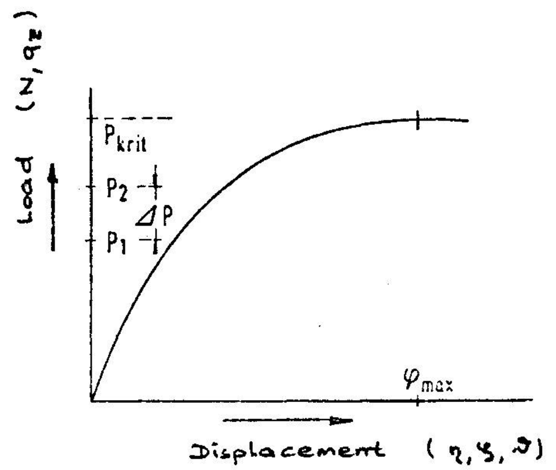

fig. 2

It was possible to put through the extensive calculations only by using a computer. The procedures were programmed for a computer with large storage capacity, type S 4004/45.

3. Numerical results

We will recognize some influences to the ultimate load of biaxial compressed solumns in fig. 3 to 7. All results are presented on a relation between the relative slenderness ratio $\bar{\lambda}$ and the relative axial force $\bar{N}$. Though they are without dimension.

The influence of the type of the residual stresses are shown in fig. 3 and 4. In fig. 3 we assume that the values of the eccentricities e_y and e_z may be one thousandth of the length of bar, (L/1000). There are no residual stresses considered in curve no. 1. In curve no. 2 we assume the greatest residual stress of compression by using a value of 0,3 $\cdot$ σ_F. In curve no. 3 we assume a compress stress 0,5 $\cdot$ σ_F. The same curve is also available if the value of compression stress is only 0,3 $\cdot$ σ_F, under the condition that the stress is zero at the point of the connection between the flange and the web.

You can see that in the field of the middle relative slenderness ratio $\bar{\lambda}$ ($\bar{\lambda} \sim$ 0,8) the reduction of the ultimate load will reach 20 percent.

In fig. 4 you see two corresponding curves. The eccentricities are assumed as e_y = L/2000 and e_z = L/1000. In this case too we get a very large reduction in the field of the middle relative slenderness ratio $\bar{\lambda}$($\bar{\lambda} \sim$ 0,8). We will notice a reduction up to 12 percent.

The effect of various eccentricities has been investigated in fig. no. 5. The curve no. 1 is valid for e_z = L/1000, e_y = 0, curve no. 2 for e_z = L/1000, e_y = L/2000. By com-

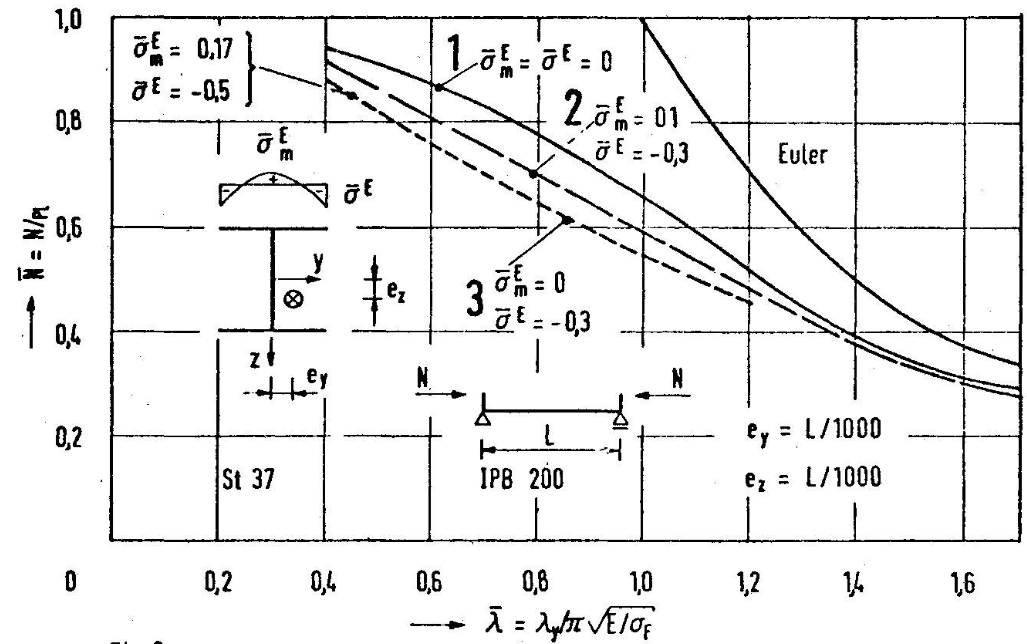

Fig. 3

Bixial loaded column influence of residual stresses

$e_y = L/1000 \quad e_z = L/1000$

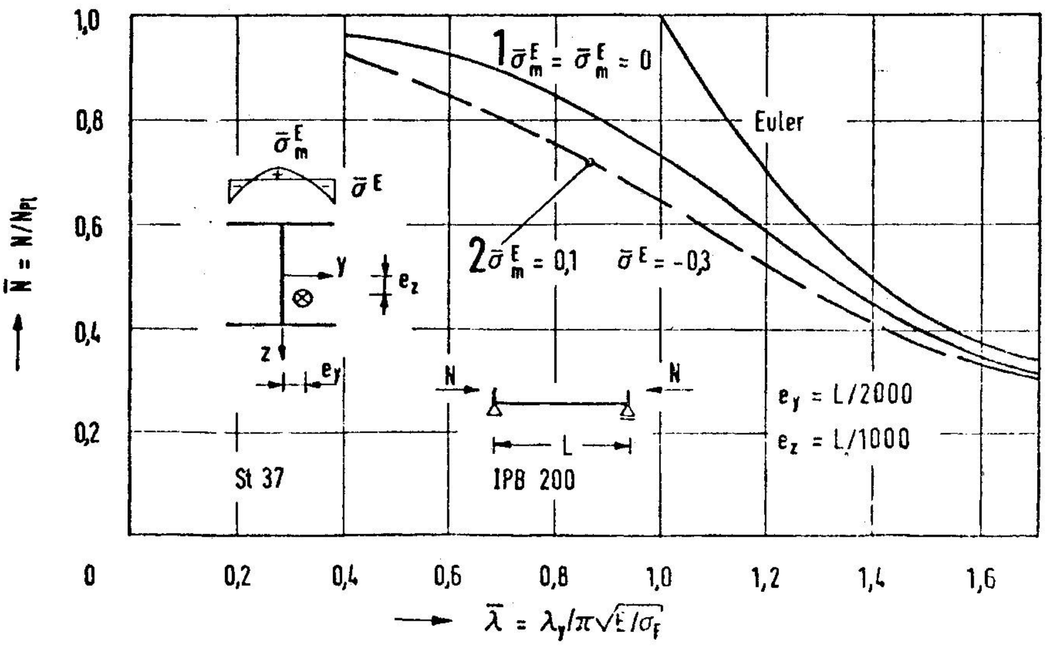

Fig. 4

Bixial loaded column influence of residual stresses

$e_y = L/2000 \quad e_z = L/1000$

Bg. 13 AK 23

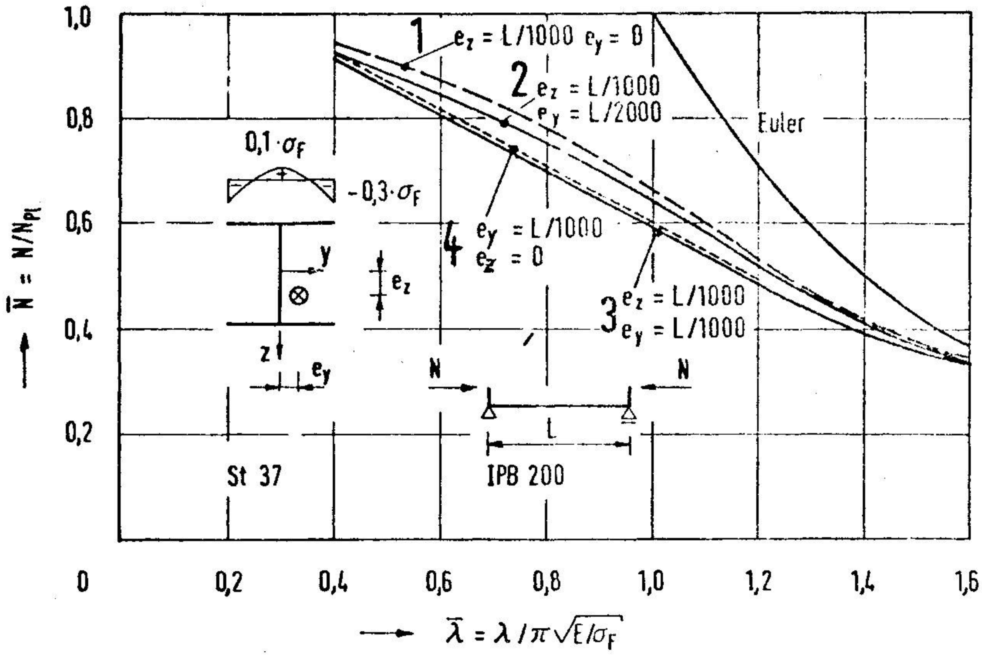

Fig. 5
Bixial loaded column influence of eccentricities

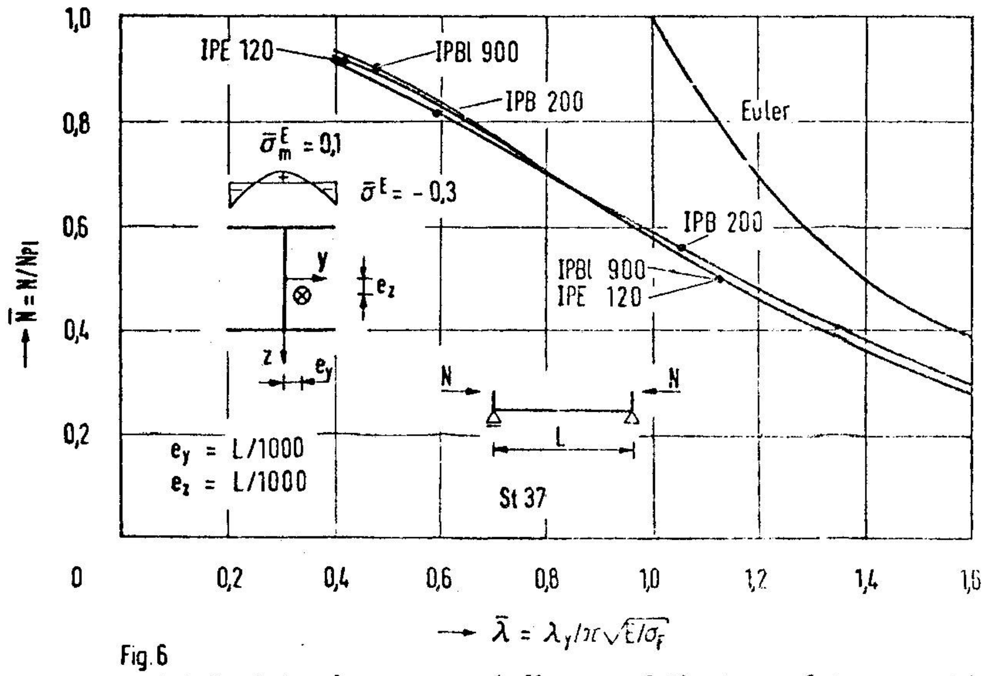

Fig. 6
Bixial loaded column influence of the type of cross-section

paring these curves we can notice a reduction of nearly 5 percent. If the eccentricity e_y increases to $e_y = L/1000$, an additional reduction is found of nearly 8 percent. (curve no. 3).

If you compare curve no. 3 to curve no. 4 you see that there is only a very small difference.

The influence of three kinds of rolled shapes is shown in fig. no. 6. There are investigated three types of characteristic sections:

- IPE 120 (small flange)
- IPB 200 (wide flange)
- IPBl 900 (high web).

The greatest values of the residual stresses are the same once in each flange. Under this condition the greatest differences amount to 5 percent.

In fig. no. 7 results of calculations are shown, where the column is loaded not only by axial forces N but also by loads q_z.

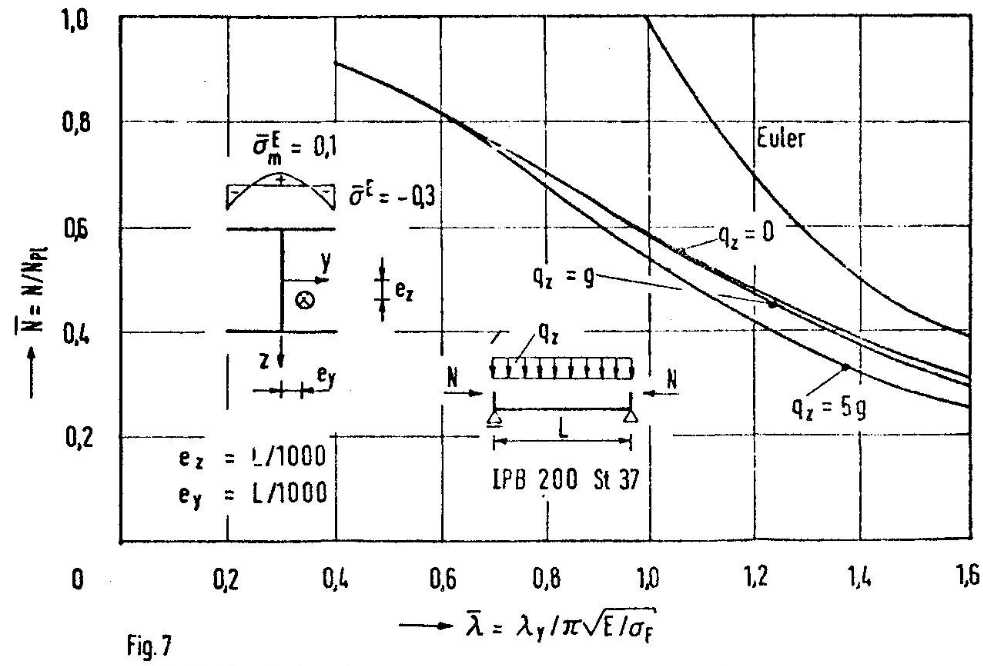

Fig. 7

Bixial loaded column influence of loads q_z

If these loads are small and if they are as great as the dead weight g, so the influence is unimportant. From $\bar{\lambda} = 0,7$ an important reduction to the ultimate load takes place if the load q_z reaches the value of 5 g. The reduction amounts to 20 %.

4. Conclusion

By the results of part 3 we can see, that the curves of the ultimate load of biaxial loaded columns are similiar to those of uniaxial loaded ones. With the assumed eccentricities the absolut values of the ultimate loads however decrease up to 10 percent. The influence of the type of cross-section is very small under the assumed residual stresses, the influence of the loads q_z has to be considered in all cases of great length of member-

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