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DEVELOPMENTS IN CZECHOSLOVAK SPECIFICATIONS FOR STEEL COLUMN DESIGN

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ABSTRACT

The current Czechoslovak design rules are based on Limit States Design philosophy. An "actual" column is considered, all initial imperfections being expressed by an equivalent initial curvature. The limiting state is given by that load which brings about an onset of yielding in the column. The revision of the Czechoslovak Specifications, which is under preparation, is likely to perform the first step from the deterministic concept to the probabilistic one. Two column curves, corresponding to two groups of steel profiles, are proposed.

1. INTRODUCTION

The current specifications for structural steel design in Czechoslovakia are based on Limit States Design philosophy, which was introduced in the entire area of civil engineering. The basis of the design concept is formulated in the document ČSN 73 0031 /1/, being common for all structural mateirals. According to it, the designer must consider two limit states :

(i) Limit State of Strength

This requires that the "maximum" loading effect shall be less or (at least) equal to the defined "minimum" strength of the structural element under consideration.

(ii) Limit State of Deformation

Requiring that the deformations, vibrations etc., corresponding to normal service conditions, shall be within permissible limits of serviceability. The objective of this contribution is to comment on the development of the criteria for the design of steel columns, related to the revision of the document ČSN 73 1401 "Structural Steel Design", which is under preparation.

2. COLUMN STRENGTH IN THE CURRENT CZECHOSLOVAK SPECIFICATIONS ČSN 73 1401

The ČSN specifications regard loading and resistance functions as independant variables, and define the "maximum" load effect and the "minimum" strength of the structural element under consideration /3/. The design procedure for steel columns is schematically demonstrated in Fig. 1.

For a particular slenderness ratio λ_1, the axial stress δ_V corresponding to the defined "maximum" axial force N_V (effect of factored loads, considering the simultaneous effect of several live loads) must be less or at least equal to the defined "minimum" load-carrying capacity of the column under consideration (curve y in Fig. 1).

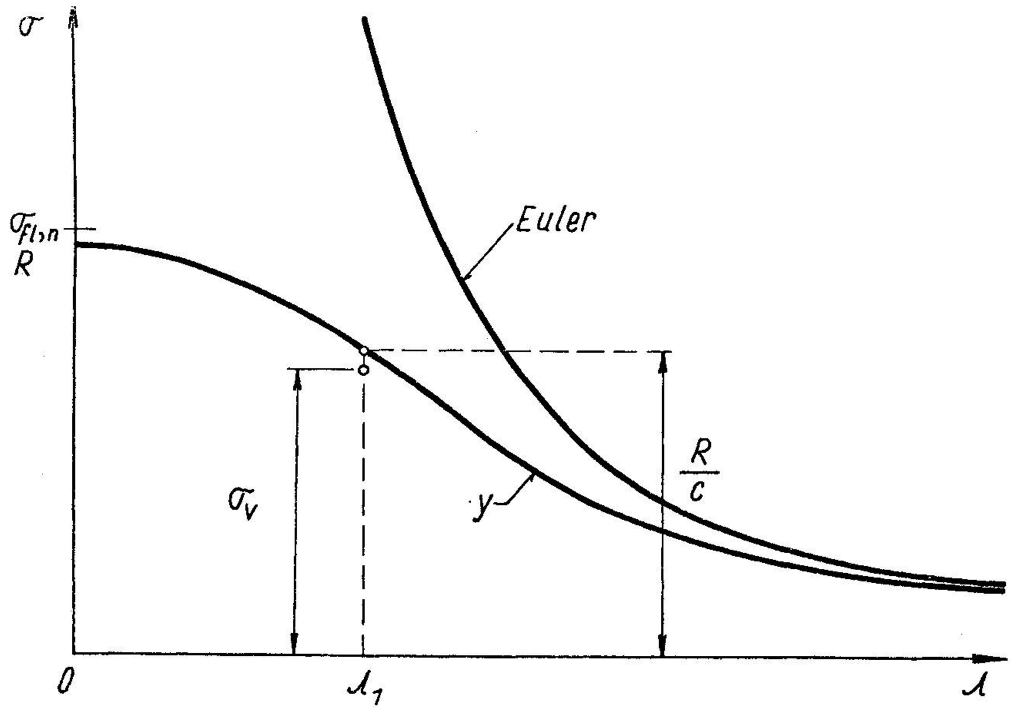

Fig. 1

The latter is related (i) to the s.c. "design stress" $R < \sigma_{fl,n}$ (where $\sigma_{fl,n}$ denotes the normative yield point stress, obtained from a statistical analysis of a population of experimental data with due regard to the selected probability, and considering the chance of underrolling of the column section), and (ii) to a "buckling coefficient" c. This can be written as follows :

$$\frac{N_V}{F} = \sigma_V \leq \frac{R}{c} = \frac{k\,\sigma_{fl,n}}{c} , \qquad (1a)$$

or

$$c\,\frac{N}{F} = c\,\sigma_V \leq R = k\,\sigma_{fl,n} . \qquad (1b)$$

F designates the area of the column cross section ; k is a non-homogeneity factor, relating R to the specified yield stress $\sigma_{fl,n}$.

3. BUCKLING COEFFICIENT c

The derivation of the "buckling coefficient" c is based (following Dutheil's concept) on the assumption of an "actual column", all initial imperfections being expressed by means of an equivalent initial curvature. The limiting state of the column is then given by that axial force which brings about the onset of yielding in the most loaded fibres of the column :

$$\bar{\sigma}_0 \left(1 + \frac{m_0}{1 - \frac{\bar{\sigma}_0}{\sigma_{cr}}} \right) = \sigma_{fl} , \qquad (2)$$

$\bar{\sigma}_0 = N_0/F$, $\sigma_{cr} = \pi^2 E/\lambda^2$, σ_{fl} denotes the yield stress and $m_o = e_o/j$; e_o designating the amplitude of the initial curvature and j the core radius of the cross section.

In the currently held edition of ČSN, the non-dimensional magnitude of m_o has been selected as a function of the second power of the slenderness ratio λ :

$$m_0 = 0.3 \left(\frac{\lambda}{100} \right)^2 = \frac{622}{\sigma_{cr}} , \qquad (3)$$

the aim being to allow for the fact that the load-carrying capacity of slender columns is more affected by initial imperfections than that of bulky ones /4/.

Then, by definition, the buckling coefficient

$$c = \frac{\sigma_{fl}}{\bar{\sigma}_0} ; \qquad (4)$$

$\bar{\sigma}_0$ follows from Eg. (2) :

$$\bar{\sigma}_0 = \frac{1}{2}\left[\sigma_{fl} + (1+m_0)\sigma_{cr}\right] - \sqrt{\frac{1}{4}\left[\sigma_{fl}(1+m_0)\sigma_{cr}\right]^2 - \sigma_{fl}\,\sigma_{cr}} . \qquad (5)$$

4. TREND OF THE REVISION OF THE CZECHOSLOVAK DESIGN CONCEPT

The strength of a steel column depends on several random variables ; such as mechanical properties of the column material, residual stresses, crookedness, rate of loading etc. While the deterministic concepts (one of which forms the basis of the currently held Czechoslovak design rules) express the effect of all variables by just one or two factors, the probabilistic approach, applied recently by ECSSA, results from a large-scale experimental program and statistical evaluation of obtained data. The column curve (or curves) in the latter case is only a mathematical description of the "boundary" line defining the "minimum" strength.

It is the authors' opinion that more attention ought to be paid to the probabilistic concept in the design of steel columns in Czechoslovakia in the near future /6, 7/. Nevertheless, it is likely that the revision of ČSN 73 1401 under preparation will perform only a first step in this direction /8, 9/ ; this being due to the lack of experimental data relating to Czechoslovak steel profiles.

The main features of the aforesaid revision, proposed by Chalupa /9/ and the Czechoslovak Permanent Committee "Steel Structure Specifications", are as follows :

(i) The revised column curves will again be based on Dutheil's concept, a second-power relationship for m_o being introduced (*). Instead of (3), however, the formula

$$m_0 = 0.26\,\frac{\sigma_{fl}}{\sigma_{cr}} = 0.26\left(\frac{\lambda}{\lambda_{fl}}\right)^2 \tag{6}$$

is going to be proposed. The introduction of $\lambda_{fl} = \pi\sqrt{E/\sigma_{fl}}$ enables the designer to use the formula for different steel grades.

(ii) Eq. (2) can be rewritten in the following way :

$$\left(\frac{\sigma_{cr}}{\bar{\sigma}_0} - 1\right)\left(1 - \frac{\bar{\sigma}_0}{\sigma_{fl}}\right) = \bar{a}. \tag{7}$$

$\bar{a}$ denotes a "buckling characteristic", reflecting the interaction of σ_{cr} with σ_{fl}, which makes it possible to introduce more than one column curve.

Two column curves are proposed in the draft of the new ČSN ; they relate to

$\bar{a}$ = 0.17 (tubes, etc.)

and

$\bar{a}$ = 0,26 (other sections).

The reader will note that the aforementioned proposal is very similar to the first and second curves of ECSSA /10/, which correspond to $\bar{a}$ = 0.16 and $\bar{a}$ = 0.27.

(iii) The buckling coefficient c is likely to be presented merely for the most common steel grade 37 (see Table 1), whose yield stress σ_{fl} = 2400 kp/cm^2.

(*) Further consideration to this point will, however, be given. The writers also hope that this question will be discussed at the Paris Colloquium.

For other steels, it can be obtained in the same table, using, however, a reduced slenderness ratio

$$\lambda' = \lambda \sqrt{\frac{\sigma_{fl}}{2400}}$$

So, for example, for steel grade 52 (σ_{fl} = 3600 kp/cm^2 and λ = 100, the value of c is to be found for $\lambda = 100 \sqrt{3600/2400} = 122$.

5. SUMMARY

The currently held Czechoslovak Specifications ČSN - Section "Column Strength" is based on Dutheil's concept, a second-power relationship for an equivalent initial curvature being used.

In the preliminary draft of the Revised Edition of ČSN, two column curves, corresponding to two groups of sections ($\bar{a}$ = 0.17 and $\bar{a}$ 0.26) are proposed. This represents the first step from the "deterministic" concept to the "statistical" approach.

Table 1

λ	Steel grade 37	
	$\bar{a}$ = 0.17	$\bar{a}$ = 0.26
20	1.01	1.01
40	1.04	1.06
60	1.12	1.17
80	1.30	1.41
100	1.66	1.82
120	2.20	2.41
140	2.86	3.13
160	3.66	3.99
180	4.58	4.97
200	5.59	6.07

In the authors' opinion, in a near future, our design rules ought to be related closer to the results of experimental and probabilistic investigations ; more attention being paid to individual variables, particularly to the effect of residual stresses.

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DEVELOPMENT OF MULTIPLE COLUMN CURVES

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ABSTRACT

The introduction of the concept of multiple column curves forms an important step in the direction of improving the method of column strength assessment. This paper presents an investigation carried out in the United States, whereby several sets of multiple column curves were developed using actually measured values of the column strength parameters. The resulting set of multiple column curves has been accepted by the Column Research Council for inclusion in the 3rd Edition of its Guide.

The paper contains the following basic sections:

1) Presentation of a deterministic investigation of the maximum strength of columns, where column curves for a large number of different structural steel shapes were developed and analyzed.
2) Presentation of a probabilistic development of multiple column curves, where the curve for each shape has been developed on the basis of statistical computations of the strength.
3) Development and comparison of the two final sets of multiple column curves resulting from the above two investigations.

1. INTRODUCTION

It is a well-known fact that the strength of a real column and the strength assigned to it by a designer may differ considerably. For several reasons this variation of the column strength has not been taken into account in the formulation of what is commonly referred to as column strength curves , basically because of the general desire of maintaining the design rules as simple as possible. This philosophy now is being reconsidered due to an increasing need for economy, efficiency, and rationale in the design of structures.

A variety of methods aimed at the implementation of more accurate means of column design can be devised. Some of these are related to the manufacture of columns, whereas others are based on theoretical developments (1, 2,). The main problem connected with the manufacture-related approaches lies in the formulation of requirements that will duly consider all pertinent factors, such as the rate of cooling after rolling, which is an extremely complex task. Most important among the theoretically based methods is the one that utilizes several column strength curves, to each of which related column curves, and it was used in a very simple form by the German specification for design of columns (DIN 4114 (1959).

Several investigations on the development and application of the multiple column curve concept have been conducted over the past few years. It is the purpose of this paper to describe the studies conducted in the United States (1,3).

2. THE CONCEPT OF MULTIPLE COLUMN CURVES

The variation of the strength of a number of different column types is strikingly illustrated by Fig. 1, which shows the results of approximately 100 tests with centrally loaded columns. The differences in column strengths are caused by the differences in column shape, steel grade, size, manufacturing method, and so on, but each test point can be predicted within an accuracy of $\pm$ 5 percent. It is evident that the use of a single column curve will significantly over- or underestimate the strength of many columns.

The essence of the multiple column curve concept therefore lies in the fact that no one column curve can represent the strength of all types of columns rationally and adequately. By introducing several curves, to each of which columns of related behavior and strength are assigned, the difference between the assessed and the actual column strength will not be completely eliminated, but rather reduced to an acceptable level. This idea is illustrated by Fig. 2, from which it may be seen that the variation of the strength of the column types assigned to, for instance, the lower of the three curves, is substantially smaller than the variation of the strength of all columns together. Whereas an increase of complexity will be the inevitable result of utilizing several column curves, significant gains may be expected in terms of accuracy and economy. The best solution will be the one where an optimum of complexity and gains has been achieved.

The studies that are reviewed in this paper have been based on two different methods of approach. The first approach deals with a deterministic investigation of the maximum strength, where column curves for a large number of different structural shapes in a variety of steel grades, manufacturing methods, sizes, and so on, have been developed. The second solution is based on a probabilistic computation of the maximum column strength, and for both methods of approach the most important column strength factors have been established

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through systematic and detailed analyses of the results. The findings have been applied towards the development of two sets of three column curves each.

3. DETERMINISTIC DEVELOPMENT OF MULTIPLE COLUMN CURVES

A total of 112 maximum strength column curves have been developed in the deterministic investigation, representing fifty-six different combinations of shape, steel grade, and so on, and with two bending axes considered for each column. An incremental, iterative computer program was developed for the maximum strength calculations (3), and the input-data used for residual stresses, yield stresses, and geometric properties were all measured values. For the initial out-of-straightness, assumed values were used, and the maximum allowable value of 1/1000 formed the bases for the development of the multiple column curves. It has been found that the theoretical column strength predictions agree with experimental column strengths to an accuracy of $\pm$ 5 percent, which must be regarded as satisfactory.

Figure 3 shows the band of all 112 column strength curves developed, using the initial out-of-straightness of 1/1000, and Fig. 4 illustrates the frequency distribution histograms of the maximum strength for a few typical nondimensional slenderness ratios. Only the upper and lower envelope curves for the band are indicated in Fig. 3, since the number and density of the curves between these two limits prevent a meaningful illustration of each separate curve. The width of the band is largest for the intermediate slenderness ratios, and tapers off towards the ends. For low slenderness ratios the variation of the maximum strength is influenced more by the variation of the yield stress than any other factor. Figure 4 indicates that for high slenderness ratios factors such as the residual stress and the yield stress have a decreasing influence, as evidenced by the pronounced kurtosis and skewness of the frequency distributions. In fact, the maximum strength of a very long column will approach the elastic (Euler) buckling load. Other investigations have confirmed these results (4,5).

The limitations on the length of this paper prevent a detailed description of the analyses of the available 112 column curves. Having decided upon a set of 3 curves as the multiple column curves, all of the column data were analyzed with regard to the most important strength factors, and each curve finally assigned to one of the three column strength categories. The results of this study are summarized in Figs. 5, 6, and 7, which show the bands of column curves that contain each of the three multiple curves. Included in these figures are also the statistical characteristics of the band of curves.

The arithmetic mean curves for the three bands of Figures 5, 6, and 7 formed the initial set of multiple column curves, and following some adjustments two further sets were developed. Figure 8 illustrates the second, and Fig. 9 the third set of curves. The multiple column curves of Fig. 9 provide a simplified solution, and from a practical standpoint, one that is easier to use. These curves also take into account the strength-raising effects of strain-hardening for short columns, by originating at the point where $P_{max}/P_y = 1.0$ and $\lambda = 0.15$. For such short columns no overall column buckling will occur.

Also included in Fig. 9 are the data on the types of column that belong to each of the three curves. It should be noted, however, that due to the limited amounts of data that were available on measured residual stresses in various column shapes, several column types have not been included in the study. This is a task of future research projects.

Detailed descriptions of the evaluations leading to the set of multiple column curves are contained in References 1 and 3, together with the mathematical representations of the curves shown in Fig. 9.

4. PROBABILISTIC DEVELOPMENT OF MULTIPLE COLUMN CURVES

The probabilistic studies of the properties of the maximum strength (1) revealed that the random variations of the maximum strength is mainly caused by the variation of the initial out-of-straightness. Based on this knowledge, it therefore stands to reason that the upper limit of the strength of a column (defined as the 2.5 percent probability level) is well described by a column curve based on an initial out-of-straightness of e/L = 1/10,000. Similarly, the lower limit is well described by a column curve for which the basic e/L is equal to 1/1000. These statements hold true for the assumption that the probability density function for the initial out-of-straightness may be described by a Type I asymptotic extreme value distribution.

It was decided to base the probabilistic set of multiple column curves on mean values for all of the column strength parameters, in particular, such that e/L = 1/1470 is the basic out-of-straightness. The use of mean values of the column strength parameters is arbitrary, since any set of data with consistent probabilistic bases may be used.

Having decided on the magnitude of the out-of-straightness, the development ceases to be of a strictly probabilistic nature. This is because in this case the arrival at a set of appropriate multiple column curves only involves the grouping of a large number of probabilistically determined maximum strength curves.

The results of the classification of the column types, together with the final set of possible multiple column curves, are shown in Fig. 10. Mathematical equations for the three curves have been determined (1), and it appears that the two sets of curves illustrated by Figs. 9 and 10 are very similar. The reason for this is of course the fact that only the magnitude of the initial out-of-straightness is different, while all the other column strength factors are the same for both sets of curves. This comparison is further clarified by Fig. 11, where the two sets of multiple column curves are shown in the same diagram.

5. SUMMARY

The following represents a brief summary of the findings presented above :

1. The differences that appear between real and assessed column strengths may be reduced significantly by the use of multiple column curves.

2. The deterministic and the probabilistic studies of the maximum strength of centrally loaded columns both have led to the development of a set of multiple column curves. Both of these are believed to represent an improvement over existing methods of column strength assessment.

3. The probabilistic and the deterministic multiple column curves differ only in that they are based on different values of the initial out-of-straightness. It is believed that the probabilistic curves are a better solution, because mean values of all column strength parameters have been used for consistency.

4. The probabilistic multiple column curves provide for somewhat higher column strengths than the deterministic ones. This is partly due to the smaller value of the initial crookedness, which has effected changes in the classification of the column types.

6. ACKNOWLEDGEMENTS

This study formed part of a research project at Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pennsylvania. The project was sponsored by the National Science Foundation and the Column Research Council. The support of these organizations is sincerely appreciated.

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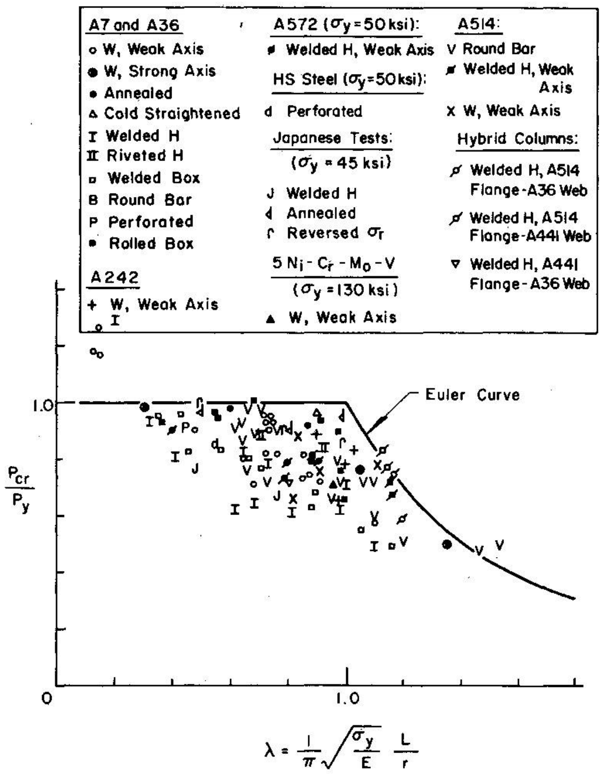

Fig. 1 Results From Approximately 100 Different Tests of Centrally Loaded Columns

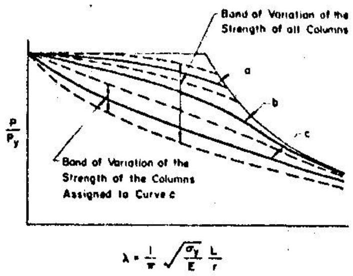

Fig. 2 A Qualitative Illustration of the Variation of Column Strength for Each Multiple Column Curve Category, Compared to the Variation of the Strength of All Columns

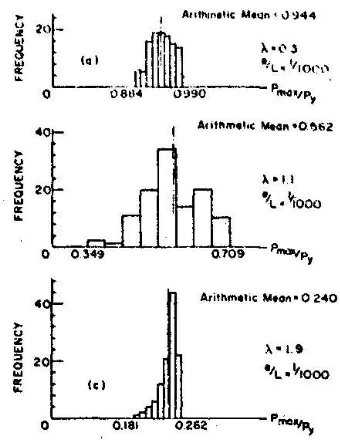

Fig. 4 Typical Frequency Distribution Histograms for the Maximum Strength of All 112 Column Curves (Initial Out-of-Straightness e/L = 1/1000)

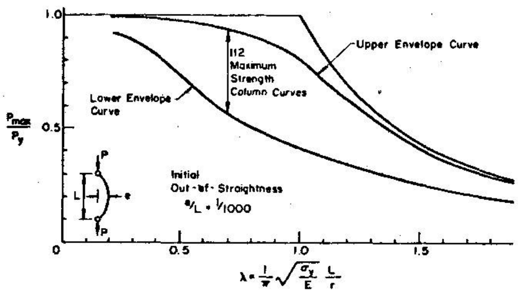

Fig. 3 The Band of All 112 Maximum Strength Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1000

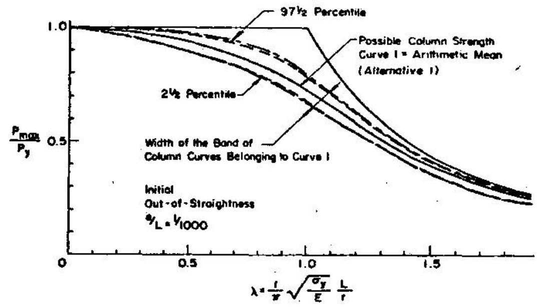

Fig. 5 Possible Column Strength Curve 1, and the Statistical Properties of the Band of Column Curves that Belong to It

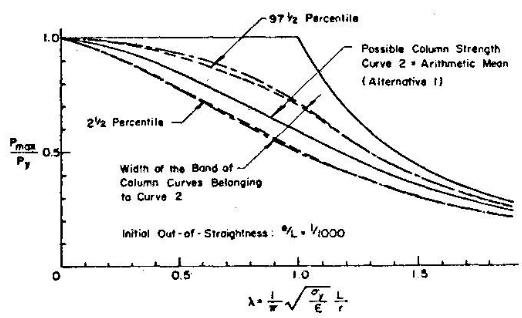

Fig. 6 Possible Column Strength Curve 2, and the Statistical Properties of the Band of Column Curves that Belong to It

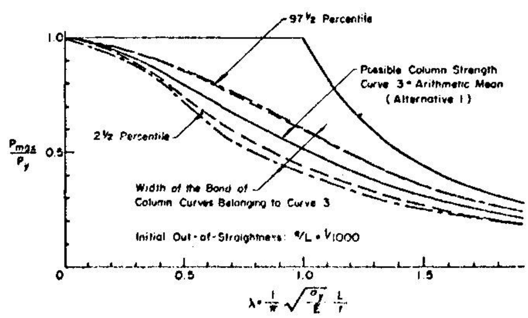

Fig. 7 Possible Column Strength Curve 3, and the Statistical Properties of the Band of Column Curves that Belong to It

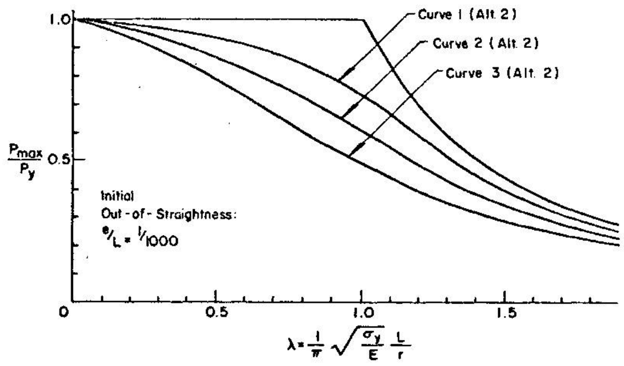

Fig. 8 Modified Maximum Strength Multiple Column Curves, Based on an Initial Out-of-Straightness of 1/1000

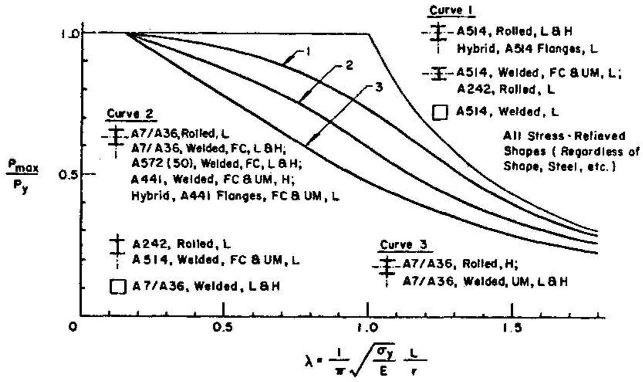

Fig. 9 Simplified Maximum Strength Multiple Column Curves (Initial Out-of-Straightness e/L = 1/1000)

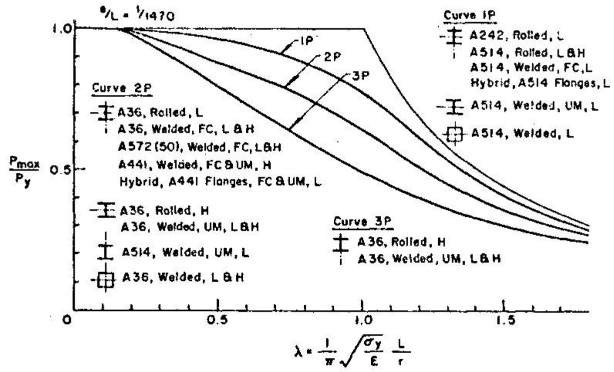

Fig. 10 Possible Maximum Strength Multiple Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1470 = Mean Value (Probabilistic Study)

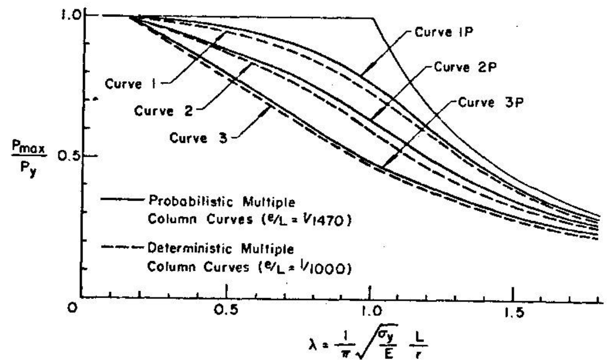

Fig. 11 A Comparison of the Two Sets of Deterministic and Probabilistic Multiple Column Curves

THE EUROPEAN COLUMN CURVES

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ABSTRACT

The column curves adopted by Commission 8 (Stability) of the European Convention of Constructional Steelwork are the result of a comprehensive program of experimental and theoretical investigations. The reduction of the load carrying capacity through geometrical imperfections of the strut, as initial out-of-straightness, and through material inhomogeneities, as residual stresses and scatter of the yield point, were systematically investigated. The theoretically predicted column curves were compared with the results of a statistical evaluation of the column tests and show close agreement.

In addition to the 3 column curves adopted by Commission 8, two more curves, an upper and a lower one are suggested for future use. The upper curve should be used for certain shapes made of high strength steels, and the lower curve for heavy shapes ("Jumbo"shapes).

1. INTRODUCTION

The theoretical analysis of the strength of steel columns guided by Commission 8 (Stability) of the European Convention of Constructional Steelwork had three major objectives:

- The systematic study of the main factors that influence the column strength, as shape of the cross section, geometric imperfections, and material inhomogeneities.
- The selection of representative column strength curves for various column types.
- The comparison of the theoretical column curves with test results.

The computer program developed for the maximum strength analysis considers the structural member with imperfections (Fig.1). Geometric imperfections are an initial curvature of any given form, and end excentricities of the axial load. Besides the axial load there can be additional small transversal loads. Residual stresses and the variation of the yield stress over a given cross section are taken into account when the relationship between axial load, moment, and elastic-plastic bending stiffness is calculated.

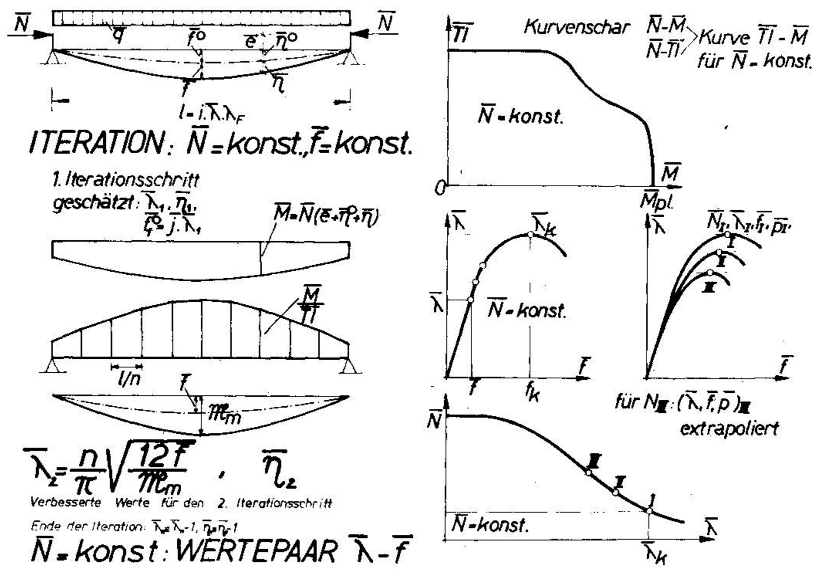

Fig.1: Maximum strength analysis of a structural member with imperfections

It is assumed that the limit of the load carrying capacity is reached, when the equilibrium between external and internal forces changes from a stable to an unstable condition. Since out of practical reasons the calculation is done for a constant axial load N, but a variable slenderness

λ, the limiting state is defined by the vertex in the curve slenderness λ versus central deflection f. For the limit slenderness λ_K the given axial load N becomes then the maximum strength.

The calculation of the column curve axial load N versus slenderness ratio λ starts in the so called "elastic" range and proceeds automatically until the yield load is reached.

The column curves are plotted in the usual non-dimensional diagram. $\bar{N}$ is the axial load N related to the yield load. $\bar{\lambda}$ is the non-dimensional slenderness ratio.

A detailed description of this maximum strength analysis is contained in Ref.(1) and (2).

2. GEOMETRIC IMPERFECTIONS

As a first step of the investigation it was analysed to which extend the unintentional load excentricity due to manufacturing methods can be simulated by an initial curvature with the shape of a half-sine wave.

The measurements on columns of the European experimental program indicated a variety of curvatures. Besides antimetric shapes, some columns had their maximum out of straightness rather in the quarter points than in the center. As Fig.2 shows, such a curvature can be approximated with sufficient accuracy by the first coefficient of a Fourier series. For a curvature with an out of straightness of $\ell/650$ near the quarter points, the substitute half sine-curve has a central bow of $\ell/1000$. The column curves calculated for both curvatures are in good agreement.

Based on this approximation, the substitute central bows for all European test columns were calculated and are shown in Fig.3 as function of the column length ℓ. The most unfavorable values for each column type vary from $\ell/530$ to $\ell/3360$. Also shown in this table are measurements on a truss bridge over the Danube: Here $\ell/1100$ was the largest eccentricity.

Since eccentricity limits are stated in several European building codes, $\ell/1000$ was choosen as reasonable assumption for the central bow of the initial curvature.

To which extend covers an initial curvature with $\ell/1000$ unintentional end eccentricities? (Fig.4) For a wide flange I section the magnitude of the end eccentricity was choosen as function of the radius of inertia i. With i/40, i/20 and i/10, these assumptions cover for this particular section an eccentricity range from 1,25 mm to 4,98 mm. Eccentricities that are unintentional can be expected to fall within the range covered by the first two curves. As Fig.4 shows, the column curve calculated with $\ell/1000$ covers most of these end eccentricities in the slenderness range of practical interest.

How does a variation and in particular a reduction of $\ell/1000$ as central bow effect the column strength. In Fig.5 column curves are calculated for $\ell/2000$, $\ell/1000$, and $\ell/500$ for a wide flange I-section without residual stresses σE, and shown with dashed lines for the same section with residual stresses. The comparison shows that the influence of the initial cur-

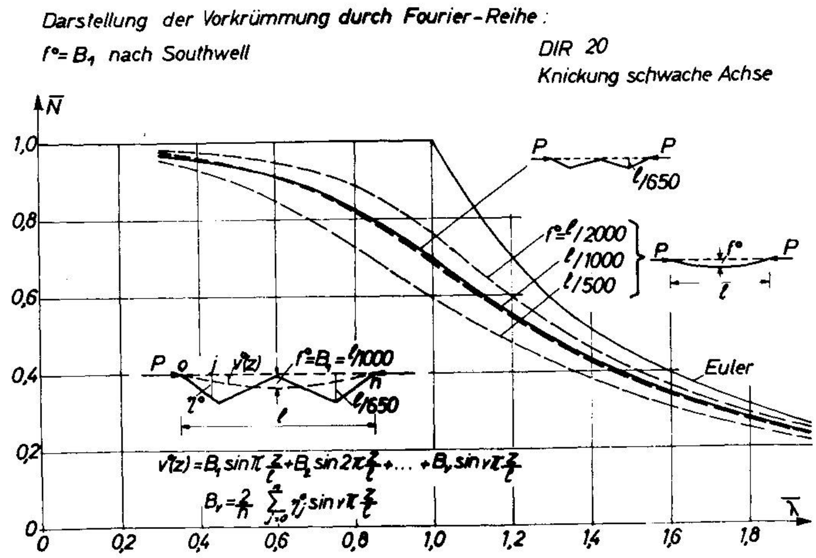

Fig.2: Approximation of an initial out-of-straightness, with maximum in the quarter points, by a Fourier series (I-shape, weak-axis bending)

IMPERFEKTIONEN DER VERSUCHSSTÄBE

Profil	Versuchs durchführung	$\bar{\sigma}^E$	$1/j = l/f°$ Minimum	Mittelwert	Maximum
IAP 150	Belgien Deutschland	≤ 0,125	590	2100	5100
IPE 160	Holland	≤0.3	2700	4400	7200
IPE 200	Belgien	0,15–0,25	1390	3800	10000
DIE 20	"	0,25–0,45	1690	3700	7600
DIR 20	"	≤0,35(0,5)	2260	5800	9500
Igeschw.	"	0,45–0,7	2280	5500	15000
□geschw.	"	≤ 1,0	3360		19000
∅gewalzt	Holland	≤0,5	1450	2800	5900
∅D/t =18	"		3050	7000	33000
∅D/t = 11	Deutschland		550	2000	3700
T 60	Holland		1740	5000	9200
TB 60	Deutschland		530	710	1100
Aufmessung Donaubrücke Ingolstadt			1100	5200	13000

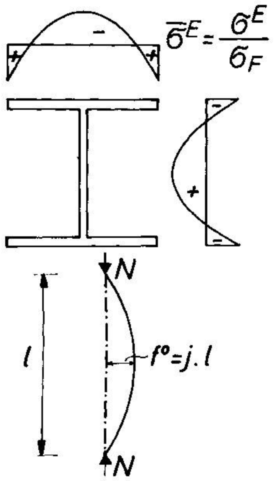

Fig.3: Initial out-of-straightness and residual stress measurements on European test columns

vature decreases with increasing residual stresses. Also, the particular slenderness ratio where the curvature has its maximum influence on the column strength has changed with increasing residual stresses from about $\bar{\lambda}=1,0$ to about $\bar{\lambda}=1,3$.

The reduction of $\ell/1000$ therefore will have a different effect on the column strength, dependent on the column type. For sections with high residual stresses the gain of column strength

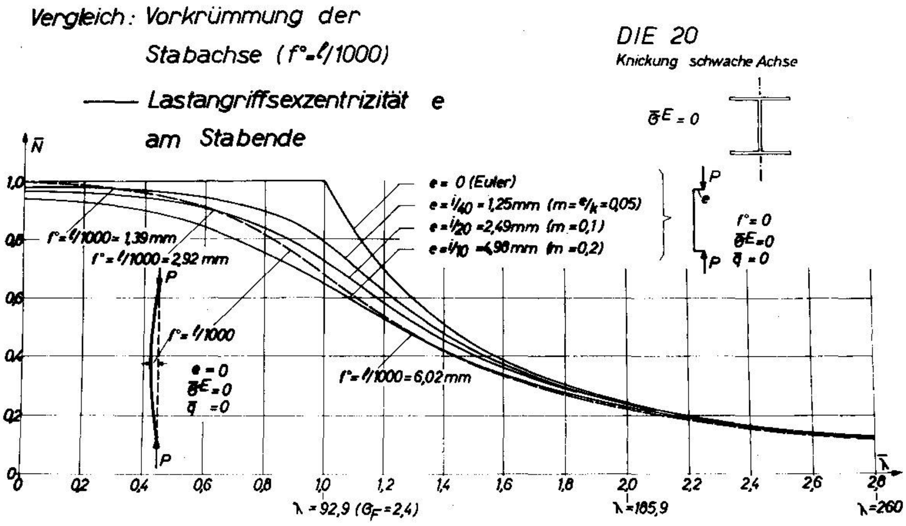

Fig.4: Effect of different geometric imperfections on the column strength (I-section, weak axis bending)

will be small, and will have its greatest influence in a slenderness range that is of no interest for the application of those sections.

All further investigations were based on a column with a sinusoidal initial curvature, with a central bow of 1/1000 of the column length.

3. SHAPE OF CROSS SECTION AND RESIDUAL STRESSES

For a particular column type -as shown in Fig.6 for I-sections- have size and geometry of the cross section no significant influence on the nondimensional column curve, if we assume the sections to be free of residual stresses.

If we consider the residual stresses typical for each of these rolled I-sections we obtain the wellknown wide scatter of column strength (Fig.7). In particular remarkable is the great reduction of the column strength with increasing residual stresses for weak axis bending. For strong axis buckling is the reduction comparably smaller for those sections that have both, compressive and tensile residual stresses in the flange. For sections with compressive stresses in the flange only the strong axis buckling is equal unfavorable as the weak axis buckling.

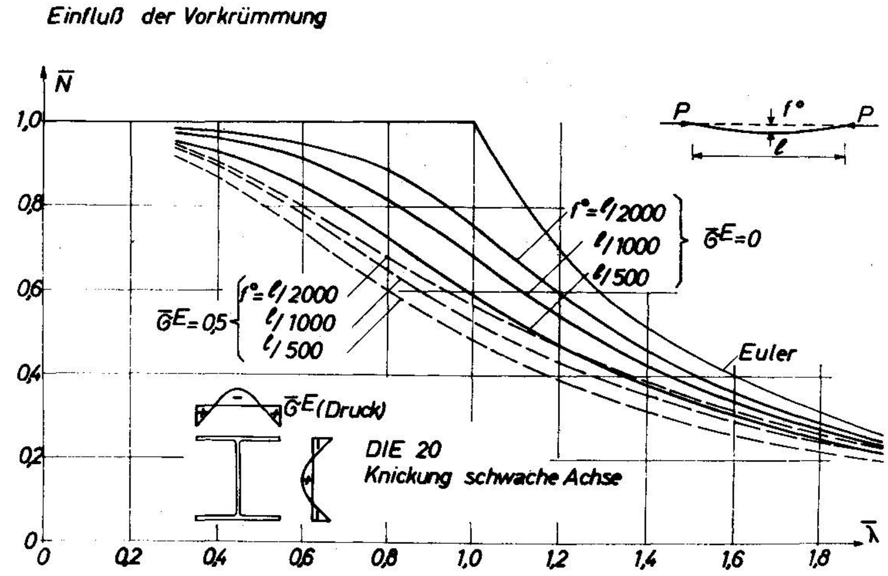

Fig.5: Effect of initial curvature, separately and in combination with residual stresses (I-shape, weak axis bending)

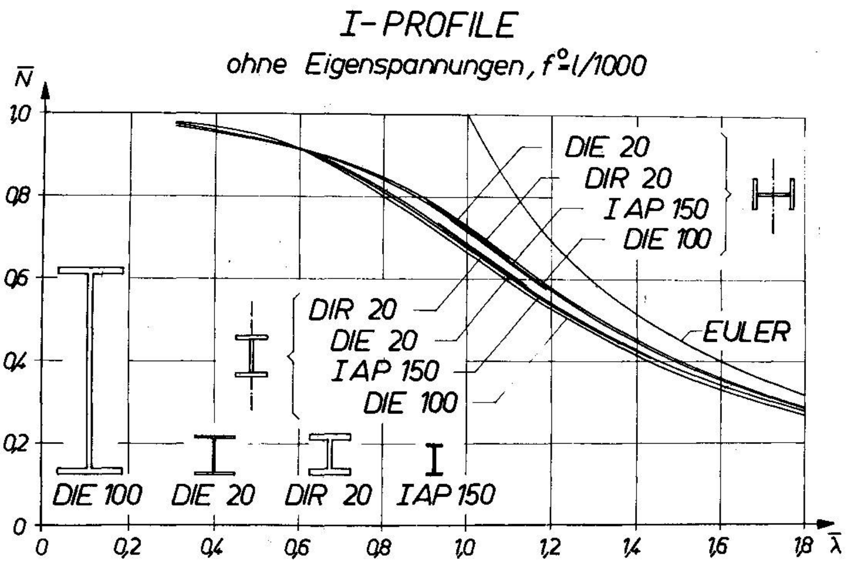

Fig.6: Column strength curves for weak and strong axis bending of I-shapes, including the effect of an initial curvature of ℓ/1000, but without residual stresses

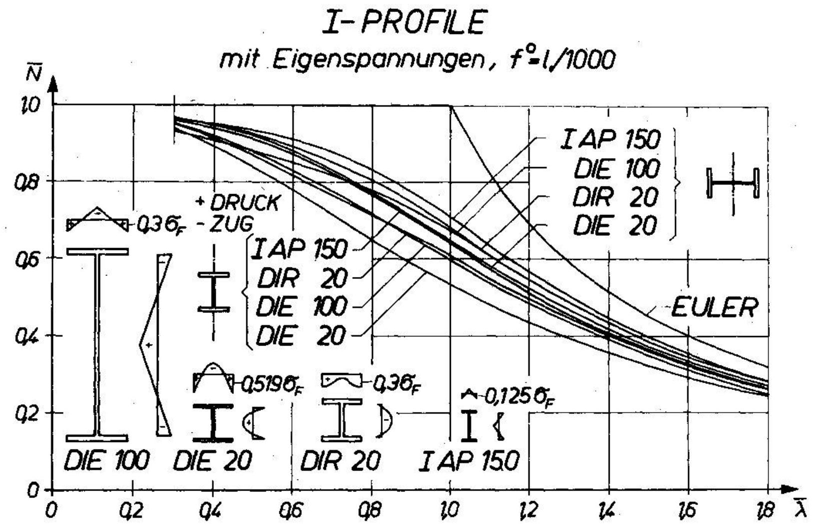

Fig.7: Column strength curves for weak and strong-axis bending of I-shapes, including the effects of an initial curvature of ℓ/1ooo and residual stresses

To obtain typical residual stress distributions for those sections most commonly used, residual stress measurements were collected and analyzed. Main sources were the measurements done by Prof.Massonnet in Liège and the results of the research done at Fritz Engineering Laboratory.

The chart in Fig.8 was developed for rolled I-sections as listed in EURONORM, with a flange thickness up to 4o mm. Since the measurements indicated clearly the dependence of magnitude and distribution of the residual stresses from the section geometry, the sections could be arranged in two groups, dependent of the heights over width ratio h/b smaller or greater than 1,2. h/b = 1,2 is the ratio, where the EURONORM sections change their geometry significantly.

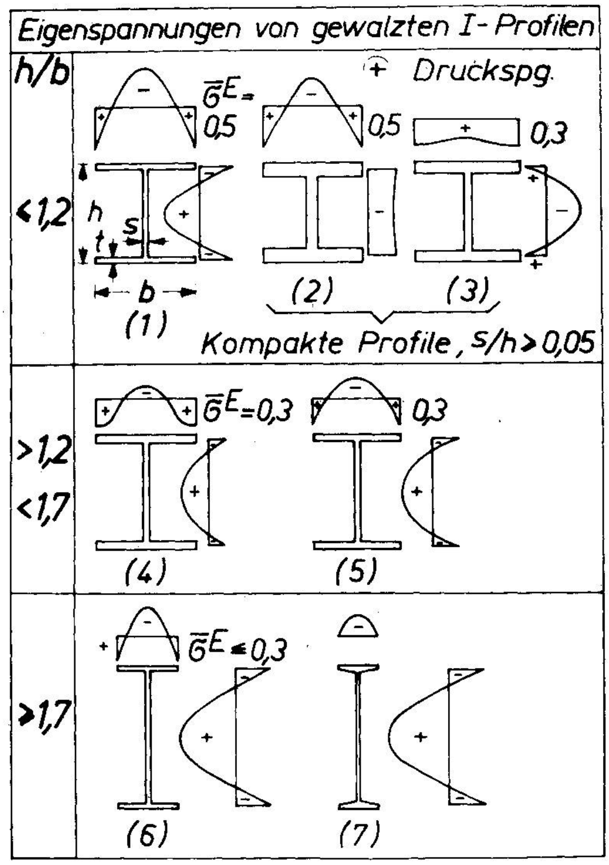

Fig.8: Typical residual stresses for rolled EURONORM I-sections

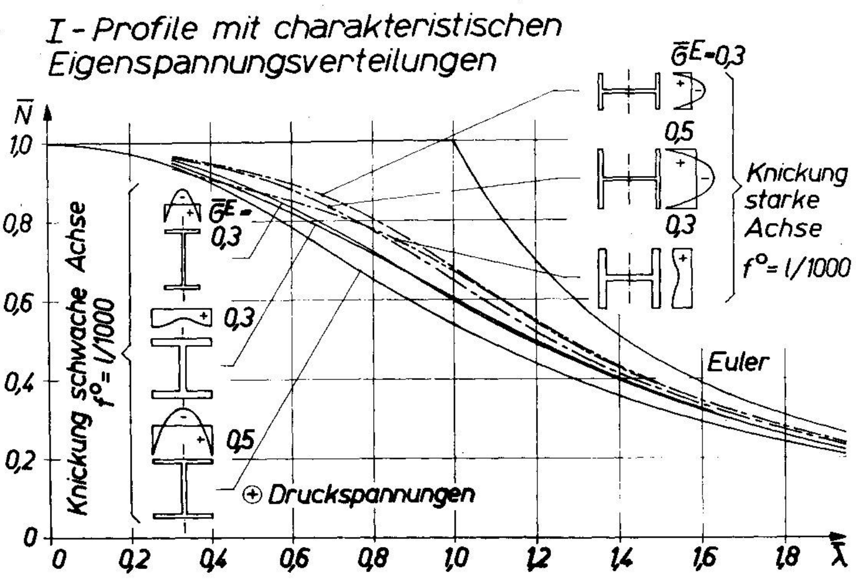

Fig.9: Column strength curves for weak and strong-axis bending of typical rolled I-sections.(Initial curvature ℓ/1ooo, residual stresses)

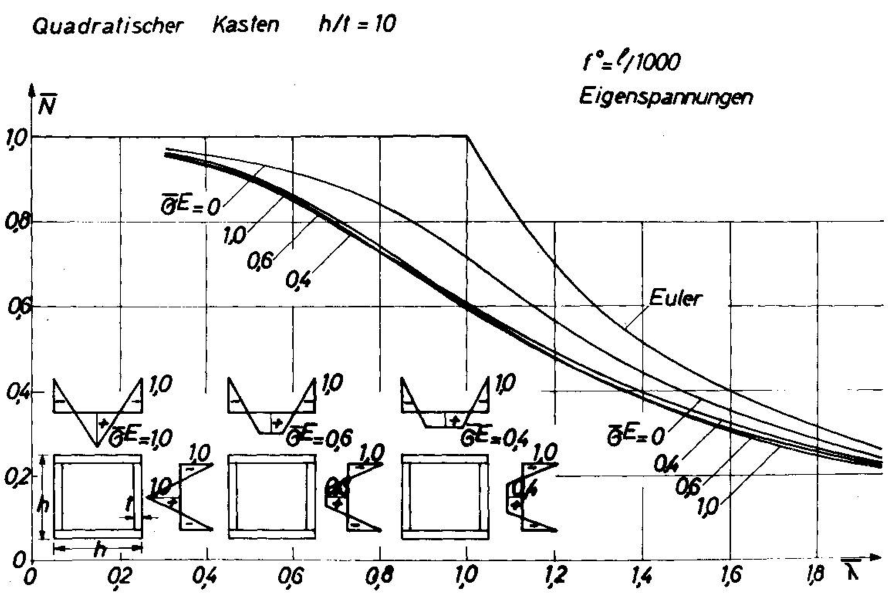

Fig.1o: Column strength curves for welded box sections, including the effects of an initial curvature of ℓ/1ooo and different residual stress patterns

The column curves for typical sections of the two groups of rolled I-sections, calculated for weak and strong-axis bending, are given in Fig.9. Since it was decided by Commission 8 to recognize a difference in column strength of more than 7%, the strength of rolled I-sections can be represented by 3 column curves.

Part of the investigations that led to the selection of representative column curves for the various sections,was to analyze the effect of a possible variation of a residual stress distribution.

For the welded box sections in Fig.1o, the magnitude of the compressive residual stresses in the flange will vary according to the actual size of the section and the amount of welding. This variation had for instance for the residual stress pattern as assumed in Fig.1o no significant influence on the column strength curves.

In Fig.11 this residual stress pattern (marked as E 2) is compared with patterns (E 5 to E 8) that have a wider zone of high tensile residual stresses and recognize the effect of different amounts of welding. They are based on theoretical investigations as outlined in (3), and correspond to box sections with the ratio of height to plate thickness h/t of 2o and 4o, each with a heavy weld (E 5 and E 7) and a light weld (E 6 and E 8). Most of the existing residual stress measurements, done for small and medium sized box sections, actually show patterns that are inbetween E 2 and E 5, E 6, E 7. Pattern E 8 with a width of the compressive stress zone of about o.9 of the section height will be found only in very wide, thinwalled sections, which are not any more subject to column buckling.

Similar investigations were done for welded I-sections, tubes, and T-sections (1) (2).

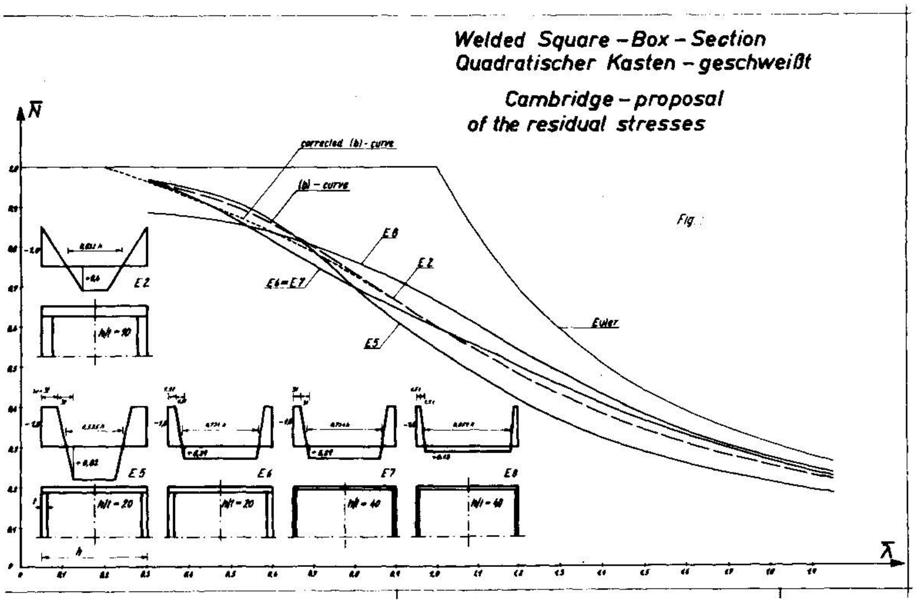

Fig.11: Column strength curves for welded box sections (Initial curvature ℓ/1ooo, residual stresses dependent of section size and amount of welding

4. REPRESENTATIVE COLUMN CURVES

The strength of the most commonly used structural sections could be related to the 3 column curves shown in Fig.12. Curve "a" is the maximum strength column curve for tubes, curve "b" the curve for the box sections, and curve "c" was calculated for the weak axis bending of wide flange I-sections. The other sections are placed in the 3 curves according to Fig.12, as described in detail in (2).

The evaluation of the nondimensional curves "a", "b", and "c" with the appropriate yield stresses is contained in (2), (4) and (5). An approximate method for the design of columns under small transversal loads, based on the curves "a", "b", and "c" is outlined in (6).

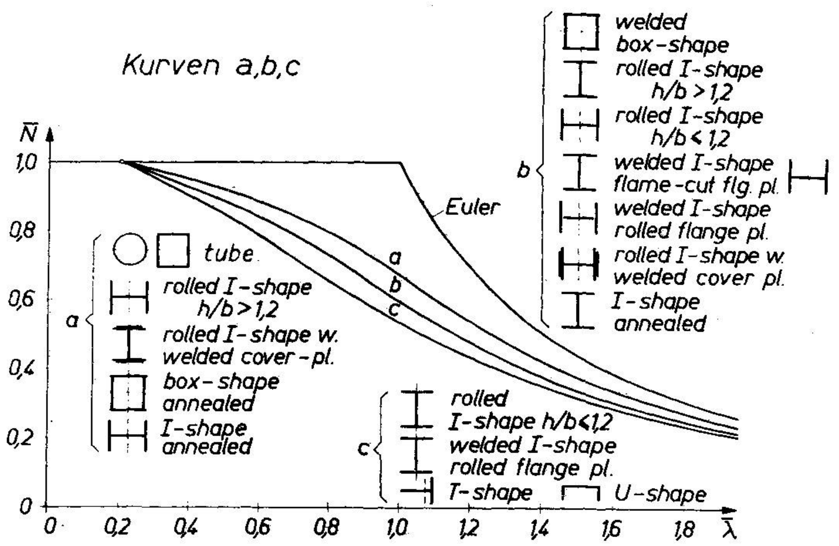

Fig.12: The column curves "a", "b", and "c" recommended by Commission 8 of ECCS

5. COMPARISON WITH TEST RESULTS

The theoretical column curves are in good agreement with the statistically evaluated European column tests (7).

Fig. 13 shows the test results for rolled tubes. The maximum strength as determined out of test results is considered to be the mean value m minus 2 times the standard deviation s. For a comparison curve "a", the theoretical curve for tubes, is evaluated with the statistically determined yield stress of the tubes, and is in good agreement with the test results.

The same good agreement can be shown between the test results for the I-sections IPE 16o and the pertinent curve "b" (Fig.14)

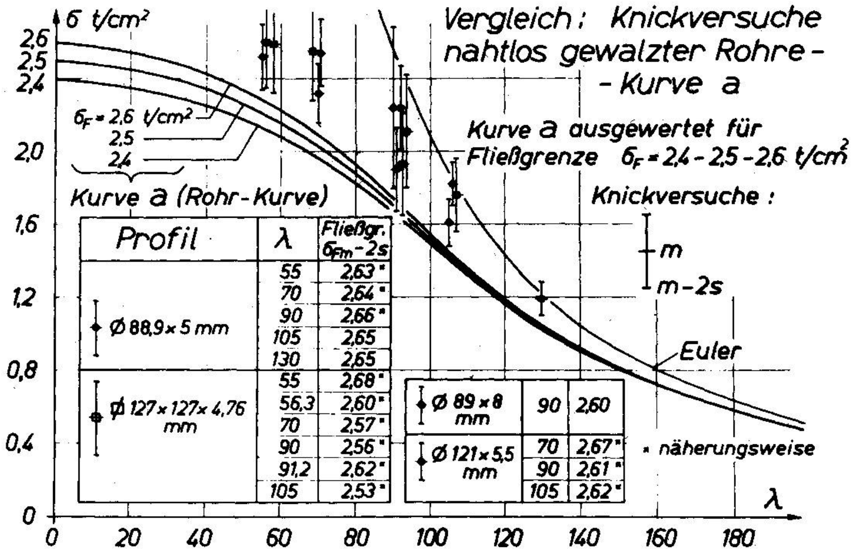

Fig.13: Statistically evaluated tests with rolled tubes and the pertinent column curve "a"

6. FUTURE DEVELOPMENTS

The recommended column curves "a", "b", and "c" are slightly conservative for rolled sections of high strength steel, since the reduced influence of the residual stresses due to the higher yield point was omitted. That the omitted gain in column strength is small for the steel-grades presently used in Europe for most rolled sections, can be shown for the case of the weak axis buckling of an I-section. The column curves in Fig.15 are calculated for gradually increasing residual stresses $\bar{\sigma}^E$. If we relate these residual stresses to the lowest and highest yield stress of steels presently used in Europe, corresponding parameters would be for instance $\bar{\sigma}^E$=o,3 for the lowest yield stress, and $\bar{\sigma}^E$=o,2 for the highest yield stress. Although the weak axis buckling is most sensitive to a variation of the residual stresses, the resulting gain of strength is too small, in order to place the section in the next higher of the column design curves "a", "b", or "c".

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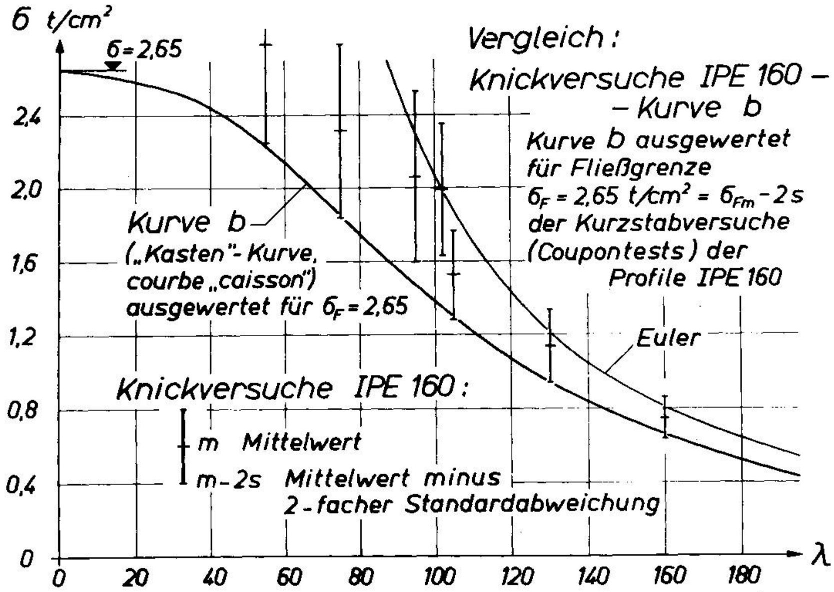

Fig.14: Statistically evaluated tests with I-sections IPE 16o and the pertinent column curve "b"

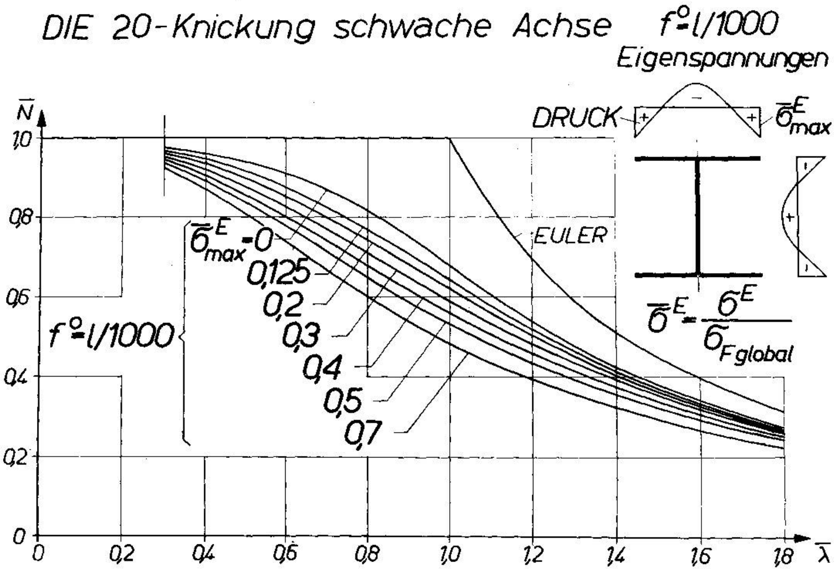

Fig.15: Column strength curves for weak-axis bending of an I-section, including the effects of an initial curvature of ℓ/1ooo and gradually increasing residual stresses

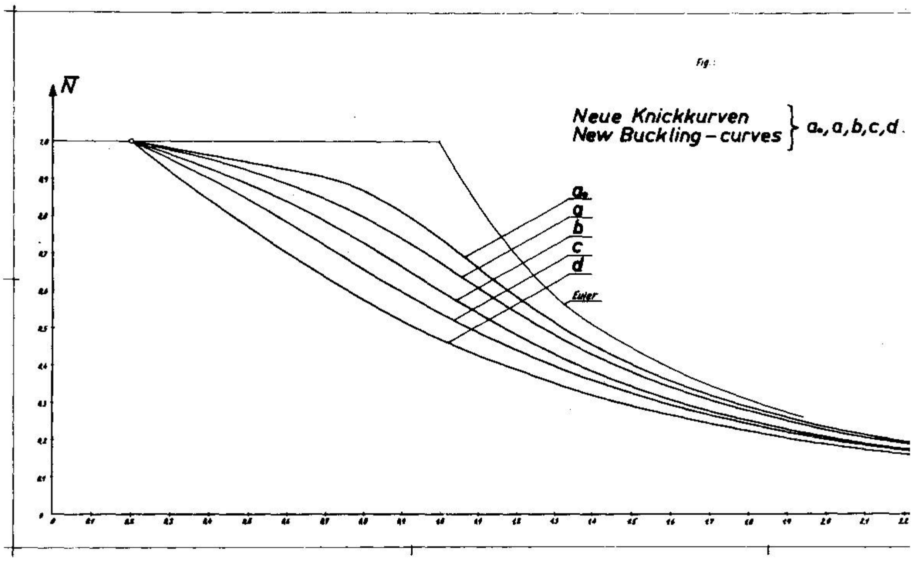

Fig.16: Suggested column curves "a^0" (for tubes of high strength steel) and "d" (for heavy sections, t > 40 mm)

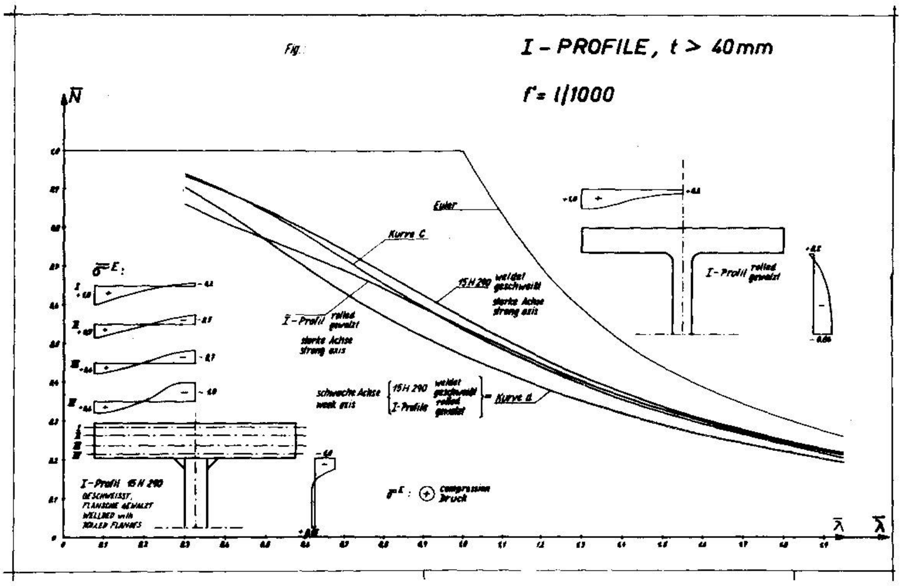

Fig.17: Column strength curves for heavy rolled and welded I-sections. (Initial curvature ℓ/1000 residual stresses)

But since steels with a much higher yield point are used more frequently for tubular members, a curve "a^o" in addition to the curves a, b, c is suggested as future design curve for these sections (Fig.16). Curve "a^o" was calculated as maximum strength curve for a tubular section without residual stresses.

Finally, as an additional provision for future developments in Europe, a curve "d" is suggested for the design of sections with a wall thickness > 4o mm. (Fig.16). Besides in the United States these heavy sections are already rolled in Great Britain, and might be introduced on the Continent in due time. The column strength curves in Fig.17 are calculated for heavy rolled and welded I-sections, and indicate the need for a design curve lower than curve "c".

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USE OF PERRY FORMULA TO REPRESENT THE NEW EUROPEAN STRUT CURVES

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ABSTRACT

The derivation of the Perry-Robertson strut formula is described, along with the other variants which have been used in codes of practice. The limitations in the derivation of the formula are noted.

The evolution of the new European strut curves is summarised, and the report shows how these curves may be represented by a modified Perry formula. The advantages of this representation are noted. Modifications to cater for welded struts and Jumbo rolled sections are described.

1. INTRODUCTION

This report is concerned with the basic design formula for pin-ended steel columns, relating failure stress to slenderness ratio.

Many such formulae are used. The resulting "strut-curves" show a remarkable variation from country to country. Some of the formulae are purely empirical, while others have a degree of theoretical justification. One of those with a more rational background is our own Perry formula, presented in 1886 by Ayrton and Perry[1]. In the form proposed by Robertson[2] (the "Perry-Robertson" formula), following his classic research in the early 1920's, this has formed the basis of British column design for over 40 years, although a variant was introduced by Godfrey[4] in 1962. A slightly different version of the Perry formula is used in France, based on the work of Dutheil[3].

Extensive column testing at Lehigh has shown that different classes of section have significantly different strut-curves[5]. The causes are partly geometrical and partly differences in the locked-in stresses. Members which suffer from severe locked-in compression in their extreme fibres undergo premature yield in these regions when loaded as struts, resulting in reduced flexural stiffness and impaired strength.

A clear indication has emerged that a better approach in codes would be to have several strut-curves for any given steel, with different classes of section allotted to different curves. Thus a Universal Column section, buckling about its minor axis, has a clearly inferior strut performance to a tube, and should be treated accordingly.

Theoretical studies have reinforced this conclusion and multiple strut-curves are likely to be adopted in new European codes. Independent studies at Graz[7] and at Cambridge[10] came up with remarkably similar proposals for such curves. The minor differences have since been ironed out. It seems likely that a Euro-Brittanic strut treatment will emerge.

Three curves are currently proposed. Their derivation has been complex and they cannot be precisely defined by simple formulae. Empirical polynomials have been devised to represent them. This report puts forward a simpler and more rational formula, based on the Perry formula, which has various advantages. The curves are not significantly altered.

Even for one class of section it would be impossible to produce <u>the</u> true strut-curve, which accurately represented the performance of any test specimen in that class. Strut performance is governed by imperfections, which vary between specimens and cause considerable scatter. The proposed new design curves aim to provide a reasonable lower bound on this scatter for each group.

2. FACTORS AFFECTING STRUT STRENGTH

2.1 Material properties

The most important material properties for strut performance are yield stress σ_y and Young's modulus E.

For design purposes the yield stress must be taken as the specification value, depending mainly on the grade of steel and also on product thickness. The range of values specified in BS.4360: Part 2: 1969 for each overall grade is as follows:

	N/mm^2
Grade 43	220 to 280
Grade 50	325 to 355
Grade 55	400 to 450

These values refer to the tensile yield stress obtained from mill-tests at a rather high strain rate. A slower and more appropriate rate of straining would give lower values. However, the compressive yield stress, which is what matters for struts, runs higher than the tensile figure for a given sample. Although these two effects tend to cancel out, we would expect to find that a significant amount of actual production would not reach the specified yield values in slowly conducted compression tests. Despite this, it is unlikely to be practical politics to do anything but adopt the BS.4360 yield values as a basis for strut design.

In I-sections the values specified effectively refer to the flange material, which always has a lower yield than the web material. This is all right, because flange properties are what matter.

Turning to E, Baker[12] records a variation from about 203 to 207 kN/mm^2 for structural steel and has come up with strong recommendation for accepting a figure* of 205 kN/mm^2. This value is likely to be used in the new British Codes. Previous codes have used values of 201 and 210 kN/mm^2. On the Continent there is some support for a high value of 214 kN/mm^2 (= 21.0 kgf/cm^2).

A further material property affecting stocky struts is the strain-hardening modulus E_s. It is seldom quoted, but is believed to be in the range E/40 to E/30. Conventional strut theories ignore the beneficial effect of strain-hardening, but its importance has been clearly demonstrated by workers in the truss field[13,14].

2.2 Imperfections

In the practical range of slenderness, where yield and instability are of comparable importance, the performance of a strut is critically affected by its imperfections, both crookedness and locked-in stress. They obviously vary a lot, and design rules must assume pessimistic values.

BS.4 specifies a straightness tolerance on rolled steel sections of L/960. On the strength of this Young[10] assumed a crookedness of L/1000 in deriving his proposed strut-curves. The Graz workers[7] adopted an identical figure. L/1000 may be a reasonable tolerance for rolled sections, but welded sections could well be more crooked than this, especially if unsymmetrical. The original Merrison appraisal rules for box-girder bridges[15] took a figure as high as L/600 to L/400 for plate stiffener combinations.

In view of the importance of initial crookedness in strut behaviour it is surprising that so little is known statistically about the values which actually occur.

Residual stresses are harder to tie down. For rolled sections the level and the pattern of the locked-in stresses inevitably varies between specimens, even from the same mill. The best documented class is the I-section[9], for which a reasonably well defined trend is apparent provided the rolling practice is taken into account. It is found that column sections tend to carry appreciable compression in the flange toes, typically approaching 100 N/mm^2, which impairs their strut performance. Beam sections do better because their flanges only contain low compressive stresses; webs may carry very high compression, sometimes approaching yield, but this is less important for column buckling.

Young[9] analysed many residual stress measurements for rolled I-sections and produced a pattern for design purposes, representing the most adverse condition

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* 205 kN/mm^2 = 20.8 x 10^3 kgf/mm^2 = 13.3 x 10^3 ton/in^2 = 29.7 x 10^6 lb/in^2.

likely to arise. His pattern depended on the flange-web area ratio, but was independent of the yield stress, suggesting that as the yield stress goes up, the relative importance of the residual stresses goes down. For any given shape of section his assumed residual stresses were independent of the absolute size or thickness. This is believed to be reasonable for the main run of I-sections, but not for the very thick "Jumbo" sections made in America, which can have very severe locked-in stresses.

Little is known about residual stresses in other types of rolled section (channel, angle, tee, bulb-flat, hollow), for which regrettably few determinations have been made. It is supposed that hot rolled hollow sections contain very low residual stresses but this fact has yet to be established.

In sections fabricated from plate, the residual stresses are more predictable in that the shrinkage forces in the welds can be estimated from the size of the weld[9]. This is of limited help, because one cannot reasonably require a designer to perform such calculations. For code purposes one must base design rules on the most adverse residual stresses for a given class of section, bearing in mind likely extremes of weld size. One difficulty is that the shape of the "tension block" in the region of the weld is not properly known. Some workers have assumed a rectangular pattern while others have taken a triangular one, leading to appreciably different strut predictions. The true pattern is probably an intermediate trapezoidal shape.

3. PERRY-ROBERTSON FORMULA

3.1 Basic derivation of Perry formula

The Perry strut formula[1] is based on the following assumptions:

(a) The strut is pinned and centrally loaded at its ends.
(b) It has an initial sinusoidal bow.
(c) There are no locked-in stresses.
(d) It behaves elastically up to failure.
(e) Failure occurs when the stress in the inner extreme fibre reaches yield*.

The resulting equation for the average applied stress σ at failure is:

$$(\sigma_E - \sigma)(\sigma_y - \sigma) = \eta\, \sigma_E \sigma \qquad (1)$$

where σ_E is the Euler stress and σ_y the yield stress.

The non-dimensional quantity η (the "Perry constant") measures the initial out-of-straightness Δ at mid-depth, and is defined thus:

$$\eta = \frac{\Delta}{c} \qquad (2)$$

in which c is the "semi-core" of the section, given by:

$$c = \frac{r^2}{y} \qquad (3)$$

where r is the radius of gyration and y is the distance from the centroid to the yielding extreme fibre.

*Note that for very unsymmetrical cross-sections it is possible, at high slenderness ratios, for tensile yield to occur at the outer extreme fibres before compressive yield is reached at the inner extreme fibres. Equation (1) does not cover this possibility.

3.2 Putting a value to η

The shape of strut-curve which results from Equation (1) is critically affected by the value taken for η, which depends of the assumed initial crookedness and which must obviously vary with the length of the strut. Robertson[2] suggested that η should be assumed proportional to the slenderness ratio, that is:

$$\eta = \alpha \frac{L}{r} \quad \text{.. .. ..} \quad (4)$$

This is equivalent to assuming that the initial out-of-straightness is proportional to the length, a reasonable assumption.

Rather than determine α from measurements of Δ, Robertson adjusted the value to give good agreement with actual strut tests. He found that the curve corresponding to $\alpha = 0.003$ formed a reasonable lower bound to the scatter of results from his own carefully conducted tests and from those of other workers.

The Perry formula, with $\eta = 0.003\ (L/r)$, became adopted for the British Standards covering bridges (BS.153) and buildings (BS.449).

In 1962 Godfrey[4] suggested a new expression $\eta = 0.3\ (L/100r)^2$ .... (5)
BS.449 adopted this change but BS.153 retains the earlier expression. Fig. 1 shows that the BS.449 version gives appreciably higher stresses at $L/r < 100$, and lower ones at $L/r > 100$ than does the BS.153 version.

Godfrey's expression (5) for η is equivalent to assuming that the initial bow Δ is preparational to L^2. The same applies to the version of the Perry formula, due to Dutheil[3], employed in the French code, which in effect uses the following expression for η:

$$\eta = 0.38 \left(\frac{\sigma_y}{250}\right)\left(\frac{L/r}{100}\right)^2 \quad \text{.. .. ..} \quad (6)$$

where the yield stress σ_y is in N/mm^2. This implies that the initial crookedness gets worse as the yield stress increases. It leads to stresses below the Godfrey curve, even for mild steel.

4. FAULTS OF THE PERRY-ROBERTSON FORMULA

The Perry-Robertson formula is the right kind of formula in that it talks about stress magnification, and it has the virtue of simplicity. It should not be regarded as a precise treatment, as it contains a certain degree of empiricism. In making the following criticisms so many years later, the author in no way wishes to belittle the great achievement of Professor Robertson in producing a strut treatment that has been widely used for nearly 50 years.

The assumption that failure occurs when yield is first reached in the extreme fibres is slightly pessimistic. The error depends of the shape factor of the section.

By using this criteria, strain-hardening is ignored, as indeed it is in most strut treatments. This makes the strut-curve dive as soon as it leaves the stess-axis, suggesting that it is never possible to achieve yield in compression, which is not true.

Robertson's adoption of a constant α in equation (4) leads to inconsistency between sections of different geometry.

From equations (2), (3) and (4):

$$\Delta = \eta c = \alpha . \frac{L}{r} . \frac{r^2}{y} \qquad \text{which gives:} \quad \frac{\Delta}{L} = \alpha . \frac{r}{y} \quad \text{.. .. .. ..} \quad (7)$$

Thus with α held constant at 0.003 Robertson's assumed crookedness Δ, as a proportion of the length, depends on r/y and so alters with the shape of the section. The variation is $2\frac{1}{2}$:1 betweeⁿ various sections. In reality one would expect the amount of initial bow to be independent of the section geometry for rolled sections and symmetrical welded ones.

The Perry equation (1) takes no account of locked-in stresses. Using the theoretical value for η, the predicted strength of a strut containing appreciable residual stresses will be too high.

Robertson partly covered this since the design value of $\eta = 0.003\ (L/r)$ was based on tests and corresponds to a fictitious initial bow, greater than L/1000. In effect he made some allowances for residaul stress effect by exaggerating the crookedness, but did not take into account the variation from one class of section to another.

5. NON-DIMENSIONAL PRESENTATION

A popular way of presenting strut data in academic work employs the non-dimensional quantities N and λ, where:

$$N = \frac{\sigma}{\sigma_y} \quad \text{and} \quad \lambda = \frac{L/r}{\pi\sqrt{E/\sigma_y}}$$

so that the Euler curve reduces to:

$$N = \frac{1}{\lambda^2}$$

This supposedly facilitates comparison between results obtained for steels of differing yield stress. There is, however, no logical reason why results obtained for widely varying steels should give identical N-λ curves.

It is perhaps not generally appreciated that the Perry-Robertson formula presented on such an N-λ plot, gives a curve which rises with the yield stress. The Dutheil version, because it makes η increases with σ_y, leads to the same N-λ curve for all strengths of steel.

6. RIGOROUS STRUT TREATMENTS

The Perry-Robertson formula is a simple approach which takes crookedness into account, but ignores locked-in stress. An equally simple type of approach is possible which allows for residual stress, but ignores crookedness[17]. What is required is a treatment which properly considers both.

Such treatments do exist, but are laborious. They are often called after Newmark[16], whose numerical integration procedure they generally employ. First one must compute moment-curvature curves for the section at various levels of axial load, taking the residual stresses into account. These M-ϕ-P curves are then used to obtain the behaviour of any given unstraight strut made of the section concerned. An iterative procedure is used to obtain the correct deflected shape corresponding to a given load P. In this manner a load-deflection curve can be generated.

Young[9,10] used a comparable finite-difference procedure.

Even these painstaking methods are not entirely rigorous, as they consider no reversal of stress, and ignore strain hardening. However, the Newmark (or Young) type of method is a valuable research tool which enables accurate (except at low L/r) strut-curves to be generated for a given section, taking into account initial bow and residual stresses.

7. THE EUROPEAN STRUT-CURVES

7.1 Evolution

The proposed new European strut-curves stem mainly from the work carried out under the late Professor Beer at Graz[7], for Commission 8 of the European Convention for Constructional Steelwork. More recently Cambridge University, supported by the Construction Industry Research and Information Association became involved[10]. The final curves have resulted from interaction between the two teams.

The procedures adopted by Schulz (at Graz) and Young (at Cambridge) were similar. A number of specific sections were chosen for study. A suitable pattern of residual stress was assumed for each, and a rigorous method (Newmark or equivalent) then employed to obtain the strut-curve. This was mostly done for mild steel with an assumed initial crookedness (Δ) of L/1000.

From the range of curves thus obtained each worker then selected a limited number for use as design curves. At Graz three such curves were eventually settled on, and at Cambridge four. In conjunction with each set a selection table or chart was provided, showing which curve to use with any given section.

The theoretical work at Graz was backed up by a massive programme of column testing, carried out in various countries[8]. Because of the variation in the imperfections from one specimen to another there was a good deal of scatter in the results, as is inevitable in strut testing. It was found that for each class of section considered, the proposed design curve formed a good lower bound to the spread of results obtained. The programme therefore provided valuable support in favour of the theoretical curves.

7.2 Plateau at low L/r

The curves as calculated did not allow for strain-hardening, and therefore started to descend immediately on leaving the stress axis. This is not in accord with the fact that a stocky member can reach its squash load, and may well exceed it. To overcome this discrepancy the Cambridge team decided to make an arbitrary adjustment in the region of low L/r, such that the curves would have an initial horizontal portion before starting to descend. The calculated strut-curves then had to be raised to join the end of this horizontal portion.

The extent of the flat part, defined by L/r < S was determined from:

$$\lambda_o = \frac{S_o}{\pi\sqrt{E/\sigma_y}} = 0.2 \quad \text{.. .. ..} \quad (8)$$

giving the following typical values for S_o:

	σ_y	S_o
Grade 43	250 N/mm^2	18
Grade 50	350 N/mm^2	15
Grade 55	450 N/mm^2	13

In arriving at the arbitrary figure of $\lambda_o = 0.2$ in equation (8) some credence was given to the notion that the plateau should extend to point where the Euler curve drawn with E replaced by E_s cuts the line $\sigma = \sigma_y$. In fact the value 0.2 corresponds to $E_s = E/25$, which may be thought a rather high value for E_s, but account must be taken of the pronounced plateaus observed in research on trussels[13,14].

7.3 The Curves

The Graz and Cambridge curves were in good general agreement except at low L/r. After a meeting in Graz to attempt to bridge the differences, it was decided to promote the three European curves but with the British plateau incorportated at low L/r. These proposals are now going forward and will, it is hoped, be adopted in the countries concerned including Britain.

The proposed curves (a,b,c) are shown in Fig.3. They result from actual computations made for mild steel, from which were then determined the non-dimensional N-λ curves given in the figure. The intention is that these non-dimensional curves, although based on mild steel, will still remain applicable as σ_y changes and be used as the basis of design for any grade of steel. The validity of so doing is discussed later.

7.4 Curve selection

Table 1 gives the agreed allocation of rolled sections to the three curves. Welded sections fabricated from plate are discussed later.

The appropriate curve for a given member depends both on the shape of section, and on the axis about which buckling occurs. The factors which push a section onto a low curve are: (a) residual compression in the extreme fibres, and (b) low value of r/y.

In some cases the curve allocation is somewhat tentative, because not enough is known about locked-in stresses. As more information comes to light, there may be some changes in the curve selection table. Even as it is now predictions will generally be an improvement over those obtained from current codes having a single strut-curve for a given steel.

7.5 Empirical formula

Young[10] has proposed the following polynominal formula to represent the non-dimensional strut-curves:

$$\lambda = \sqrt{\frac{C_o}{N} + C_1 + C_2N + C_3N^2} \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (9)$$

Values of the four coefficients, which provide a close fit to the computed European N-λ curves, are listed in Table 2.

This note puts forward an alternative to equation (9).

8. APPLICATION OF PERRY FORMULA TO THE EUROPEAN CURVES

8.1 Required changes

It is proposed that the Perry formula (1) should continue as a basis for strut design. Its method of application will need to be modified to provide a range of curves to cover different section groups and to incorporate a horizontal plateau at low L/r.

8.2 Adjustment of α

A range of curves can be provided by varying α in expression (4). A section which performs well as a strut (low residual stresses, high r/y) should be accorded a low value of α, while a poor performer (unfavourable residual stress, low r/y) should be given a high α.

If there were no residual stresses and if first yield caused immediate failure, the appropriate α would be, from (7): $\alpha = (\Delta/L).(y/r)$. Taking an initial crookedness of L/1000, this becomes $\alpha = 0.001\ (y/r)$ .. .. .. (10).

Ideal values of α range from 0.001 to 0.003. They do not form a true basis for design, because residual stresses have been ignored. For nearly all sections the adverse effect of locked-in stress will more than cancel out the conservative nature of the first yield failure criterion, and increased values of α will be needed.

8.3 Formation of plateau

The need for a plateau up to $L/r = S_o$ (see section 7.2), can be conveniently provided by replacing equation (4) with the following:

$$L/r < S_o \;\ldots\ldots\ldots\ldots\; \eta = 0$$

$$L/r > S_o \;\ldots\ldots\ldots\ldots\; \eta = \alpha\left(\frac{L}{r} - S_o\right) \quad \ldots \ldots \ldots \ldots \ldots \quad (11)$$

The strut is fictitiously taken as being initially straight if its length is less than S_o; it it is longer than this, its initial bow is taken as proportional to $(L - rS_o)$. S_o is given by (8).

Fig. 1 shows the resulting curves for mild steel struts with various values of α, compared with the present British Standards.

8.4 Representation of the European curves

Using the method just described, the Perry strut formula (1) can be readily employed to represent the new European strut-curves. Summarizing:

(i) The basic formula remains as given by equation (1).
(ii) η is now obtained from (11). S_o is assigned the value given by (8).
(iii) For each curve a value of α is selected, to obtain a good fit.

It is suggested that the following values of α should be adopted:

	α
Curve a	0.0020
Curve b	0.0035
Curve c	0.0055

The resulting modified Perry curves for mild steel are shown in Fig. 2, where they may be compared with the European curves plotted from the polynominal expressions (9). The agreement is acceptable.

Some might argue that in the important range $L/r = 40$ to 100 the accuracy of representation of the two upper curves (a,b) could be improved by a slight increase in the values adopted for α. Bearing in mind the many uncertainties in strut prediction and the doubts about the assumed imperfections, the author considers that this would suggest a degree of accuracy that does not really exist and that it would be more sensible to adopt the "round-number" values listed above.

9. TREATMENT OF HIGH YIELD STRESS STEELS

The European proposals consist of a set of three non-dimensional N-λ curves. They result from computations performed on mild steel struts, but are intended to give the necessary σ-L/r curves for design in any steel.

In fact there is no reason why, for a given section, the strut-curves for different grades of steel should all lie on top of each other when shown on an N-λ plot. Even when residual stresses are ignored, the true N-λ curve tends to become raised as the steel gets stronger. This is apparent from Fig. 3.

Use of the same N-λ curve for all yield stresses, would imply increasing initial crookedness as the yield stress goes up, for which there is no justification.

When residual stresses are introduced, the effect becomes more pronounced. It is believed that the absolute level of locked-in compressive stress in a memeber of given section is largely independent of the yield stress of the steel. As the yield stress goes up, the relative importance of the residual stresses therefore goes down. This suggests that the N-λ curve appropriate to a certain section in mild steel will be even further over-safe when employed for design in high yield. This contention is supported by computations performed by Young[10], which show that a Universal Column section (buckling about yy) in Grade 55 steel has a significantly higher N-λ curve than the same section in Grade 43, eqivalent to a rise from his curve C to curve B.

Young, in the Cambridge proposals[10], envisaged that this effect of yield stress would be taken into account in the curve selection procedure; his selection chart enabled a higher curve to be used, when the yield stress was sufficiently high. This suggestion did not find favour with the Europeans, as it was thought to only bring benefit for very strong steels. With Grade 50 steel the improvement, in N-λ terms, was not enough to permit a rise to the next design curve up.

The European N-λ curves (Fig. 3) have therefore gone forward as a proposed basis for design, without any allowance being made for yield stress in the selection table (see Table 1). This arrangement, if finally adopted, will penalise members made of higher yield steels.

The proposed adaption of the Perry formula, summarized in 8.4, attempts to overcome this difficulty and does not penalise the stronger steels to much. Taking a given value for α automatically makes the N-λ curve move up with increasing yield stress - as it should. The proposed values for α (0.0020, 0.0035, 0.0055) have been chosen so as to fit the European curves (a,b,c), when these are applied to mild steel members. For higher yield steels the European treatment becomes increasingly over-safe, whereas the procedure of 8.4 preserves reasonable accuracy. This is apparent from Fig. 3 which shows N-λ plots for three grades of steel, based on the proposed Perry treatment with α = 0.0020, compared with the (unvarying) European curve a. Even so, the theoretical results obtained by Young[10] suggest that for members containing unfavourable residual stresses, as for example a Universal Column buckling about yy, the Perry treatment will still tend to penalise the stronger steels a little.

10. SECTIONS FABRICATED FROM PLATE

The proposed new strut-curves, used in conjunction with Table 1, directly cover rolled sections including I-sections reinforced with flange cover-plates. Members built up from plate, such as welded I- and box-sections are less straightforward. The locked-in stresses caused by welding are not properly understood, the exact pattern in the vicinity of the weld being uncertain. The appropriate strut-curves are therefore not yet clearly determined, but it is apparent that the curves developed for rolled sections do not quite have the right shape.

Theoretical results by Young[10], although based on an over-idealized pattern of residual stress, indicate a characteristic difference between rolled and welded strut-curves. This is shown in Fig. 4, which compares the strut-curve for rolled Universal Column section buckling about yy (i.e. curve c), with those which are believed to be typical for welded sections of similar shape. The essential point is the depression of the welded curves in the earlier part of their range. This is governed by the severity of the residual compressive stress at the toes of the flanges, which will depend on the weld heat input relative to the area of the section.

The European proposals cope with this situation by utilizing for the welded sections a lowered curve based on a fictitiously reduced yield stress. The reduction in σ_y ought strictly to be related to the size of the welds, but in view of the various uncertainties a uniform reduction is made. Thus lightly welded sections tend to be penalized and heavily welded ones favoured. The proposed yield reductions translated into British terms would be as follows, the figure for grade 55 being the author's own extrapolation:

	assumed reduction in σ_y
Grade 43	15 N/mm²
Grade 50	20 "
Grade 55	25 "

Although the compressive residual stress is largely independent of the yield stress for a given size of weld, it is still reasonable to have more reduction with the stronger steels, because the welds will tend to be bigger.

The class of curve to be used, with suitably reduced yield stress, is as follows:

	Curve
Welded I-sections (buckling about xx)	b
Welded I-sections (buckling about yy)	c
Welded box sections	b

In the case of "heavily welded" boxes a further modification is proposed, but it is thought that this will not affect practically designed columns.

It is interesting to note, that when the flanges of a welded I-section are known to be flame-cut from plate instead of being rolled flats, higher stresses are permissible because of favourable tension induced in the flange toes. In this case curve b may be used for xx or yy buckling without any reduction to σ_y.

11. JUMBO SECTIONS

Work at Lehigh[6] has clearly shown that very massive "Jumbo" I-sections (with say 60 mm flanges) can contain far worse residual stresses than do sections of normal thickness. The locked-in compression at the toes can approach yield. Column tests have shown these sections to have an impaired column capacity.

It has been suggested that when the flange thickness exceeds 40 mm, the next lower strut curve should be used. For a Jumbo column buckling about yy a new curve below c would become necessary. In anticipation of Jumbo rolling on this side of the Atlantic, an appropriate curve has been included in the relevent figures of this report, computed for $\alpha = 0.0080$. At present only two section in the British book quality for this curve, if 40 mm is in fact to be the change-over thickness.

A sudden jump to a lower curve could sometimes lead to anomalies, and it might be thought preferable to have a sliding scale for α when the thickness passes to 40 mm.

12. ADVANTAGES OF THE PERRY FORMULA

The advantages of the modified Perry formula are:

(a) It is simpler than the polynomial expression currently proposed.
(b) σ may be expressed in terms of L/r, as well as L/r in terms of σ.
(c) With suitable α values, it fits the agreed curves.
(d) High yield steels are not penalised as much as in the current proposals.
(e) Extra curves may be added by selecting suitable values of α.

13. DESIGN DATA

Design curves and tables are given in the author's full report[18].

Fig. 1 compares existing British Standards with the present proposals. The spread of the proposed curves embraces the present B.S. curves. One hopes that any loss in economy for sections allocated to a low curve will be offset by revisions of load factors.

14. CONCLUSIONS

(1) Several strut curves are necessary for diffining types of section.
(2) The European curves provide a suitable basis for strut design.
(3) The modified Perry formula represents these curves simply and conveniently.
(4) It does not penalise high strength steels as heavily as does the N-λ form.
(5) The selection table (table 1) may be revised as knowledge improves.
(6) More information is needed on crookedness and locked-in stresses.
(7) Further work is needed on welded fabricated struts.

ACKNOWLEDGEMENT

The author would like to pay tribute to the late Professor Beer for his work towards a unified basis of strut design in Europe. He would also like to thank Mr. P.K. Clarkson for plotting the strut-curves in this reoprt. This paper was shortened for publication here by Mr. C.D. Bradfield.

Table 1

CURVE SELECTION TABLE FOR ROLLED SECTIONS

Axis	x-x		y-y	
SECTION	European curve	α	European curve	α
Universal column	b	.0035	c	.0055
Universal beam	a	.0020	b	.0035
UC or UB with cover-plates	b	.0035	a	.0020
Channel or tee	c	.0055	c	.0055
Angle (any axis)	c	.0055		
Round tube	a	.0020	a	.0020
Rectangular Hollow Section	a	.0020	a	.0020

Notes:
1. The curve allocation is generally in accordance with the European proposals.
2. The allocation for angles is the author's own suggestion, pending more information.
3. Universal columns with flanges thicker than 40 mm to have α = 0.0080 for yy buckling.
4. For welded I- and box-sections refer to Section 10.

Table 2

COEFFICIENTS FOR USE IN POLYNOMIAL EXPRESSION (9)

$$\lambda = \sqrt{C_0/N + C_1 + C_2N + C_3N^2}$$

Curve	C_0	C_1	C_2	C_3
a	+1.0	-0.61	+1.29	-1.64
b	+0.92	-0.51	+0.43	-0.80
c	+0.92	-0.39	-0.74	+0.25

Table 3

ASSUMED YIELD STRESSES FOR STRUT DESIGN

Hot Rolled Sections

BS 4360 Grade	assumed σ_y (N/mm^2)	thickness range (mm)
43	225 240 255	40 to 63 16 to 40 < 16
50	340 355	16 to 63 < 16
55	410 430 450	40 to 63 16 to 40 < 16

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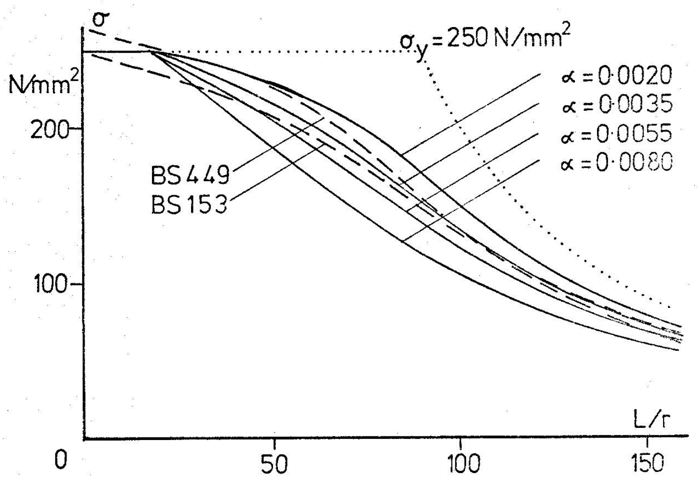

Fig. 1 Comparison of current British design curves with the new proposals.

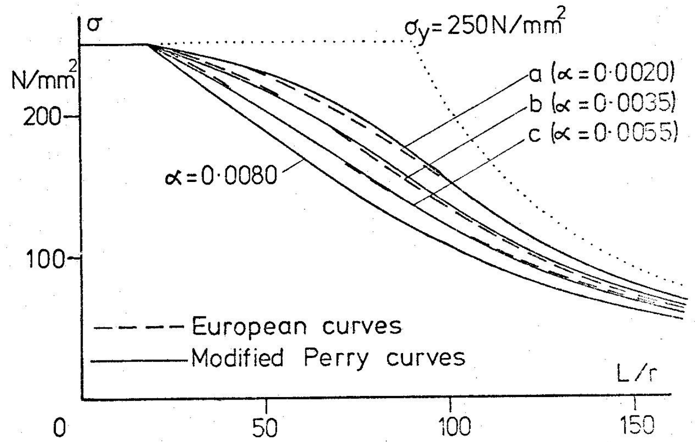

Fig. 2 Comparison of the modified Perry curves, plotted for mild steel, with the corresponding European curves. The lowest curve is

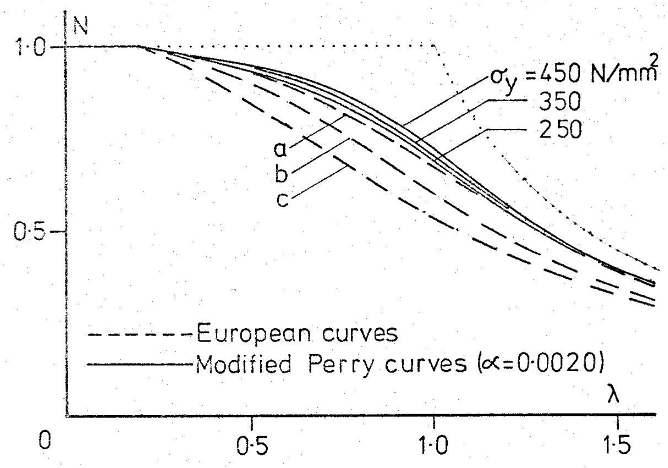

Fig. 3 Non-dimensional plot showing the increasing difference between the appropriate Perry curves and European curve a as the yield stress increases. The comparison would be similar for curves b and c.

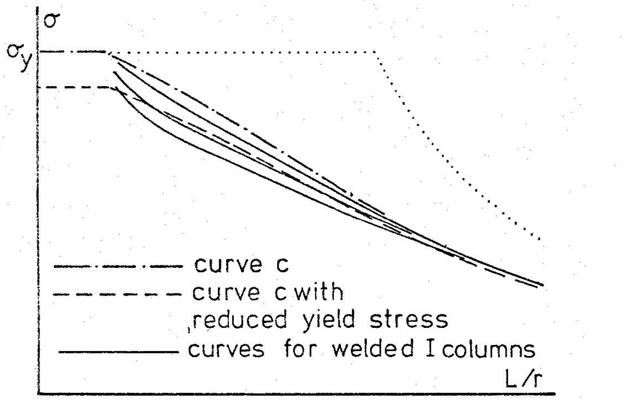

Fig. 4 Treatment of welded columns.

APPLICATION OF THE BUCKLING CURVES OF THE EUROPEAN CONVENTION FOR CONSTRUCTIONAL STEELWORK TO FRAME COLUMNS

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ABSTRACT

This paper is an attempt to develop a method for the application of the European buckling curves to frame columns.

For this purpose results of the effective length method are compared with results of a second order elastic-plastic thoery.

The investigation includes the following steps :

(1) Approximate solution for the buckling strength of a hinged column with geometrical imperfections only.

(2) Evaluation of the necessary "representative" geometrical imperfection for a satisfactory agreement with the European buckling curves.

(3) Approximate solution for the ultimate strength of a two-hinged frame with geometrical imperfections.

(4) Comparison between the results of the effective length method and the approximate ultimate strength of the two-hinged frame.

The investigation shows that the effective length method with application of the European buckling curves is a reasonable approach for the determination of buckling loads for frame columns.

1. INTRODUCTION

Commission 8 of the European Convention for Constructional Steelwork has proposed new buckling curves for hinged Columns on the basis of very thorough experimental and statistical investigations, including the influence of all kind of imperfections /1/. It is very likely that these curves shall be adopted by several European countries in the near future.

The column with hinges on both ends, however, is not found very often in actual structures. Therefore it is necessary to ask how the "European buckling curves" can be used for other structural components, such as columns in frames.

It is obvious that one convenient way to do this is to determine the "effective length" of the frame column, calculate the slenderness-ratio and take the corresponding critical stress from the buckling curves. One does not know, however, whether this is a reasonable approach, because the imperfections -especially the geometrical ones- can be very different from those in two-hinged columns.

The purpose of this paper is to help to decide whether the "effective length" method is acceptable or not. The decision would be very easy if similar curves as the European buckling curves would be available for frames. It is very clear, however, to everybody who has studied the work which has led to the European curves that an enormous expensive and time consuming work is necessary to establish such curves. Therefore an approximate method to answer the question mentioned above is shown in the following four steps of investigation.

2. PROCEDURE OF INVESTIGATION

The first step is to establish buckling curves for a hinged column by an approximate ultimate strength method which includes one parameter only, to cover the influence of all possible imperfections. This parameter shall be called the "representative imperfection".

In step two the value of the representative imperfection is chosen for each group of cross-sections such as to match the resulting curves with the corresponding European buckling curves a, b, and c very closely. With this one gets an idea of the order of magnitude of the representative imperfection to be assumed for a frame structure.

In step three an approximate solution for the buckling strength of a frame with one characteristic (representative) geometrical imperfection is developed, using the same simplifying assumptions as for the determination of the ultimate strength in step one.

Finally, in step four a numerical example of the discussed frame will be evaluated. The results of this calculation will be compared with the results obtained by using the effective length method in connection with the European buckling curves. This comparison will lead to a conclusion about the possibility of the application of the European buckling curves to frame columns.

3. APPROXIMATE SOLUTION FOR THE BUCKLING STRENGTH OF A HINGED COLUMN WITH GEOMETRICAL IMPERFECTIONS ONLY

To establish ultimate strength curves for a hinged column, and later on for a frame, a second order elastic-plastic theory with the following assumptions is used /2/, /3/.

a) The ultimate strength is obtained by a failure mechanism with a sufficient number of plastic hinges.

b) Spread of plastification is neglected.

c) The influence of axial thrust on the plastic moment capacity and on the bending stiffness (i. e. on the deformation) is taken into account.

A column with an initial sinusoidal out of straigthness as "representative imperfection" is considered (see fig. 1) :

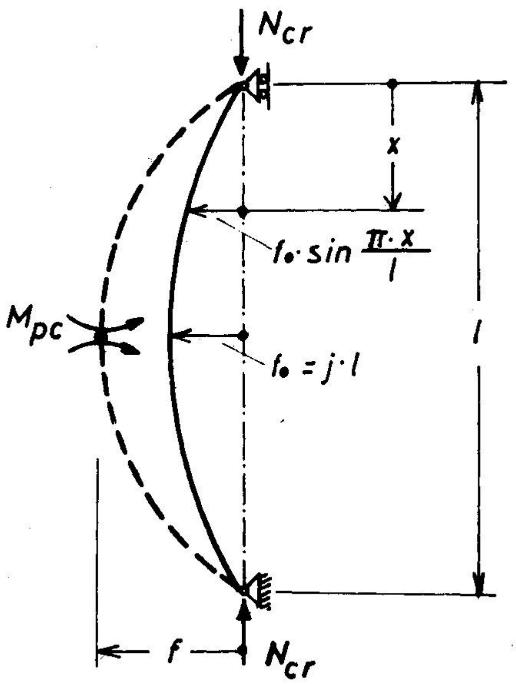

fig. 1 : Column with sinusoidal initial out of straightness.

This column failes when a plastic hinge has performed at mid-length. At this instant the condition for equilibrium can be expressed as :

$$N_{cr} \cdot f = M_{PC} \tag{1}$$

Right before this moment, in accordance with assumption b), the column behaves completely elastic, so that the deformation f is :

$$f = f_o \frac{1}{1 - \frac{N_{cr}}{N_E}} \tag{2}$$

where $N_E = \pi^2 \; EI/l^2$, the critical Euler load.

Using the same symbols as Beer /1/ does, namely :

$$\bar{N} = \frac{N}{\sigma_F \cdot F} = \frac{N}{\sigma_y \cdot A}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_F} \quad \text{with} \quad \lambda_F = \pi \sqrt{\frac{E}{\sigma_F}} \quad \text{and} \quad \lambda = \frac{l}{i} = \frac{l}{r} \, ,$$

applying Eq. (2) to Eq. (1) and rearranging the following equation results :

$$\bar{N} = 2 . \alpha . \frac{1}{j} . \frac{i}{h} . \frac{1}{\lambda_F} . \frac{1}{\bar{\lambda}} . \frac{M_{PC}}{M_P} . (1 - \bar{N} . \bar{\lambda}^2) \qquad (3)$$

with α = shape factor = plastic modulus/elastic modulus

i = r = radius of gyration

M_P = plastic moment

M_{PC} = $F(\bar{N})$ = plastic moment reduced by axial force $\bar{N}$

j = characteristic value for the representative imperfection (see fig. 1)

For numerical computations it is more convenient to express $\bar{\lambda}$ as a function of $\bar{N}$, i. e. write Eq. (3) as :

$$\bar{\lambda} = - \frac{1}{4\alpha . \frac{1}{j} . \frac{i}{h} . \frac{1}{\lambda_F} . \frac{M_{PC}}{M_P}} + \sqrt{\left(\frac{1}{4 . \frac{1}{j} . \frac{i}{h} . \frac{1}{F} . \frac{M_{PC}}{M_P}} \right)^2 + \frac{1}{\bar{N}}} \qquad (4)$$

M_{PC}/M_P can be taken as a function of $\bar{N}$ from the interaction diagram fig. 2 for various cross sections.

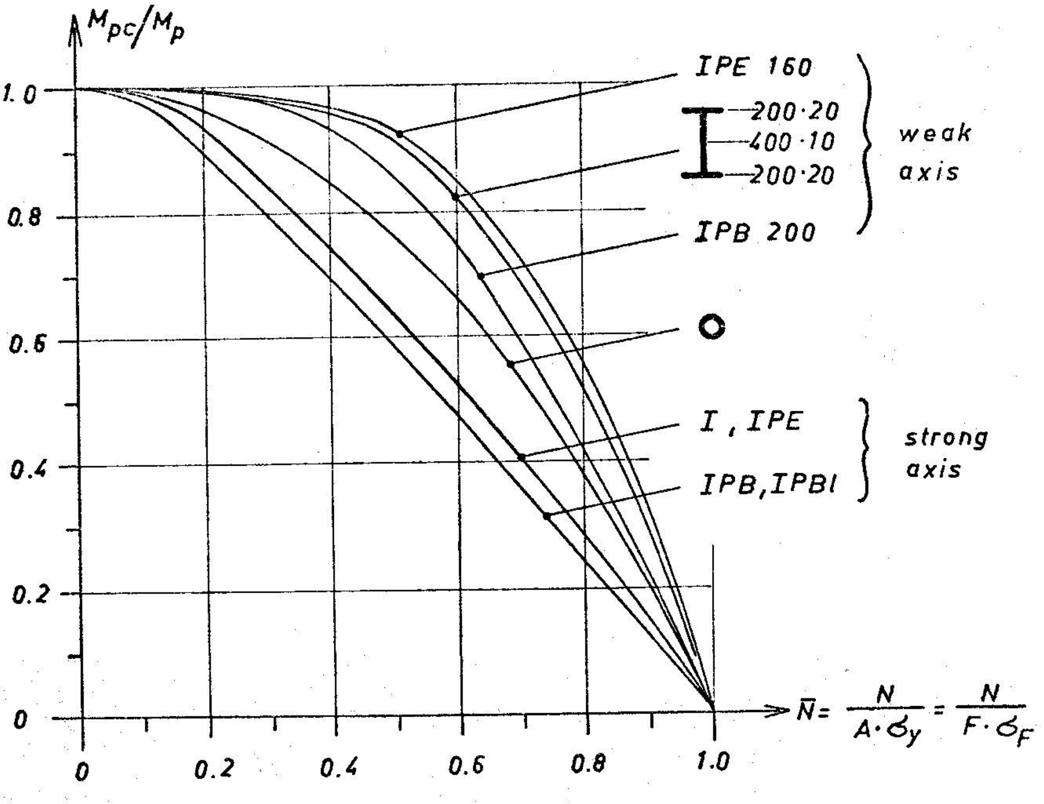

fig. 2 : Interaction diagram M_{pc}/M_p – $\bar{N}$

It is interesting to note that by assuming an ideal straight bar, i. e. $j \Rightarrow 0$, Eq. (4) leads to the critical Euler load :

$$\lim_{j \to 0} \bar{\lambda} = \sqrt{\frac{1}{\bar{N}}} \quad , \qquad \text{or} \qquad \bar{\lambda}^2 = \frac{1}{N}$$

$$\frac{\lambda^2}{\pi^2 \cdot \frac{E}{\sigma_F}} = \frac{1}{\frac{N_{cr}}{\sigma_F \cdot F}}$$

$$N_{cr} = \frac{\pi^2 \cdot E}{\lambda^2} \cdot F = \pi^2 \frac{EI}{l^2}$$

4. EVALUATION OF THE NECESSARY ORDER OF MAGNITUDE FOR THE REPRESENTATIVE GEOMETRICAL IMPERFECTIONS TO BE ASSUMED

Eq. (4) has been evaluated numerically for the six cases shown in table 1.

Table 1 : investigated cross sections

Case	Cross section and bending direction	Corresponding European buckling curve
1a	IPE 160, strong axis bending	a
1b	◎ 8 5/8" (D/t = 219,1/5,9)	
2a	IPB 200 (DIN 20), strong axis bending	b
2b	IPE 160, weak axis bending	
3a	IPB 200, weak axis bending	c
3b	I — 200 · 20, — 400 · 10, — 200 · 20 welded, weak axis bending	

The $\bar{\lambda}$ — $\bar{N}$ curves for each case were plotted for various values of j. Only those curves which fit best to the corresponding European buckling curves are shown in fig. 3, 4 and 5

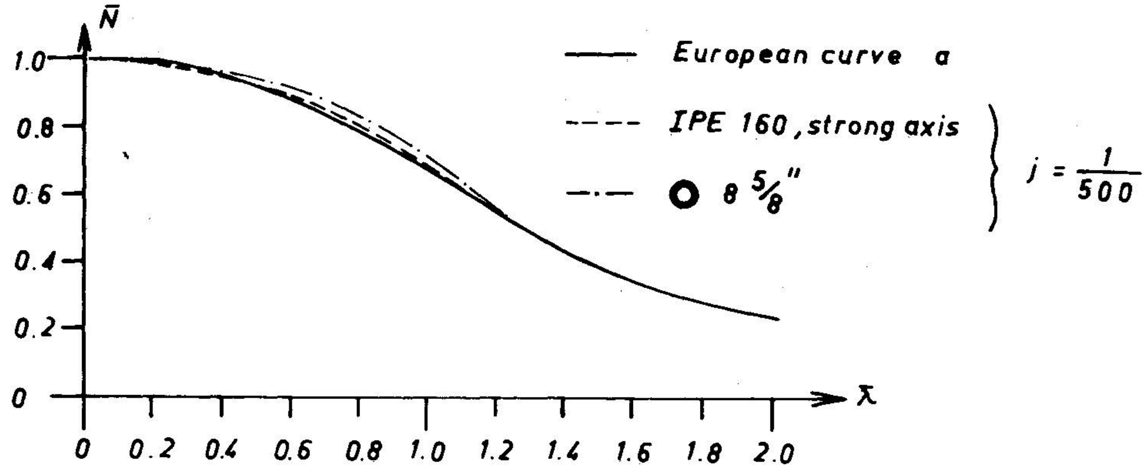

fig. 3 : European buckling curve a compared with approximate theory

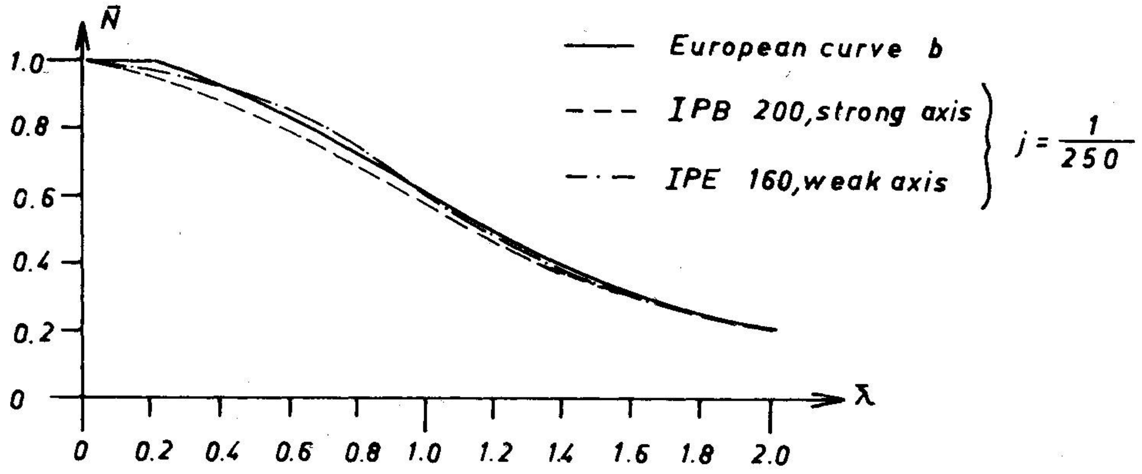

fig. 4 : European buckling curve b compared with approximate theory

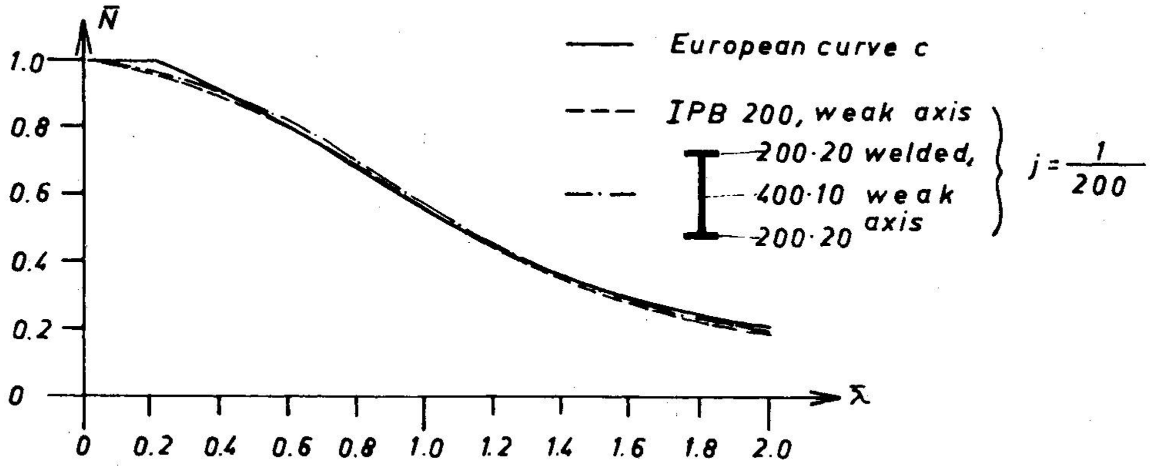

fig. 5 : European buckling curve c compared with approximate theory

From these figures it can be seen that the values in table 2 for the representative imperfections should be assumed, when applying the shown approximate method to calculate the ultimate strength of a centrally compressed hinged strut.

Case for European buckling curve	Characteristic value j for the representative imperfection
a	1/500
b	1/250
c	1/200

These values for j and the shown approximate method lead to buckling curves which are in satisfactory agreement with the more exact European curves, which include the influence of all kinds of imperfections, especially those of residual stresses. It is reasonable to use these values -at least their order of magnitude- also for frames, though the configuration of the assumed geometrical imperfection might be different from the sinusoidal one.

5. APPROXIMATE SOLUTION FOR THE BUCKLING STRENGTH OF A TWO-HINGED FRAME WITH GEOMETRICAL IMPERFECTIONS

The ultimate strength of the frame shown in fig. 6.a is calculated by the same method and with the same assumptions used in chapter 3.

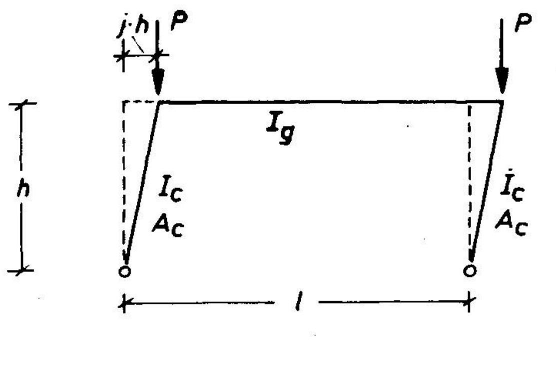

fig. 6.a : portal-frame with geometr. imperfection

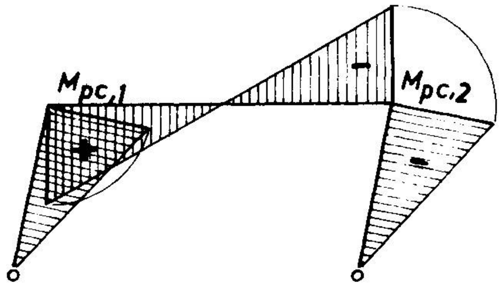

fig. 6.b : moment distribution at ultimate load

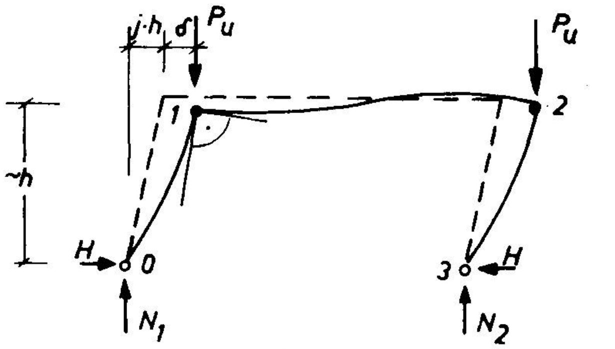

fig. 6.c : elastic-plastic deformation at the instant of failure

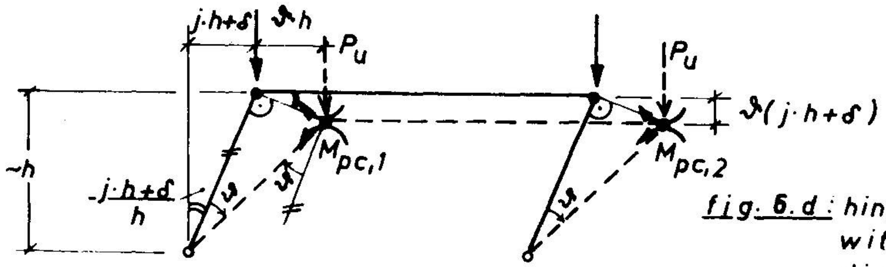

fig. 6.d : hinge mechanism with virtual displacements

Instead of crooked columns an initial displacement j.h of the column tops is assumed. The frame shall be designed in such a manner that the moment capacity of the columns is less than that of the girder. Therefore the plastic hinges shall performe in the columns.

Using the principle of virtual displacements the following work equation can be derived from fig. 6.d :

$$(M_{PC,1} + M_{PC,2}) \cdot \theta = 2\, P_u \cdot \theta \cdot (j \cdot h + \delta)$$

This leads to Eq. (5) for the ultimate load :

$$\boxed{P_u = \frac{M_{PC,1} + M_{Pc,2}}{2\,(j \cdot h + \delta)}} \qquad (5)$$

This equation still contains three unknowns : the two axial forces N_1 and N_2 (since $M_{PC,1}$ and $M_{PC,2}$ depend on these quantities), and the elastic-plastic deformation δ (see fig. 6.c).

N_1 and N_2 are obtained from fig. 6.c by using the equilibrium conditions $\Sigma M_3 = 0$ and $\Sigma M_o = 0$:

$$N_1 = \frac{2\,P_u}{1}\left[\frac{1}{2} - (j \cdot h + \delta)\right] \qquad (6.a)$$

$$N_2 = \frac{2\,P_u}{1}\left[\frac{1}{2} + (j \cdot h + \delta)\right] \qquad (6.b)$$

The deformation δ can be obtained by using the condition for continuity at the location of the last plastic hinge at the moment of failure. This is point 1 in Fig. 6.c, because $N_2 > N_1$, i. e. $M_{PC,2} < M_{PC,1}$, so that the plastic hinge at point 2 has to perform firstly.

The condition of continuity at point 1 is :

$$\phi_{10} = \phi_{12}, \text{ (equal nod rotations)} \tag{7}$$

where
$$\phi_{10} = \frac{\delta}{h} - \delta'_{10} \cdot \frac{M_{PC,1} \cdot h}{EI_c} \tag{8.a}$$

$$\phi_{12} = \frac{1}{EI_g} \cdot (\alpha'_{12} \cdot M_{PC,1} - \beta'_{12} \cdot M_{PC,2}) , \tag{8.b}$$

with the "stability function"

$$\alpha'_{10} = \frac{\sin \varepsilon - \varepsilon . \cos \varepsilon}{\varepsilon^2 . \sin \varepsilon} \tag{9.a}$$

$$\varepsilon = h\sqrt{\frac{N_1}{EI_c}} \tag{9.b}$$

and $\alpha'_{12} \cong \frac{1}{3}$, $\beta'_{12} \cong \frac{1}{6}$ (because $N_{12} = H \cong 0$). (9.c)

Applying Eq. (8.a) and (8.b) to (7) and rearranging, the following equation for δ results :

$$\boxed{\delta = \alpha'_{10} \cdot \frac{M_{PC,1} \cdot h^2}{EI_c} + \frac{1 \cdot h}{EI_g} \cdot \left(\frac{1}{3} M_{PC,1} - \frac{1}{6} M_{PC,2}\right)} \tag{10}$$

With the four equations (5), (6.a), (6.b), and (10) which describe the failure condition of the frame the four unknown quantities P_u, N_1, N_2 and δ can be calculated by a trial and error procedure. This can be done conveniently by means of a computer programme. (Starting with $P_u = 0.8 . A_c . \sigma_F$ and $\delta = 0$, experience shows that a "hand computation" takes about three hours until the results for P_u converge sufficiently after about 5 steps.)

6. COMPARISON BETWEEN THE RESULTS OF THE APPROXIMATE METHOD AND THE "EFFECTIVE LENGTH METHOD" (USING THE EUROPEAN BUCKLING CURVES) FOR A TWO-HINGED FRAME..

6.1. Results of the approximate method

The equations of chapter 5 were solved for the following numerical example :

girder span : 1 = 12 m

girder section : IPE 600 : Ig = 92 080 cm^4

column height : h = 10 m, 8 m, 6 m and 5 m

column section : IPB 360 : I_c = 43 190 cm^4
A_c = 181 cm^2
i_c = 15,5 cm
s_x = 2 400 cm^3
α = 1,12

steel grade : St.52 (σ_F = 3 600 kp/cm^2 = 51.2 Ksi,
E = 2 100 Mp/cm^2)

The initial displacement as the representative imperfection was assumed to
j . h = h/250,

because the rolled section IPB 360 with H/b = 1.2 belongs to the European buckling curve b.

The results for the different column heights are shown in table 3 :

Table 3 : Results of approximate ultimate load theory

column height h [m]	10	8	6	5
ultimate load P_u [Mp = 1000 kp]	168.8	243.7	370.2	453.9

6.2. Results of the "effective length method"

In order to determine the effective length the alignment charts of the CRC-Guide to Design Criteria for Metal Compression Members are used (see also /4/).

$$\text{With} \quad G_A = \frac{I_c/h}{I_g/h} = \frac{43\ 190\ /h}{92\ 080/12}$$

and $G_B = \infty$ (frictionless pin at the column foot)

the following results are obtained (see table 4).

Table 4 : Results of effective length method

h [m]	G_A	K = β	$\lambda = \beta h/i_c$	$\bar{\lambda} = \frac{\lambda}{\pi\sqrt{\frac{E}{\sigma_F}}}$	$\bar{N}$ from European buckling curve b	$P_{cr} = \bar{N} \cdot A_c \cdot_F$ Mp
10	0.56	2.20	141.9	1.86	0.236	153.8
8	0.70	2.24	115.6	1.52	0.335	218.3
6	0.94	2.35	90.9	1.19	0.486	316.7
5	1.12	2.38	76.8	1.01	0.592	385.7

The results of tables 3 and 4 are plotted in Fig. 7.

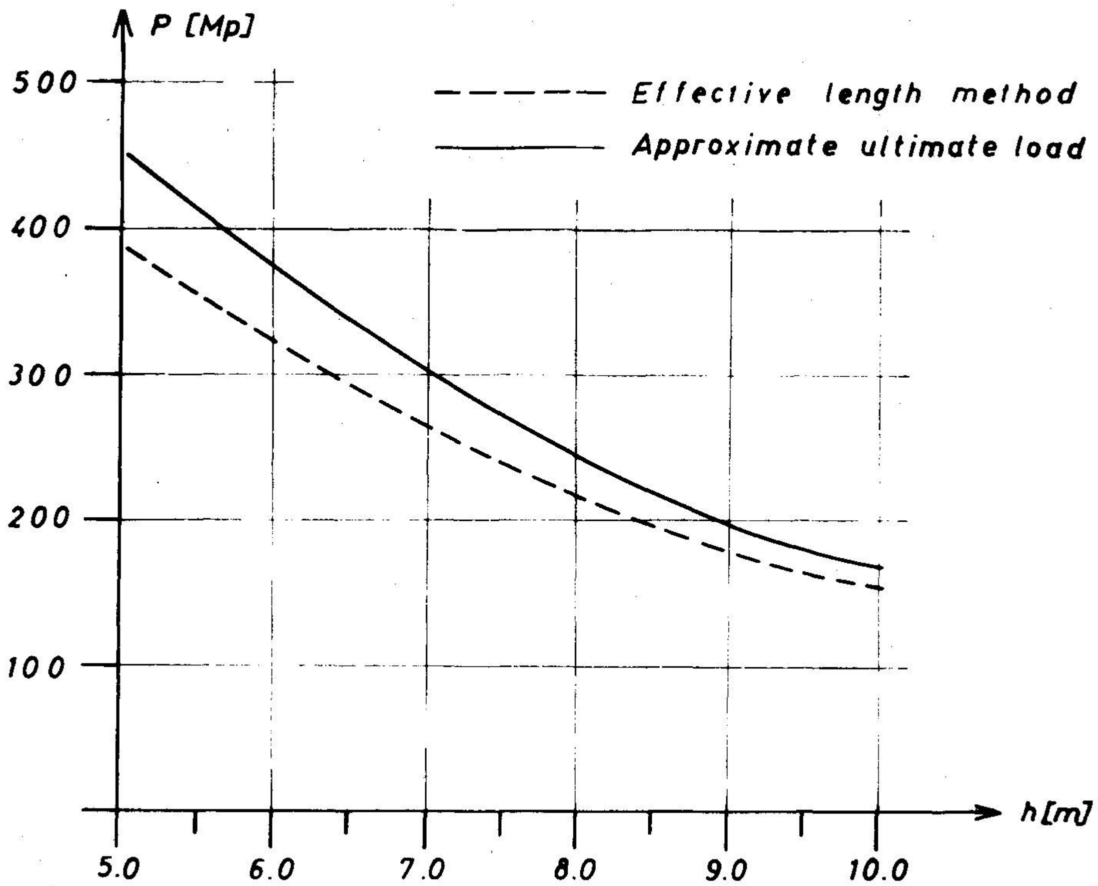

fig. 7: Comparison between effective length method (European buckling curve b) and approximate ultimate load for a two-hinged portal-frame.

The comparison shows that the results of the effective length method with application of the European buckling curves are a little smaller than the results of an approximate ultimate load method. Thus the effectiv length method seems to be -at least for the discussed example of a two-hinged frame- on the safe side.

7. CONCLUSION

The investigations of this paper show that the effective length method with application of the new European buckling curves is a reasonable approach for the determination of critical buckling loads of single-story frames - though it might in some cases be too far on the safe side. Further research work has to be done in order to decide whether this method is suitable for multi-story frames also. It seems to the author that for some other simple structures, loading cases or boundary conditions of structural components the shown procedure is a helpful and not too unconvenient way to decide this open question.

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EMPIRICAL FORMULATION OF MULTIPLE COLUMN STRENGTH CURVES

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ABSTRACT

A procedure for the empirical formulation of a related set of steel column strength curves is presented. The formulas are developed in terms of correction functions to idealized critical load strengths. The method yields a family of column curves by altering a single coefficient. Its value, perhaps, lies more in the logic of the approach and the "way of thinking" that is introduced rather than in its feasibility for design application.

If strain hardening is neglected, the upper bound of column strength for structural steel with an assumed bilinear elastic-plastic stress-strain curve is either the yield load or the Euler Load. The proposed procedure embodies a certain rationality in that it "corrects" the upper bound strength. Two correction functions are introduced: (1) A function of column slenderness, C_L; and (2) A function of shape, column type, and fabrication process, C_S, as determined by the band of test results plotted against slenderness for a particular type of column. For a given column curve the shape function, C_S, will be a numerical constant.

For dimensionless plots the slenderness ratio, L/r, is replaced by the slenderness function, λ,

$$\lambda = \frac{L}{r}\frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}} \qquad (1)$$

and the Euler load,

$$P_e = \frac{P_y}{\lambda^2} \qquad (2)$$

The reduction in strength from the idealized (upper bound) strength of a "perfect" structural steel column depends on a multiplicity of factors, amply discussed in other colloquium papers, and nearly always reaching a sharp peak when the Euler Load is equal to the yield load, or when λ is equal to unity.

In the proposed procedure the column strength is represented by one of the following two formulas:
for $\lambda \leqslant 1$,

$$P = (1 - C_S C_{L_1}) P_y \qquad (3)$$

for $\lambda \geqslant 1$,

$$P = (1 - C_S C_{L_2}) P_e \qquad (4)$$

Alternatively, introducing Eq. 2, Eq. 4 may be written,
for $\lambda > 1$,

$$P = \frac{1}{\lambda^2}(1 - C_S C_{L_2}) P_y \qquad (5)$$

Thus the idealized strength of a structural steel column is used as a first approximation, corrected downward by C_S, a function of shape, fabrication process, plate thickness etc.,

$$C_S = 1 - \frac{P_{cy}}{P_y}, \text{ where } P_{cy} \text{ is the column strength when } \lambda = 1.$$

The shape function, C_S, is a single numerical coefficient for any given column strength curve, determined as indicated above. In some cases one curve may be indicated for a certain range of thickness, such as flange thickness of a rolled W shape, and another curve for a different range of thickness. In this case C_S may be made a variable, thus providing direct interpolation between curves and avoiding a sudden jump in column selection.

The numerical application of the procedure will be illustrated by closely approximating the median curve "b" recently recommended* by the European Convention of Constructional Steelwork. The curve is plotted on Figure 1 and the empirical formulation is based on four control points a, b, c, and d, as shown. Location c establishes

$$C_S = 0.4013^*$$

Letting C_{L1}, for $\lambda<1$ be represented by the quadratic,

$$C_{L1} = A + B\lambda + C\lambda^2 \qquad (6)$$

Passing Eq. 6 through control points a, b, and c,

$$C_{L1} = -0.1295 + 0.5270\lambda + 0.6025\lambda^2 \qquad (7)$$

For the region $\lambda>1$, the formulation for C_{L2} is taken as

$$C_{L2} = D + \frac{E}{\lambda} + \frac{F}{\lambda^2} \qquad (8)$$

The plot of curve "b" for $\lambda>1$, in Fig. 1, made use of control points c and d, with the added requirement that the slope be continuous at c with the curve for $\lambda<1$. Slope continuity requires that,

$$2D + 3E + 4F = \frac{2}{C_S} - B - 2C \qquad (9)$$

which gives for curve b

$$C_L = +0.0232 + \frac{0.7018}{\lambda} + \frac{0.2750}{\lambda^2} \qquad (10)$$

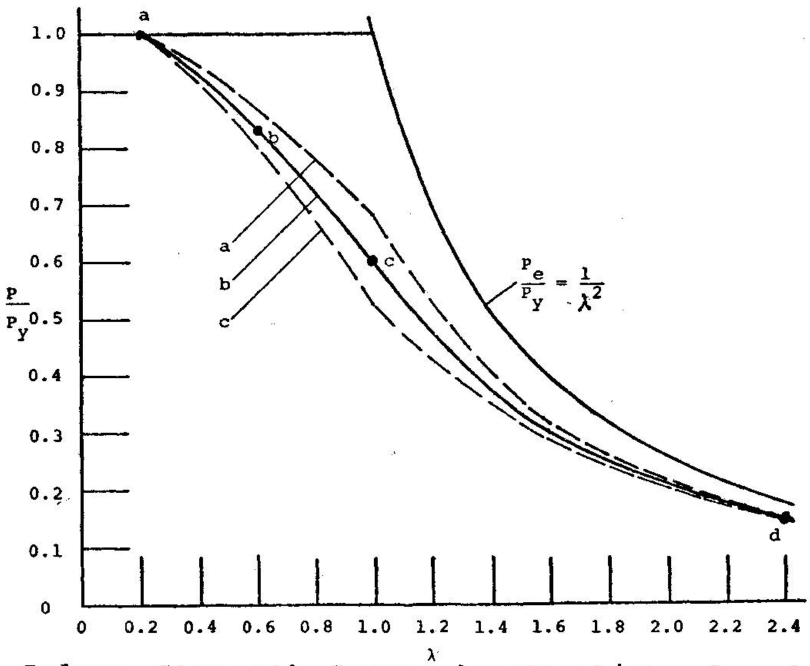

Fig. 1. Column Strength Curves by Equations 3 and 5

*As per letter from Dr. Gerald Schulz, 12 December, 1972.

A tabular comparison of the European Curve b values and those obtained from Equations 3, 5, 7, and 10 follows.

Values of P/P_y

λ	original curve b	approx-imation
0.2	1.0000	1.0000
0.4	0.9250	0.9287
0.6	0.8380	0.8380
0.8	0.7270	0.7280
1.0	0.5987	0.5987
1.2	0.4809	0.4718
1.4	0.3831	0.3741
1.6	0.3078	0.3014
1.8	0.2502	0.2468
2.0	0.2070	0.2055
2.2	0.1746	0.1735
2.4	0.1483	0.1482

For $\lambda<1$ the maximum deviation in the above tabulation is about 0.4%, for $\lambda>1$, about 2.4%. Better agreement for $\lambda>1$ could be obtained by permitting a very small slope discontinuity at $\lambda=1$.

Although a family of curves may be obtained simply by changing a single numerical coefficient, C_S, it may be seen by Eq. 9 that the condition for slope continuity is not independent of C_S, The result of changing C_S, alone, to obtain approximations of European Curves "a" and "c" is shown by the dashed lines on Figure 1, The differences in these curves is as much as 3%. Better agreement could be obtained, and slope discontinuities eliminated, by introducing new equations for C_L, but then the simple interrelationship would be lost. There are, of course, many other empirical equations that might be introduced within the overall framework of the procedure.

In summary, a method for systematising the formulation of empirical column strength curves for structural steel has been presented. The procedure is based on the concept of correcting the upper bound critical load and the effects of length are separated from the effects of shape and fabrication.