Application of the buckling curves of the European convention for constructional steelwork to frame columns

APPLICATION OF THE BUCKLING CURVES OF THE EUROPEAN CONVENTION FOR CONSTRUCTIONAL STEELWORK TO FRAME COLUMNS

Udo Vogel
o. Prof., Dr.-Ing.
Technische Universität - Berlin

ABSTRACT

This paper is an attempt to develop a method for the application of the European buckling curves to frame columns.

For this purpose results of the effective length method are compared with results of a second order elastic-plastic thoery.

The investigation includes the following steps :

(1) Approximate solution for the buckling strength of a hinged column with geometrical imperfections only.

(2) Evaluation of the necessary "representative" geometrical imperfection for a satisfactory agreement with the European buckling curves.

(3) Approximate solution for the ultimate strength of a two-hinged frame with geometrical imperfections.

(4) Comparison between the results of the effective length method and the approximate ultimate strength of the two-hinged frame.

The investigation shows that the effective length method with application of the European buckling curves is a reasonable approach for the determination of buckling loads for frame columns.

1. INTRODUCTION

Commission 8 of the European Convention for Constructional Steelwork has proposed new buckling curves for hinged Columns on the basis of very thorough experimental and statistical investigations, including the influence of all kind of imperfections /1/. It is very likely that these curves shall be adopted by several European countries in the near future.

The column with hinges on both ends, however, is not found very often in actual structures. Therefore it is necessary to ask how the "European buckling curves" can be used for other structural components, such as columns in frames.

It is obvious that one convenient way to do this is to determine the "effective length" of the frame column, calculate the slenderness-ratio and take the corresponding critical stress from the buckling curves. One does not know, however, whether this is a reasonable approach, because the imperfections -especially the geometrical ones- can be very different from those in two-hinged columns.

The purpose of this paper is to help to decide whether the "effective length" method is acceptable or not. The decision would be very easy if similar curves as the European buckling curves would be available for frames. It is very clear, however, to everybody who has studied the work which has led to the European curves that an enormous expensive and time consuming work is necessary to establish such curves. Therefore an approximate method to answer the question mentioned above is shown in the following four steps of investigation.

2. PROCEDURE OF INVESTIGATION

The first step is to establish buckling curves for a hinged column by an approximate ultimate strength method which includes one parameter only, to cover the influence of all possible imperfections. This parameter shall be called the "representative imperfection".

In step two the value of the representative imperfection is chosen for each group of cross-sections such as to match the resulting curves with the corresponding European buckling curves a, b, and c very closely. With this one gets an idea of the order of magnitude of the representative imperfection to be assumed for a frame structure.

In step three an approximate solution for the buckling strength of a frame with one characteristic (representative) geometrical imperfection is developed, using the same simplifying assumptions as for the determination of the ultimate strength in step one.

Finally, in step four a numerical example of the discussed frame will be evaluated. The results of this calculation will be compared with the results obtained by using the effective length method in connection with the European buckling curves. This comparison will lead to a conclusion about the possibility of the application of the European buckling curves to frame columns.

3. APPROXIMATE SOLUTION FOR THE BUCKLING STRENGTH OF A HINGED COLUMN WITH GEOMETRICAL IMPERFECTIONS ONLY

To establish ultimate strength curves for a hinged column, and later on for a frame, a second order elastic-plastic theory with the following assumptions is used /2/, /3/.

a) The ultimate strength is obtained by a failure mechanism with a sufficient number of plastic hinges.

b) Spread of plastification is neglected.

c) The influence of axial thrust on the plastic moment capacity and on the bending stiffness (i. e. on the deformation) is taken into account.

A column with an initial sinusoidal out of straigthness as "representative imperfection" is considered (see fig. 1) :

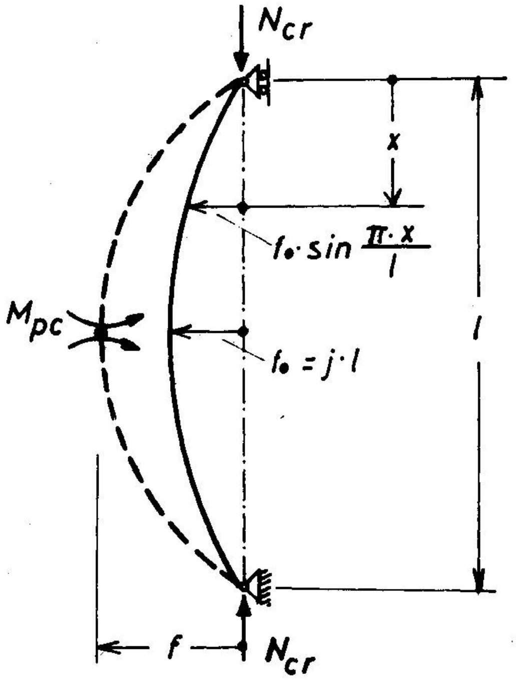

fig. 1 : Column with sinusoidal initial out of straightness.

This column failes when a plastic hinge has performed at mid-length. At this instant the condition for equilibrium can be expressed as :

$$N_{cr} \cdot f = M_{PC} \tag{1}$$

Right before this moment, in accordance with assumption b), the column behaves completely elastic, so that the deformation f is :

$$f = f_o \frac{1}{1 - \frac{N_{cr}}{N_E}} \tag{2}$$

where $N_E = \pi^2 \; EI/l^2$, the critical Euler load.

Using the same symbols as Beer /1/ does, namely :

$$\bar{N} = \frac{N}{\sigma_F \cdot F} = \frac{N}{\sigma_y \cdot A}$$

$$\bar{\lambda} = \frac{\lambda}{\lambda_F} \quad \text{with} \quad \lambda_F = \pi\sqrt{\frac{E}{\sigma_F}} \quad \text{and} \quad \lambda = \frac{1}{i} = \frac{1}{r} ,$$

applying Eq. (2) to Eq. (1) and rearranging the following equation results :

$$\bar{N} = 2.\alpha.\frac{1}{j} . \frac{i}{h} . \frac{1}{\lambda_F} . \frac{1}{\bar{\lambda}} . \frac{M_{PC}}{M_P} . (1 - \bar{N}. \bar{\lambda}^2) \qquad (3)$$

with α = shape factor = plastic modulus/elastic modulus

i = r = radius of gyration

M_P = plastic moment

M_{PC} = $F(\bar{N})$ = plastic moment reduced by axial force $\bar{N}$

j = characteristic value for the representative imperfection (see fig. 1)

For numerical computations it is more convenient to express $\bar{\lambda}$ as a function of $\bar{N}$, i. e. write Eq. (3) as :

$$\bar{\lambda} = - \frac{1}{4\alpha . \frac{1}{j} . \frac{i}{h} . \frac{1}{\lambda_F} . \frac{M_{PC}}{M_P}} + \sqrt{\left(\frac{1}{4 . \frac{1}{j} . \frac{i}{h} . \frac{1}{F} . \frac{M_{PC}}{M_P}}\right)^2 + \frac{1}{\bar{N}}} \qquad (4)$$

M_{PC}/M_P can be taken as a function of $\bar{N}$ from the interaction diagram fig. 2 for various cross sections.

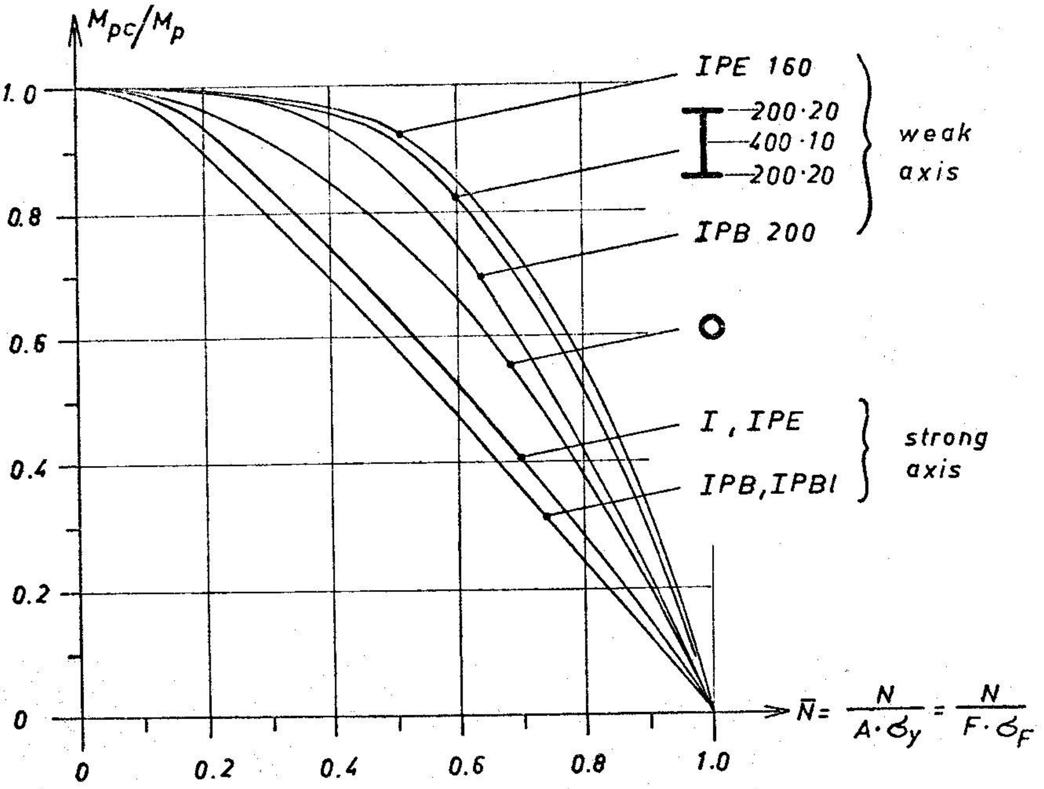

fig. 2 : Interaction diagram M_{pc}/M_p — $\bar{N}$

It is interesting to note that by assuming an ideal straight bar, i. e. $j \Rightarrow 0$, Eq. (4) leads to the critical Euler load :

$$\lim_{j \to 0} \bar{\lambda} = \sqrt{\frac{1}{\bar{N}}} \quad , \qquad \text{or} \qquad \bar{\lambda}^2 = \frac{1}{N}$$

$$\frac{\lambda^2}{\pi^2 \cdot \frac{E}{\sigma_F}} = \frac{1}{\frac{N_{cr}}{\sigma_F \cdot F}}$$

$$N_{cr} = \frac{\pi^2 \cdot E}{\lambda^2} \cdot F = \pi^2 \frac{EI}{l^2}$$

4. EVALUATION OF THE NECESSARY ORDER OF MAGNITUDE FOR THE REPRESENTATIVE GEOMETRICAL IMPERFECTIONS TO BE ASSUMED

Eq. (4) has been evaluated numerically for the six cases shown in table 1.

Table 1 : investigated cross sections

Case	Cross section and bending direction	Corresponding European buckling curve
1a	IPE 160, strong axis bending	a
1b	◎ 8 5/8" (D/t = 219,1/5,9)	
2a	IPB 200 (DIN 20), strong axis bending	b
2b	IPE 160, weak axis bending	
3a	IPB 200, weak axis bending	c
3b	 — 200 · 20 / — 400 · 10 / — 200 · 20; welded, weak axis bending	

The $\bar{\lambda}$ — $\bar{N}$ curves for each case were plotted for various values of j. Only those curves which fit best to the corresponding European buckling curves are shown in fig. 3, 4 and 5

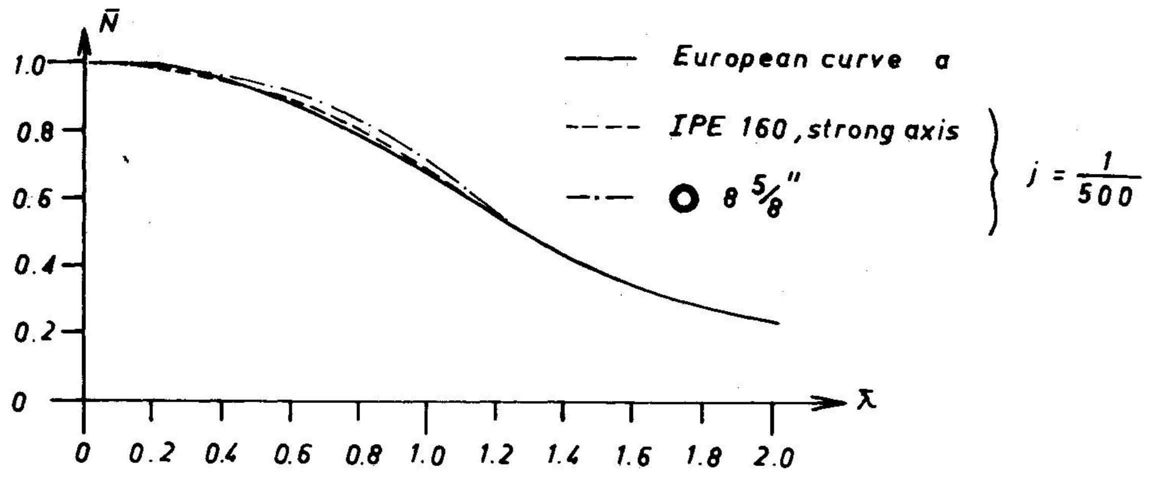

fig. 3 : European buckling curve a compared with approximate theory

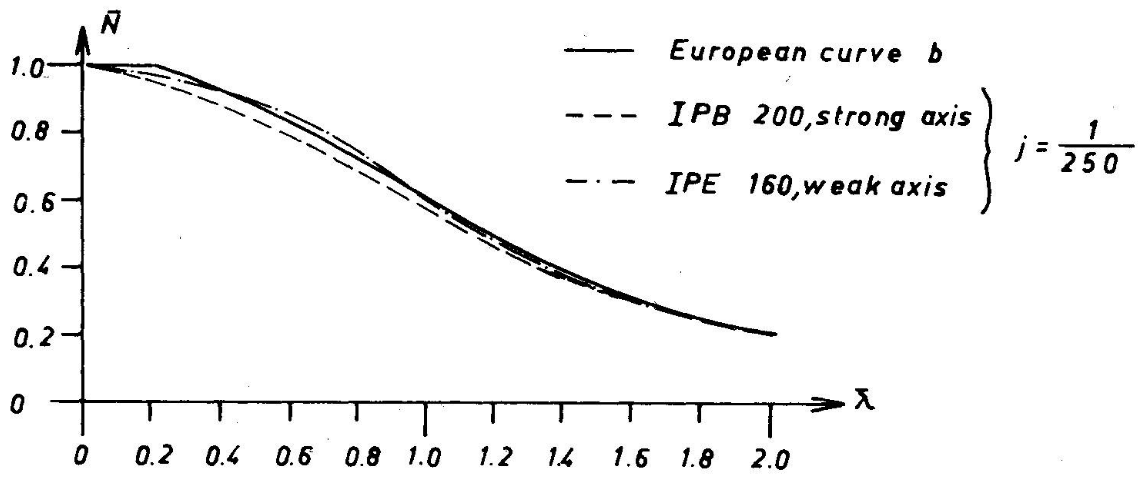

fig. 4 : European buckling curve b compared with approximate theory

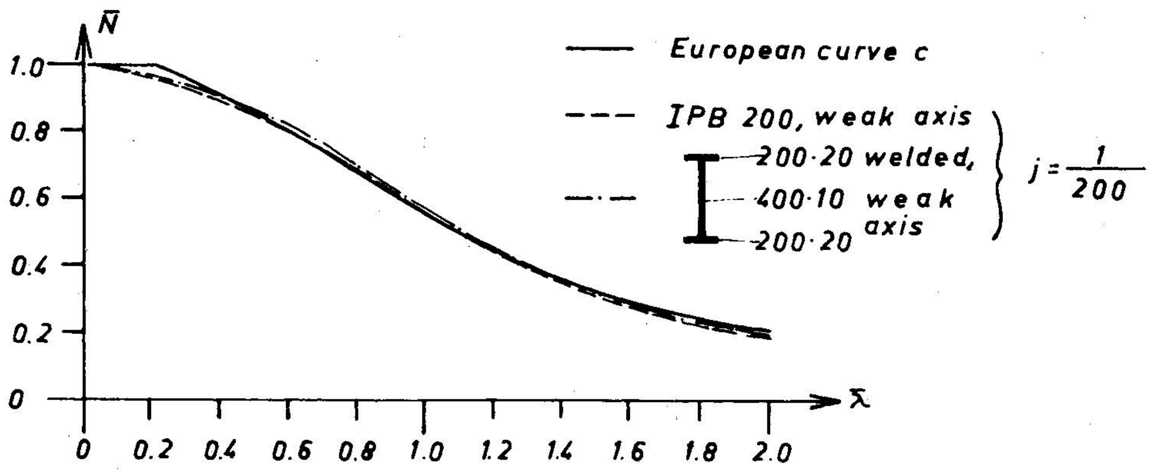

fig. 5 : European buckling curve c compared with approximate theory

From these figures it can be seen that the values in table 2 for the representative imperfections should be assumed, when applying the shown approximate method to calculate the ultimate strength of a centrally compressed hinged strut.

Case for European buckling curve	Characteristic value j for the representative imperfection
a	1/500
b	1/250
c	1/200

These values for j and the shown approximate method lead to buckling curves which are in satisfactory agreement with the more exact European curves, which include the influence of all kinds of imperfections, especially those of residual stresses. It is reasonable to use these values -at least their order of magnitude- also for frames, though the configuration of the assumed geometrical imperfection might be different from the sinusoidal one.

5. APPROXIMATE SOLUTION FOR THE BUCKLING STRENGTH OF A TWO-HINGED FRAME WITH GEOMETRICAL IMPERFECTIONS

The ultimate strength of the frame shown in fig. 6.a is calculated by the same method and with the same assumptions used in chapter 3.

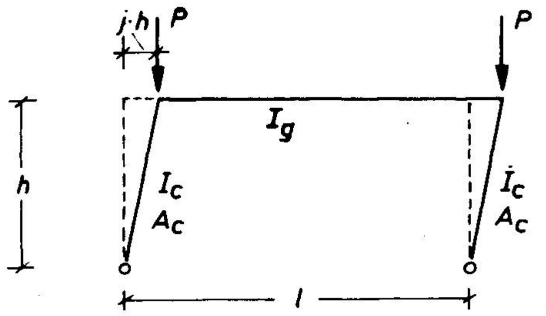

fig. 6.a : portal-frame with geometr. imperfection

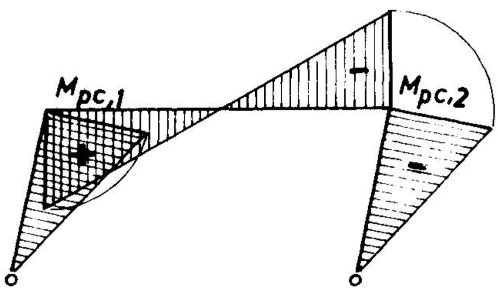

fig. 6.b : moment distribution at ultimate load

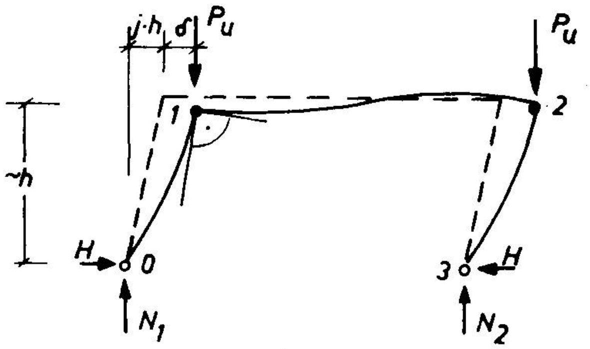

fig. 6.c : elastic-plastic deformation at the instant of failure

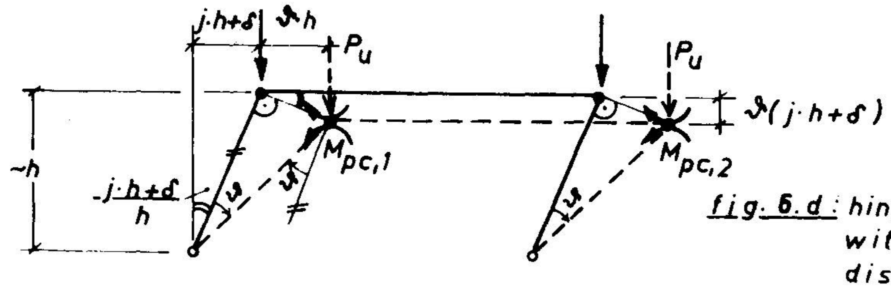

fig. 6.d : hinge mechanism with virtual displacements

Instead of crooked columns an initial displacement j.h of the column tops is assumed. The frame shall be designed in such a manner that the moment capacity of the columns is less than that of the girder. Therefore the plastic hinges shall performe in the columns.

Using the principle of virtual displacements the following work equation can be derived from fig. 6.d :

$$(M_{PC,1} + M_{PC,2}) \cdot \theta = 2\,P_u \cdot \theta \cdot (j \cdot h + \delta)$$

This leads to Eq. (5) for the ultimate load :

$$\boxed{P_u = \frac{M_{PC,1} + M_{Pc,2}}{2\,(j \cdot h + \delta)}} \qquad (5)$$

This equation still contains three unknowns : the two axial forces N_1 and N_2 (since $M_{PC,1}$ and $M_{PC,2}$ depend on these quantities), and the elastic-plastic deformation δ (see fig. 6.c).

N_1 and N_2 are obtained from fig. 6.c by using the equilibrium conditions $\Sigma M_3 = 0$ and $\Sigma M_o = 0$:

$$N_1 = \frac{2\,P_u}{1}\left[\frac{1}{2} - (j \cdot h + \delta)\right] \qquad (6.a)$$

$$N_2 = \frac{2\,P_u}{1}\left[\frac{1}{2} + (j \cdot h + \delta)\right] \qquad (6.b)$$

The deformation δ can be obtained by using the condition for continuity at the location of the last plastic hinge at the moment of failure. This is point 1 in Fig. 6.c, because $N_2 > N_1$, i. e. $M_{PC,2} < M_{PC,1}$, so that the plastic hinge at point 2 has to perform firstly.

The condition of continuity at point 1 is :

$$\phi_{10} = \phi_{12}, \text{ (equal nod rotations)} \quad (7)$$

where
$$\phi_{10} = \frac{\delta}{h} - \delta'_{10} \cdot \frac{M_{PC,1} \cdot h}{EI_c} \quad (8.a)$$

$$\phi_{12} = \frac{1}{EI_g} \cdot (\alpha'_{12} \cdot M_{PC,1} - \beta'_{12} \cdot M_{PC,2}) , \quad (8.b)$$

with the "stability function"

$$\alpha'_{10} = \frac{\sin \varepsilon - \varepsilon . \cos \varepsilon}{\varepsilon^2 \cdot \sin \varepsilon} \quad (9.a)$$

$$\varepsilon = h\sqrt{\frac{N_1}{EI_c}} \quad (9.b)$$

and $\alpha'_{12} \cong \frac{1}{3}$, $\beta'_{12} \cong \frac{1}{6}$ (because $N_{12} = H \cong 0$). (9.c)

Applying Eq. (8.a) and (8.b) to (7) and rearranging, the following equation for δ results :

$$\boxed{\delta = \alpha'_{10} \cdot \frac{M_{PC,1} \cdot h^2}{EI_c} + \frac{1 \cdot h}{EI_g} \cdot \left(\frac{1}{3} M_{PC,1} - \frac{1}{6} M_{PC,2}\right)} \quad (10)$$

With the four equations (5), (6.a), (6.b), and (10) which describe the failure condition of the frame the four unknown quantities P_u, N_1, N_2 and δ can be calculated by a trial and error procedure. This can be done conveniently by means of a computer programme. (Starting with $P_u = 0.8 . A_c . \sigma_F$ and $\delta = 0$, experience shows that a "hand computation" takes about three hours until the results for P_u converge sufficiently after about 5 steps.)

6. COMPARISON BETWEEN THE RESULTS OF THE APPROXIMATE METHOD AND THE "EFFECTIVE LENGTH METHOD" (USING THE EUROPEAN BUCKLING CURVES) FOR A TWO-HINGED FRAME..

6.1. Results of the approximate method

The equations of chapter 5 were solved for the following numerical example :

girder span : $1 = 12$ m

girder section : IPE 600 : $Ig = 92\ 080\ cm^4$

column height : $h = 10$ m, 8 m, 6 m and 5 m

column section : IPB 360 : I_c = 43 190 cm^4
A_c = 181 cm^2
i_c = 15,5 cm
s_x = 2 400 cm^3
α = 1,12

steel grade : St.52 (σ_F = 3 600 kp/cm^2 = 51.2 Ksi,
E = 2 100 Mp/cm^2)

The initial displacement as the representative imperfection was assumed to
j . h = h/250,

because the rolled section IPB 360 with H/b = 1.2 belongs to the European buckling curve b.

The results for the different column heights are shown in table 3 :

Table 3 : Results of approximate ultimate load theory

column height h [m]	10	8	6	5
ultimate load P_u [Mp = 1000 kp]	168.8	243.7	370.2	453.9

6.2. Results of the "effective length method"

In order to determine the effective length the alignment charts of the CRC-Guide to Design Criteria for Metal Compression Members are used (see also /4/).

With $G_A = \dfrac{I_c/h}{I_g/h} = \dfrac{43\ 190\ /h}{92\ 080/12}$

and $G_B = \infty$ (frictionless pin at the column foot)

the following results are obtained (see table 4).

Table 4 : Results of effective length method

h [m]	G_A	K = β	$\lambda = \beta h/i_c$	$\bar{\lambda} = \dfrac{\lambda}{\pi\sqrt{\dfrac{E}{\sigma_F}}}$	$\bar{N}$ from European buckling curve b	$P_{cr} = \bar{N} \cdot A_c \cdot {}_F$ Mp
10	0.56	2.20	141.9	1.86	0.236	153.8
8	0.70	2.24	115.6	1.52	0.335	218.3
6	0.94	2.35	90.9	1.19	0.486	316.7
5	1.12	2.38	76.8	1.01	0.592	385.7

The results of tables 3 and 4 are plotted in Fig. 7.

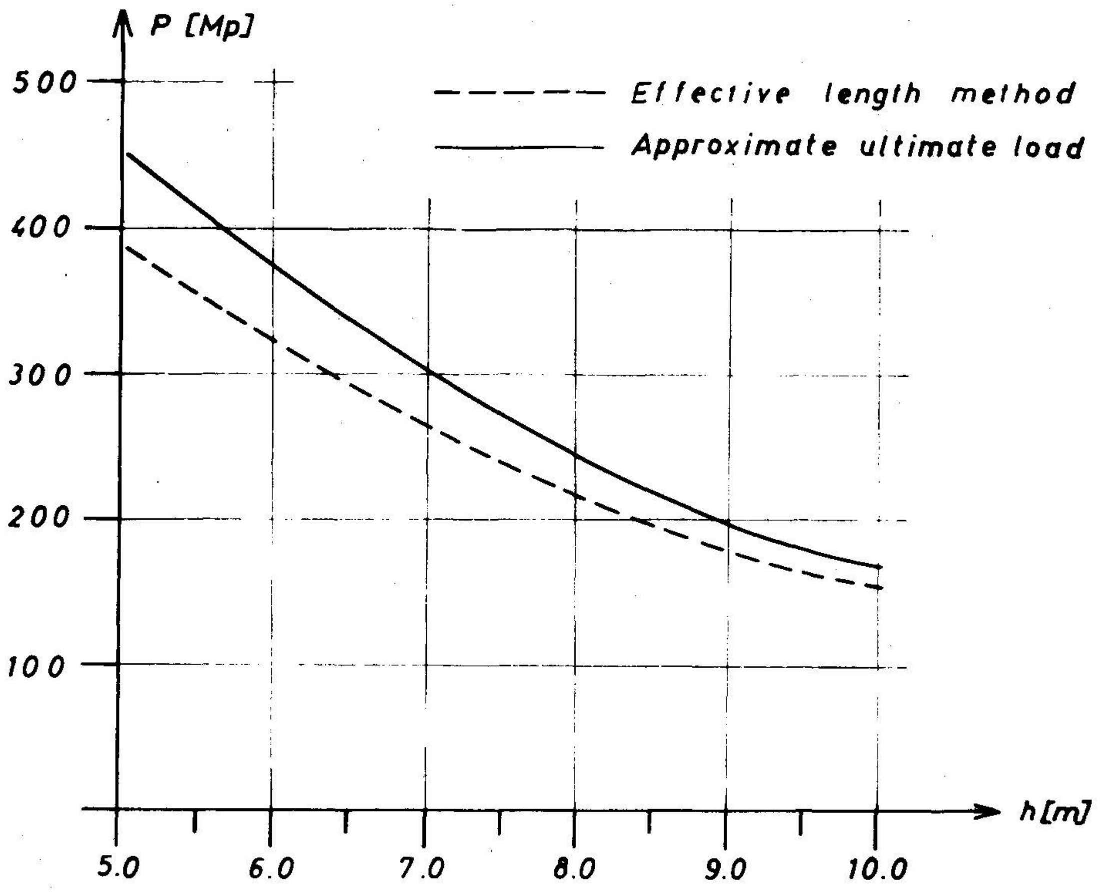

fig. 7: Comparison between effective length method (European buckling curve b) and approximate ultimate load for a two-hinged portal-frame.

The comparison shows that the results of the effective length method with application of the European buckling curves are a little smaller than the results of an approximate ultimate load method. Thus the effectiv length method seems to be -at least for the discussed example of a two-hinged frame- on the safe side.

7. CONCLUSION

The investigations of this paper show that the effective length method with application of the new European buckling curves is a reasonable approach for the determination of critical buckling loads of single-story frames - though it might in some cases be too far on the safe side. Further research work has to be done in order to decide whether this method is suitable for multi-story frames also. It seems to the author that for some other simple structures, loading cases or boundary conditions of structural components the shown procedure is a helpful and not too unconvenient way to decide this open question.

R E F E R E N C E S

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