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DETERMINATION OF THE ELASTIC LIMITS FOR BUCKLING ANALYSIS

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ABSTRACT

This paper presents the results of a statistical research on the yield points to introduce in the buckling curves. If an even degree of safety should be obtained for all the components of a structure, whether in tension or in compression, a higher yield point than the minimum guaranteed tensile limit must be adopted for the buckling curves.

This conclusion was accepted by the "Convention Européenne" for its column strength curves.

RESUME

Ce rapport présente les résultats d'une recherche statistique sur les limites élastiques à introduire dans les courbes de flambement. Si l'on veut obtenir une sécurité homogène pour toutes les pièces d'une structure, tant tendues que comprimées, une limite élastique plus élevée que celle minima garantie en traction, doit être adopté pour les courbes de flambement.

Cette conclusion a été acceptée par la "Convention Européenne" dans ses courbes de flambement.

The buckling curves recommended by Commission No. 8 of the "Convention Européenne de la Construction Métallique" (CECM) represent a substantial achievement by that organization, illustrated by the fact that the failure stresses given thereon are some 20% higher than those of the German Standard DIN 4114 for the majority of ordinary sections over the range of slenderness with which constructors are mainly concerned.

This outcome is based on:

- a very extensive programme of experimental research into the mechanical properties of the steel and the buckling strength of columns with varying cross section and slenderness ratio;
- an equally comprehensive programme of theoretical research into buckling of columns with or without geometrical and structural imperfections, undertaken on computers on which the buckling phenomenon has been simulated in a certain way to reproduce the loading tests;
- the "statistical-probabilistic" philosophy of safety adopted by the CECM, which not only led to a rational interpretation of the theoretical and experimental results but was used to specify the research programmes themselves.

DEFINITION OF PROBLEM

A significant aspect of the buckling curves, related in part to the probabilistic theory of safety, is their termination, for slenderness ratio 0, at points sometimes higher than the allowable elastic limits of 24 and 36 kg/mm^2 for steels E24 and E36 (previously designated as A37 and A52).

It is perhaps without precedent that values higher than the guaranteed minimum elastic limits should be adopted for standards, or recommendations, as allowable structural failure stresses.

There are three reasons for this conclusion:

a) the experimental curve, connecting points situated at two Standard Deviations SD below the mean values curve, ends at 26 kg/mm^2;

b) the elastic limit of open walled sections is higher in overall compression than in tension, the compression limit being the failure stress at slenderness ratio 0. Both the mean value derived from these tests and that value minus two SD are higher in overall compression than in tension - as per EURONORM (1) - this latter value (mean M - 2 SD) being the failure stress according to the failure criterion of Commission No. 1 of the "Convention Européenne". If this guaranteed tensile level is 24 for A37 and 36 kg/mm^2 for A52, it is logical to adopt higher values for the buckling curves at slenderness ratio 0, since these are directly affected by the overall elastic limit in compression and not by that in tension;

c) an even degree of safety should be maintained for all components, whether in tension or in compression, constituting a structure.

This last point, however, requires a much longer explanation.

HOMOGENEOUS SAFETY AND FAILURE CRITERION

The failure criterion adopted by Commission No. 1 of the "Convention Européenne" is the following: the stress or failure load of a member is the measured mean value for a series of specimens minus two SD. If the sta tistical distribution is a normal one of the "Gauss-Laplace" type (as is the case for the CECM measurements), there is a 97.7% probability that the value question will be under the member's strength and a 2.3% one that it will be over.

The CECM experimental programme has been precisely elaborated so that members were tested under conditions very close to those found in actual structures, and in sufficient number to obtain mean ultimate loads and Standard Deviations possessing statistical validity for the whole European production.

The designer wishing to maintain an even degree of safety in the various elements of a structure must base his calculations on the failure loads of the various component parts, all such loads having the same probability of being exceeded (or of not being reached) irrespective of whether the loading is in tension, compression or bending.

According to the failure criterion of Commission No. 1 - buckling load equal to the mean minus two SD - the probability of not reaching this value is 2.3% and must be the same for all components so as to avoid wasting material in some part of the structure without rationally adding to its general safety.

Under these conditions, it is fairly easy to check that the elastic limit used for the buckling curves must be higher than that of the tensile specimens (and also higher than that adopted for designing parts in simple tension); in other words, if 24 kg/mm^2 is guaranteed as mean minus two SD, the elastic limit to be used for establishing the buckling curves must be higher than 24 kg/mm^2 in order to preserve the failure probability of 2.3% for compressed members.

INVESTIGATION OF STATISTICAL VALUES FOR ULTIMATE BUCKLING LOADS

The ultimate buckling load P of a member is given to the structural designer by functions of the type:

$$P = P\ (R,\ T,\ F,\ A,\ \lambda) \qquad (1)$$

where:

R = elastic limit in kg/mm^2

T = residual stress at a given point (at the edges of the flanges, for example) in kg/mm^2

F = initial deflection of the member per °/oo of its length

A = section area in mm^2

λ = slenderness ratio

Functions (1) can be determined either experimentally or more rapidly by means of a computer by simulating load tests under various initial conditions [2].

R, T, F, A are not exactly known, though their probable distribution can be found, and a normal distribution of the Gauss-Laplace type can be assumed with average values $\overline{R}$, $\overline{T}$, $\overline{F}$, $\overline{A}$ and Standard Deviations r, t, f, a. The slenderness ratio λ, on the other hand, can be assumed, at least at its first approximation, exactly known, since any variation of A (due in general, more to wall thickness differences than to wall dimensions) has negligible effect on the radius of gyration. The member's length is an obviously known factor.

The mean value $\bar{P}$ of the buckling load, its Standard Deviation and the value $\bar{\bar{P}}$ of this load, to which reference is made after the adopted failure criterion, will be:

$$\bar{P} = P\ (\bar{R}, \bar{T}, \bar{F}, \bar{A}, \lambda) \tag{2}$$

$$p^2 = (\frac{\partial P}{\partial R}\, r)^2 + (\frac{\partial P}{\partial T}\, t)^2 + (\frac{\partial P}{\partial F}\, f)^2 + (\frac{\partial P}{\partial A}\, a)^2 \tag{3}$$

$$\bar{\bar{P}} = \bar{P} - 2p = P\ (\bar{R}, \bar{T}, \bar{F}, \bar{A}, \lambda) - $$

$$- 2 \sqrt{(\frac{\partial P}{\partial P}\, r)^2 + (\frac{\partial P}{\partial T}\, t)^2 + (\frac{\partial P}{\partial F}\, f)^2 + (\frac{\partial P}{\partial A}\, a)^2} \tag{4}$$

We would be tempted, knowing (1), to calculate (4) as follows:

$$\bar{\bar{P}}_1 = P\ (\bar{R} - 2r, \bar{T} + 2t, \bar{F} + 2f, \bar{A} - 2a, \lambda) \tag{5}$$

but we would find: $\bar{\bar{P}}_1 < \bar{\bar{P}}$.

This appears evident when we reflect that the 2.3% probability of having a column with a failure load below $\bar{\bar{P}}$ (4) is equal to that of having a column with an elastic limit below $(\bar{R} - 2r)$, with any T, F and A, or a column with $T > (\bar{T} + 2t)$ and any R, F, A, or else $F > (\bar{F} + 2f)$ and any R, T, A, or, finally, $A < (\bar{A} - 2a)$ and any R, T, F.

The "composite" probability of having simultaneously $R < \bar{R} - 2r$, $T > \bar{T} + 2t$, $F > \bar{F} + 2f$, $A < \bar{A} - 2a$ is the product of four 2.3% degrees of probability that each event should occur independently of the three others, this is equal to: $(2.3 \times 10^{-2})^4 = 28 \times 10^{-8}$ which is well below the 2.3% probability that $\bar{\bar{P}}$ (4) be smaller than the member's buckling load, assuming fairly reasonably that the variables R, T, F, A are statistically independent.

It follows that, in order to obtain $\bar{\bar{P}}$, it will be necessary to introduce at (1) the values R_1, T_1, F_1, A_1 so that:

$$\bar{\bar{P}} = P\ (R_1, T_1, F_1, A_1, \lambda) = \bar{P} - 2p \tag{6}$$

with

$$R_1 = \bar{R} - \alpha r,\ T_1 = \bar{T} + \alpha t,\ F_1 = \bar{F} + \alpha f,\ A_1 = \bar{A} - \alpha a \tag{7}$$

where $\alpha < 2$.

This coefficient α has been determined with sufficient accuracy in various conditions as follows. $\bar{\bar{P}}$ can be calculated by means of (4) or by developing in series (6), we can write (the terms in the bracket are all positive, taking into account the signs r, t, f, a and the partial derivates - see Table II):

$$\bar{\bar{P}} = P\ (R_1, T_1, F_1, A_1, \lambda) = P\ (\bar{R}, \bar{T}, \bar{F}, \bar{A}, \lambda) -$$

$$- \alpha (\frac{\partial P}{\partial R}\, r + \frac{\partial P}{\partial T}\, t + \frac{\partial P}{\partial F}\, f + \frac{\partial P}{\partial A}\, a) \tag{8}$$

and by comparing (4) and (8):

$$2p = \alpha \left(\frac{\partial P}{\partial R} r + \frac{\partial P}{\partial T} t + \frac{\partial P}{\partial F} f + \frac{\partial P}{\partial A} a\right) \qquad (9)$$

By squaring and taking account of (3):

$$4p^2 = 4 \left\{ \left(\frac{\partial P}{\partial R} r\right)^2 + \left(\frac{\partial P}{\partial T} t\right)^2 + \left(\frac{\partial P}{\partial F} f\right)^2 + \left(\frac{\partial P}{\partial A} a\right)^2 \right\} =$$

$$= \alpha^2 \left(\frac{\partial P}{\partial R} r + \frac{\partial P}{\partial T} t + \frac{\partial P}{\partial F} f + \frac{\partial P}{\partial A} a\right)^2 =$$

$$= \alpha^2 \left\{ \left(\frac{\partial P}{\partial R} r\right)^2 + \left(\frac{\partial P}{\partial T} t\right)^2 + \left(\frac{\partial P}{\partial F} f\right)^2 + \left(\frac{\partial P}{\partial A} a\right)^2 \right\} +$$

$$+ \alpha^2 \left\{ 2 \frac{\partial P}{\partial R} \frac{\partial P}{\partial T} rt + 2 \frac{\partial P}{\partial R} \frac{\partial P}{\partial F} rf + 2 \frac{\partial P}{\partial R} \frac{\partial P}{\partial A} ra + \right.$$

$$\left. + 2 \frac{\partial P}{\partial T} \frac{\partial P}{\partial F} tf + 2 \frac{\partial P}{\partial T} \frac{\partial P}{\partial A} ta + 2 \frac{\partial P}{\partial F} \frac{\partial P}{\partial A} fa \right\} \qquad (10)$$

All the terms of (10) being positive or equal to zero, we obtain additional confirmation of the fact that $\alpha < 2$. Indeed, in the last of the expressions for $4p^2$ set down in (10) above, we recognize the value of p^2 in the first quantity between the braces; let us designate the second quantity between the braces γ^2. We thus have:

$$4p^2 = \alpha^2(p^2 + \gamma^2) \quad \text{then} \quad \alpha = 2\left(\frac{1}{1 + \gamma^2/p^2}\right)^{1/2} \qquad (11)$$

TABLE I - Mean values $\bar{R}$, $\bar{T}$, $\bar{F}$, $\bar{A}$ and their SD r, t, f, a in % and in absolute values for calculating α ($\bar{\bar{R}} = \bar{R} - 2r = 24$ kg/mm^2)

$\bar{R}$	r		$\bar{T}$	t		$\bar{F}$	f		$\bar{A}$	a	
kg/mm²	%	kg/mm²	kg/mm²	%	kg/mm²		%		mm²	%	mm²
26,70	5	1,35	3,33	10	0,33	10^{-3}	10	10^{-4}	2 010	2	40
26,70	5	1,35	3,33	15	0,50	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	2	40
26,70	5	1,35	3,33	15	0,50	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	3	60
26,70	5	1,35	3,33	20	0,66	10^{-3}	20	2×10^{-4}	2 010	3	60
26,70	5	1,35	3,33	20	0,66	10^{-3}	20	2×10^{-4}	2 010	4	80
28,25	7,5	2,12	3,53	10	0,35	10^{-3}	10	10^{-4}	2 010	2	40
28,25	7,5	2,12	3,53	15	0,53	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	2	40
28,25	7,5	2,12	3,53	15	0,53	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	3	60
28,25	7,5	2,12	3,53	20	0,70	10^{-3}	20	2×10^{-4}	2 010	3	60
28,25	7,5	2,12	3,53	20	0,70	10^{-3}	20	2×10^{-4}	2 010	4	80
30,00	10	3,00	3,75	10	0,375	10^{-3}	10	10^{-4}	2 010	2	40
30,00	10	3,00	3,75	15	0,56	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	2	40
30,00	10	3,00	3,75	15	0,56	10^{-3}	15	$1,5 \times 10^{-4}$	2 010	3	60
30,00	10	3,00	3,75	20	0,75	10^{-3}	20	2×10^{-4}	2 010	3	60
30,00	10	3,00	3,75	20	0,75	10^{-3}	20	2×10^{-4}	2 010	4	80

EXPERIMENTAL DATA

We have adopted in the Table I values of r, t, f, a and $\bar{R}$, $\bar{T}$, $\bar{F}$, $\bar{A}$. Those of $\bar{R}$ have been selected so that, for deviations of 5%, 7.5% and 10%

of $\overline{R}$, the elastic limit $\bar{R} = \overline{R} - 2r$ amounts to 24 kg/mm^2. The actual value of r, derived from CECM stub column tests 1 was 7.5% of $\overline{R}$, thus giving an $\bar{R}$ value always above 24 kg/mm^2. A Standard Deviation of about 10% would seem to result from a Belgian statistical investigation [4].

For $\overline{T}$ and $\overline{F}$ and their Standard Deviations t, f (10%, 15% and 20%), we have adopted values equal to or below those experimentally ascertained by the "Convention Européenne" or other research workers. A SD smaller than that actually found has been adopted, since the resulting value of α is then greater and this would further penalise the mean values $\overline{R}$ as can be seen in Table III. We have even found values of t above 30% of $\overline{T}$ (figure 3, reproduced after [3], figure 2a).

The nominal section IPE 160 has been used for $\overline{A}$. The deviation a% of 4% has been experimentally obtained but the deviations of 2% and 3% have also been adopted for this investigation.

The partial derivatives have been extracted, for different slenderness ratios, from figures in report [2] and more precisely:

$\frac{\partial P}{\partial R}$ from the curve with initial deflection F = 1/1,000 (fig. 2)

$\frac{\partial P}{\partial T}$ by comparing the curves for F = 1/1,000 of both figures with and without residual stresses T (fig. 1 and 2)

$\frac{\partial P}{\partial F}$ by comparing the curves for F = 1/1,000 and F = 1/500 (fig. 2)

$\frac{\partial P}{\partial A}$ limit buckling stresses obtained from the curve with F = 1/1,000 (fig. 2)

These derivates are recorded in Table II.

TABLE II - Partial derivates of buckling load P of the column in relation to: elastic limit R, residual stress T, initial deflection F, section area A

λ	$\frac{\partial P}{\partial R}$ mm²	$\frac{\partial P}{\partial T}$ mm²	$\frac{\partial P}{\partial F}$ kg	$\frac{\partial P}{\partial A}$ kg/mm²
		$\overline{R}$ = 26,70 kg/mm²		
0	+ 2 010	0	0	+ 26,70
55	+ 1 350	— 580	— 2 950 × 10³	+ 23,20
75	+ 905	— 580	— 4 850 × 10³	+ 19,75
95	+ 455	— 330	— 4 300 × 10³	+ 15,90
105	+ 135	— 83	— 3 750 × 14³	+ 14,15
		$\overline{R}$ = 28,25 kg/mm²		
0	+ 2 010	0	0	+ 28,25
55	+ 1 350	— 562	— 3 400 × 10³	+ 24,30
75	+ 905	— 482	— 5 100 × 10³	+ 20,50
95	+ 455	— 241	— 4 300 × 10³	+ 16,25
105	+ 135	— 83	— 3 700 × 10³	+ 14,30
		$\overline{R}$ = 30 kg/mm²		
0	+ 2 100	0	0	+ 30,00
55	+ 1 430	— 652	— 3 900 × 10³	+ 25,50
75	+ 975	— 650	— 5 450 × 10³	+ 21,30
95	+ 460	— 190	— 4 550 × 10³	+ 16,55
105	+ 150	— 93	— 3 650 × 10³	+ 14,40

CALCULATIONS AND RESULTS ANALYSIS

The values of α and $R_1 = \bar{R} - \alpha r$ have been calculated for various slenderness ratios, for different combinations of the Standard Deviations r, t, f, a and for three values of $\bar{R}$ (26.70 - 28.25 - 30 kg/mm^2). The results, given in Table III, indicate that:

a) α increases when the SD t% and f% are reduced and the value of $R_1 = \bar{R} - \alpha r$ to be adopted for the buckling curves is thus also reduced; this is the reason for chosing t and f smaller than the actual values.

b) α increases with r% and $R_1 = \bar{R} - \alpha r$ decreases to equality with $\bar{R}$; for the cases referred to in Table III, however, $\bar{\bar{R}} = \bar{R} - 2r$ being fixed at 24 kg/mm^2, r% increases with $\bar{R}$ and R_1 follows suit (see column 6 of Table III).

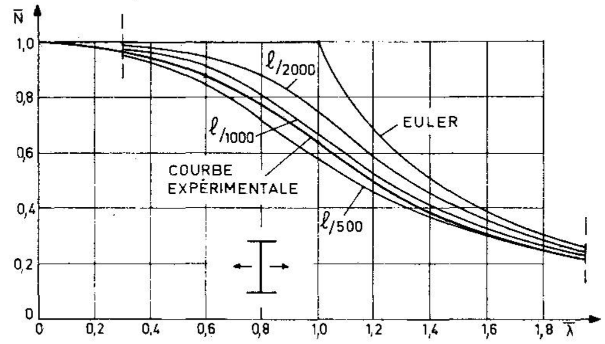

Fig. 1 - Theoretical curves, non-dimensional, for IPE 160, without residual stresses. Buckling about the minor axis

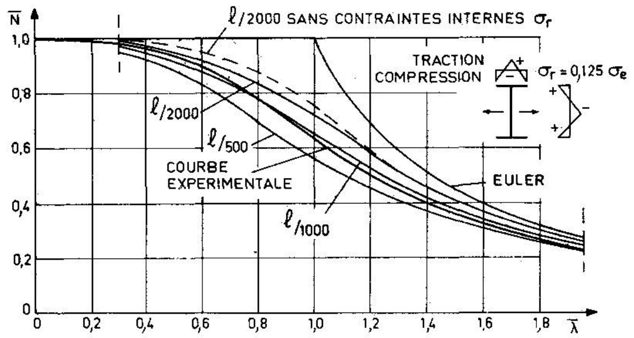

Fig. 2 - Theoretical curves, non-dimensional, for IPE 160, with residual stresses.

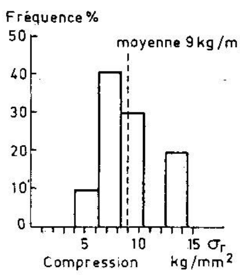

Fig. 3 - Diagram of distribution of residual stresses σ_r measured in flanges of sections.

c) when a% increases, α can either increase or decrease.

d) the R_1 values, in the last column of Table III, are the elastic limits to be used for the buckling curves which give loads with a 2.3% probability of not being reached. R_1 varies with the slenderness ratio, but this variation is fairly small, especially in the slenderness ratio range between 50 and 90 which concerns the constructor.

For steel with r% = 7.5%, i.e. with the most probable value of the SD (according to 4 , fig. 8, 9, 10, etc.) R_1 is almost consistently higher than 25.5 kg/mm^2 for $\lambda > 55$. For r% = 5% we have $R_1 > 25$ kg/mm^2 and for steel with r% = 10% we then have $25.5 < R_1 < 26.5$ kg/mm^2 for $\lambda > 55$.

There results an R_1 value systematically and clearly above $\bar{R} = \bar{\bar{R}} - 2r = 24$ kg/mm .

If $\bar{\bar{R}}$ is above 24 kg/mm^2, as the tests of the "Convention Européenne" have confirmed, R_1 will be even higher and be above the values given in Table III for the equality between r%, t%, f% and a%.

The values of R_1 adopted by the "Convention Européenne" for establishing the buckling curves are sufficiently near the values of Table III, in particular those found for steel with $\bar{\bar{R}} = 28.25$ kg/mm^2 and r% = 7.5%.

It is worth recalling that the overall compression values of $\bar{\bar{R}}$ found experimentally have been consistently higher than those measured in tension for open sections.

TABLE III

λ (1)	r/t/f/a % (2)	α (3)	$\bar{\bar{R}}$ kg/mm^2 (4)	$\bar{R} = \bar{\bar{R}} - 2r$ kg/mm^2 (5)	$R_1 = \bar{\bar{R}} - \alpha r$ kg/mm^2 (6)
0	5/10/10/2	1,54	26,70	24	24,62
0	5/15/15/2	1,54	26,70	24	24,62
0	5/15/15/3	1,46	26,70	24	24,73
0	5/20/20/3	1,46	26,70	24	24,73
0	5/20/20/4	1,42	26,70	24	24,78
0	7,5/10/10/2	1,64	28,25	24	24,78
0	7,5/15/15/2	1,64	28,25	24	24,78
0	7,5/15/15/3	1,54	28,25	24	25,--
0	7,5/20/20/3	1,54	28,25	24	25,--
0	7,5/20/20/4	1,48	28,25	24	25,11
0	10/10/10/2	1,70	30,—	24	24,9
0	10/15/15/2	1,70	30,—	24	24,9
0	10/15/15/3	1,61	30,—	24	25,18
0	10/20/20/3	1,61	30,--	24	25,18
0	10/20/20/4	1,54	30,—	24	25,38
55	5/10/10/2	1,28	26,70	24	24,97
55	5/15/15/2	1,21	26,70	24	25,06
55	5/15/15/3	1,19	26,70	24	25,09
55	5/20/20/3	1,15	26,70	24	25,15
55	5/20/20/4	1,16	26,70	24	25,14
55	7,5/10/10/2	1,39	28,25	24	25,29
55	7,5/15/15/2	1,33	28,25	24	25,44
55	7,5/15/15/3	1,27	28,25	24	25,55
55	7,5/20/20/3	1,23	28,25	24	25,65
55	7,5/20/20/4	1,21	28,25	24	25,69
55	10/10/10/2	1,49	30,--	24	25,53
55	10/15/15/2	1,43	30,--	24	25,72
55	10/15/15/3	1,36	30,--	24	25,92
55	10/20/20/3	1,31	30,--	24	26,07
55	10/20/20/4	1,27	30,—	24	26,20
75	5/10/10/2	1,15	26,70	24	25,15
75	5/15/15/2	1,09	26,70	24	25,23
75	5/15/15/3	1,09	26,70	24	25,22
75	5/20/20/3	1,06	26,70	24	25,27
75	5/20/20/4	1,08	26,70	24	25,24
75	7,5/10/10/2	1,26	28,25	24	25,58
75	7,5/15/15/2	1,19	28,25	24	25,73
75	7,5/15/15/3	1,16	28,25	24	25,79
75	7,5/20/20/3	1,12	28,25	24	25,88
75	7,5/20/20/4	1,12	28,25	24	25,88
75	10/10/10/2	1,36	30,--	24	25,92
75	10/15/15/2	1,28	30,--	24	26,16
75	10/15/15/3	1,23	30,—	24	26,31
75	10/20/20/3	1,18	30,—	24	26,46
75	10/20/20/4	1,16	30,—	24	26,53
95	5/10/10/2	1,11	26,70	24	25,21
95	5/15/15/2	1,07	26,70	24	25,25
95	5/15/15/3	1,11	26,70	24	25,21
95	5/20/20/3	1,09	26,70	24	25,23
95	5/20/20/4	1,13	26,70	24	25,18
95	7,5/10/10/2	1,17	28,25	24	25,77
95	7,5/15/15/2	1,12	28,25	24	25,88
95	7,5/15/15/3	1,12	28,25	24	25,87
95	7,5/20/20/3	1,10	28,25	24	25,93
95	7,5/20/20/4	1,12	28,25	24	25,88
95	10/10/10/2	1,24	30,--	24	26,27
95	10/15/15/2	1,29	30,--	24	26,44
95	10/15/15/3	1,16	30,—	24	26,52
95	10/20/20/3	1,13	30,--	24	26,61
95	10/20/20/4	1,13	30,--	24	26,61
105	5/10/10/2	1,22	26,70	24	25,05
105	5/15/15/2	1,21	26,70	24	25,06
105	5/15/15/3	1,27	26,70	24	25,—
105	5/20/20/3	1,25	26,70	24	25,--
105	5/20/20/4	1,29	26,70	24	24,95
105	7,5/10/10/2	1,18	28,25	24	25,76
105	7,5/15/15/2	1,16	28,25	24	25,78
105	7,5/15/15/3	1,22	28,25	24	25,67
105	7,5/20/20/3	1,20	28,25	24	25,70
105	7,5/20/20/4	1,25	28,25	24	25,60
105	10/10/10/2	1,15	30,--	24	26,56
105	10/15/15/2	1,13	30,--	24	26,62
105	10/15/15/3	1,17	30,--	24	26,49
105	10/20/20/3	1,15	30,--	24	26,54
105	10/20/20/4	1,20	30,--	24	26,41

CONCLUSIONS

For the reasons given above, and in particular for preserving an even degree of safety over the various parts of a structure, whatever the type of loading, the "Convention Européenne" has deemed it appropriate to adopt, for relatively thin sections (thickness < 20 mm), elastic limits higher than the minimum values guaranteed by conventional tensile tests.

This decision, which may appear audacious and strange at first sight, does not introduce an entirely new principle in structural design. Indeed, this same principle has been applied for a long time, if not from the inception, when we are dealing with applied loads, and there seems to be no valid reason for not accepting it for strength of materials.

When loads act separately, we take from each the maximum value that can be anticipated. When several statistically independent loads work in unison, we apply reduction coefficients to the maximum loads adopted when each was acting on its own. In the same way, it is fairly logical in a complex phenomenon depending on several independent variables (R, T, F and A) such as buckling to admit that these variables do not all possess simultaneously the most unfavourable values amounting to the minimum or maximum values imposed by inspection regulations, or adopted for investigating less complex phenomena. For phenomenon less complex than buckling, it does, however, appear that a single variable can play a determining role which must be counteracted by taking a mean value less 2 Standard Deviations and not merely less $\alpha < 2$.

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