IV

Finite Element Analysis of Beam Shear Problems

Analyse du cisaillement d'une poutre par la méthode des éléments finis

Finite Elemente Berechnung von Schubproblemen in Balken

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SUMMARY
A non-linear finite element method of analysis was used to predict the behaviour of reinforced concrete beams. Special consideration was paid to the assumptions on change in stiffness of concrete due to cracking and bond. The results were compared with test results. The method of analysis used appeared to predict the behaviour of the beams subjected to shear and bending in terms of pattern and propagation of cracks, mode of failure and the shear at the failure.

RESUME
L'étude du comportement des poutres en béton armé est faite à l'aide d'une analyse non linéaire par la méthode des éléments finis. Les hypothèses concernant les changements de la rigidité du béton dues à la fissuration et à l'adhérence sont considérées avec soin. Les résultats analytiques et expérimentaux sont comparés. En appliquant la méthode décrite, on peut prédire le comportement d'une poutre soumise à la flexion et au cisaillement. On peut prédire la formation et la propagation des fissures ainsi que le mode de ruine.

ZUSAMMENFASSUNG
Eine nichtlineare Finite Elemente Berechnung wird durchgeführt, um das Verhalten von Stahlbetonbalken zu untersuchen. Den Annahmen über Steifigkeitsänderungen infolge Reissen des Betons sowie Verbund wird besondere Beachtung geschenkt. Den Ergebnissen werden Versuchsresultate gegenübergestellt. Die verwendete Methode gestattet die Voraussage des Verhaltens von Balken unter Biegung und Querkraft. Insbesondere können das Rissbild und die Rissentwicklung sowie die Art des Versagens verfolgt werden.

1. INTRODUCTION

1.1 Nature of the problem

The plastic behavior demonstrated by beams failing under the influence of bending and shear is complex and diverse, because it is affected by a broad variation in geometry of beams, distribution of reinforcing steel and boundary conditions. For confident application of plastic theory information is desirable for the behavior of beams at various stages of loading. The finite element analyses are one way of providing useful information. However, the idealization for the known inelastic behavior of concrete and a gradual destruction of bond between steel and concrete to be used in such methods of analysis is still in the process of development.

1.2 Scope

An attempt was made by a parallel study of analyses and tests for refinement for the method of analysis and better understanding of the behavior of the beams. A non-linear finite element method of analysis with several assumptions was used to predict the behavior of beams with rectangular cross section, containing flexural reinforcement only and subjected to bending and shear. The predicted results were compared with test results.

2. FINITE ELEMENT MODEL

2.1 Non-linear analysis

The finite element program used for this study was a two dimensional program which was a part of a general non-linear analysis program referred as COMPOSITE III and developed by Kokubu, Yamada and Sakurai (1). In this two dimensional analysis program conventional constant strain triangular elements and truss elements are used. Non-linear behavior is idealized by piece-wise linear analysis and modification of stiffness of elements according to stress or strain conditions as appropriate. The increments for applied loads can be prescribed to any required magnitude including zero. When the losses of stresses occur in an element due to cracking or crushing of concrete, the element stresses are transformed into nodal forces for the nodes connected to that element. In the next load increment, after modification of the stiffness of that element as required, the calculated set of the nodal forces is applied to the structure. Thus the relief of stresses in that element and redistribution of those stresses to the neighboring elements are approximated.

Four assumptons were made for this study. Those assumptions concerned : (1) stiffness of concrete, (2) failure criterion for concrete, (3) change in stiffness due to cracking, and (4) bond between steel and concrete.

2.2 Stiffness of concrete

The material stiffness matrix is defined by a shear modulus G and a bulk modulus K in this finite element program. The two moduli used for this study were identical to the tangent moduli given by Kupfer et al. for each load increment. If the moduli G and K are transformed into the modulus of elasticity E for a case of uniaxial compression, the resulting stress strain relationship is as indicated in Fig.1. If cracking occur in the concrete element, the procedure of this section is over-ridden by the provision given in Section 2.3.

2.3 Change in stiffness due to cracking

When the first cracking occurs, one of the axes of orthogonality is made coincident to the direction of the crack. Then, the coefficients in the material stiffness matrix related to the stress component which caused cracking are made equal to zero. If an element containing a crack is subjected to shear stress, displacements parallel to the crack may occur. The resistance against such displacements may be approximately expressed by the shear modulus, if the relation between the displacement and the stress is known. However, information is not sufficient for this behavior, and hence, it was assumed that the shear modulus was not changed due to cracking. This behavior implies that the interfaces of a crack are capable of transferring the same magnitude of shear as uncracked concrete. When the second crackig occurs, the remaining coefficients in the material stiffness matrix are made practically equal to zero (one thousandth of the original values).

2.4 Failure criterion

The criteria for crushing and cracking of concrete are identical to those proposed by Kupfer et al., and are expressed in terms of principal stresses (2). A graphical representation is given in Fig.2.

2.5 Bond

The strength and behavior of beams failing under the influence of shear and bending was reported to vary with the bond characteristics of flexural reinforcement, and the reasons were explained by Kani (3), (4) in connection with the stress trajectories. Among the factors related to bond and influencing those trajectories, the variation in stress in flexural reinforcement with the distance from the section of maximum moment and distribution of flexural cracks were considered to be more significant. Therefore, for this analysis flexural cracks were forced to occur in the spacing approximately equal to those observed in actual beams, but the reinforcing steel was rigidly connected to the nodes of concrete elements without use of bond elements between the steel and the concrete. The flexural cracks were allowed to occur in only predetermined groups of elements by prescribing a very high value as fictitious tensile strength of concrete for the elements where cracking should be prevented. The elements for which cracking was prevented are indicated in Fig.3 with shades.

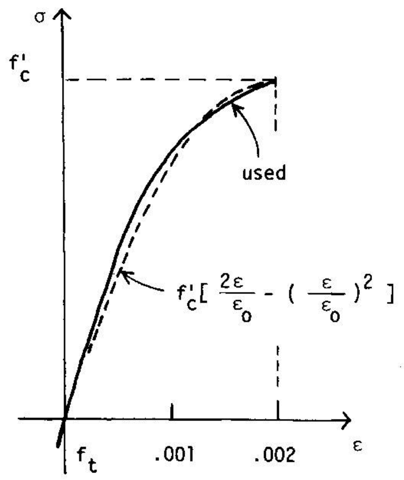

FIG.1 STIFFNESS OF CONCRETE

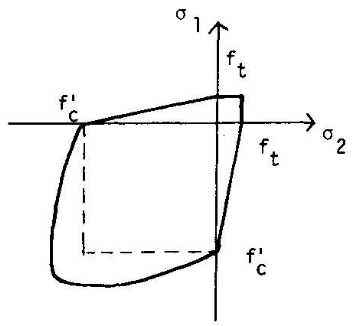

FIG.2 FAILURE CRITERION FOR CONCRETE

2.6 Structural model

The structural model used for this analysis is an idealization of a beam with rectangular cross section 15 cm in width, 25 cm in depth and 210 cm in length. The distance between the compression surface and the centroid of the flexural reinforcement, or effective depth, was 22 cm. The beam was simply supported and subjected to two symmetrical concentrated loads. The distance between the point of application of the load and the support, or shear span, was 66 cm. The finite element mesh layout and the boundary conditions are shown in Fig.3.

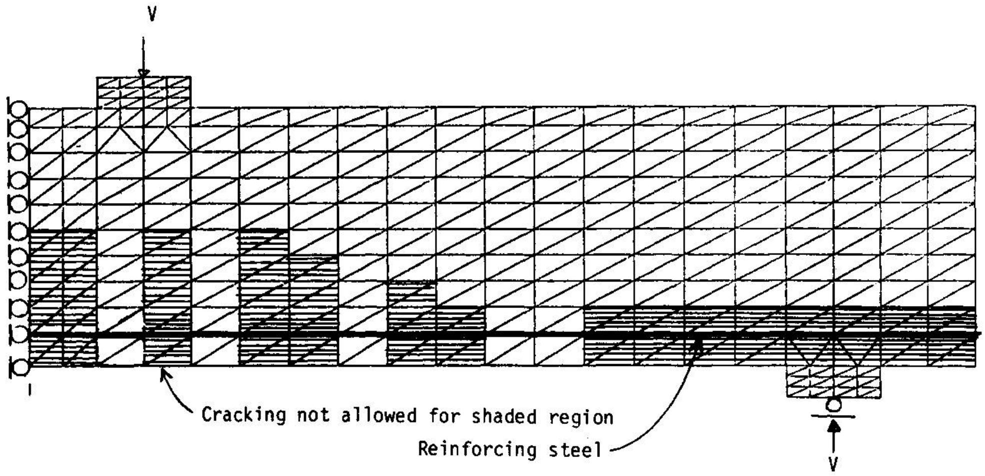

FIG.3 FINITE ELEMENT IDEALIZATION

3. COMPARISON OF PREDICTED AND OBSERVED BEHAVIOR

3.1 Loading

The load level, or applied shear was expressed as fractions of V_{flex}, where V_{flex} is a calculated shear which would cause flexural failure. When the concrete strength was 27 Mpa and the yield point of the flexural reinforcement was equal to 400 Mpa, and with the amount of flexural reinforcement equal to 11.6 cm^2consisting of 3-Φ22 mm bars (3 percent of steel ratio), the shear V_{flex} was 115 KN. The loads were applied in 16 steps, or with an increment of 0.044 V_{flex}.

The progressive propagation of cracks may not be predicted if the external loads are kept to increase at each increment of analysis, because of the provision used for this analysis in order to take into account of redistribution of stresses due to crushing and cracking of concrete as stated in Section 2.1. A particular analysis concerning this problem is to be reported in Section 3.7.

3.2 Cracking

Development of cracks with increasing load is shown in Fig.4 for the applied shear equal to 0.18, 0.44 and o.53 V_{flex}. Cracks are denoted by outlining the triangular elements where cracking was predicted, and by lines representing the directions of the cracks in those diagrams. Flexural cracks in the tension surface of the beam were allowed to occur at only prescribed locations as explained in Section 2.5. At a shear equal to 0.44 V_{flex} inclined cracks appeared above the flexural cracks in a section about the beam effective depth away from the point of application of the load, at about the middle of the shear span, and in a section about the effective depth away from the support and at a level about one third the beam depth from the tension side of the beam. Those cracks were to increase in number at higher loads, but those inclined cracks remained disconnected from each other. Long inclined cracks observed in beam tests were not predicted by this analysis. At a shear equal to 0.53 V_{flex}, cracking were predicted in extensive portion of the top surface of the beam. At a section about the beam effective depth away from the point of application of the load the uncracked portion of concrete was significantly reduced due to development of inclined cracks developing from the flexural cracks as well as cracks developed on top surface of the beam.

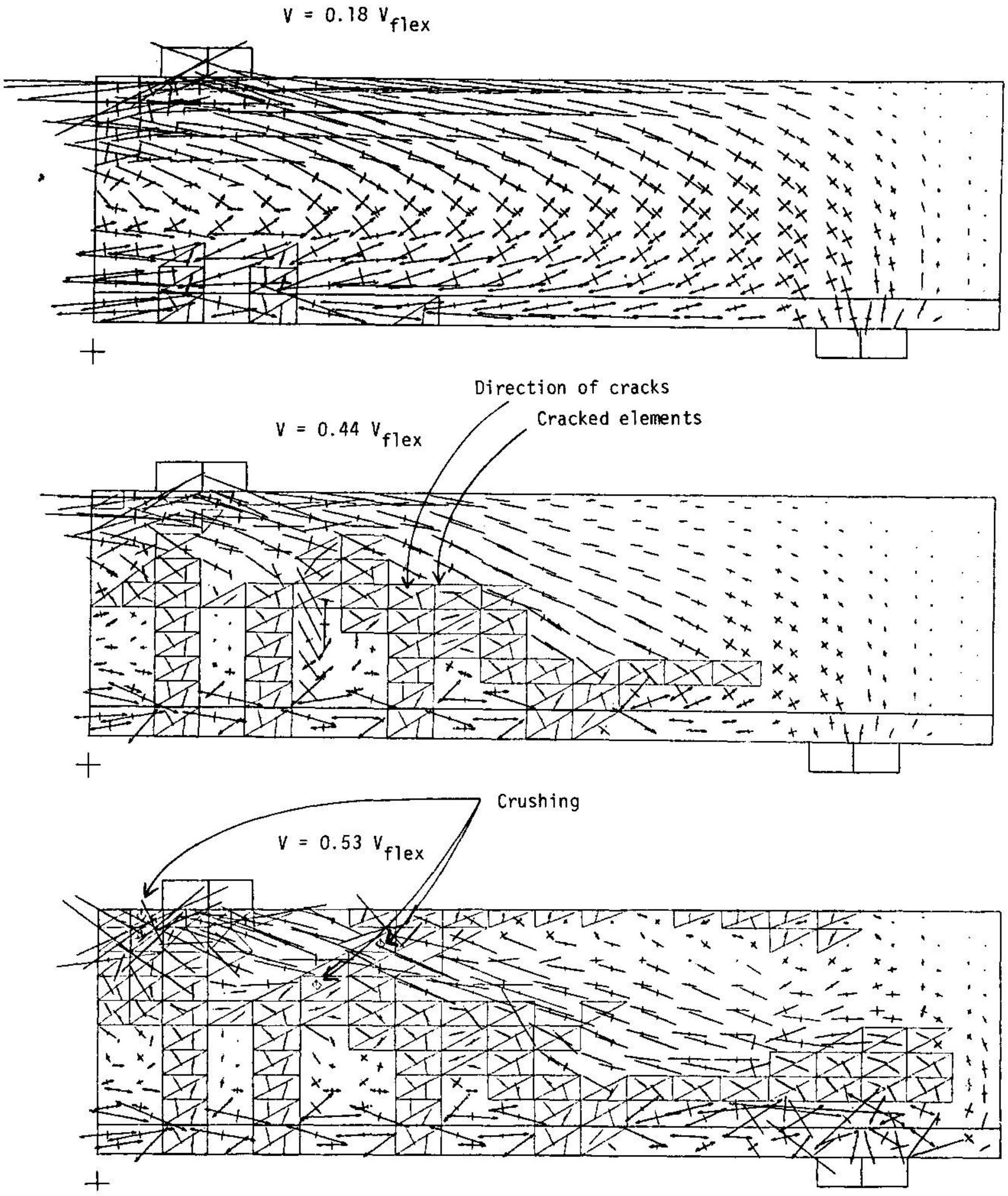

FIG.4 PREDICTED PRINCIPAL STRESSES, CRACKING AND CRUSHING

3.3 Stress trajectory

At a shear equal to 0.18 V_{flex} cracking was locally contained in the region of maximum moment, and the most portion of the beam remained elastic. The compressive and tensile stress trajectories indicated in Fig.4 followed smooth curves which resembled arches and hanging arches. At a shear equal to 0.44 V_{flex} the compressive stress trajectories dominated in the middle of the shear span and in the compression zone in midspan sections. At a higher load directions of the trajectories flattened in both the upper and lower portions of the beam as apparent in the diagram for 0.53 V_{flex} .

3.4 Crushing of concrete

The predicted crushing of concrete is denoted by a mark which has a shape of small rhombus in Fig.4 for a shear equal to 0.53 V Crushing was predicted in two sections in the beam. The first section was located adjacent to the loading plate and in the region of the maximum bending moment. The second section was situated about a beam effective depth away from the loading plate and in the direction of the reduced moment. In this section crushing was predicted in the narrowest path for the compressive stress trajectories.

3.5 Effect of bond

If the gradual breakage of bond between reinforcing steel and concrete was not taken into account in this method of analysis, the predicted cracking and trajectories are as indicated in Fig.5. Since the reinforcing bars were rigidly connected to the surrounding concrete elements, all of these concrete elements cracked.

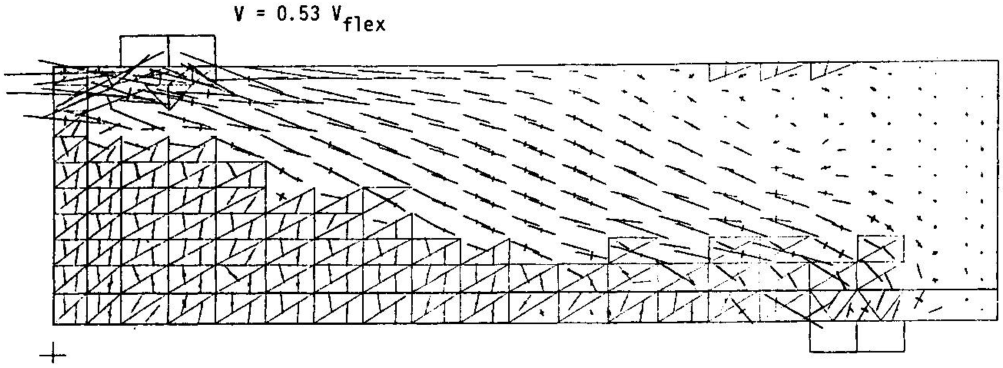

FIG.5 BEAM WITH CRACKING ALLOWED FOR ALL ELEMENTS

3.6 Compressive principal stress

For a more quantitative representation of the stress trajectories plot was made in Fig.6 for the distribution of the compressive principal stress along the vertical cross section of the beam located at 0.54, 1.18, 1.82 and 2.74 times the beam effective depth apart from the position of loading and for a section without shear. Three curves in each section represent stresses for applied shears equal to 0.18, 0.44 and 0.53 V_{flex} resspectively. At the lowest shear the distribution of this stress was as expected by elementary beam theory. However, at higher loads the distribution became markedly different. At the section closest to the position of the load a concentration of the stress occurred near the top surface of the beam. In contrast, at the next section moved toward the support the peak value for the stress occurred some distance apart from the top of the beam. At the sections closer to the support the distribution is more uniform and the peak gradualy shifted downward. The manner in which those stresses are distributed is understandable by referring the pattern of cracking.

3.7 Progressive propagation of cracks

So far report has been made for the analyses where the external loads were increased in increments at each increment of analysis. In order to predict the possible progressive propagation of cracks, one series of analysis was made where the external load was not increased after it reached 0.44 V_{flex}. For the following increments of analysis the loads applied to the structure were only those resulting from stress redistribution. The results after three repetitions of such analyses are indicated in Fig.7. The results demonstrated that a crack propagated along the flexural reinforcement from the bottom end of an inclined crack to the end of the beam without an increase in the external load when the applied shear reached 0.44 V_{flex}.

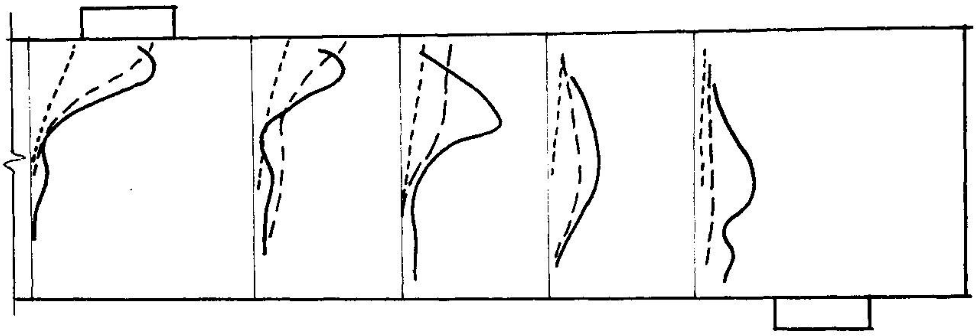

FIG.6 PREDICTED DISTRIBUTION OF COMPRESSIVE PRINCIPAL STRESSES FOR SHEAR 0.18, 0.44 AND 0.53 V_{flex}

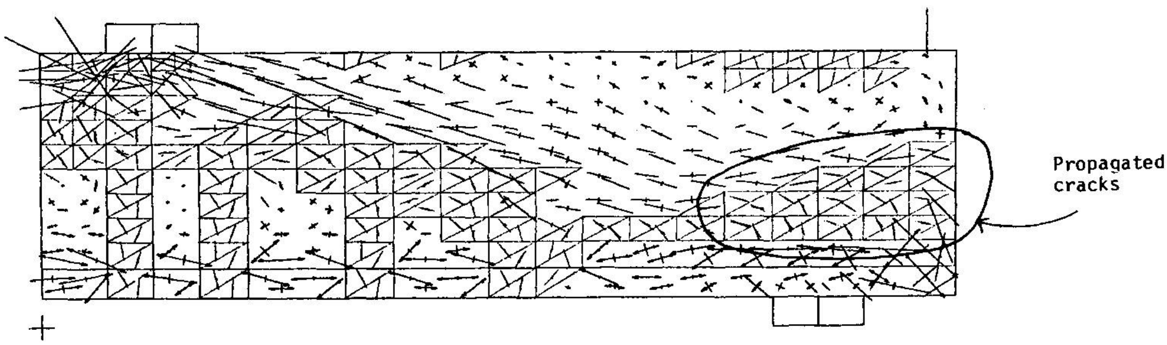

FIG.7 PREDICTED PROGRESSIVE PROPAGATION OF CRACKS ALONG REINFORCING BARS AT A SHEAR 0.44 V_{flex}

3.8 Failure of beam

In the beam failed in test the inclined crack propagated along the reinforcing bars and at the seemingly same instance crushing occurred in the upper portion of this inclined crack as well as cracks in the top surface of the beam. The shear at failure was 0.39 V_{flex}. The mode of failure appeared to coincide with the prediction described in Section 3.7 and indicated by Fig.7. The prediction for the shear at failure was 0.44 V_{flex}, or 12 percent higher than the measured.

The results predicted with monotonically increasing shear higher than 0.44 V_{flex} and reported in Section 3.2 through 3.6 are probably more indicative for the cases where the propagation of crack along flexural reinforcement was prevented by some measure. For the latter cases failure could occur due to crushing of concrete in the compression zone at cross sections some distance apart from the point of application of the load. This crushing may be attributed to the reduction in dimension of the path for compressive stress trajectories due to development of flexural and inclined cracks and cracks in the top surface of the beam.

4. CONCLUSIONS

The analytical and experimental study reported here was concerned with the shear failure of the beams with a rectangular cross section, without web reinforcement and where the ratio of shear span and beam effective depth was equal to three. Within the scope of this study the following conclusions appeared relevant.

1. The finite element method of analysis used here appeared to predict the behavior of the beams failing under the influence of shear and bending in terms of crack pattern, mode of failure and the shear at failure, if adequate considerations are made for bond between flexural reinforcement and concrete, criterion for crushing and cracking of concrete, and progressive propagation of cracks by the procedure given in this study.
2. The suppression of occurrence of flexural cracks at prescribed locations used in this study is a convenient practice to substitute a difficult but very important problem of idealizing the non-uniform bond between steel and concrete.
3. The finite element method of analysis used in this study enabled prediction of progressive development of cracks along the flexural reinforcement near the failure load by a few increments of analysis where the external load was set equal to zero and only the forces resulting from the stresses relieved from concrete elements which failed were considered.

ACKNOWLEDGMENT

This study was supported by the Grant-in-Aid for Scientific Research No.246118 from the Japanese Ministry of Education.

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