Computational aspects of a general purpose program for the analysis of creep and shrinkage in concrete structures

Autor(en): **Haas, W.**

Objekttyp: **Article**

Zeitschrift: **IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen**

Band (Jahr): **31 (1978)**

PDF erstellt am: **13.09.2024**

Persistenter Link: https://doi.org/10.5169/seals-24930

IABSE
AIPC
IVBH

COLLOQUIUM on:
"INTERFACE BETWEEN COMPUTING AND DESIGN IN STRUCTURAL ENGINEERING"
August 30, 31 - September 1, 1978 - ISMES - BERGAMO (ITALY)

Computational Aspects of a General Purpose Program for the Analysis of Creep and Shrinkage in Concrete Structures

Programme général pour le calcul des effets du fluage et du retrait du béton, du point de vue de la programmation

Computeraspekte in einem allgemeinen Programm zur Berechnung der Answirkungen des Kriechens und Schwindens im Betonbau

W. HAAS
Dr. - Eng.
Rechen-und Entwicklungsinstitut für EDV im Bauwesen - RIB e.V.
Stuttgart, German Federal Republic

Summary

Two examples are presented which show the requirements of a general - purpose program for the analysis of creep and shrinkage in concrete structures. The impact of the different construction phases and of the different design standards on the formulation of the stress-strain law and on the data and program organization are discussed. Results of the two calculations are presented.

Résumé

Deux exemples montrent la capacité qu' un programme pour le calcul des effets du fluage et du retrait du béton devrait avoir. On tient compte de l' influence de la réalisation de la construction en plusieurs étapes et avec différents systèmes statiques sur le comportement des matériaux et sur l'organisation du programme. Des résultats de deux calculs sont présentés.

Zusammenfassung

Die Anforderungen an ein Programm zur Berechnung der Auswirkungen des Kriechens und Schwindens werden an je einem Beispiel aus dem Hoch - und Brückenbau erörtert. Dabei werden die Auswirkungen der Baupraxis, z. B. der abschnittsweisen Herstellung des Bauwerks in Bauzuständen, auf die Formulierung des Stoffgesetzes und die Daten - und Programmorganisation erörtert. Für zwei Beispiele werden abschliessend einige Berechnungsergebnisse erläutert.

1. INTRODUCTION

With regard to the design of structural analysis programs including the effects of time-dependent behaviour of concrete (e.g. creep and shrinkage) attention is usually restricted to the mechanical and numerical aspects, such as the form of suitable stress-strain laws or the development of stable and accurate time-integration schemes.

In considering these undoubtedly important problems one often neglects the organizational aspects of the task resulting, for example, from the fact that concrete structures are always erected in a sequence of construction phases with often different structural systems, or, that cross-sections may be subsequently supplemented by concrete or prestressing cables. Moreover, the dead weight and the prestressing are usually set up in stages corresponding to the construction phases. This loading history may continue over the course of several years.

The first purpose of this contribution consists of discussing some stress-strain laws for the time-dependent behaviour of concrete from the point of view mentioned above. In addition this paper describes some organizational aspects of programs for the analysis of the effects of creep and shrinkage in concrete structures in conjunction with two example problems. Results of both examples are discussed.

2. STRESS-STRAIN LAW FOR THE TIME-DEPENDENT BEHAVIOUR OF CONCRETE

As most design codes [1], [2], [3], [4] include statements on uniaxial stress-strain behaviour only, the present discussion is restricted to one-dimensional stress-strain laws. In addition shrinkage, being independent of stresses, will not be taken into account in the following equations.

In formulating a stress-strain law for the creep behaviour of concrete, the validity of the priciple of superposition [5], [6] or a linear stress-strain law [7] is assumed. This assumption, which appears to be valid for stress levels up to about 30 % of the ultimate strength, is very useful for the analysis. This assumption and the assumption that the cross-sections of beams will remain plane, for example, result in a linear stress distribution over the cross-section.

Based on this assumption, the linear stress-strain law can be expressed as Stieltjes' integral [8]:

$$\varepsilon(t) = \int_0^t \frac{1}{E(\tau)}\left[1+\varphi(t,\tau)\right]d\sigma(\tau) \tag{1}$$

Here ε denotes the strain, σ the stress, E the time-dependent elasticity modulus, φ the creep function, τ the time at which the stress variation $d\sigma(\tau)$ occurs and t the time of observation.

The integral representation of equation (1) is unsuitable for implementation into a program due to the large amounts of data involved [9], [10].

An extremely simple stress-strain law can be obtained if the equation for a constant stress σ_0 applied at time t_0.

$$\varepsilon(t) = \sigma_0 \frac{1+\varphi(t,t_0)}{E(t_0)} \tag{2}$$

is also used for time-dependent stress histories $\sigma(t)$.

This results in the procedure of the effective modulus E_{eff}, which is based on the following equations:

$$\varepsilon(t) = \frac{\sigma(t)}{E_{eff}} \tag{3}$$

with

$$E_{eff} = \frac{E(t_o)}{1+\varphi(t,t_o)} \tag{4}$$

With this equation it is possible to calculate the effects of creep and shrinkage as in an elastic structure with the modulus of elasticity $E = E_{eff}$. However, as can be seen from equation (4) the effective modulus E_{eff} depends on the time of loading t_o and thus each loading case of the loading history will have its own effective modulus E_{eff}. If, as is the case in the two examples to be described later, the loading history consists of a large number of loading cases with different t_o the procedure becomes complex and uneconomical. Apart from this, the procedure also becomes inaccurate if a considerable change in stress $\Delta\sigma$ occurs between the loading time t_o and the time of observation t and the concrete ages considerably [11].

A similarly simple analysis of the effects of creep and shrinkage for one loading case in one step can be done with Trost's ϱ-formula [6].

$$\varepsilon(t) = \frac{\sigma(t_o)}{E(t_o)}\left[1+\varphi(t,t_o)\right] + \frac{\sigma(t)-\sigma(t_o)}{E(t_o)}\left[1+\varrho\varphi(t,t_o)\right] \tag{5}$$

The relaxation factor ϱ allows to take into account the influence of the aging of concrete more accuratly than by the effectiv modulus method. For this reason the procedure is also known in the USA as the "age-adjusted effective modulus method".

The great advantage of equations (3) and (5) is that they allow to calculate the stress resultants and displacements at the time of observation t for one loading case in one step (one-step procedures). However, if the various construction phases consisting of different structural systems are to be considered, the changes in stresses and deformations within each construction phase must be computed using the respective structural system. For this reason, the total changes in the stress resultants and the deformations up to the time t must not be calculated in one step (i. e., using one particular structural system). In such cases the above-mentioned advantages of the equations (3), (5) may no longer be relevant, and thus a rate type stress-strain law is preferable.

Let us now take a closer look at the differential equation (6) of the aging three-parameter solid (Fig. 1).

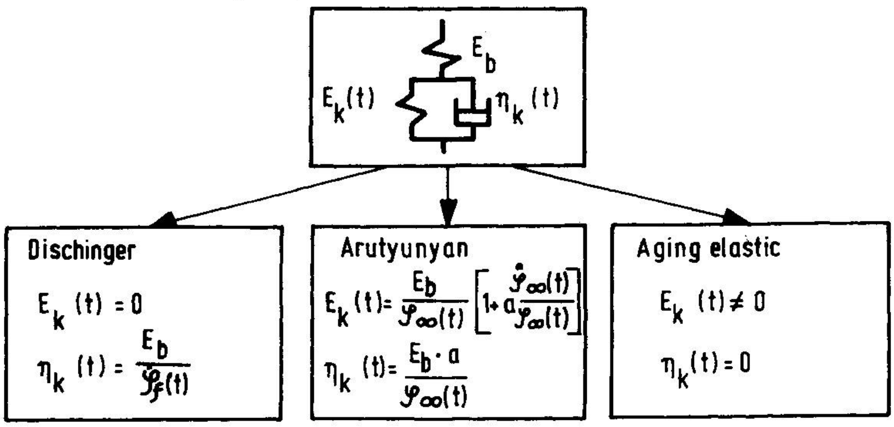

Fig. 1 Some creep laws which can be represented by the aging three-parameter solid

This model consists of a spring E_b and a Kelvin element with a dime-dependent spring $E_k(t)$ and a time-dependent dashpot $\eta_k(t)$:

$$\dot{\sigma}\left(1 + \frac{E_k + \dot{\eta}_k}{E_b}\right) + \ddot{\sigma}\,\frac{E_k}{E_b} = \dot{\varepsilon}\,(E_k + \eta_k) + \ddot{\varepsilon}\,\eta_k \qquad (6)$$

If we assume the functions $E_k(t)$, $\eta_k(t)$ to be piecewise constant, this differential equation can be transformed to an integro-differential equation suitable for numerical integration [10].

Up to now the aging three-element model has been used almost exclusively in the limited form of Arutyunyan's creep law [13], [14]. This creep law assumes the parameters $E_k(t)$ and $\eta_k(t)$ to be interconnected by the following equation:

$$a = \frac{\eta_k}{E_k + \dot{\eta}_k} = \text{const.} \qquad (7)$$

a has the dimension of time and is usually called "retardation time".

The creep law of Arutyunyan yields the following equations for the creep function

$$\varphi(t,\tau) = \varphi_\infty(\tau) \cdot f(t-\tau) \qquad (8)$$

with

$$f(t-\tau) = 1 - e^{-\frac{t-\tau}{a}} \qquad (9)$$

In many cases, actual creep curves cannot be represented by eqs. (8), (9) with sufficient accuracy due to the exponential expression in eq. (9) [11].

These difficulties, however, can be avoided by omitting the assumption of a constant retardation time a; in this case the functions of $E_k(t)$ and $\eta_k(t)$ can be determined independently. This results in a large degree of adaptability to a large variety of creep laws (cf. Fig. 1).

In the special case

$$E_k(t) = 0 \qquad (10)$$

$$\eta_k(t) = \frac{E_b}{\dot{\varphi}_f} \qquad (11)$$

we obtain Dischinger's creep law (rate of creep method) which yields the following equation for the creep function.

$$\varphi(t,\tau) = \varphi_f(t) - \varphi_f(\tau) \qquad (12)$$

In this equation creep is described in terms of a flow function φ_f which is irreversible upon stress removal.

The equation for the creep function laid down in the prestressed concrete guidelines [3] and the CEB recommendations [1]

$$\varphi(t,\tau) = \varphi_f(t) - \varphi_f(\tau) + \varphi_v(t-\tau) \qquad (13)$$

is obtained from e. g. (12) by adding the another term $\varphi_v(t-\tau)$ that is completely reversible when the stress has been removed, and which is referred to as "delayed elasticity".

This creep law, too, can be represented by means of the aging, three-parameter solid with an accuracy perfectly adequate for practical requirements. Delayed elasticity with a final value, as prescribed by the guidelines for prestressed concrete, of $0.4 E_b$ can be taken into account if $E_K(t)$ is suitably determined. Also the development of creep in time can be approximated to a sufficient degree of accuracy by appropriately choosing the viscosity function $\eta_K(t)$ [15].

3. CREEP AND SHRINKAGE ANALYSIS OF THE "HYPOHOCHHAUS"

The "Hypohochhaus" is a 137 m high administration building presently under construction in Munich. In this building the vertical loads from all storeys are carried by a prestressed supporting floor half-way up the building which itself is supported, by four stairway towers (cf. Figs. 2 and 3).

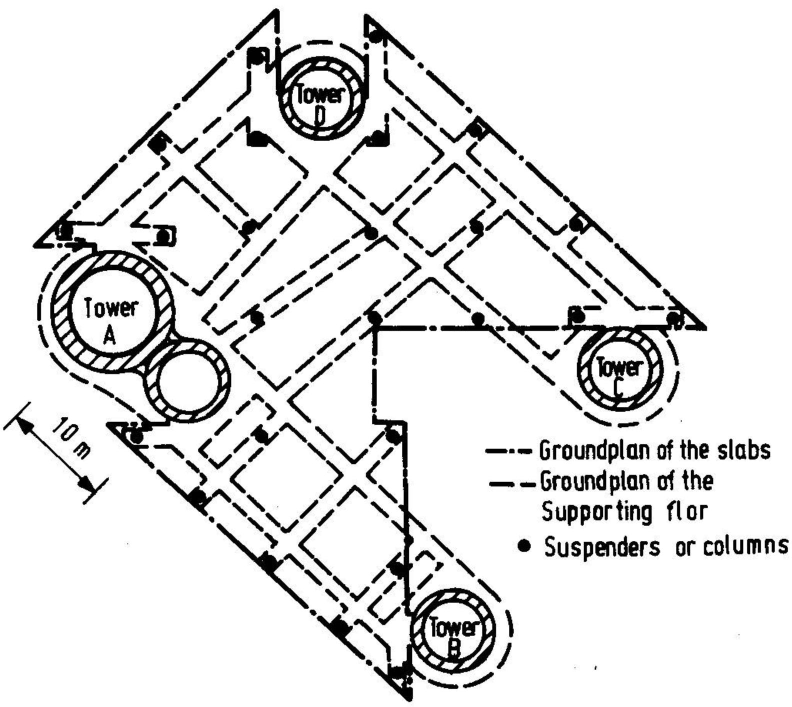

Fig. 2 Ground plan of the "Hypohochhaus"

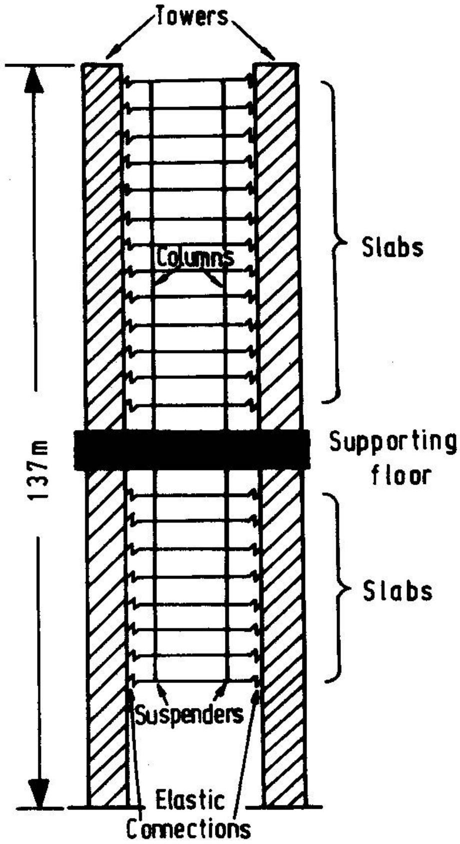

Fig. 3 Sketch of the structural behaviour of the "Hypohochhaus"

Thus, the slabs above the supporting floor rest on it, while the slabs below the supporting floor are suspended. In horizontal direction, however, the slabs are linked to the towers by means of elastic connections.

The slabs are constructed of lightweight concrete, to keep their dead-weight low; but the towers and the prestressed supporting floor are built of normal concrete. The considerable differences in the creep and shrinkage behaviour of these two materials (cf. Fig. 4) cause large changes of stresses,the history of which had to be calculated.

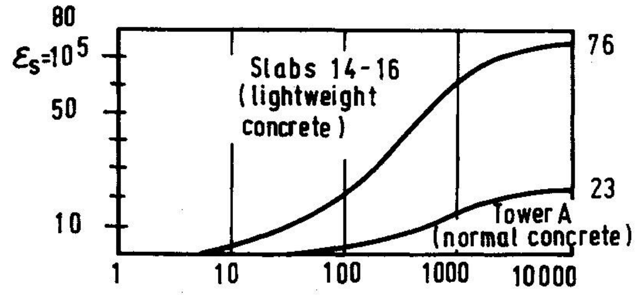

Fig. 4 Shrinkage of normal and lightweight concrete

For the creep and shrinkage analysis the structure is idealized as a space frame with 224 beams, 242 joints, 126 rigidity conditions and 36 hinged joints with elastic connections. The supporting floor is idealized as a grid structure. The 32 lightweight concrete slabs are grouped into nine blocks and each block is represented by a system of four beams, forming a cross. The moment of inertia of these beams I_2 is assumed to be very large and the cross-sectional area is determined, such that the total stiffness of a slab block connecting the four towers is represented correctly.

These simplifications are justified, since the main purpose of the analysis is not the determination of the state of stress in the slabs, but the history of internal forces in the elastic connections, the towers and the prestressed supporting floor.

The building is erected in a sequence of seven construction phases with different structural systems. The different slab blocks are constructed and linked to the staircase towers at four different instants of time.

The loading history consists of 8 permanent loading cases "dead and live loads" and 3 loading cases due to prestressing, all of which are applied at different times. The prestressing includes a total of 124 tendons in different positions. One cross-section could contain up to 48 tendons.

The creep and shrinkage analysis of this example is based on Arutyunyan's creep law. By distorting the time scale it is possible to approximate the prescribed material curves for the normal concrete and the lightweight concrete with sufficient accuracy using the exponential expression in equations (8) and (9). The significant differences in the creep and shrinkage characteristics of the different materials and the extremely varying effective thicknesses imply that the maximum stresses do not occur at $t \rightarrow \infty$ but at an intermediate point in time which is to be determined.

Therefore, the entire histories of the internal forces and deformations have to be computed in order to be able to find their maximum values.

These calculations are carried out for two cases: one, shrinkage alone, and two, shrinkage plus permanent load. Some typical results are shown in Figs. 5 and 6.

Fig. 6 indicates the remarkable fact that the force in the elastic connection of the first slab block above the supporting floor due to shrinkage alone is higher than the force due to shrinkage plus permanent load. The reason for this lies in the fact that the tensile stresses in the connections due to shrinkage of the slabs are considerably reduced by the compressive stresses induced by the permanent loads.

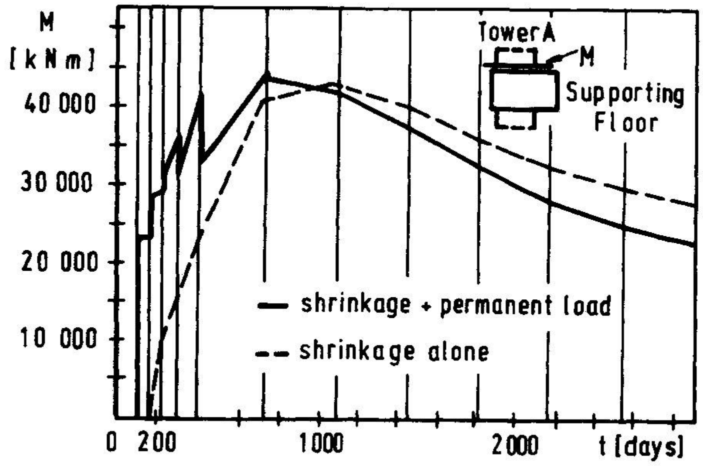

Fig. 5 Bending moment in tower A at the height of the upper surface of the supporting floor

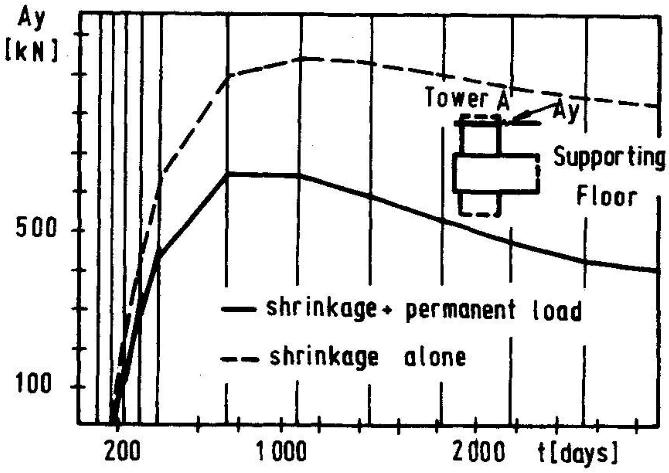

Fig. 6 Force Ay in the elastic connection of the first slab block above the supporting floor at tower A

4. CREEP AND SHRINKAGE ANALYSIS OF THE KOCHER VALLEY BRIDGE

The Kocher Valley Bridge is a prestressed-concrete motorway bridge at present under construction as part of the extension of the Weinsberg - Nuremberg motorway between Schwäbisch Hall and Künzelsau. Its maximum height above the valley floor is 185 m.

The structural system of the bridge consists of a frame structure, the four central columns of which are rigidly connected with the superstructure. The regular width of the spans of the superstructure is 138 m (cf. Fig. 7).

The six-lane highway is supported by a box girder 6,5 m high with widely overhanging plates. The superstructure is constructed in stages by cantilevering out using an advancing girder. In this procedure the box girder section without the lateral plates (i.e., the core cross-section) is first constructed by cantilevering out in both directions from the tops of the piers. As soon as one section extending from one centre of a span to the centre of the next span has been completed, one begins to concrete the lateral plates on both sides of the core structure, the so-called "follow-up" structure, thus completing the final cross-section of the bridge. These concrete plates are braced against the

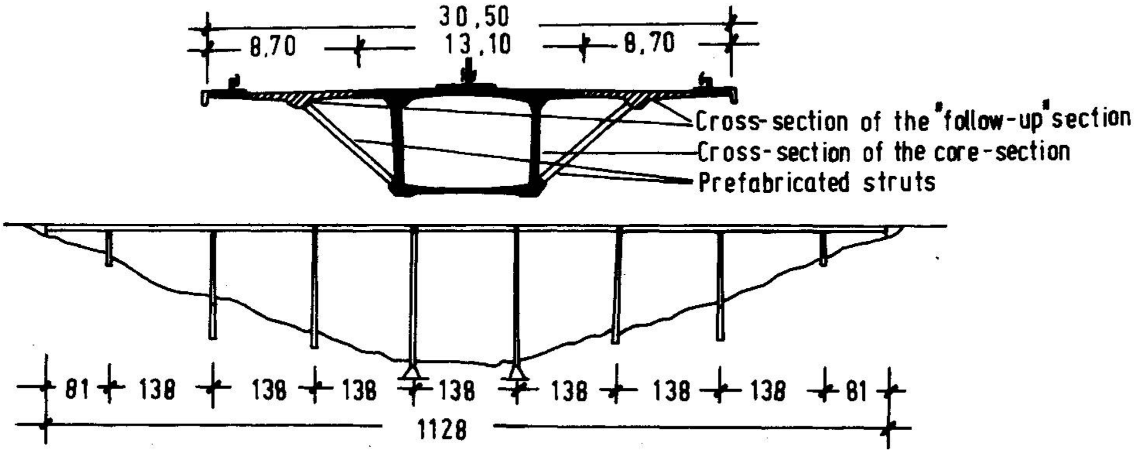

Fig. 7 Cross-section and side view of the Kocher Valley Bridge

core by means of prefabricated struts. Simultaneaously, the next section in the cantilever construction of the core proceeds as before. Due to the short period of time available, the construction of the superstructure is advanced, starting from both abutments simultaneously.

The delayed addition of the "follow-up" cross-section to the core cross-section causes stress redistributions in the cross-section ("cross-sectional creep") and deformations which, in turn, produce stress redistributions in the entire structural system ("system creep"). Both types of cree-effects can, in this case, no longer be considered independently [14]. The construction of the bridge in 14 construction phases with different structural systems also yields stress redistributions in the total structure. In order to keep these effects to a minimum, so-called joint-expansion moments and forces are applied by means of presses to the coupling joints before these are closed. These joint-expansion moments and forces provide the internal stresses which would be present if all loads had been applied right from the start to the final structural system. But as the bridge contains concrete of varying ages this measure cannot completely suppress system creep. In addition, the temporal progression of the deflections has to be calculated in order to determine the required camber of the bridge.

In calculating the effects of creep and shrinkage it is necessary to analyse half of the bridge only as the structural problem is symmetric. It is idealized as a plane frame with 13 beams. Each of these beams may have varying cross-sectional properties along its axis and each cross-section may be a composite cross-section. With this extraordinarily efficient beam we are able to account for the gradual increase in cross-sectional values due to the concreting-on of the "follow-up" cross-section, and the successive prestressing and injecting of the tendons.

The following loading cases are considered:

- 32 loading cases of dead load due to the weight of the core cross-section
- 32 loading cases of dead load due to the weight of the "follow-up" cross-section
- 39 loading cases for various positions of the cantilevering out scaffold and the advancing girder
- 9 loading cases due to joint expansion
- 9 loading cases related to pavement, railing etc.
- 41 loading cases due to prestressing of 175 tendons in the core cross-section and 90 tendons in the "follow-up" cross-section

These loading cases were applied at 42 different instants of time.

The analysis was carried out with the improved three-parameter solid. The creep curves of the prestressed concrete guidelines were used as input data by a preprocessor to compute the time dependent material characteristic values of the three-parameter solid.

The integration in time of the stresses and displacements was carried out in 90 time steps. It is interesting to note that 83 steps were required to describe accurately the different construction phases and the loading history and only 7 steps to determine the changes in the internal forces and displacements during the period beginning with the opening of the bridge to traffic (t = 1120 days) up to the assumed date of termination (t = 12.000 days).

Fig. 8 shows the changes of the normal forces in the core cross-section and the "follow-up" cross-section versus time.
It indicates that the "follow-up" cross-section gradually carries a considerable proportion of the load.

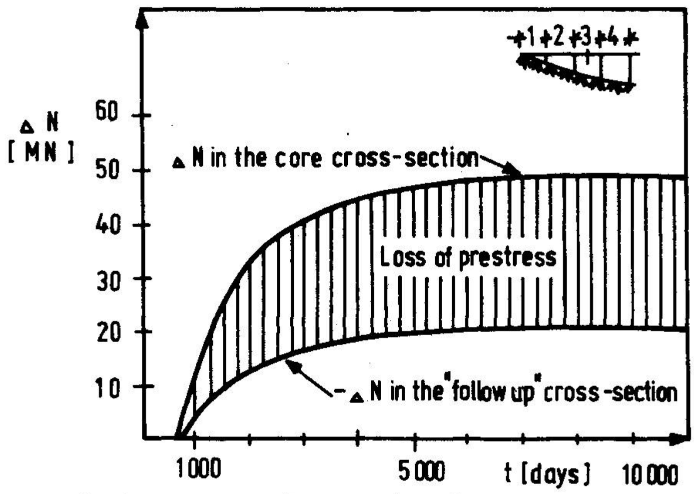

Fig. 8 Changes of the normal forces in the core cross-section and the "follow-up" cross-section in the middle of the third span of the bridge

In Fig. 9 the normal forces and their changes at $t \to \infty$ are shown. 68 % of the normal force at $t \to \infty$ in the "follow-up" structure are caused by the effects of creep and shrinkage. Another result of creep and shrinkage is a reduction of the normal force in the core of 50 MN. Of this figure, 29 MN are prestress losses and 21 MN have been taken over by the "follow-up" cross-section.

	Core	"Follow-up"
Normal forces $t \to \infty$	- 82	- 31
Changes in the normal forces, $t \to \infty$	+ 50	- 21

Fig. 9 Normal forces and their changes in the middle of the third bridge span (unit = MN)

Fig. 10 shows the changes of the bending moment at the middle of the third span. The inflections at the beginning of the curve are due to the different construction phases during this period. The total change in the bending moment ΔM = 12.1 MN at the time $t \rightarrow \infty$ is about 5 % of the bending moment due to the total dead weight of the superstructure on the final system. Thus, the expansion of the joints has greatly reduced system creep.

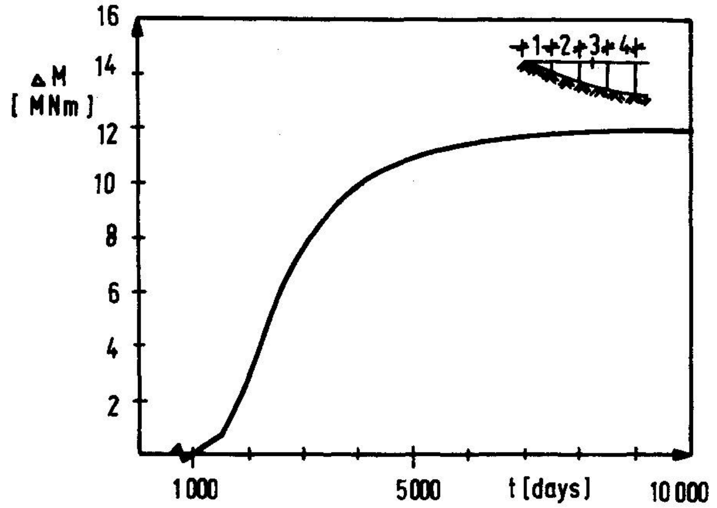

Fig. 10 Changes of the bending moment in the middle of the third span of the bridge

REFERENCES

1. COMITÉ EUROPÉEN DU BETON - CEB. Time Dependent Behaviour of Concrete. State of Art Report, 1973

2. ACI COMMITTEE 209/II. Prediction of Creep, Shrinkage and Temperature Effects in Concrete Structures. ACI - SP27. Designing for Effects of Creep, Shrinkage and Temperature. Detroit, 1971.

3. Richtlinien für Bemessung und Ausführung von Spannbetonbauteilen, Fassung Juni 1973

4. DIN 1045 - Beton- und Stahlbetonbau, Bemessung und Ausführung, Ausgabe Januar 1972.

5. MC HENRY, D.: A New Aspect of Creep in Concrete and its Application to Design. Proc. ASTM 43 (1943)

6. TROST, H.: Auswirkungen des Superpositionsprinzips auf Kriech- und Relaxationsprobleme bei Beton und Stahlbeton. Beton- und Stahlbetonbau 61 (1967), H. 10, 11.

7. ONAT, E. T.: Description of Mechanical Behaviour of Inelastic Solids Proc. of the 5th U.S. National Congress of Applied Mechanics, S. 421.

8. BAŽANT, Z. P.: Theory of Creep and Shrinkage in Concrete Structures. A Precis of Recent Developments. Mechanics Today, Vol. 2. New York, Toronto, Oxford, Sydney, Braunschweig. Pergamon Press Inc. 1975.

9. ZIENKIEWICZ, O. C., WATSON, M.: Some Creep Effects in Stress Analysis with Particular Reference to Concrete Pressure Vessels. Nuclear Engineering and Design 4 (1966) S. 406.

10. HAAS, W.: Über ein für die EDV geeignetes Verfahren zur Erfassung des Kriechens und Schwindens von Beton. Dissertation, Universität Stuttgart, 1974.

11. BAŽANT, Z. P., NAJJAR, L. J.: Comparison of Approximate Linear Methods for CONCRETE CREEP. Proc. ASCE, J. Struct. Div. 99 (1973) No. St9, S. 1851.

12. BAŽANT, Z. P.: Prediction of Concrete Creep, Effects using Age-Adjusted Effective Modulus Method. ACI Journal 69 (1972), S. 212.

13. ARUTYUNYAN, N. Kh.: Some Problems in the Theory of Creep in Concrete Structures. Pergamon Press,1966.

14. SCHADE, D., HAAS, W.: Elektronische Berechnung der Auswirkungen von Kriechen und Schwinden bei abschnittsweise hergestellten Verbundstabwerken. Heft 244 des DAfStb, Berlin 1975.

15. HAAS, W.: Über das Anwendungsspektrum des alternden Drei-Elemente-Modells. To be published.