Development of multiple column curves

DEVELOPMENT OF MULTIPLE COLUMN CURVES

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ABSTRACT

The introduction of the concept of multiple column curves forms an important step in the direction of improving the method of column strength assessment. This paper presents an investigation carried out in the United States, whereby several sets of multiple column curves were developed using actually measured values of the column strength parameters. The resulting set of multiple column curves has been accepted by the Column Research Council for inclusion in the 3rd Edition of its Guide.

The paper contains the following basic sections:

1) Presentation of a deterministic investigation of the maximum strength of columns, where column curves for a large number of different structural steel shapes were developed and analyzed.
2) Presentation of a probabilistic development of multiple column curves, where the curve for each shape has been developed on the basis of statistical computations of the strength.
3) Development and comparison of the two final sets of multiple column curves resulting from the above two investigations.

1. INTRODUCTION

It is a well-known fact that the strength of a real column and the strength assigned to it by a designer may differ considerably. For several reasons this variation of the column strength has not been taken into account in the formulation of what is commonly referred to as column strength curves , basically because of the general desire of maintaining the design rules as simple as possible. This philosophy now is being reconsidered due to an increasing need for economy, efficiency, and rationale in the design of structures.

A variety of methods aimed at the implementation of more accurate means of column design can be devised. Some of these are related to the manufacture of columns, whereas others are based on theoretical developments (1, 2,). The main problem connected with the manufacture-related approaches lies in the formulation of requirements that will duly consider all pertinent factors, such as the rate of cooling after rolling, which is an extremely complex task. Most important among the theoretically based methods is the one that utilizes several column strength curves, to each of which related column curves, and it was used in a very simple form by the German specification for design of columns (DIN 4114 (1959).

Several investigations on the development and application of the multiple column curve concept have been conducted over the past few years. It is the purpose of this paper to describe the studies conducted in the United States (1,3).

2. THE CONCEPT OF MULTIPLE COLUMN CURVES

The variation of the strength of a number of different column types is strikingly illustrated by Fig. 1, which shows the results of approximately 100 tests with centrally loaded columns. The differences in column strengths are caused by the differences in column shape, steel grade, size, manufacturing method, and so on, but each test point can be predicted within an accuracy of ± 5 percent. It is evident that the use of a single column curve will significantly over- or underestimate the strength of many columns.

The essence of the multiple column curve concept therefore lies in the fact that no one column curve can represent the strength of all types of columns rationally and adequately. By introducing several curves, to each of which columns of related behavior and strength are assigned, the difference between the assessed and the actual column strength will not be completely eliminated, but rather reduced to an acceptable level. This idea is illustrated by Fig. 2, from which it may be seen that the variation of the strength of the column types assigned to, for instance, the lower of the three curves, is substantially smaller than the variation of the strength of all columns together. Whereas an increase of complexity will be the inevitable result of utilizing several column curves, significant gains may be expected in terms of accuracy and economy. The best solution will be the one where an optimum of complexity and gains has been achieved.

The studies that are reviewed in this paper have been based on two different methods of approach. The first approach deals with a deterministic investigation of the maximum strength, where column curves for a large number of different structural shapes in a variety of steel grades, manufacturing methods, sizes, and so on, have been developed. The second solution is based on a probabilistic computation of the maximum column strength, and for both methods of approach the most important column strength factors have been established through systematic and detailed analyses of the results. The findings have been applied towards the development of two sets of three column curves each.

3. DETERMINISTIC DEVELOPMENT OF MULTIPLE COLUMN CURVES

A total of 112 maximum strength column curves have been developed in the deterministic investigation, representing fifty-six different combinations of shape, steel grade, and so on, and with two bending axes considered for each column. An incremental, iterative computer program was developed for the maximum strength calculations (3), and the input-data used for residual stresses, yield stresses, and geometric properties were all measured values. For the initial out-of-straightness, assumed values were used, and the maximum allowable value of 1/1000 formed the bases for the development of the multiple column curves. It has been found that the theoretical column strength predictions agree with experimental column strengths to an accuracy of $\pm$ 5 percent, which must be regarded as satisfactory.

Figure 3 shows the band of all 112 column strength curves developed, using the initial out-of-straightness of 1/1000, and Fig. 4 illustrates the frequency distribution histograms of the maximum strength for a few typical nondimensional slenderness ratios. Only the upper and lower envelope curves for the band are indicated in Fig. 3, since the number and density of the curves between these two limits prevent a meaningful illustration of each separate curve. The width of the band is largest for the intermediate slenderness ratios, and tapers off towards the ends. For low slenderness ratios the variation of the maximum strength is influenced more by the variation of the yield stress than any other factor. Figure 4 indicates that for high slenderness ratios factors such as the residual stress and the yield stress have a decreasing influence, as evidenced by the pronounced kurtosis and skewness of the frequency distributions. In fact, the maximum strength of a very long column will approach the elastic (Euler) buckling load. Other investigations have confirmed these results (4,5).

The limitations on the length of this paper prevent a detailed description of the analyses of the available 112 column curves. Having decided upon a set of 3 curves as the multiple column curves, all of the column data were analyzed with regard to the most important strength factors, and each curve finally assigned to one of the three column strength categories. The results of this study are summarized in Figs. 5, 6, and 7, which show the bands of column curves that contain each of the three multiple curves. Included in these figures are also the statistical characteristics of the band of curves.

The arithmetic mean curves for the three bands of Figures 5, 6, and 7 formed the initial set of multiple column curves, and following some adjustments two further sets were developed. Figure 8 illustrates the second, and Fig. 9 the third set of curves. The multiple column curves of Fig. 9 provide a simplified solution, and from a practical standpoint, one that is easier to use. These curves also take into account the strength-raising effects of strain-hardening for short columns, by originating at the point where $P_{max}/P_y = 1.0$ and $\lambda = 0.15$. For such short columns no overall column buckling will occur.

Also included in Fig. 9 are the data on the types of column that belong to each of the three curves. It should be noted, however, that due to the limited amounts of data that were available on measured residual stresses in various column shapes, several column types have not been included in the study. This is a task of future research projects.

Detailed descriptions of the evaluations leading to the set of multiple column curves are contained in References 1 and 3, together with the mathematical representations of the curves shown in Fig. 9.

4. PROBABILISTIC DEVELOPMENT OF MULTIPLE COLUMN CURVES

The probabilistic studies of the properties of the maximum strength (1) revealed that the random variations of the maximum strength is mainly caused by the variation of the initial out-of-straightness. Based on this knowledge, it therefore stands to reason that the upper limit of the strength of a column (defined as the 2.5 percent probability level) is well described by a column curve based on an initial out-of-straightness of e/L = 1/10,000. Similarly, the lower limit is well described by a column curve for which the basic e/L is equal to 1/1000. These statements hold true for the assumption that the probability density function for the initial out-of-straightness may be described by a Type I asymptotic extreme value distribution.

It was decided to base the probabilistic set of multiple column curves on mean values for all of the column strength parameters, in particular, such that e/L = 1/1470 is the basic out-of-straightness. The use of mean values of the column strength parameters is arbitrary, since any set of data with consistent probabilistic bases may be used.

Having decided on the magnitude of the out-of-straightness, the development ceases to be of a strictly probabilistic nature. This is because in this case the arrival at a set of appropriate multiple column curves only involves the grouping of a large number of probabilistically determined maximum strength curves.

The results of the classification of the column types, together with the final set of possible multiple column curves, are shown in Fig. 10. Mathematical equations for the three curves have been determined (1), and it appears that the two sets of curves illustrated by Figs. 9 and 10 are very similar. The reason for this is of course the fact that only the magnitude of the initial out-of-straightness is different, while all the other column strength factors are the same for both sets of curves. This comparison is further clarified by Fig. 11, where the two sets of multiple column curves are shown in the same diagram.

5. SUMMARY

The following represents a brief summary of the findings presented above :

1. The differences that appear between real and assessed column strengths may be reduced significantly by the use of multiple column curves.

2. The deterministic and the probabilistic studies of the maximum strength of centrally loaded columns both have led to the development of a set of multiple column curves. Both of these are believed to represent an improvement over existing methods of column strength assessment.

3. The probabilistic and the deterministic multiple column curves differ only in that they are based on different values of the initial out-of-straightness. It is believed that the probabilistic curves are a better solution, because mean values of all column strength parameters have been used for consistency.

4. The probabilistic multiple column curves provide for somewhat higher column strengths than the deterministic ones. This is partly due to the smaller value of the initial crookedness, which has effected changes in the classification of the column types.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

1. Bjorhovde, Reidar
   DETERMINISTIC AND PROBABILISTIC APPROACHES TO THE STRENGTH OF STEEL COLUMNS
   Ph.D. dissertation, Lehigh University, Bethlehem, Pennsylvania, U.S.A. May 1972.

2. Beedle, L. S.
   DUCTILITY AS A BASIS FOR STEEL DESIGN
   Engineering Plasticity, ed. by J. Heyman and F. A. Leckie, Cambridge University Press, 1968.

3. Bjorhovde, Reidar, and Tall, Lambert
   MAXIMUM COLUMN STRENGTH AND THE MULTIPLE COLUMN CURVE CONCEPT
   Fritz Engineering Laboratory Report No. 337.29, Lehigh University Bethlehem, Pennsylvania, U.S.A., October 1971.

4. Tall, L.
   RECENT DEVELOPMENTS IN THE STUDY OF COLUMN BEHAVIOR
   Journal, Institution of Engineers, Australia, Vol. 36, No. 12, December 1964.

5. Batterman, R. H., and Johnston, B. G.
   BEHAVIOR AND MAXIMUM STRENGTH OF METAL COLUMNS
   Journal, ASCE Struct. Div., Vol. 93, No. ST2, April 1967.

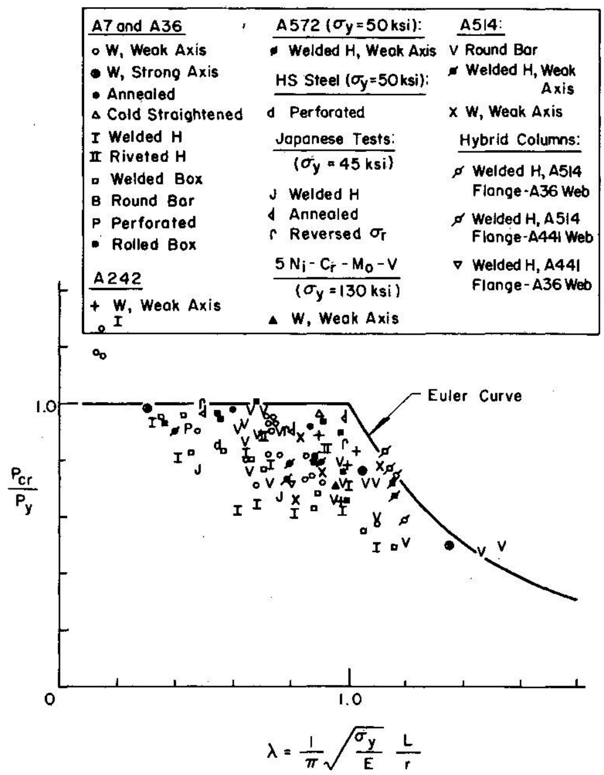

Fig. 1 Results From Approximately 100 Different Tests of Centrally Loaded Columns

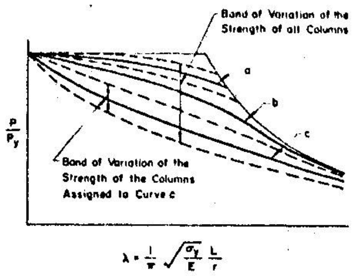

Fig. 2 A Qualitative Illustration of the Variation of Column Strength for Each Multiple Column Curve Category, Compared to the Variation of the Strength of All Columns

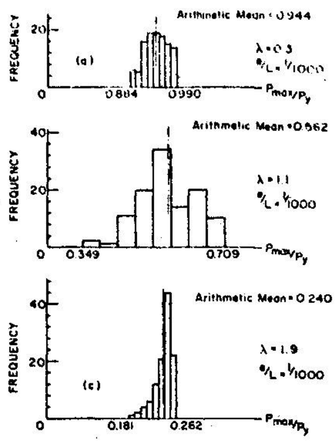

Fig. 4 Typical Frequency Distribution Histograms for the Maximum Strength of All 112 Column Curves (Initial Out-of-Straightness e/L = 1/1000)

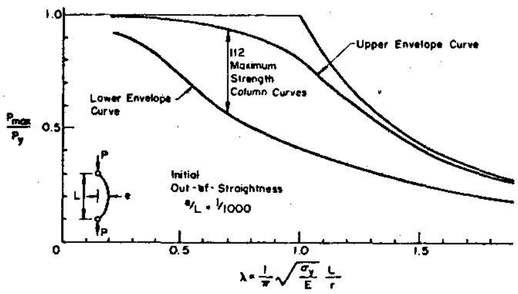

Fig. 3 The Band of All 112 Maximum Strength Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1000

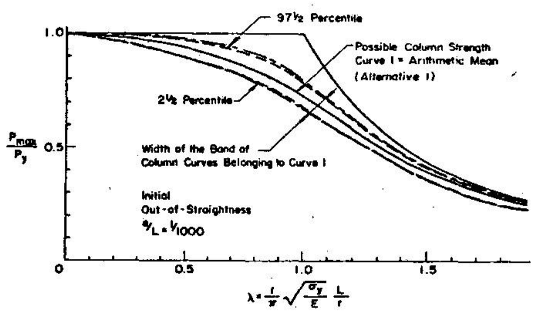

Fig. 5 Possible Column Strength Curve 1, and the Statistical Properties of the Band of Column Curves that Belong to It

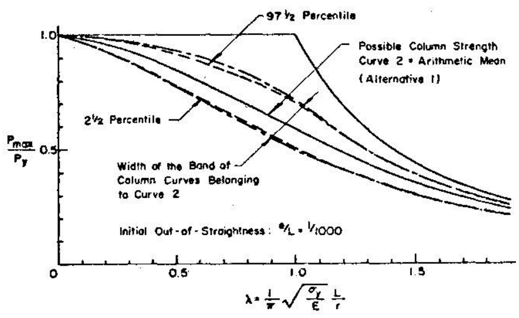

Fig. 6 Possible Column Strength Curve 2, and the Statistical Properties of the Band of Column Curves that Belong to It

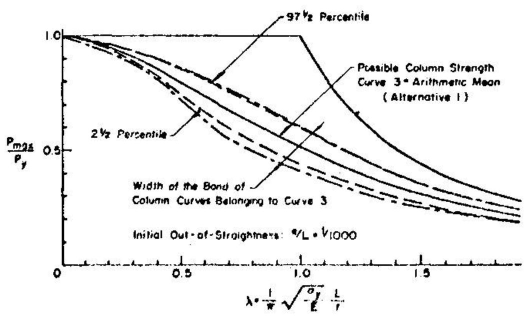

Fig. 7 Possible Column Strength Curve 3, and the Statistical Properties of the Band of Column Curves that Belong to It

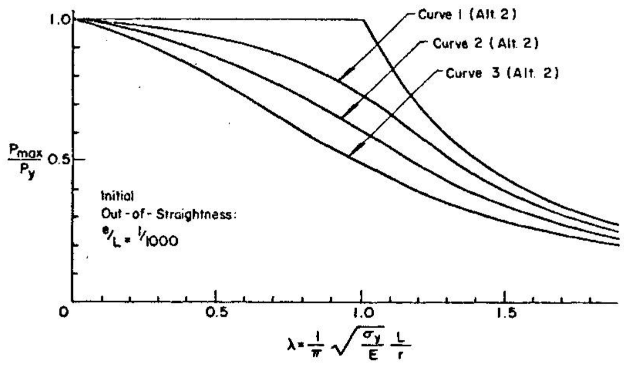

Fig. 8 Modified Maximum Strength Multiple Column Curves, Based on an Initial Out-of-Straightness of 1/1000

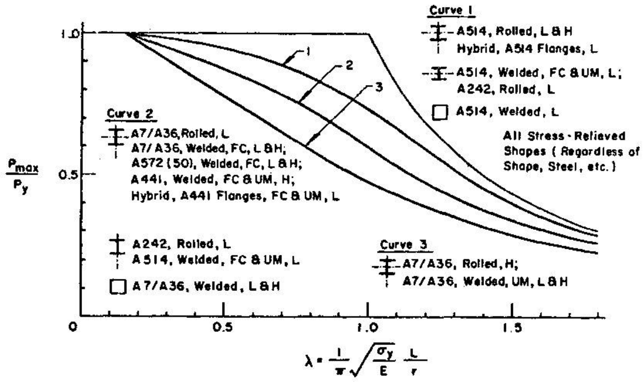

Fig. 9 Simplified Maximum Strength Multiple Column Curves (Initial Out-of-Straightness e/L = 1/1000)

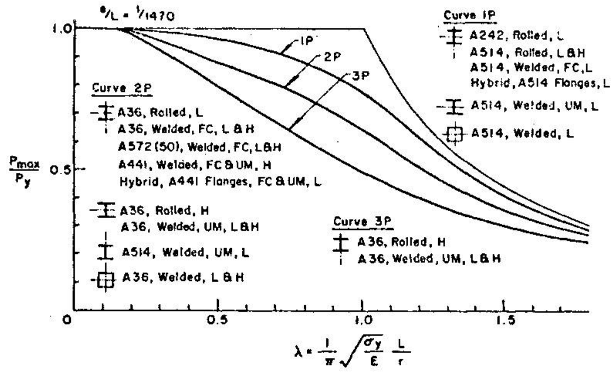

Fig. 10 Possible Maximum Strength Multiple Column Curves, Based on an Initial Out-of-Straightness e/L = 1/1470 = Mean Value (Probabilistic Study)

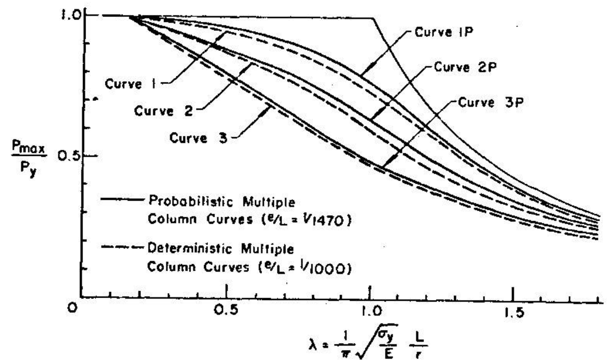

Fig. 11 A Comparison of the Two Sets of Deterministic and Probabilistic Multiple Column Curves