The European column curves

THE EUROPEAN COLUMN CURVES

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ABSTRACT

The column curves adopted by Commission 8 (Stability) of the
European Convention of Constructional Steelwork are the re-
sult of a comprehensive program of experimental and theore-
tical investigations. The reduction of the load carrying ca-
pacity through geometrical imperfections of the strut, as
initial out-of-straightness, and through material inhomo-
geneities, as residual stresses and scatter of the yield
point, were systematically investigated. The theoretically
predicted column curves were compared with the results of
a statistical evaluation of the column tests and show close
agreement.

In addition to the 3 column curves adopted by Commission 8,
two more curves, an upper and a lower one are suggested
for future use. The upper curve should be used for certain
shapes made of high strength steels, and the lower curve
for heavy shapes ("Jumbo"shapes).

1. INTRODUCTION

The theoretical analysis of the strength of steel columns guided by Commission 8 (Stability) of the European Convention of Constructional Steelwork had three major objectives:

The systematic study of the main factors that influence the column strength, as shape of the cross section, geometric imperfections, and material inhomogeneities.

The selection of representative column strength curves for various column types.

The comparison of the theoretical column curves with test results.

The computer program developed for the maximum strength analysis considers the structural member with imperfections (Fig.1). Geometric imperfections are an initial curvature of any given form, and end excentricities of the axial load. Besides the axial load there can be additional small transversal loads. Residual stresses and the variation of the yield stress over a given cross section are taken into account when the relationship between axial load, moment, and elastic-plastic bending stiffness is calculated.
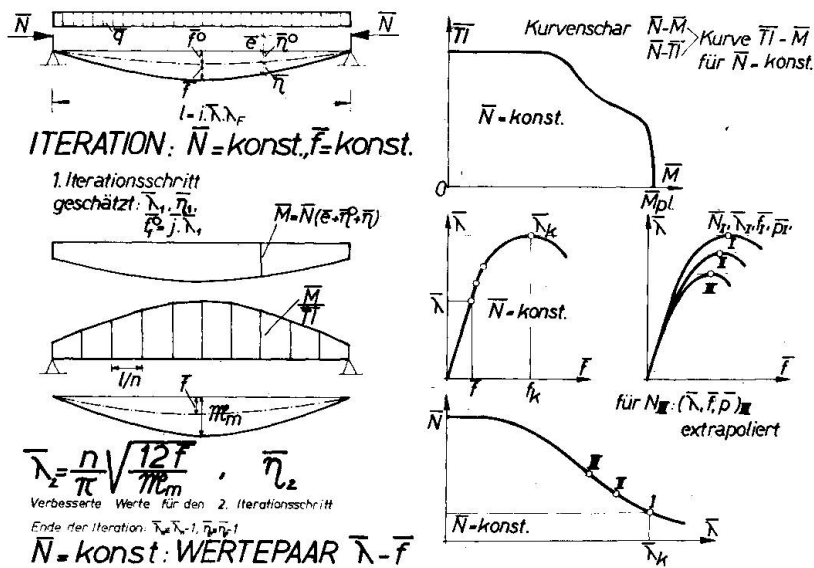

Fig.1: Maximum strength analysis of a structural member with imperfections

It is assumed that the limit of the load carrying capacity is reached, when the equilibrium between external and internal forces changes from a stable to an unstable condition. Since out of practical reasons the calculation is done for a constant axial load N, but a variable slenderness

λ, the limiting state is defined by the vertex in the curve slenderness λ versus central deflection f. For the limit slenderness λ_K the given axial load N becomes then the maximum strength.

The calculation of the column curve axial load N versus slenderness ratio λ starts in the so called "elastic" range and proceeds automatically until the yield load is reached.

The column curves are plotted in the usual non-dimensional diagram. $\bar{N}$ is the axial load N related to the yield load. $\bar{\lambda}$ is the non-dimensional slenderness ratio.

A detailed description of this maximum strength analysis is contained in Ref.(1) and (2).

2. GEOMETRIC IMPERFECTIONS

As a first step of the investigation it was analysed to which extend the unintentional load excentricity due to manufacturing methods can be simulated by an initial curvature with the shape of a half-sine wave.

The measurements on columns of the European experimental program indicated a variety of curvatures. Besides antimetric shapes, some columns had their maximum out of straightness rather in the quarter points than in the center. As Fig.2 shows, such a curvature can be approximated with sufficient accuracy by the first coefficient of a Fourier series. For a curvature with an out of straightness of $\ell/650$ near the quarter points, the substitute half sine-curve has a central bow of $\ell/1000$. The column curves calculated for both curvatures are in good agreement.

Based on this approximation, the substitute central bows for all European test columns were calculated and are shown in Fig.3 as function of the column length ℓ. The most unfavorable values for each column type vary from $\ell/530$ to $\ell/3360$. Also shown in this table are measurements on a truss bridge over the Danube: Here $\ell/1100$ was the largest eccentricity.

Since eccentricity limits are stated in several European building codes, $\ell/1000$ was choosen as reasonable assumption for the central bow of the initial curvature.

To which extend covers an initial curvature with $\ell/1000$ unintentional end eccentricities? (Fig.4) For a wide flange I section the magnitude of the end eccentricity was choosen as function of the radius of inertia i. With i/40, i/20 and i/10, these assumptions cover for this particular section an eccentricity range from 1,25 mm to 4,98 mm. Eccentricities that are unintentional can be expected to fall within the range covered by the first two curves. As Fig.4 shows, the column curve calculated with $\ell/1000$ covers most of these end eccentricities in the slenderness range of practical interest.

How does a variation and in particular a reduction of $\ell/1000$ as central bow effect the column strength. In Fig.5 column curves are calculated for $\ell/2000$, $\ell/1000$, and $\ell/500$ for a wide flange I-section without residual stresses σE, and shown with dashed lines for the same section with residual stresses. The comparison shows that the influence of the initial cur-

Darstellung der Vorkrümmung durch Fourier-Reihe :

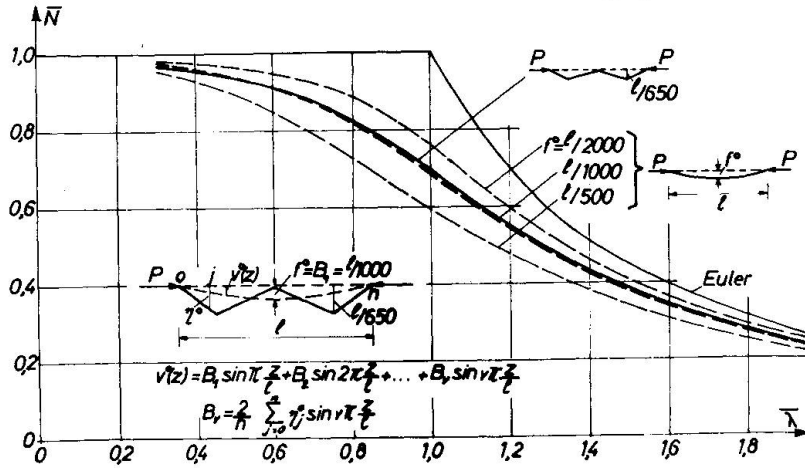$f° = B_1$ nach Southwell

DIR 20
Knickung schwache Achse

Fig.2: Approximation of an initial out-of-straightness, with maximum in the quarter points, by a Fourier series (I-shape, weak-axis bending)

IMPERFEKTIONEN DER VERSUCHSSTÄBE

Profil	Versuchs-durchführung	$\bar{\sigma}^E$	1/j = l/f°		
			Minimum	Mittelwert	Maximum
IAP 150	Belgien Deutschland	≤0,125	590	2100	5100
IPE 160	Holland	≤0,3	2700	4400	7200
IPE 200	Belgien	0,15-0,25	1390	3800	10000
DIE 20	"	0,25-0,45	1690	3700	7600
DIR 20	"	≤0,35(0,5)	2260	5800	9500
Igeschw.	"	0,45-0,7	2280	5500	15000
□geschw.	"	≤1,0	3360		19000
∅gewalzt	Holland	≤0,5	1450	2800	5900
∅D/t=18	"		3050	7000	33000
∅D/t=11	Deutschland		550	2000	3700
T60	Holland		1740	5000	9200
TB60	Deutschland		530	710	1100
Aufmessung Donaubrücke Ingolstadt			1100	5200	13000

$$\bar{\sigma}^E = \frac{\sigma^E}{\sigma_F}$$
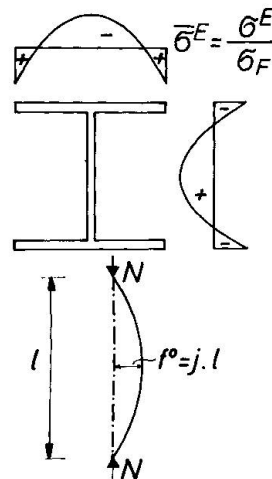
$f° = j \cdot l$

Fig.3: Initial out-of-straightness and residual stress measurements on European test columns

vature decreases with increasing residual stresses. Also,
the particular slenderness ratio where the curvature has its
maximum influence on the column strength has changed with in-
creasing residual stresses from about $\bar\lambda=1,o$ to about $\bar\lambda=1,3$.

The reduction of $\ell/1ooo$ therefore will have a different effect
on the column strength, dependent on the column type. For sec-
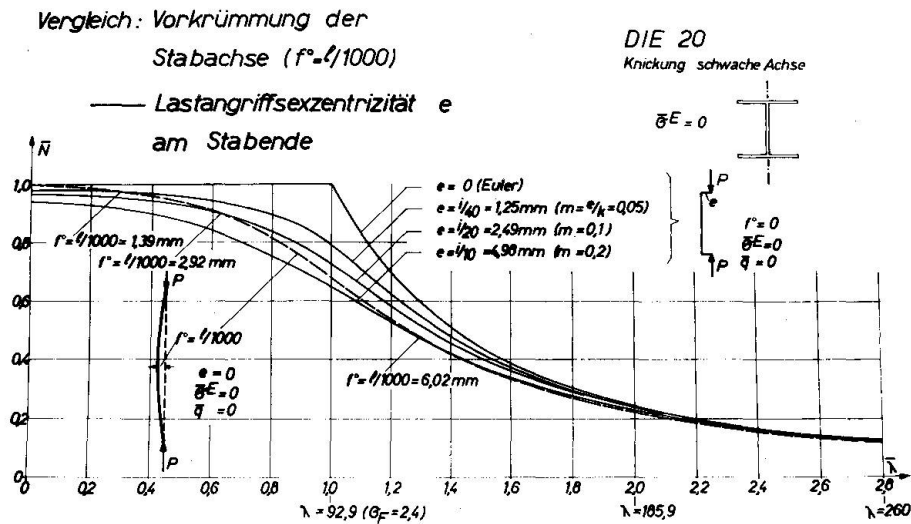tions with high residual stresses the gain of column strength

Fig.4: Effect of different geometric imperfections on
the column strength (I-section, weak axis bending)

will be small, and will have its greatest influence in a
slenderness range that is of no interest for the application
of those sections.

All further investigations were based on a column with a
sinusoidal initial curvature, with a central bow of 1/1ooo of
the column length.

3. SHAPE OF CROSS SECTION AND RESIDUAL STRESSES

For a particular column type -as shown in Fig.6 for I-sections-
have size and geometry of the cross section no significant
influence on the nondimensional column curve, if we assume
the sections to be free of residual stresses.

If we consider the residual stresses typical for each of
these rolled I-sections we obtain the wellknown wide scatter
of column strength (Fig.7). In particular remarkable is the
great reduction of the column strength with increasing re-
sidual stresses for weak axis bending. For strong axis buck-
ling is the reduction comparably smaller for those sections
that have both, compressive and tensile residual stresses in
the flange. For sections with compressive stresses in the flan-
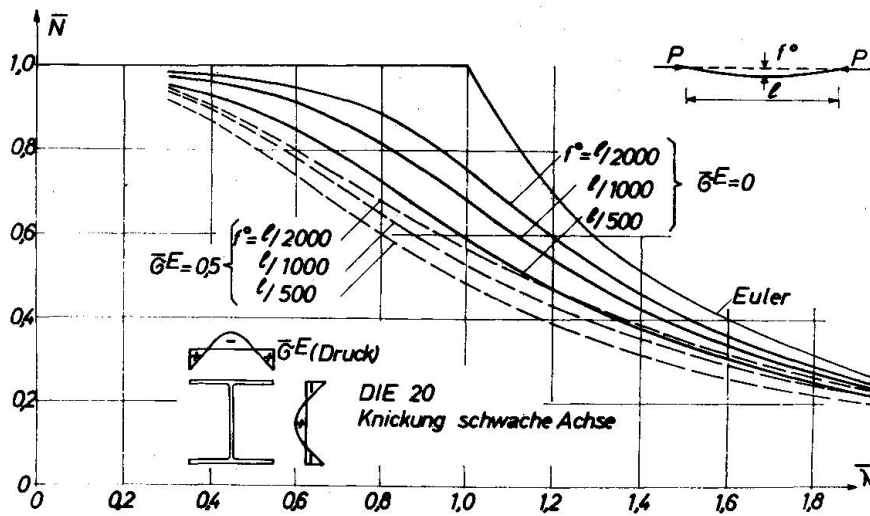ge only the strong axis buckling is equal unfavorable as the
weak axis buckling.

Fig.5: Effect of initial curvature, separately and in combination with residual stresses (I-shape, weak axis bending)
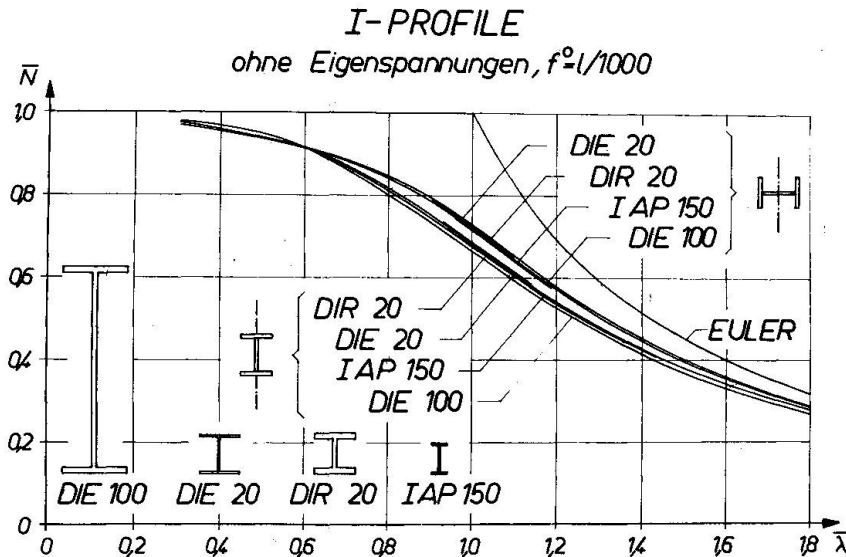

Fig.6: Column strength curves for weak and strong axis bending of I-shapes, including the effect of an initial curvature of ℓ/1ooo, but without residual stresses
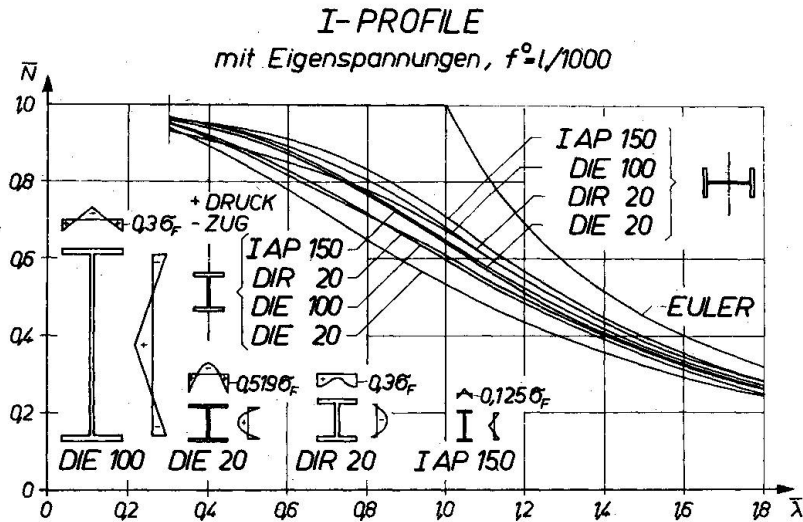
Fig.7: Column strength curves for weak and strong-axis
bending of I-shapes, including the effects of an
initial curvature of ℓ/1000 and residual stresses

To obtain typical resi-
dual stress distributions
for those sections most
commonly used, residual
stress measurements were
collected and analyzed.
Main sources were the
measurements done by
Prof.Massonnet in Liège
and the results of the
research done at Fritz
Engineering Laboratory.

The chart in Fig.8 was
developed for rolled
I-sections as listed
in EURONORM, with a
flange thickness up to
40 mm. Since the measure-
ments indicated clearly
the dependence of magni-
tude and distribution of
the residual stresses from
the section geometry, the
sections could be arran-
ged in two groups, de-
pendent of the heights
over width ratio h/b
smaller or greater than
1,2. h/b = 1,2 is the
ratio, where the EURO-
NORM sections change their
geometry significantly.
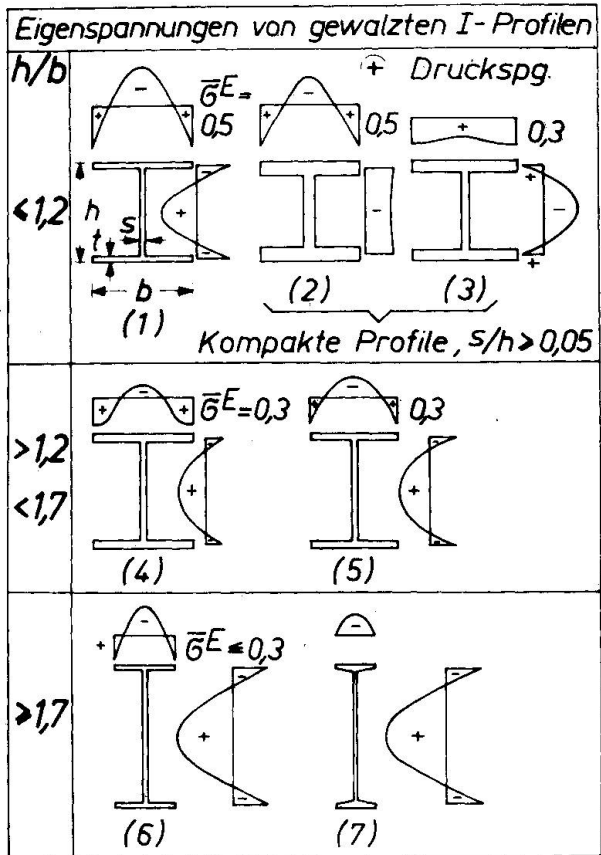
Fig.8: Typical residual stresses
for rolled EURONORM I-sec-
tions

I - Profile mit charakteristischen
Eigenspannungsverteilungen
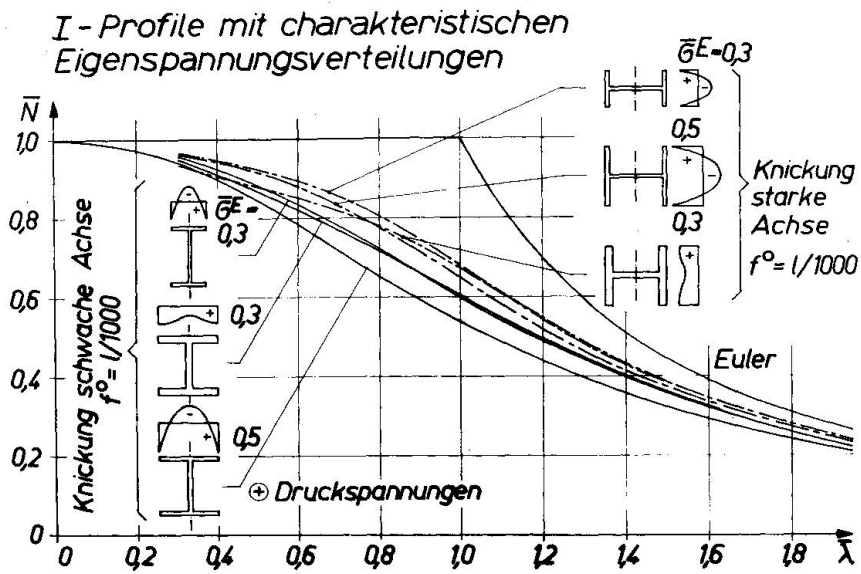

Fig.9: Column strength curves for weak and strong-axis
bending of typical rolled I-sections.(Initial
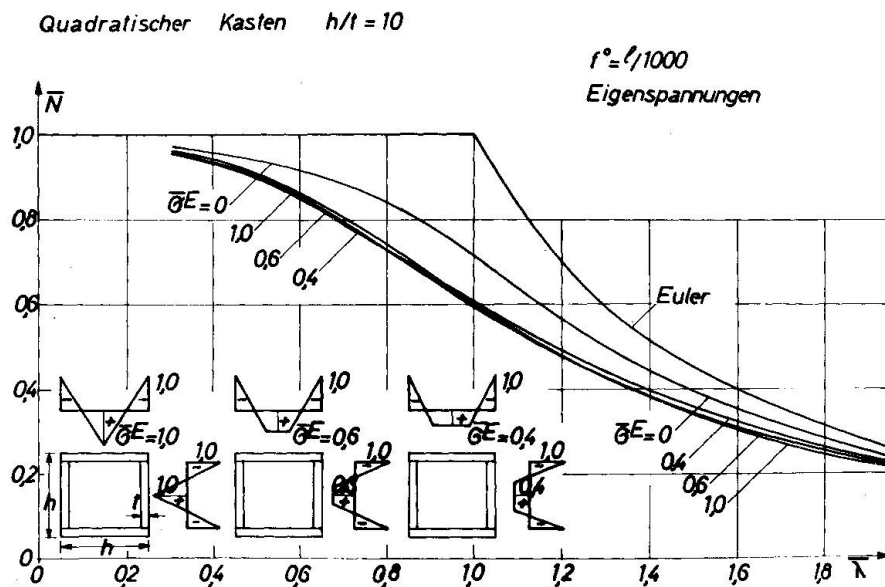curvature ℓ/1ooo, residual stresses)

Fig.1o: Column strength curves for welded box sections,
including the effects of an initial curvature
of ℓ/1ooo and different residual stress patterns

The column curves for typical sections of the two groups of rolled I-sections, calculated for weak and strong-axis bending, are given in Fig.9. Since it was decided by Commission 8 to recognize a difference in column strength of more than 7%, the strength of rolled I-sections can be represented by 3 column curves.

Part of the investigations that led to the selection of representative column curves for the various sections,was to analyze the effect of a possible variation of a residual stress distribution.

For the welded box sections in Fig.1o, the magnitude of the compressive residual stresses in the flange will vary according to the actual size of the section and the amount of welding. This variation had for instance for the residual stress pattern as assumed in Fig.1o no significant influence on the column strength curves.

In Fig.11 this residual stress pattern (marked as E 2) is compared with patterns (E 5 to E 8) that have a wider zone of high tensile residual stresses and recognize the effect of different amounts of welding. They are based on theoretical investigations as outlined in (3), and correspond to box sections with the ratio of height to plate thickness h/t of 2o and 4o, each with a heavy weld (E 5 and E 7) and a light weld (E 6 and E 8). Most of the existing residual stress measurements, done for small and medium sized box sections, actually show patterns that are inbetween E 2 and E 5, E 6, E 7. Pattern E 8 with a width of the compressive stress zone of about o.9 of the section height will be found only in very wide, thinwalled sections, which are not any more subject to column buckling.

Similar investigations were done for welded I-sections, tubes, and T-sections (1) (2).
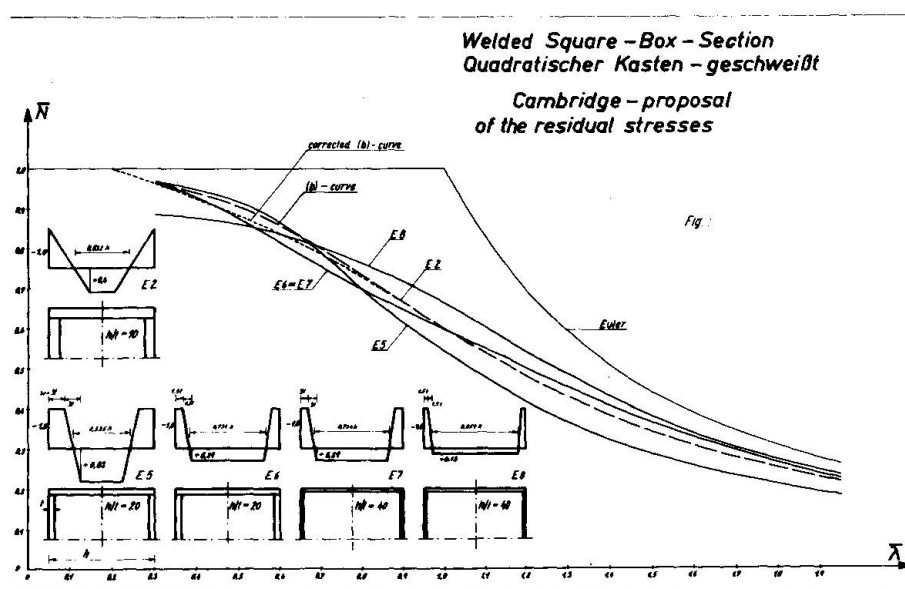

Fig.11: Column strength curves for welded box sections
(Initial curvature ℓ/1ooo, residual stresses
dependent of section size and amount of welding

4. REPRESENTATIVE COLUMN CURVES

The strength of the most commonly used structural sections could be related to the 3 column curves shown in Fig.12. Curve "a" is the maximum strength column curve for tubes, curve "b" the curve for the box sections, and curve "c" was calculated for the weak axis bending of wide flange I-sections. The other sections are placed in the 3 curves according to Fig.12, as described in detail in (2).

The evaluation of the nondimensional curves "a", "b", and "c" with the appropriate yield stresses is contained in (2), (4) and (5). An approximate method for the design of columns under small transversal loads, based on the curves "a", "b", and "c" is outlined in (6).
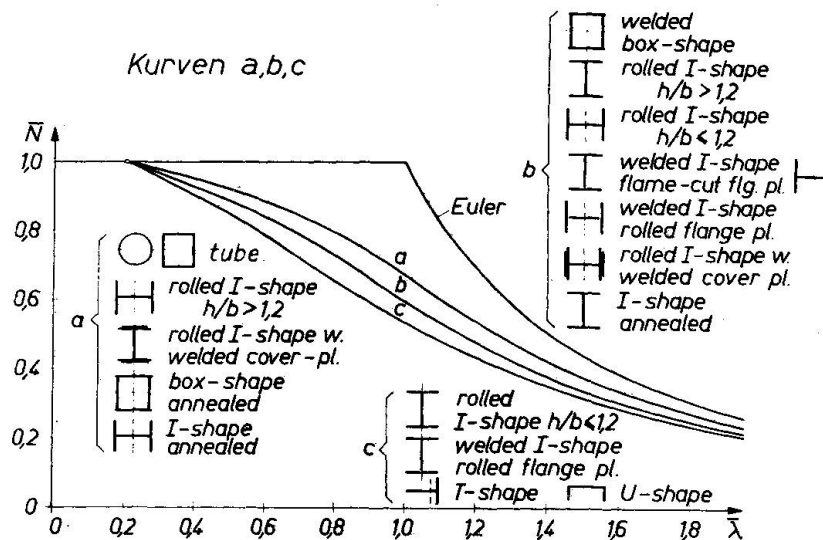
Fig.12: The column curves "a", "b", and "c" recommended by Commission 8 of ECCS

5. COMPARISON WITH TEST RESULTS

The theoretical column curves are in good agreement with the statistically evaluated European column tests (7).

Fig. 13 shows the test results for rolled tubes. The maximum strength as determined out of test results is considered to be the mean value m minus 2 times the standard deviation s. For a comparison curve "a", the theoretical curve for tubes, is evaluated with the statistically determined yield stress of the tubes, and is in good agreement with the test results.

The same good agreement can be shown between the test results for the I-sections IPE 16o and the pertinent curve "b" (Fig.14)

394

Vergleich: Knickversuche
nahtlos gewalzter Rohre -
- Kurve a

Kurve a ausgewertet für
Fließgrenze $\sigma_F = 2{,}4 - 2{,}5 - 2{,}6$ t/cm²

$\sigma_F = 2{,}6$ t/cm²
2,5
2,4

Kurve a (Rohr-Kurve)

Knickversuche:

$\dashv m$

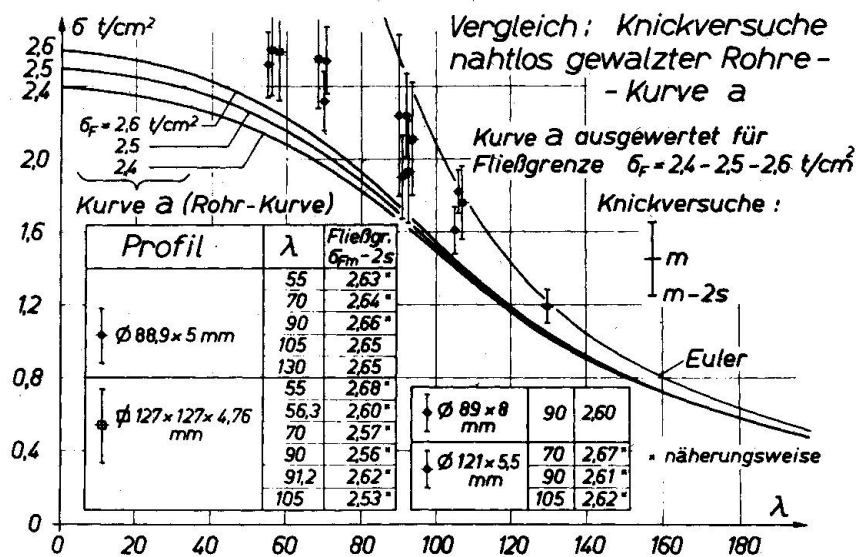$\dashv m-2s$

Euler

* näherungsweise

Profil	λ	Fließgr. $\sigma_{Fm}-2s$
$\varnothing\,88{,}9 \times 5$ mm	55	2,63 *
	70	2,64 *
	90	2,66 *
	105	2,65
	130	2,65
$\varnothing\,127 \times 127 \times 4{,}76$ mm	55	2,68 *
	56,3	2,60 *
	70	2,57 *
	90	2,56 *
	91,2	2,62 *
	105	2,53 *

	λ	
$\varnothing\,89 \times 8$ mm	90	2,60
$\varnothing\,121 \times 5{,}5$ mm	70	2,67 *
	90	2,61 *
	105	2,62 *

Fig.13: Statistically evaluated tests with rolled tubes
and the pertinent column curve "a"

6. FUTURE DEVELOPMENTS

The recommended column curves "a", "b", and "c" are slightly conservative for rolled sections of high strength steel, since the reduced influence of the residual stresses due to the higher yield point was omitted. That the omitted gain in column strength is small for the steel-grades presently used in Europe for most rolled sections, can be shown for the case of the weak axis buckling of an I-section. The column curves in Fig.15 are calculated for gradually increasing residual stresses $\bar{\sigma}^E$. If we relate these residual stresses to the lowest and highest yield stress of steels presently used in Europe, corresponding parameters would be for instance $\bar{\sigma}^E = o{,}3$ for the lowest yield stress, and $\bar{\sigma}^E = o{,}2$ for the highest yield stress. Although the weak axis buckling is most sensitive to a variation of the residual stresses, the resulting gain of strength is too small, in order to place the section in the next higher of the column design curves "a", "b", or "c".
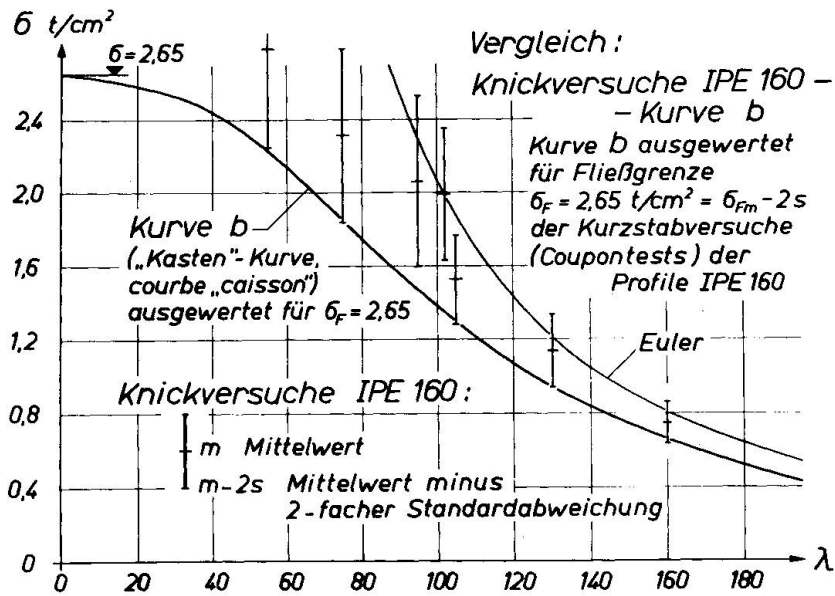
395

Fig.14: Statistically evaluated tests with I-sections
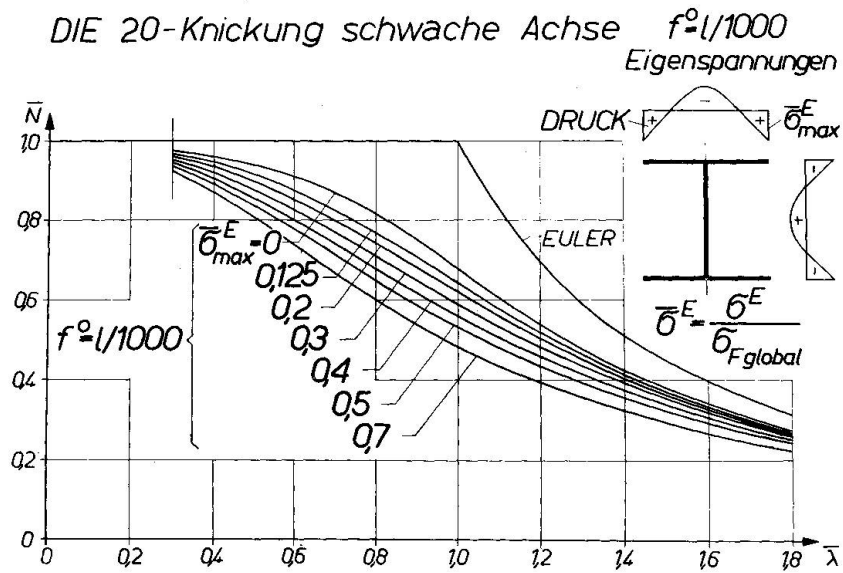IPE 160 and the pertinent column curve "b"

Fig.15: Column strength curves for weak-axis bending
of an I-section, including the effects of an
initial curvature of ℓ/1ooo and gradually in-
creasing residual stresses

Fig.16: Suggested column curves "a⁰" (for tubes of high
strength steel) and "d" (for heavy sections,
t > 4o mm)

Fig.17: Column strength curves for heavy rolled and
welded I-sections. (Initial curvature ℓ/1ooo
residual stresses)

But since steels with a much higher yield point are used more frequently for tubular members, a curve "a^O" in addition to the curves a, b, c is suggested as future design curve for these sections (Fig.16). Curve "a^O" was calculated as maximum strength curve for a tubular section without residual stresses.

Finally, as an additional provision for future developments in Europe, a curve "d" is suggested for the design of sections with a wall thickness > 4o mm. (Fig.16). Besides in the United States these heavy sections are already rolled in Great Britain, and might be introduced on the Continent in due time. The column strength curves in Fig.17 are calculated for heavy rolled and welded I-sections, and indicate the need for a design curve lower than curve "c".

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