Some remarks regarding buckling curves

SOME REMARKS REGARDING BUCKLING CURVES

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ABSTRACT

Remarks are given in connection with the buckling resp.instability design curves for pin-ended struts relating the effective stiffness of the cross-section with the influence of residual stresses and of unavoidable geometrical imperfections.

Correlation between buckling and instability design curves is shown. Instability and buckling curves for the struts made of material with Ramberg-Osgood stress-strain diagrams are discussed and the problem of composite struts and of the calculation of load-carrying sistems are mentioned. The need for the typification of the shapes of the stress-strain diagrams for different materials, shapes of the cross-section and the distribution of residual stresses is emphasized.

INTRODUCTION

Under the guidance of the late Professor Beer there have been determined in Graz, on the basis of extensive analyses, three European curves. Some changes have been done lately for the region of very small slendernesses according to the suggestion of the team from Cambridge. This one parallelly made the British version of curves. Regarding the buckling curves of Graz there is an important addition for the simple consideration of dead and/or wind load of very slender struts. Graz has one more additional proposition, for one higher lying curve for high-strength steel, and lower lying for "jumbo" profiles. Dwight suggests in his present report an interesting simplification of the presentation of curves. Also in the USA an extensive study of multiple curves is going on, both deterministic and probabilistic, as Bjorhovde and Tall will report. We might expect on this colloquium also other important reports regarding strut curves and, of course, possible different views.

In the invitation for this colloquium Professor Beer expressed a very justified wish that there would be reached an unified view about the buckling curves. This would be very important not only for the treatment of pin-ended struts, but especially for struts in a system and so for the determination of the buckling and the instability limit state of the load-carrying system as a whole. It might come very soon to an agreement about the basic curves for compression members made of steel, because an extensive research work has been already done in this field. Or it might not come so soon. There should also be done much more about the composite sections (steel - concrete) and regarding struts made of alluminium alloys the finite propositions are still expected. That is why it would be extremely useful to come, at least, to an agreement about the basic assumptions.

In the following some remarks, which should contribute to a better further progress, are given.

FLEXURAL STIFFNESS, IMPERFECTIONS

For numerical simulation computer programs, which have the decisive part in research of compression instability, the data about the effective flexural stiffness of cross-sections due to the influence of nonelasticity are very important. The effective flexural stiffness depends on the shape of the cross-section, on the shape of stress-strain diagram of the material, on its cross-section nonhomogenity, and on residual stresses. It would be necessary to make a corresponding selection of the typical cross-sections regarding their geometrical shape and the distribution of residual stresses, taking into account different possible technologies of production (I-profiles, box sections, tubes, other sections).

While typifying the sections, only the ratios of dimensions are important. Fig. 1 shows both extreme profiles and a kind of the average section for narrow and wide flange European I-profiles.

The typification of curved stress-strain diagrams (i.e. of the alluminium alloys) is possibile when using one single parameter with the dimensionless form of the Osggod- Ramberg's equation, if we omit the classical definition of the yield stress $\sigma_{0,2}$. We replace it with, for example σ_{2}^{0} , which represents the stress when the plastic strains are equal to the elastic ones (Fig. 2). Fig. 3 shows a choice of the dimensionless curves, the shapes of which are given with only one parameter n. It should not be a problem nowadays to introduce the registration of all necessary parameters, which determine the basic mechanical behaviour of material, into the routine testing. For mild steel also the strain and the tangent modulus at the beginning of strain hardenin should be included. In the stress-strain diagram for concrete we'd better decide, if possible, for only one curve out of different propositions according to Fig. 4. This is important for the treatment of the composite cross-sections.

The determination of typical dimensionless distribution of residual stresses for typical sections would especially help in quicker application of research made till now about the load-carrying capacity of industrially produced compression members. Here team work with metallurgists would be useful. The important thing is, first, to determine the normal technology and then the various possibilities in the technology, and not before this to determine the corresponding distribution of residual stresses. Such a way is, of course, more reliable than that with incidentally taken specimens. The question what should be taken for typical residual stresses, those out of normal technology or those out of irregular processing, but the most unfavourable, is of special consideration.

The use of the load-shortening diagrams obtained by stub column test, will be well exploited, when we analyse them in comparison with computed diagrams at the consideration of the influence of residual stresses and nonhomogenity. The agreement with computed diagrams should exist also in full tension tests and in tension tests of single strips. For the consideration of the separate influence of nonhomogenity in the strength of the section also the parallel tests in stress-relieved state are useful.

Geometrical imperfections of compression members can be more easily controlled than structural imperfections. Here we have a possibility of variations from the ideal perfect strut regarding the sections and length to those imperfections, which are not permitted according to the definitions of standards of tolerances of measures and shapes.

BUCKLING AND INSTABILITY CURVES

The possibility that in compression members the most inconvenient structural and geometrical imperfections will appear simultaneously, is of course small. But from the viewpoint of safety we need such curves of instability, which represent the minimum guaranteed instability limit load with the consideration of the most inconvenient state of structural imperfections and the most inconvenient, but still permitted, geometrical imperfections. At the same time, with the help of the appropriate buckling curves (without geometrical imperfections), also the determination of the buckling load of compressed members is possible (with consideration of normal structural imperfections). In such a way we can have always the survey about the region of the possible actual state (Fig. 5).
I do not know if in the present situation there is necessary to think much more about the probability of the appearance of different intermediate possibilities. But it would be very useful to gather sistematically the statistical data about the possible geometrical imperfections of the struts in the systems, where the question of the probability of the simultaneous appearance of the most unfavourable geometrical imperfections is more important.

Previous buckling curves in inelastic region, which at the higher slendernesses pass on to the Euler's curve and are connected with a variable coeficient of safety, are, in fact, principally identical with new instability curves, which take into account the initial crookedness of struts and the constant factor of safety. The variable coeficient of safety at the buckling curve somehow includes the influence of the initial crookedness. The relations are shown in Fig. 6. In Fig.7 there are shown the present buckling rules for compression members in the USA and West Germany, translated in the instability curves with the constant factor of safety.

There is always and advantage to have a dimensionless presentation of the buckling and instability curves.

To determine the buckling load of the linear systems, we need effective flexural stiffness of the cross-sections (or the effective modulus of elasticity), dependent on the axial force in the individual struts. The corresponding relations are given in Fig.8.

We can rely on the fact that in the course of time it will be necessary to have still more dimensionless buckling and instability curves, even interlacing ones. This is to be expected because of the different shapes of cross-sections, stress-strain diagrams and distribution of residual stresses and also because of the different influence of the initial crookedness on the instability limit load at different strength of the material, and smaller effect of residual stresses in high strength material.

STRUTS FROM RAMBERG-OSGOOD MATERIAL

In the following figures from 9 to 12 there is shown how the instability curves for compressed members of rectangular cross-section with initial crookedness 1/1000 depend on the strength degree of the materials at different shapes of stress-strain diagram, expressed by the parameter n. This numerical experimentation with the help of a computer was made with the assumption that there is no strain reversal. Fig.13 shows the comparison of the T curves without strain reversal with the R curves, where strain reversal is taken into account. We can see in Fig.14 the corresponding diagram for the determination of the effective flexural stiffness of the rectangular cross-section, as a function of the dimensionless axial force and bending moment. There is also evident the limited region of R curves, which are computed according to the assumption that the axial force is constant, while the bending moment increases. Fig.15 reminds us of the fact that in materials with the low parameter n of the stress-strain diagram exists considerable difference between the buckling loads according to tangent modulus concept and the buckling loads according to reduced modulus concept for different cross-sections. Although, as it is well known, the actual load-carrying capacity is somewhere in-between, the question is whether it will not be worthwhile, sometimes to take into account the increased carrying capacity because of strain reversal. When n is high (Fig.16), the differences are considerably smaller. However, also here the question exists whether the use of the increased buckling loads above $\bar{N} = 1$, is convenient in the region of very small slenderness, because here the unreversal shortening of the strut becomes substantial. May be we should pay attention sometimes also to a serviceability limit state, defined with an appropriate limit of the unreversal deformation.

COMPOSITE STRUTS

The suitability of the dimensionless presentation of the bugkling curves, also for the struts with composite cross-sections (steel - concrete), can be well seen in Fig.17. The curves for all composite cross-sections are always between the lower curve for plane concrete and the upper one for plane mild steel (here without influence of residual stresses). Of course in $\bar{N}$ and $\bar{\lambda}$ there are involved parameters, which take into account the shape of single cross-sections and the properties of single materials. And in Fig.18 there is shown the diagram for the determination of effective flexural stiffness for a concrete filled tube. It is intended for changing compressed axial force and bending moment without strain reversal. According to so many variations regarding possible cross-sections and material properties, the question of the typical composite cross-sections is even more important.

LOAD CARRYING SYSTEMS

At the end, we might discuss very shortly the question of buckling and instability of the linear systems. We can say in connection with buckling that it is no problem for any multistory plane frame to get automatically the shape of the buckling deformation and the buckling safety factor with the help of computers after Vianello's method, that is with the iteration and the use of the Ist order theory. With given relations of effective flexural stiffness for the given cross-section and with the consideration of structural imperfections, if necessary, we can automatically take into account the influence of inelasticity.

While the buckling calculations of plane frames with the help of computer represent today a routine work (input of data as in the STRESS program), the calculation of the elasto-plastic instability of multistory frames, loaded vertically and horizontally, is nowadays possible, but still very unpractical and expensive, because the inelasticity changes in cross-sections, both for the iteration of a given loading and for the increase in loading. That is why this way of computing is intended today before all for research work and especially for the evaluation of different approximate methods like the 2nd order plastic hinge theory, the Merchant's formulae, and interaction formulae for beam-columns. It is evident that the degree of the accuracy of the approximate methods can be well evaluated only with the help of a more precise method.

But the most important basis for the elasto-plastic calculation are the data about the variation of the effective stiffness of the cross-sections. And so we must return to the appeal for the necessity of unification of these data which is even more unavoidable especially at the additional consideration of biaxial bending, torsion and plate buckling. In this way it would be possible to reliably compare the results of the calculation of instability behaviour of complicated structures carried out with different numerical methods in different places. And this would enable considerably quicker progress in the spreading of knowledge in the field of compression instability.

CONCLUSION

The appropriate unification of stress-strain diagrams, cross-sections and the corresponding distribution of residual stresses give a general value of the data about the effective stiffness, which are the most important basis for the determination of the buckling or instability limit load of any linear load carrying system.

ACKNOWLEDGEMENT

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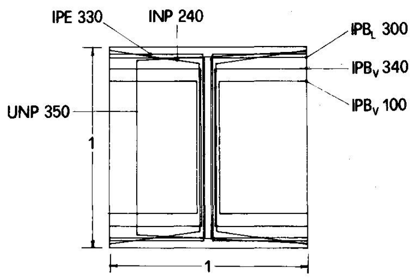


Fig.1

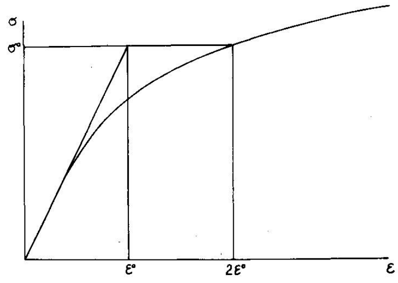


FIg.2

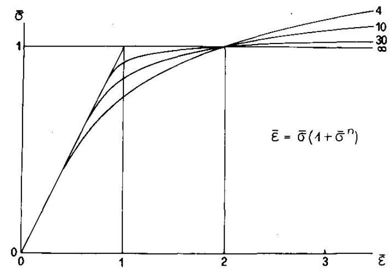


Fig.3

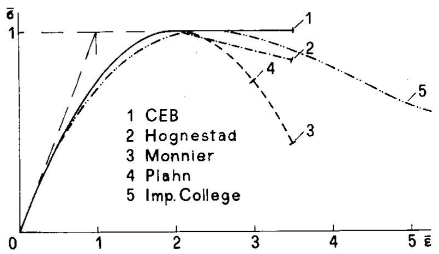


Fig.4

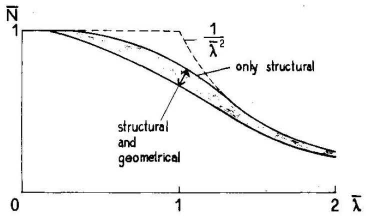


Fig.5

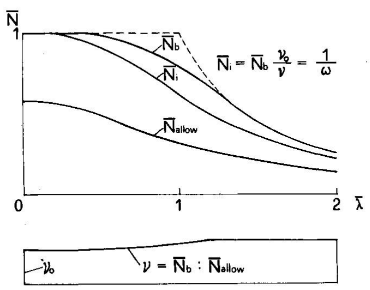


Fig.6

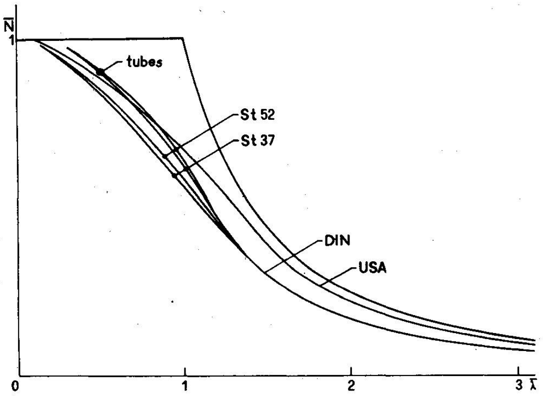
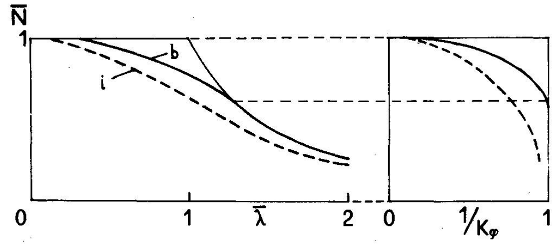


Fig.7



$$1/K_\varphi = \frac{J_{eff}}{J} = \frac{E_{eff}}{E} = \bar{N}_{b,i}\,\bar{\lambda}^2 = f(\bar{N}_{b,i})$$

Fig.8

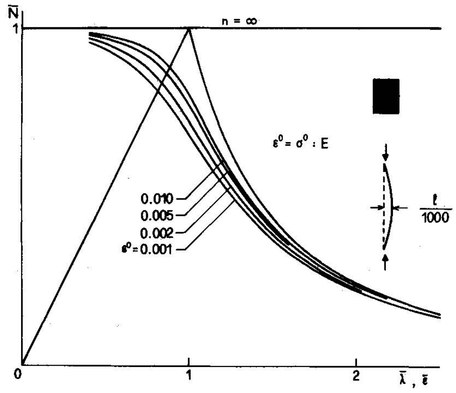


Fig.9

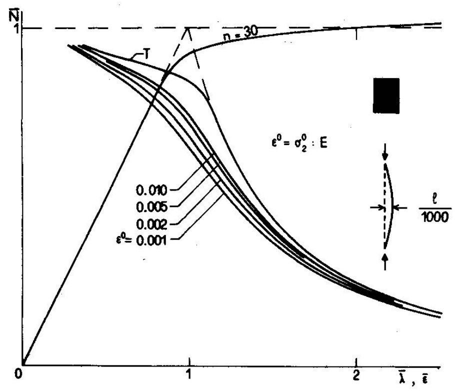


Fig.10

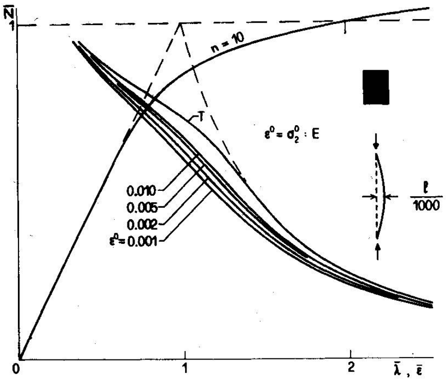


Fig.11

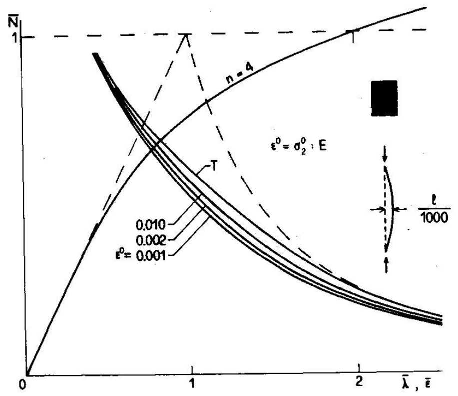


Fig.12

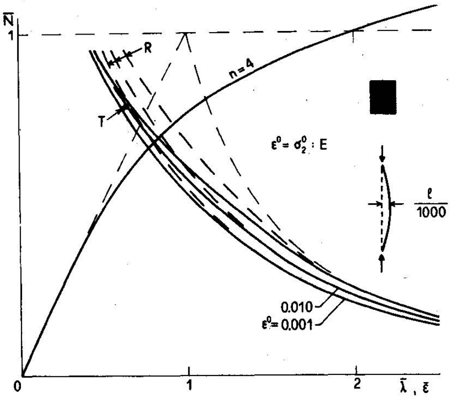


Fig.13

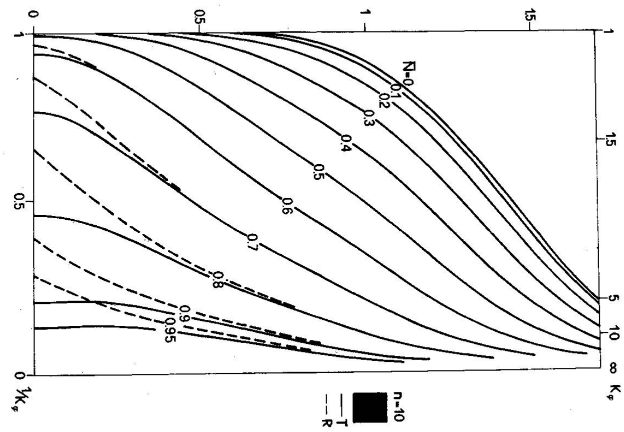


Fig.14

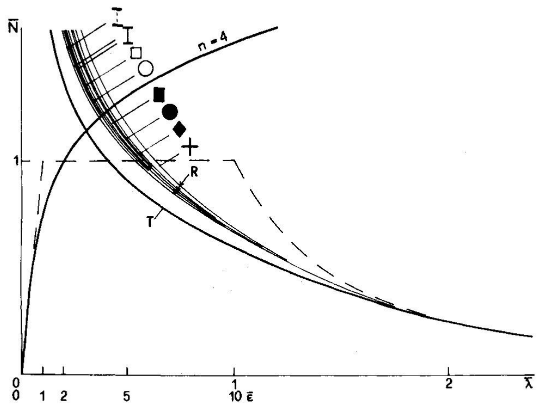


Fig.15

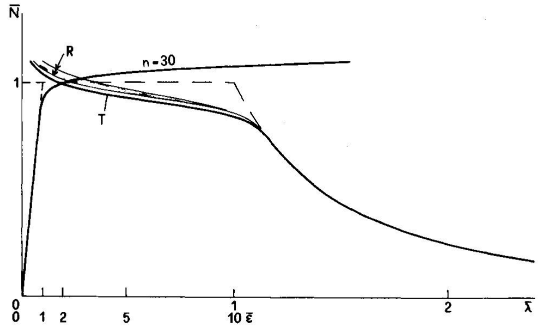


Fig.16

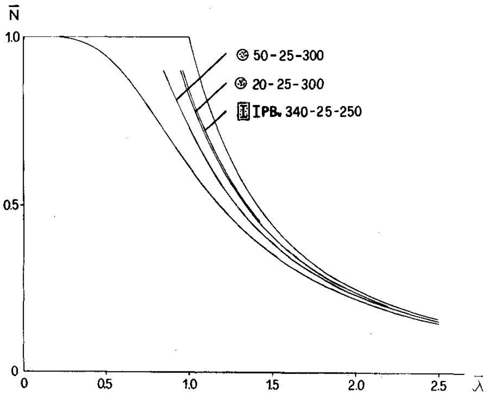


Fig.17

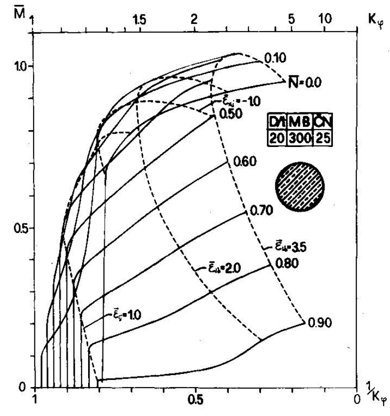


Fig.18