Centrally compressed built-up struts

CENTRALLY COMPRESSED BUILT-UP STRUTS

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ABSTRACT

This paper presents the results of both an experimental and theoretical research on built-up compact struts.

Channels and unequal angles back to back are considered with different slendernesses and type of connectors.

The experimental results are compared with the ones obtained using the C.E.C.M. buckling curves for:

1) welded connectors
2) tightened bolted connections
3) untightened bolted connections
4) hotgalvanized or painted elements.

A numerical approach allowing for elastic unloading processes is finally presented.

1. Introductory Remarks

So far as the authors know, built-up struts are still designed with the theory of elastic equilibrium bifurcation for the fasterners as well as the whole strut (fig.1).

It is normally assumed that subjected to the critical load the equilibrium configuration and, in particular, the deflection f_{gc} that characterises overall collapse, will be indeterminate. In this case, the fasteners (e.g. batten plates) are designed (fig.2) for a deflection f_{1c} that will provoke the local failure of the most compressed of the chords. This means that initial out-of streightness in the axis and the load eccentricities will have no influence, nor will the transversally distributed loads (dead load, wind etc).

For simple struts, however, this concept was given up about twenty years ago, and replaced by another, which follows the behaviour of the strut step by step as the loads increase, taking into account realistic values of the geometrical and mechanical imperfections as well as real transversal loads.

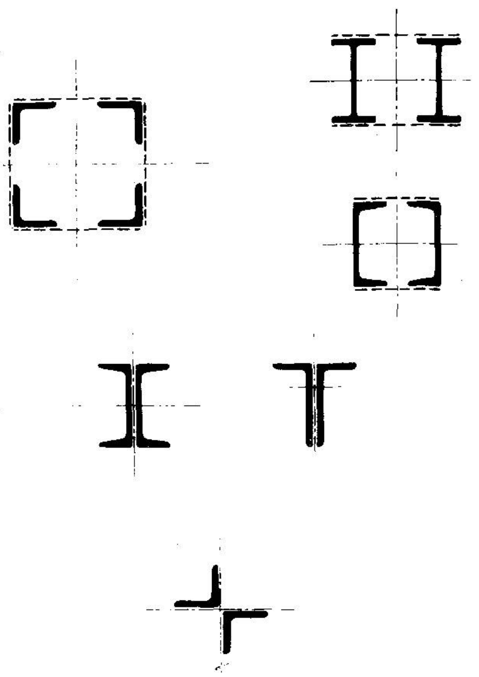


FIG.1

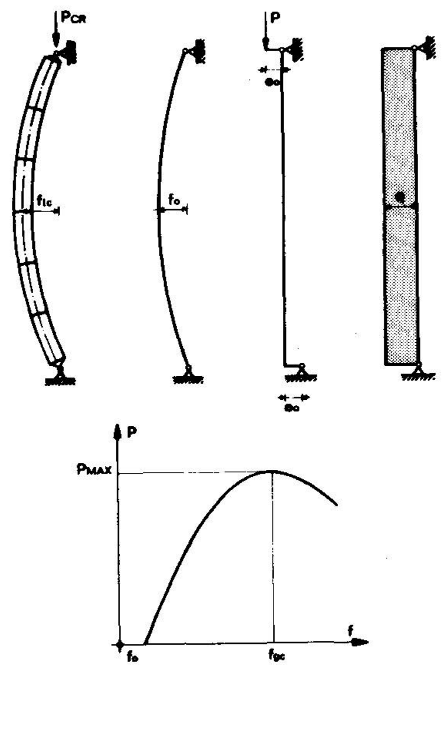


FIG.2

This leads to a curve P=P (f) which is characterised by a well defined maximum, and therefore by a value f_{gc} of the deflection which characterises overall collapse for that maximum.

Both P_{MAX} and f_{gc} are greatly influenced by a number of factors that are present during loading. They are: mechanical characteristics (residual stresses σ_0) geometrical imperfections (the initial out-of-straightness of the axis f_o and load eccentricity e_o) loads q distributed along the axis of the strut (forces linked to the volume or the surface - dead weight, wind, dynamic forces).

So it may be said that:

$$f_{gc} = f_{gc}(P_{MAX}, \sigma_o, e_o, f_o, q, \bar{\sigma})$$

where σ_o, f_o, e_o and q must be worked out beforehand, on a statistical basis as well as the yield point $\bar{\sigma}$.

The load P_{MAX} is less than that calculated without σ_o, f_o, e_o and q but being much more realistic, a safety factor may be adopted for these axially loaded struts that is the same as for tensioned bars. To sum up this new concept, then overall collapse deflection is no longer indeterminate, and can in fact be worked out quantitatively by a clear calculation process.

This kind of approach, when applied to simple struts of different cross section has lead to the definition of the European Curves $\sigma = \sigma(\lambda)$. If reference is now made in particular to built-up struts it will be seen that there need be no guarantee that the fasteners along the strut remain efficient until, between one fastener and the next, the failure of one of the component struts. All that is required now is that neither of the following conditions arises separately:

a) failure of a component strut between one fastener and the next when $f < f_{gc}$,

b) failure of a fastener along the strut when $f < f_{gc}$.

This represents a different way of looking at the situation. The design of the fastener no longer depends on the local design of the component strut, but both depend on the overall behaviour of the structure.

This overall behaviour has only been studied within the limits of the theory of bifurcation.

Other more worthwile approaches are being looked into, but this, of course, is not easy.

It is particularly unrealistic to use calculation methods that do not take into account the unloading processes during lateral buckling of the strut. The component strut furthest from the original line of the axis may even become tensioned rather than compressed.

2. Experimental Results

The behaviour of one particular class of built-up compact struts was studied with back to back separators.

Two sets of experiments have so far been carried out. The first (see figs.3 and 4) used back to back 140UNP channels 15

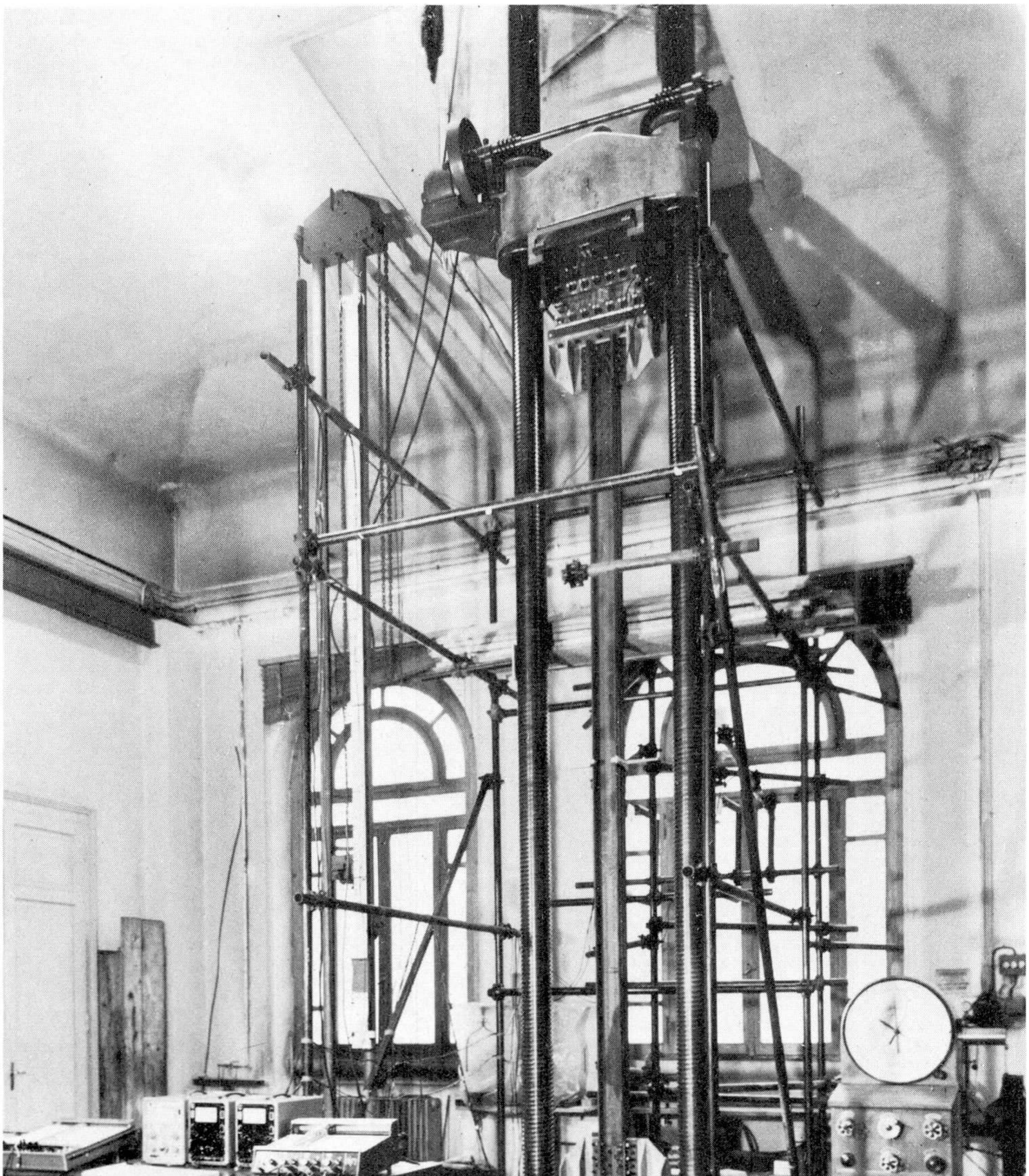


FIG.3

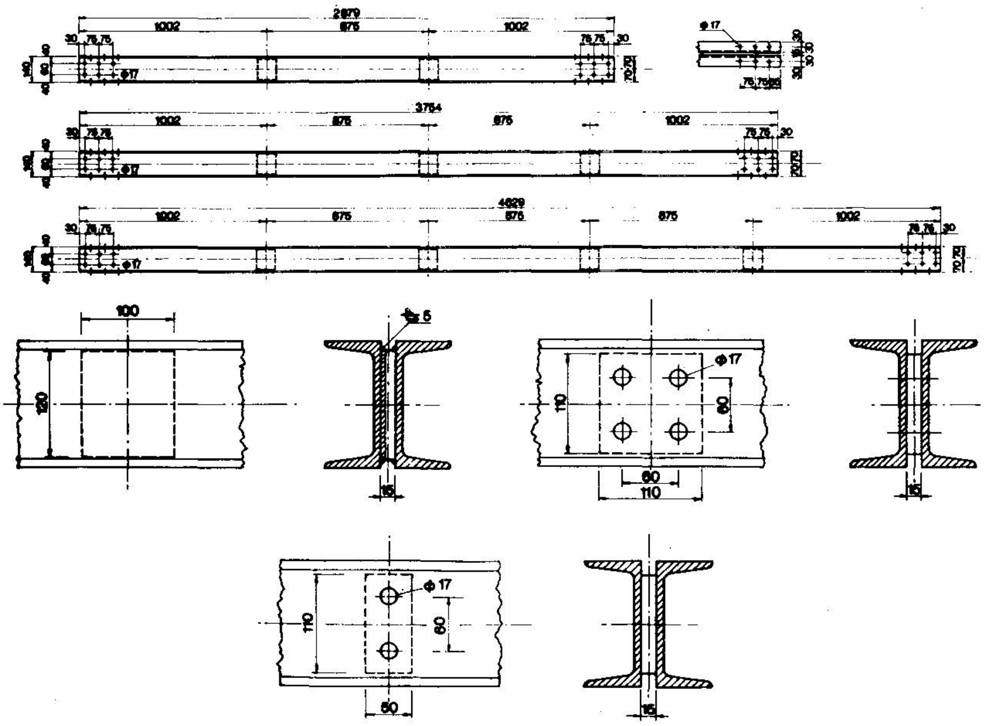

2879
1002
875
1002
3754
4629
100
120
ϕ 17
110
60
15

FIG.4

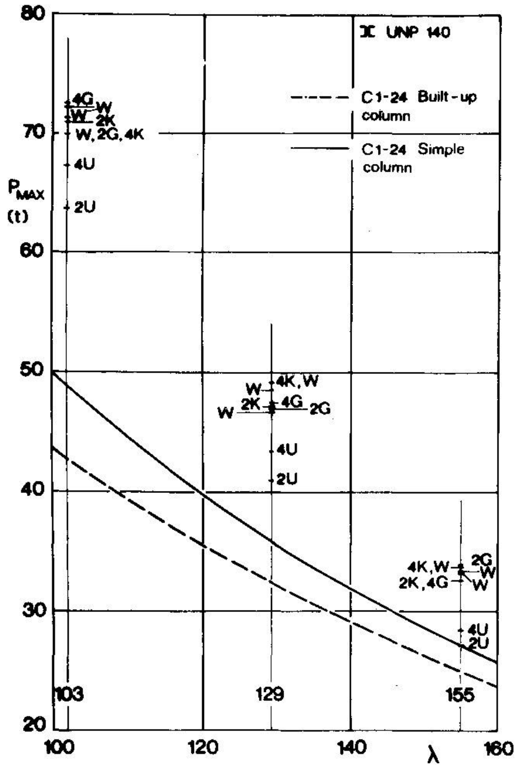

UNP 140
C1-24 Built-up column
C1-24 Simple column
PMAX (t)
λ
4G
W
2K
W, 2G, 4K
4U
2U
4K, W
4G
2G
103
129
155
80
70
60
50
40
30
20
100
120
140
160

FIG.5

mm apart, fastened at 2,3 or 4 intermediate points with solid washers or packings. The specified yield point was $\bar{\sigma}$ =24Kg/mm^2. The total slenderness ratios, depending on the number of fastners, were 103,129, and 155, while the local ratio (between packings) was 50. The end fasteners (24 shear resistent sections with Ø16 bolts of type 10 K) were all the same and designed for the ultimate load of the struts at zero slenderness. The intermediate fasteners were of the following kinds:

	Connections	Symbol
	welds	W
bolts	4 Ø 16 of type 10 K, tightened	4K
	2 " " " 10 K, "	2K
	4 " " " 8 G, "	4G
	2 " " " 8 G, "	2G
	4 " " " 10 K, untightened	4U
	2 " " " 10 K, "	2U

The experimental results are given in fig.5 and are compared with the curve $P_{MAX} = P_{MAX}(\lambda)$ (maximum load depending on the slenderness of the simple strut) deduced from the European Curve C1-24.

The struts with untightened fasteners (in which the settlement of the bolt in its hole becomes significant) were the least successful. It also became clear that the European curve C1-24 for simple struts, at least for high slendernesses was not safe enough while the dashed curve referring to an ideal slenderness $\lambda_{id} = \sqrt{(\lambda^2 + \lambda_1^2)}$ certainly is.

The second set of experiments was on (fig.6) unequal angles 5″× 3″× 5/16″ with 2,3 and 4 intermediate fasteners.

The specified yield point was $\bar{\sigma}$ =36 K/mm^2. The total slendernesses, depending on the number of fasteners, were 97,117 and 137, while the local slenderness was 50.

The end connections were this time designed for the real capacities of the strut and were made of the same kind of bolts used for the intermediate fasteners. These latter were of the following kinds:

	Connections	Symbol
	welds	W
bolts	2 Ø 24 of type 10 K, tightened	2K
	1 " " " 70 K "	1K
	2 " " " 5 D "	2D
	1 " " " 5 D "	1D

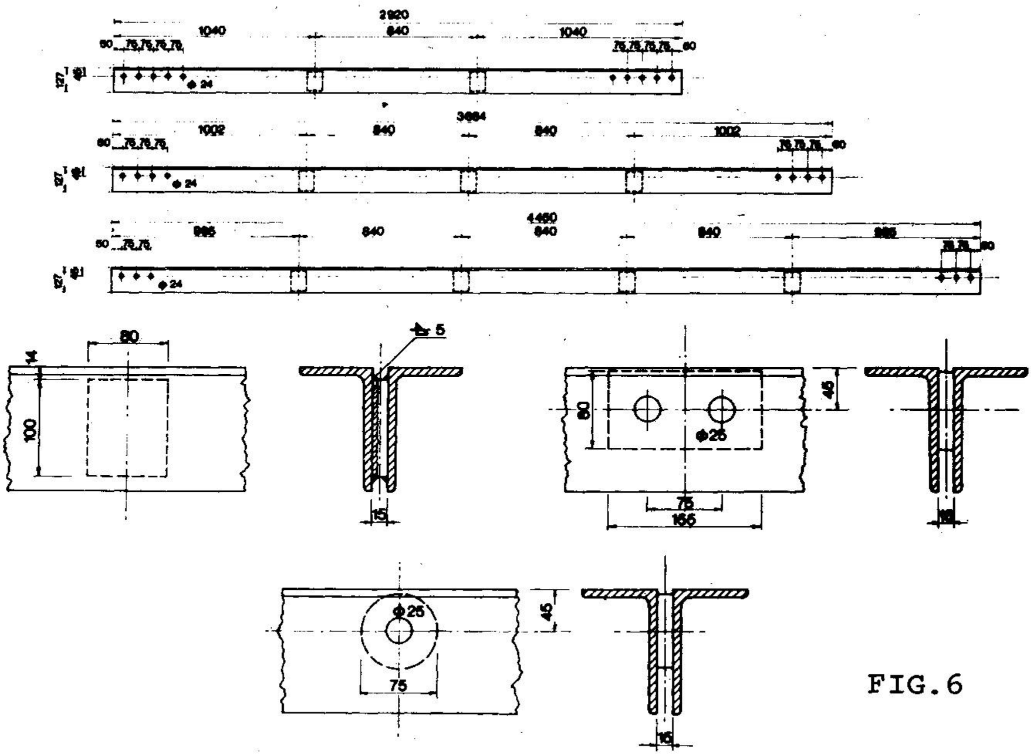
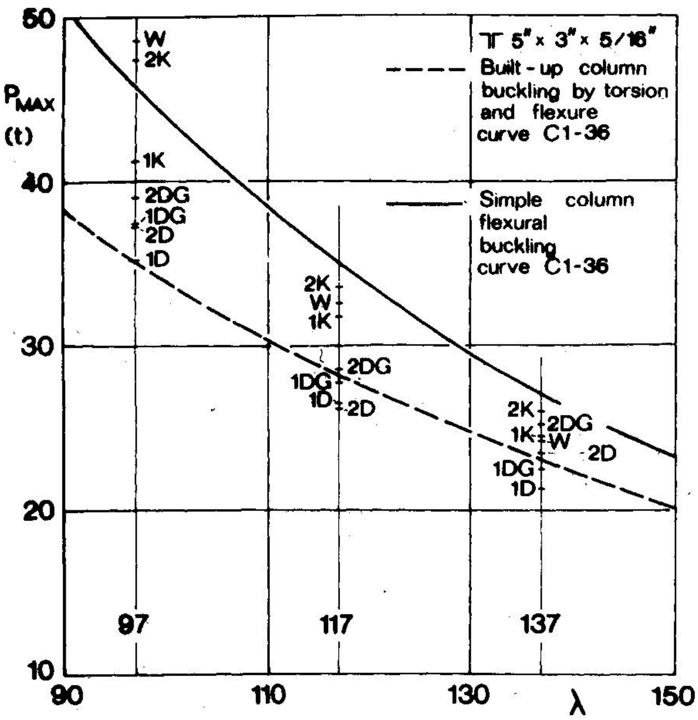


FIG.6



P_MAX (t)
λ
T 5" x 3" x 5/16"
Built-up column buckling by torsion and flexure curve C1-36
Simple column flexural buckling curve C1-36
97
117
137

FIG.7

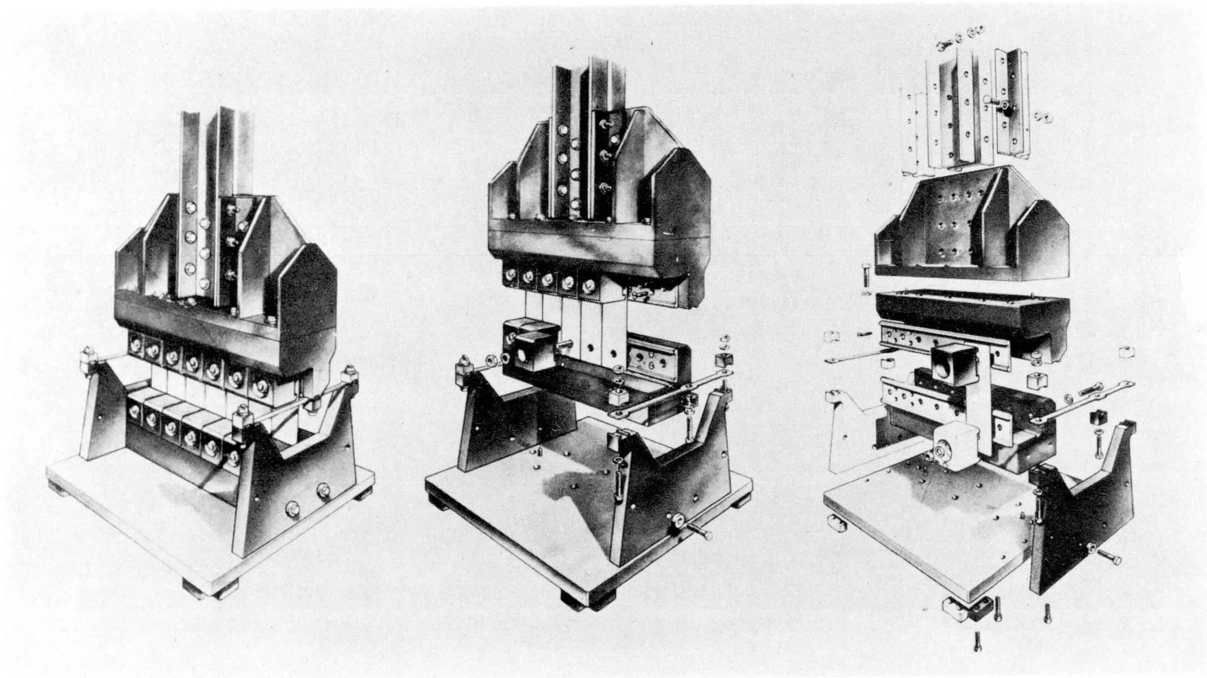


FIG.9

The fasteners shown by the letter G were for hot galvanized struts and bolts.

The experimental results are given in fig.7 and are compared to the curve $P_{MAX} = P_{MAX}(\lambda)$ deduced from the European Curve C1-36. Clearly the curve cannot be used for our purposes. Here two different effects come together, the first being the unfavourable influence of flexural-torsional instability the second being the greater or lesser stiffness of the intermediate fasteners. Even reference to the dashed curve, which corrects the slenderness by taking into account the above mentioned effects in the elastic range, does not guarantee safety in all situations. The authors consider that if normal safety margins are to be respected, welded joints or high strength friction type bolts are essential.

To sum up:

a) Built-up struts with washers or packings can only be considered as perfectly solid if their design assumes the ideal slenderness.
b) If the cross-section of the struts is not orthogonally symmetrical to the plain of deflection, and so flexural-torsional instability arises, it is no longer possible to make direct reference to the European Curves C1-24 and C1-36.
c) The stiffer are the end connections, the better is the performance of built-up struts.
d) The forces acting on the intermediate fasteners are less than those allowed for by the theory of bifurcation. The design must therefore pay particular attention to the qualitative aspects of constructional detailing, stressing stiffness rather than strength of the intermediate fasteners.

3. Test Equipment

The hydraulic press used for experiments had a pair of fixtures for the test struts to the machine. These make up a cylindrical elastic hinge which allows the end section of the strut to rotate around an axis when loaded, without friction but with a known elastic moment. The test equipment can be used on compressed structural elements of up to 7m in length, and the fixtures have a capacity of 100 metric tons.

These elastic hinges eliminate friction, since the end-hinged system (fig.8), adopted by many researchers, has been abandoned in favour of a continuous beam. In this way the test strut constitues the intermediate element, and the ends always remain within the elastic range thus allowing the end sections of the test strut to rotate, bringing into play an elastic end moment. By using very flexible elements at the ends to transmit the axial load, the elastic end moments can be greatly reduced. In this way the effective length of the test piece is not much less than the distance between the intermediate supports and a further advantage is that the transversal reactions of the supports are quite small compared to the axial loads. Since the members at the ends never leave the elastic range, the moment applied at the ends of the test piece can always be measured. The equipment is shown in fig.9.

The calculation method for establishing the effective length of the test piece is given in fig.10. This, as can easily be seen, is suitable for determining the critical loads corresponding to a symmetrical deformation.

The calculation results establish the effective length for a strut in these test conditions. The diagram in fig.10 shows the distance L between the axes of the elastic hinges as x-coordinate. The y-coordinate gives, for different values of the moment of inertia I of the test piece, the ratio between its actual slenderness λ and the slenderness λ_E it would have if hinged at the ends of span L.

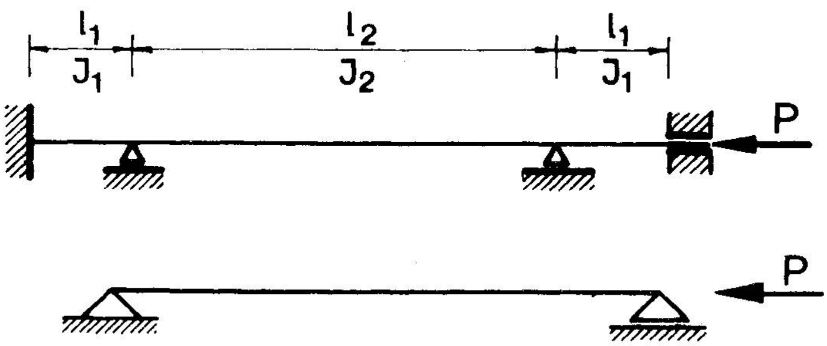


FIG.8

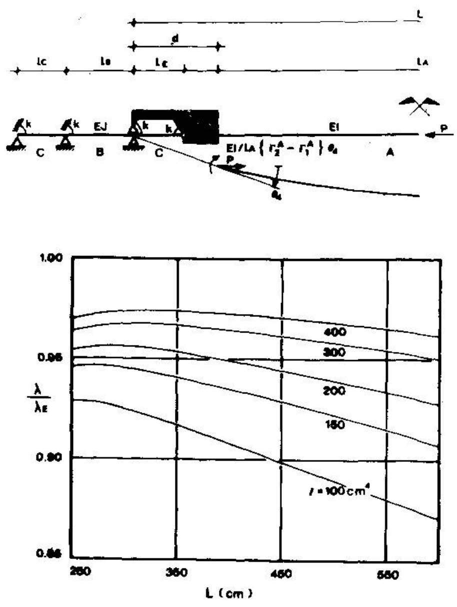


FIG.10

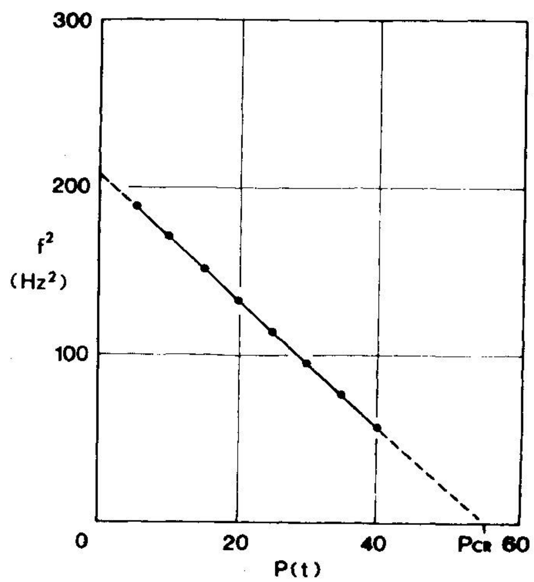


FIG.11

The fixtures were given a series of checks and calibrations to verify their static behavior, both when and when not connected to the test machine. Deflection test were carried out in the absence of axial loads. These showed that the actual position of the axis of rotation coincided with the theoretical position. They also checked the flexural stiffness, and calibrated the measuring equipment for bending moments.

The experimental value of the flexural stiffness was 298.6 metric ton cm/rad and agreed very well with the theory based on the model of fig.10: K_T =299.4 metric tons cm/rad. A further series of tests was carried out to verify the effective lengths of a set of struts with the same moment of inertia but different lengths. The dynamic method was used to find Euler critical load experimentally (fig.11).This was then compared with the theory. The theory turned out to be only 2% lower than the experimental results, so the calculation criteria may be considered precise enough for all practical purposes.

4. Numerical Approach

A numerical approach for calculating the bearing capacity of a built-up strut should allow for:

a) establishing compatibility of displacements at the intermediate and end connections in order to calculate the equilibrium configuration. This is possible, in principle, if solution techniques are used which assume that the axial load is an indipendent variable.

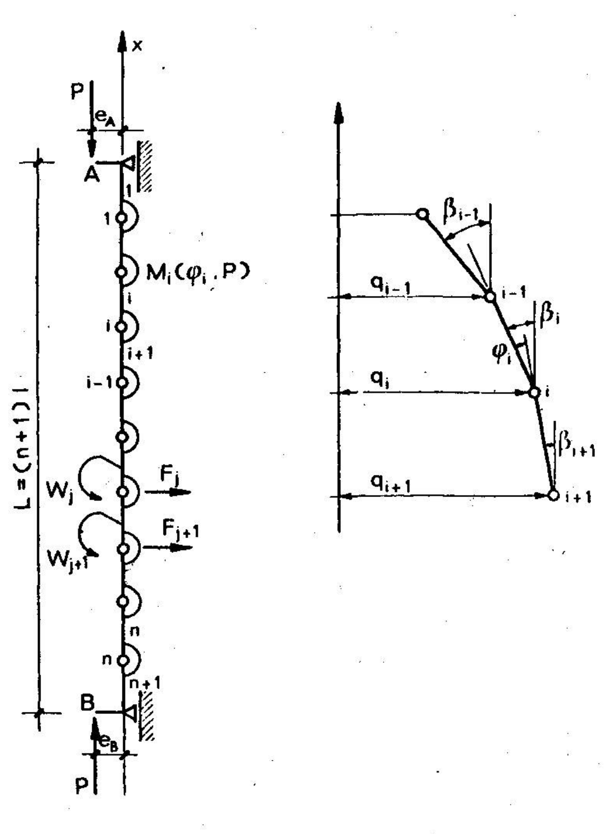


FIG.12

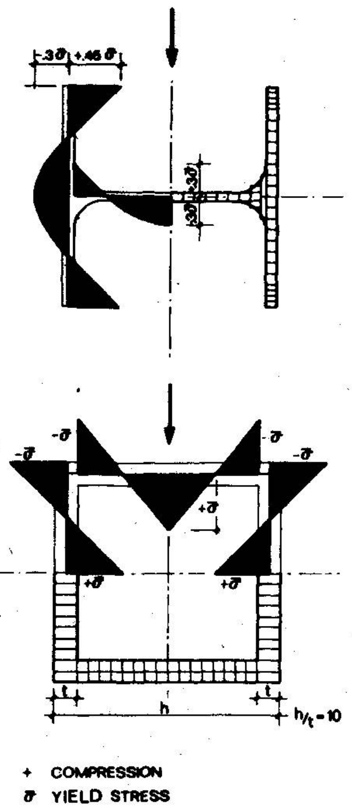
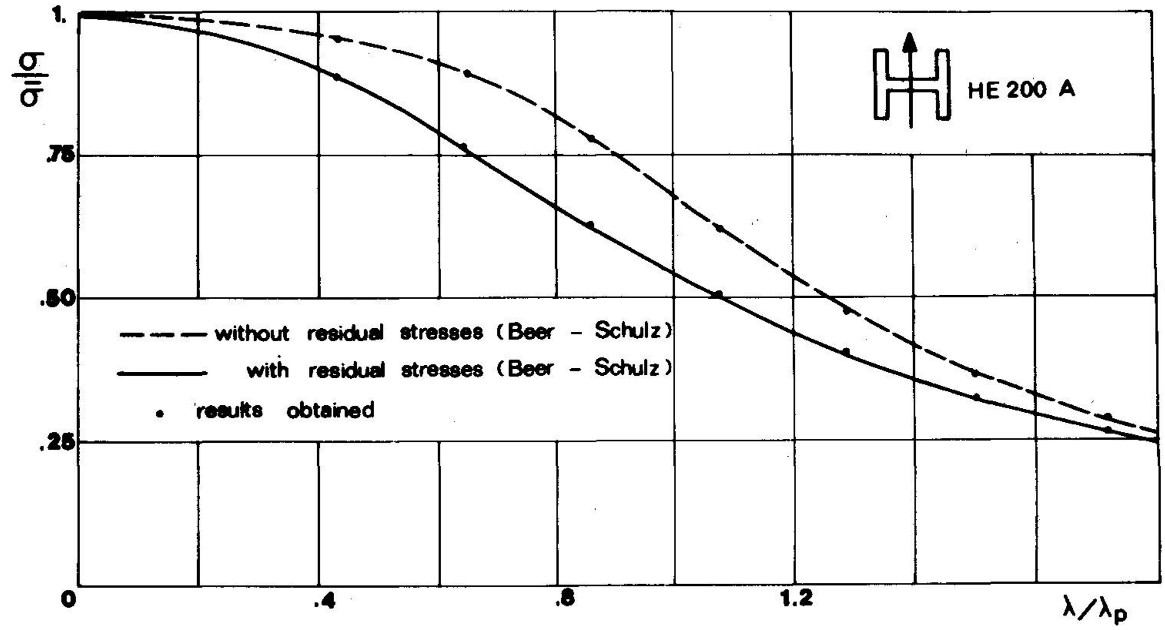
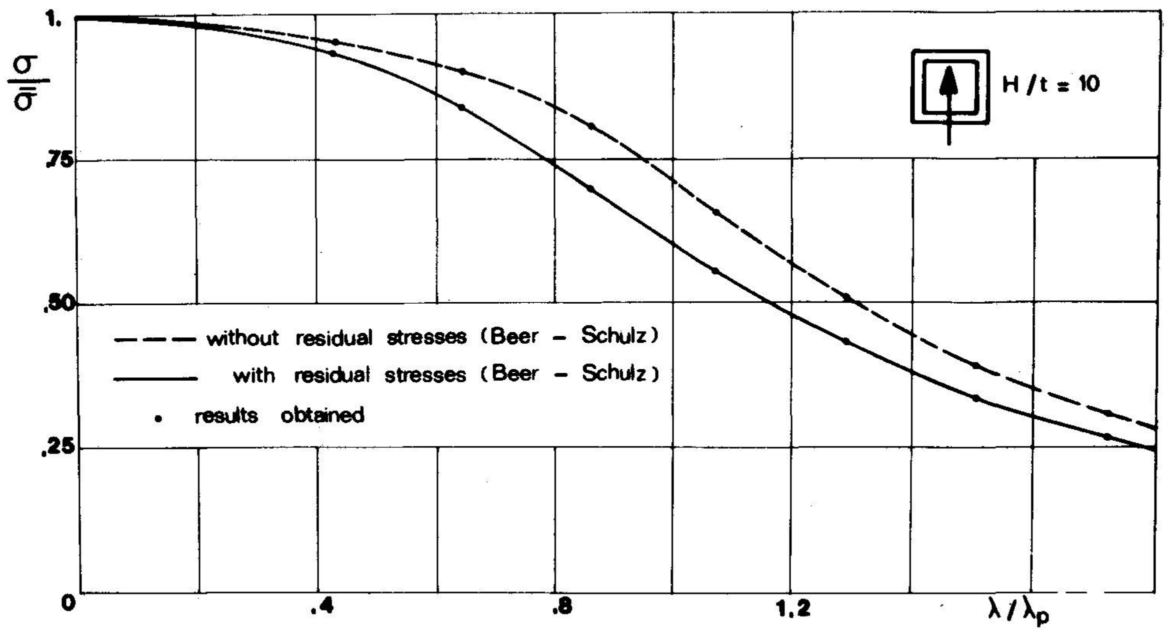


FIG.13



1.
.75
.50
.25
0
.4
.8
1.2
λ/λp
HE 200 A
without residual stresses (Beer - Schulz)
with residual stresses (Beer - Schulz)
results obtained

FIG.14



1.
.75
.50
.25
0
.4
.8
1.2
λ/λp
H/t = 10
without residual stresses (Beer - Schulz)
with residual stresses (Beer - Schulz)
results obtained

FIG.15

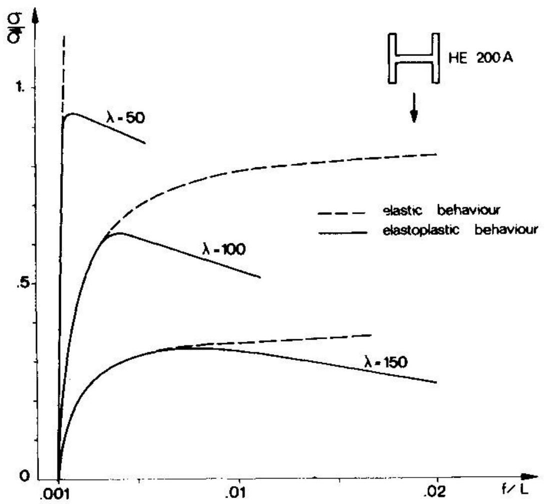


FIG.16

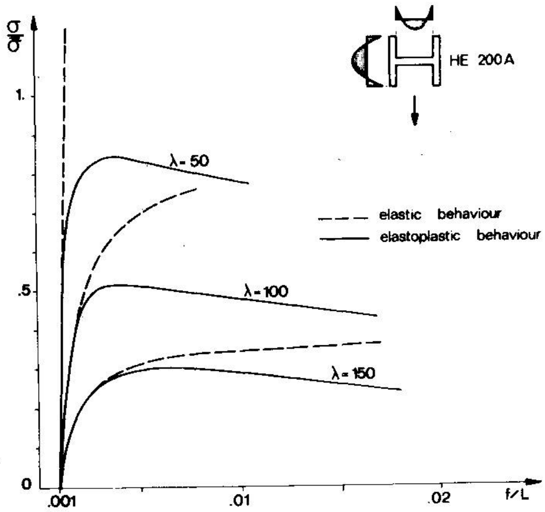


FIG.17

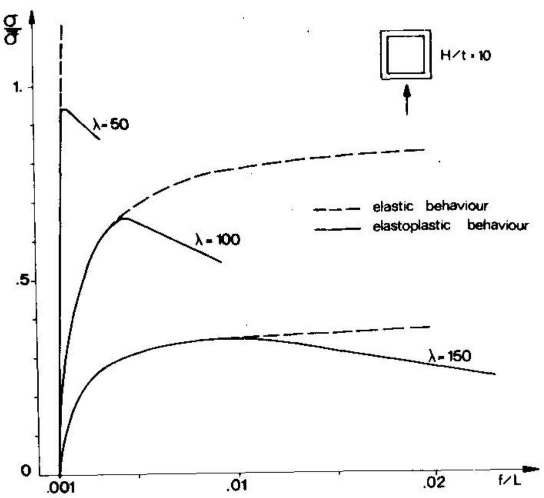


FIG.18

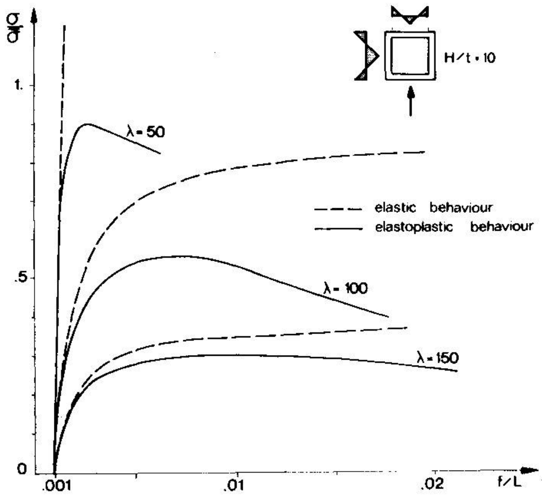
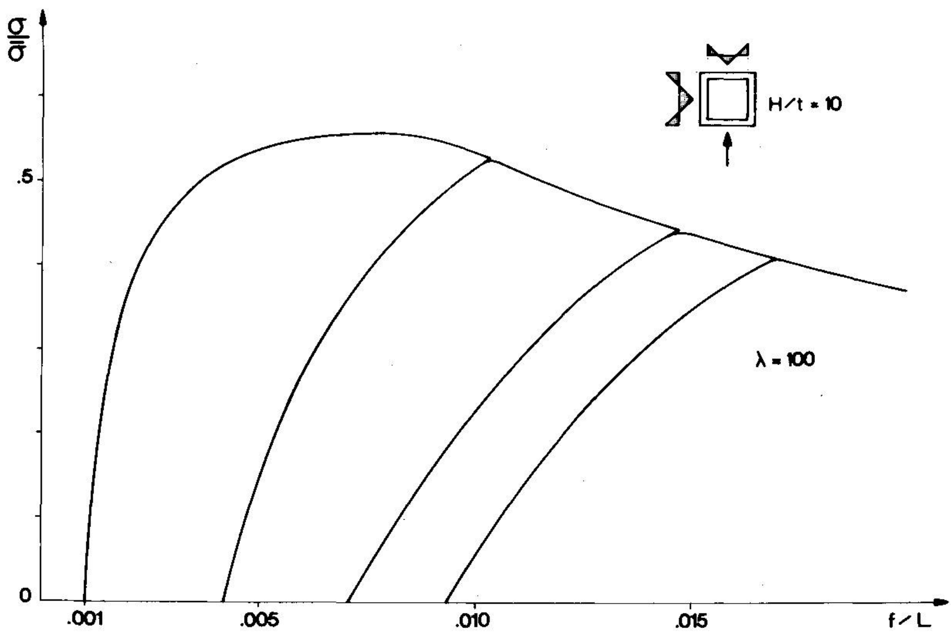
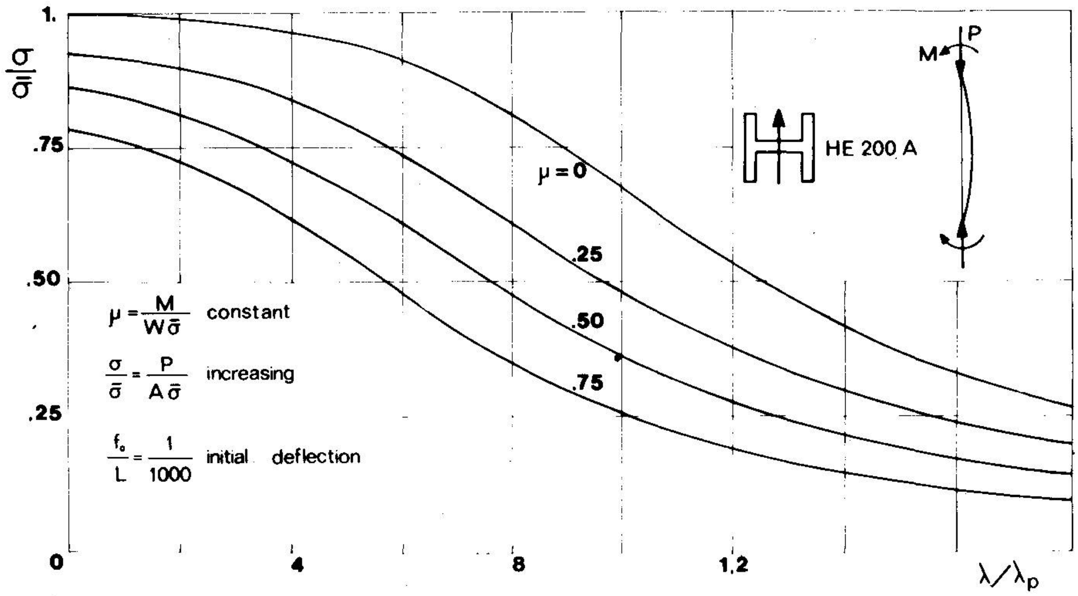


FIG.19



$\frac{\sigma}{\bar{\sigma}}$
.5
0
H/t = 10
λ = 100
.001
.005
.010
.015
f/L

FIG.20



1.
$\frac{\sigma}{\bar{\sigma}}$
.75
.50
.25
0
4
8
1,2
λ/λ_p
μ = 0
.25
.50
.75
HE 200 A
M
P
$\mu = \frac{M}{W\bar{\sigma}}$ constant
$\frac{\sigma}{\bar{\sigma}} = \frac{P}{A\bar{\sigma}}$ increasing
$\frac{f_o}{L} = \frac{1}{1000}$ initial deflection

FIG.21

b) global or local elastic unloadings of the chords because as the axial load increases one of the chords might become tensioned instead compressed. This unloading is possible if the problem is posed in incremental terms, and the equilibrium configuration that corresponds to the load $P+\Delta P$ is calculated by starting from the configuration corresponding to P .

Since the non-holonomy of the moment-curvature law cannot be left out, a calculation method was developed that respected points a) and b) in order to study the behaviour of built-up struts. This method has been proved for simple struts, and must now be extended to built-up columns.

The principles of the method are (fig.12):

a) the strut is reduced to a model with a finite number of degrees of freedom and made up of rigide parts and elementary cells in which all the flexibility, both axial and flexural, is concentrated.
b) the equilibrium equations are written in non dimensional form through the equivalence of both the Euler critical load and the limit elastic bending moment of the beam and of the model.
c) the relative rotations φ_i of the parts of the model are assumed as the unknowns depending on the applied load σ_N: $\varphi_i = \varphi_i(\sigma_N)$
d) the problem is reduced to incremental form by differentiating the equilibrium equation with respect to the independent variable σ_N .
e) integration of the system of differential equations is obtained by a technique widely used for dynamic problems based on a modification of Euler-Cauchy method, starting from the initial configuration.

So far, the mathematical program has been checked against some known results. In particular, HE200A and box struts, with or without residual stresses were considered (fig.13) and the results compared with those obtained by Beer and Schulz (see figs. 14 and 15).

The axial load-deflection laws for different slendernesses were computed, also for loads decreasing as the deflection increases (fig.16,17,18,19).

Overall unloading was also studied (fig.20). Finally the maximum axial load carried by struts subject to constant bending moments and increasing axial loads was calculated (fig.21).

5. Details of the Calculation Method.

With reference to fig.12 the equilibrium equations are:

$$1)\quad \{M\} - P\ell\,[C]\{\varphi\} = [C]\left(\ell\{F^*\} + P\{e\}\right)$$

where are:

M_i bending moment,
P axial load,

ℓ length of the parts,
φ_i relative rotations,
F_i^* generalised external forces,
e_i generalised eccentricities,
q_i transversal displacements,
$[C]$ a matrix defined by the relation: $\{q/\ell\} = [C]\{\varphi\}$.

The following quantities are defined:

$\bar{\sigma}$ yield point,
A area of the cross section,
ρ radius of inertia of the cross section,
h distance of the most compressed fibres from the centroid of the cross section,
λ slenderness,
$\lambda_P = \pi\sqrt{E/\bar{\sigma}}$.

The following non dimensional quantities are defined:

$$\{\mu\} = \left\{\frac{M}{\bar{M}}\right\}, \quad \sigma_N = \frac{P}{\bar{\sigma}A}, \quad \{\Phi\} = \left\{\frac{\varphi}{\bar{\varphi}}\right\},$$

$$\{f^*\} = \left\{\frac{F^*\ell}{\bar{M}}\right\}, \qquad \{e^*\} = \left\{\frac{e\,h}{\rho^2}\right\}.$$

The first eigenvalue β_E of the problem:

$$\left([I] - \frac{P\ell}{k}[C]\right)\{\varphi\} = 0,$$

defines the parameter:

$$\beta = \beta_E \frac{\lambda^2}{\lambda_P^2}.$$

Because of the equivalence of both the limit elastic moment and the Euler critical load of the strut as well as the model equations (1) can be written in the non dimensional form:

$$2)\quad \{\mu\} - \sigma_N\,\beta\,[C]\{\Phi\} = [C]\left(\{f^*\} + \sigma_N\{e^*\}\right).$$

Differentiation with respect to σ_N gives:

$$3)\quad \left\{\frac{d\Phi}{d\sigma_N}\right\} = \left([D] - \beta\sigma_N[C]\right)^{-1}\left(\beta[C]\{\Phi\} + [C]\{e^*\} - \left\{\frac{\partial\mu}{\partial\sigma_N}\right\}\right)$$

with:

$$d_{ik} = \begin{cases} 0 & \text{for } i \neq k \\ \dfrac{\partial\mu_i}{\partial\Phi_i} & \text{for } i = k \end{cases}$$

Starting from the configuration A corresponding to σ_N the solution is obtained by using the iterative formula:

$$\{\Phi_B^{(n+1)}\} = \{\Phi_A\} + \frac{1}{2}\Delta\sigma_N\left(\left\{\frac{d\Phi_A}{d\sigma_N}\right\} + \left\{\frac{d\Phi_B^{(n)}}{d\sigma_N}\right\}\right).$$

The integration step is automatically regulated and becomes smaller and smaller as the number of iterations needed for the required accuracy increases. When the step becomes very small or the derivative (3) is very great the loading process stops. At this point constant axial load is assumed, the relative rotation at the middle is increased and the second equilibrium configuration is found by iteration. After which the integration method is taken up again, making $\Delta \sigma_N < 0$ in formula (3) of the derivatives $\{\partial \mu / \partial \sigma_N\}$.
If overall unloading is required, $\Delta \Phi_i$ must also be negative in formula (3) of the derivatives as well as $\Delta \sigma_N$.

Of course the function $\mu = \mu(\Phi, \sigma_N)$ must be calculated at each attempt. This is done by iteration, starting from the characteristics of the cross section. For this purpose in order to speed up the process for ideally elastic-plastic material, the neutral axis can be obtained by a method which take into account the variation of the boundary of the plastic region of the cross section.

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