Similitude of Brittle Fracture of Structural Concrete

Similitude de rupture fragile en béton armé

Ähnlichkeit bei sprödem Betonversagen

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SUMMARY
On three examples of brittle fracture of structural concrete - i.e. bending shear, punching shear, shear in joints between prefabricated floor slabs - it is demonstrated that the absolute size of the member influences the ultimate shear stress. This influence is explained by linear elastic fracture mechanics.

RÉSUMÉ
Par trois exemples de rupture fragile en béton armé - effort tranchant en flexion, poinçonnement de dalles, effort tranchant dans les joints entre éléments de plancher prefabriqués - il est démontré que les dimensions absolues des éléments influencent les tensions de rupture en cisaillement. Cette influence est expliquée par la mécanique de rupture linéaire-élastique.

ZUSAMMENFASSUNG
An drei Beispielen für spröden Bruch von Stahlbetonbauteilen - Biegeschubbruch, Durchstanzen, Schubversagen von Fugen zwischen Fertigteildeckenplatten - wurde gezeigt, dass die absolute Grösse des Bauteils die Bruchschubspannung beeinflusst. Dieser Einfluss wurde mithilfe der linear-elastischen Bruchmechanik erklärt.

1. INTRODUCTION

With the aid of modern computational methods and facilities it is possible to
analyse all kinds of structures under almost all loading conditions. Expecially
those structures do not give problems which exhibit elastic behaviour in the
service state and plastic, or more general ductile, behaviour in the limit state.
Most of the concrete structures are designed in such a way that they behave as
mentioned. There are only a few cases where brittle fracture or almost brittle
fracture occurs. But just these cases are the ones which should receive at-
tention, because they can be dangerous if the governing parameters are not well
recognized. This is true of structural design and also of analysis if the
engineer is not aware of possible influences which may reduce the safety of the
structure.

In the following only one aspect will be discussed in relation to three examples
of structures, namely, the size effect in connection with shear failure of beams
and slabs, punching shear of slabs, and shear transfer across joints between
precast hollow core floor slabs. Common practice is that an average shear stress
is calculated and compared with a limiting value which is a certain fraction of
the tensile strength of concrete. This is done at finite element level, but
also for the structure as a whole. Sometimes the size effect is taken into
account by an empirical reduction factor. In the following it is tried to show
that fracture mechanics concepts could be applied to explain the influence of
the absolute size of the structure on the limit load.

2. EXAMPLE OF BRITTLE FRACTURE

Brittle fracture of concrete structures is the exceptional case and is normally
avoided by appropriate reinforcement or prestressing. But sometimes, because
of economical advantages, reinforcement is ignored and the concrete alone has
to be responsible for the loading capacity. The first example of this is a
beam or slab without shear reinforcement. Fig. 1 shows the loading system and
the crack pattern after failure. The common behaviour of such a beam is that
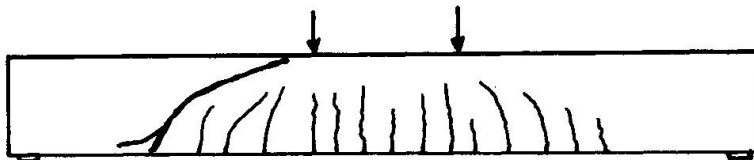
Fig. 1. Loading system and crack pattern of a beam after shear failure

the first cracks occur due to bending. Subsequently the cracks become longer
until the extension of one inclined crack leads to failure because of low com-
pressive strength or low tensile strength. Failure is "explosive", without ad-
vance announcement by excessive deformation.

A similar example is punching shear of slabs which is shown in Fig. 2.
Fracture occurs when a concentrated load which acts on a reinforced concrete
slab reaches the loading capacity. After the formation of radial and circular

Fig. 2.   Punching shear of concrete slab

cracks, sudden fracture occurs without announcement. The difference between
bending shear and punching shear lies in the manner of stressing and the accom-
panying shape of the fractured element, which is a plane in the first case and
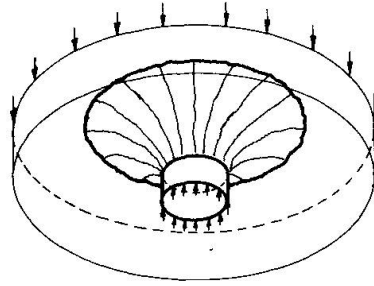a cone in the second.

The third example is taken from a precast concrete building in which the floors
consist of prestressed hollow core slabs. The slabs are laid side by side and
are connected by mortal-filled joints. In most cases a thin topping concrete
layer is applied in order to ensure horizontal stiffness and vertical load
distribution. Actually, this topping delays the work on the building site and
is also an inconvenient feature when the building has to be demolished. There-
fore, hollow core slab floors without topping are being investigated in regard
to the shear behaviour of the joints under in-plane forces. A typical example
of a joint is shown in Fig. 3. Shear stresses in a test specimen are indicated
in Fig. 4 exhibiting stress peaks at the ends of the joint. If the joint mortar
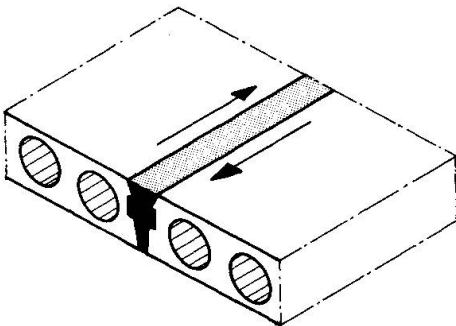
Fig. 3.   Hollow core slab with   Fig. 4.   Shear stresses and crack
          mortar joint   in the joint
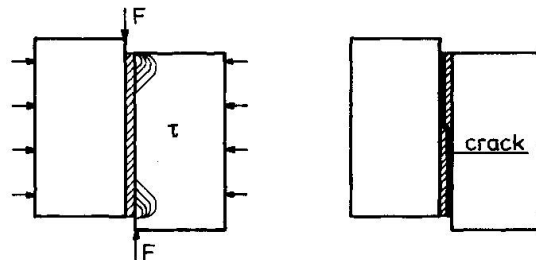
is a brittle material, a crack will start from the end of the joint as soon as
the peak stress reaches a certain maximum value and adhesion between mortar and
concrete fails. Because this is true for a short joint length and for a large
joint length the question arises as to the limiting shearing force F as a
function of the joint length. In the case of a plastic material and in the case
of a friction mechanism between the slabs, F is linear function of the length,
but in the case of a brittle material this is not expected.

3. KNOWLEDGE FROM EXPERIMENTAL INVESTIGATIONS

Much experimental work has been done on bending shear in beams and slabs.
The results show a distinct dependence of the ultimate average shear stress on
the absolute depth of the beam. Fig. 5 shows the results of various researchers
(for details see [1]).
Results of punching shear tests on slabs of various thickness are very scarce,
but the few available ones also show a decrease of the punching shear stress
with increasing slab thickness, Fig. 6.
In the Stevin Laboratory at Delft research is being carried out on joints be-
tween hollow core slabs and also indicates a nonlinear dependence of shear
force on the joint length. The results have not yet been fully worked out,
however.
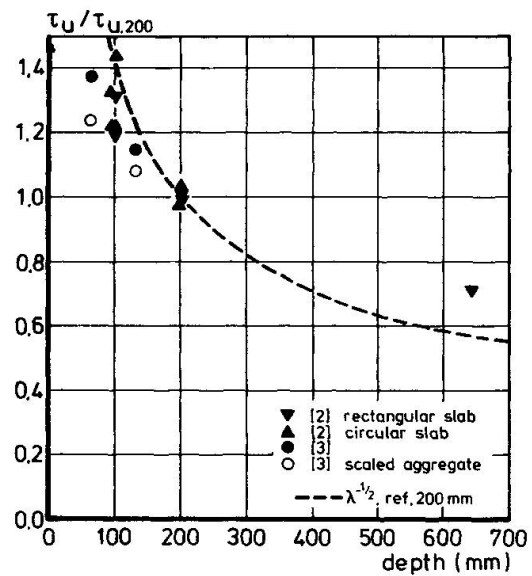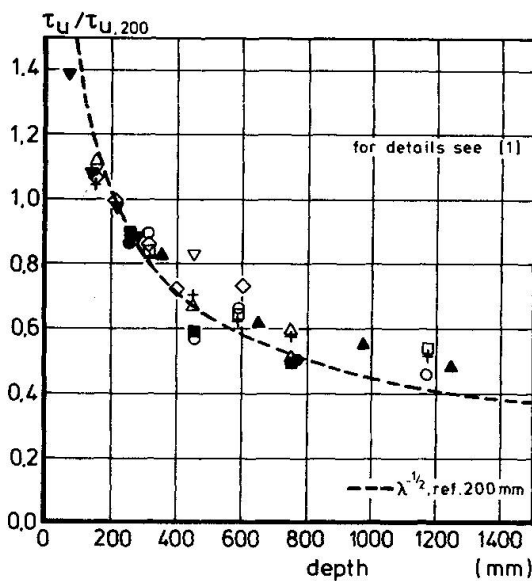
Fig. 5.   Relative ultimate shear stress   Fig. 6.   Relative punching shear
          vs. depth of the beam   stress vs. slab thickness

A common feature of all the experimental results is a significant influence of
the absolute size of the specimens on the ultimate average shear stress. This
influence is not explicable with the strength of material concept, for which the
absolute dimensions do not matter. Therefore fracture mechanics will be applied,
which could show a way out of the difficulty.

4. FRACTURE MECHANICS AND CONCRETE

Linear static fracture mechanics as originated by Griffith [4] considers the
surface energy at a crack (energy demand) and the elastic energy release around
a crack. The condition for a stable crack is that the energy demand is greater
than the energy release.
From this condition the maximum nominal stress in an infinite slab with a cen-
tral crack of length 2a before failure, is

$$\sigma = \sqrt{\frac{2E\gamma}{\pi a}} = \frac{1}{\sqrt{a}} \sqrt{\frac{2E\gamma}{\pi}} \qquad (1)$$

For two specimens of the same material the factor $2E\gamma/\pi$ is constant and
therefore the maximum nominal stress σ depends on the absolute value of the
crack length. The larger the crack, the lower is σ.
Another approach in fracture mechanics is the determination of the stress field
around a crack [5].

$$\sigma = \frac{K}{\sqrt{2\pi a}} \cdot f(\ldots) \qquad (2)$$

where K is the stress intensity factor and f a non-dimensional function which
takes the geometry of the specimen or structural member and the loading con-
dition into account. Failure will occur if K is equal to a critical value K_c
which is a material constant. Thus, eq. (2) can be rewritten as

$$\sigma = \frac{1}{\sqrt{a}} \cdot \frac{K_c}{\sqrt{2\pi}} \cdot f(\ldots) \qquad (3)$$

which gives essentially the same result as eq. (1).

Until now, fracture mechanics has been widely used in mechanical, ship and
airplane engineering for failure analysis of metal structures. The application
of fracture mechanics to concrete is rather limited.
At micro-level it is used for the treatment of the influence of sustained [6]
and short-time loading [7] on the strength of concrete, at macro-level for the
strength of a beam [8] and the failure criterion of a large dam [9]. All these
examples relate to plain concrete.

In general, linear elastic fracture mechanics presumes elastic behaviour until
failure, i.e., there is no plastic deformation near the crack tip. Concrete
does not exactly satisfy this condition; there are always microcracks around
the aggregate particles and also around the tip of the discrete crack. But if
this microcracking zone is small compared with the size of the crack, linear
elastic fracture mechanics may be applied [10].
Whereas plain concrete always develops brittle failure in a tensile stress
field, this is not true of reinforced concrete, where the tensile forces are
taken over by the reinforcement and ductile behaviour occurs. Only in cases
where the concrete is responsible for the loading capacity does brittle failure
occur. This is one condition for the application of fracture mechanics to rein-
forced concrete. Another is that there is a decisive macrocrack which deter-
mines the behaviour of the structure. A third is the same as for plain concrete,
namely that the microcracked zone is small compared with the visible crack.
These requirements are fulfilled in the three examples given in Chapter 2.

5. SIMILITUDE OF BRITTLE FRACTURE OF REINFORCED CONCRETE

Stress intensity factors of members with discrete reinforcing bars are not
available in the literature and no attempt will here be made to establish them.
The configuration most closely related to the shearing and punching problems
is a beam with an inclined crack treated by Bowie [11] . In regard to the joint
between slab units there is a solution by Erdogan[12] and subsequent papers
by several authors [13, 14, 15] . All these solutions have the general form as
given in eq. (3). This means that the square root of the absolute crack length
is inversely proportional to the failure stress. If the crack pattern in two
members of different size (scale factor λ) is similar, the failure shear stress
is also inversely proportional to the square root of the scale factor λ

$$\tau_u \sim \lambda^{-\frac{1}{2}} \tag{4}$$

The occurrence of similar crack patterns in scaled beams has been recognized
[16] and has been observed in current research on joints. So it is not sur-
prising that eq. (4) can yield good results, as illustrated by the dotted line
in Fig. 5 and 6. Punching shear (Fig. 6) does not appear to conform so closely,
but experimental results are admittedly very scarce. So far as joints are con-
cerned, the experiments have not yet been completed and worked out.

6. FINITE ELEMENT ANALYSIS

Bazant states in the introductory report [17] that the analysis of crack pro-
pagation leads to different results for different choices of the finite element
mesh if the strength of material approach is used. He gives a possible solution
by means of an energy criterion which is applied in each single element.
Argyris, Faust and Willam [18] also mention fracture mechanics, but prefer a
stress-strain concept (J-integral) than the linear elastic fracture mechanics
which actually can predict only the limit state and not the state of stable
crack growth.

Gergely and White [19] also emphasize the fracture mechanics approach and show
the result of a finite element analysis of a beam on the basis of fracture me-
chanics. Crack formation and deflection of the beam show close agreement with
experimental verification.
It is not completely clear whether in these calculations fracture mechanics
have been applied to the single element only or also to the whole structure.
As can be concluded from the foregoing chapters, it should be required that
also - in the case of brittle failure - the whole structure should be checked
by means of fracture mechanics and, as a first step, by linear elastic fracture
mechanics. A comparative calculation for similar beams (as in Fig. 1) could
show the feasibility of such a finite element analysis.

7. CONCLUSIONS

Advanced mechanics of reinforced concrete implies also fracture mechanics concepts which can be applied at micro-level (finite elements, inhomogeneous structure of concrete) but also at macro-level. To demonstrate the applicability at macro-level, three examples of brittle failure have been considered: bending shear in beams, punching shear in slabs, and in-plane shear in joints between hollow core slabs. Using linear elastic fracture mechanics it was found that

$$\tau_u \sim \lambda^{-\frac{1}{2}}$$

where τ_u is the ultimate average stress and λ the scaling factor between similar structures. This relation was confirmed by experimental results. In the future, stress-intensity factors should be derived for reinforced structures which allow a quantitative analysis, not merely a comparative one.
It should be emphasized that fracture mechanics can only be applied to reinforced concrete in the case of brittle fracture. Normally those cases are avoided by appropriate reinforcing, but if they can occur, fracture mechanics should be used.
If brittle failure of concrete is analysed by means of finite element programs, fracture mechanics should be applied not only within single elements but also to the whole structure or a major part of it.

8. NOTATION

a   half crack length

d   effective depth

E   Young's modulus

F   force

K   stress-intensity factor

V   shear force

γ   surface energy

λ   scale factor

σ   normal stress

τ   shear stress
           (nominal, computed acc. to CEB-FIP)

subscripts:

c   critical

U   ultimate

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