Modelling of viscoelastic properties of reinforced concrete

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Simulation des propriétes viscoélastiques du béton armé

Modellierung der viskoelastischen Eigenschaften des bewehrten Betons

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SUMMARY
The authors have developed a method of analysis allowing for the non-homogeneity of the structural material and the location of the reinforcement. It deals with the so-called Layer Finite Strip Method. The material characteristics of each strip are different and the material of the strips may even be anisotropic. The physical properties of the material are simulated on the basis of the linear theory of viscoelasticity and the theory of reinforcement. Using the method of identification, the influence of cracks was included in the material characteristics.

RÉSUMÉ
Les auteurs du mémoire ont élaboré une méthode de calcul permettant de prendre en considération la nonhomogénéité des matériaux et de l'armature. Il s'agit d'une méthode de bandes finies où les caractéristiques des bandes individuelles diffèrent et les matériaux peuvent être anisotropiques aussi. Les propriétes physiques des matériaux sont simulées selon la théorie linéaire de visco-élasticité et la théorie de l'armature. Les auteurs ont appliqués la méthode d'identification pour dériver les caractéristiques du matériau dans la zône sollicitée à la traction et à la compression en tenant compte de l'effect des fissures.

ZUSAMMENFASSUNG
Die Autoren entwickelten eine Berechnungsmethode, die die Nichthomogenität des Konstruktionsmaterials und der verwendeten Bewehrung berücksichtigt. Es handelt sich um die Methode der finiten geschichteten Streifen, wobei die Materialcharakteristiken in den einzelnen Streifen verschieden sind. Das Material kann in den Streifen auch anisotrop sein. Die physikalischen Eigenschaften des Materials sind auf Grund der linearen viskoelastischen Theorie und der Bewehrungstheorie modelliert.
Mittels der Methode wurden die Charakteristiken des Grundmaterials in der Zug- und Druckzone mit integraler Berücksichtigung des Einflusses von Rissen abgeleitet.

1. INTRODUCTION

We will analyse the state of stress and strain of layered two-dimensional structures. By deriving of the basic equations we considered the material viscoelastic and the displacements small and continues. The analysed structure is imaginary divided into finite number of layers. Each layer has different material properties, but constant within the region of particular layer.

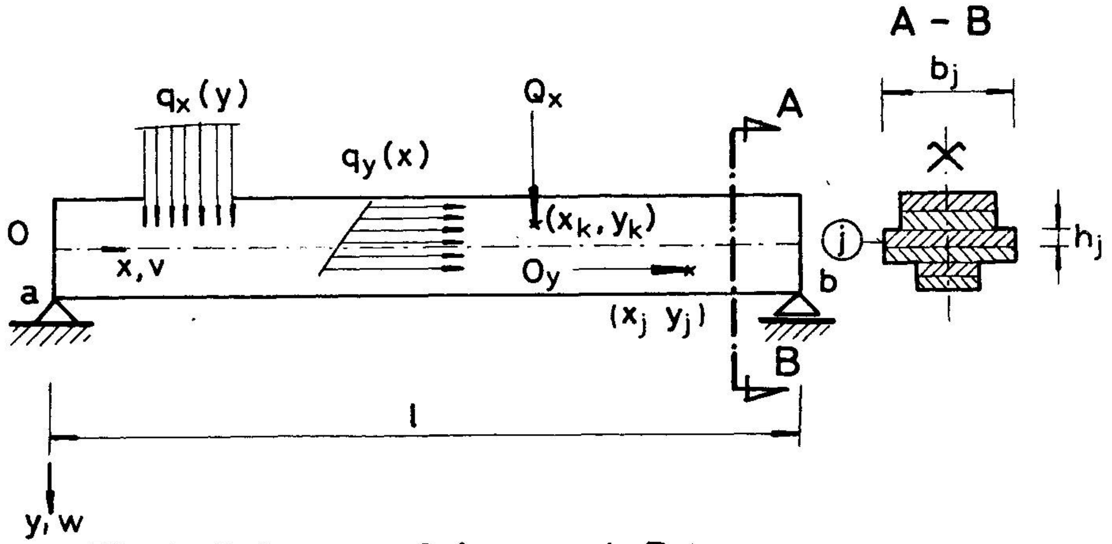

Fig.1. Scheme of Layered Beam

Material properties of each layer were simulated using the rheological models and the theory of reinforcement 1 - 3 .

2. LAYER FINITE STRIP METHOD /LSFM/

From the mathematical point of view is our problem formulated as follows: We have to find the set of vector-functions $\{\delta_m(x,y,t)\}$ for each $m = 1, 2, 3, \dots, N$, where N is a total number of layers.

This set of vector functions $\{\delta_m(x,y,t)\}$ and corresponding stress $\{\sigma_m(x,y,t)\}$ satisfies the equilibrium conditions and also given boundary conditions. The applied method is based on the well hnown idea, to minimize the energetical potencial of the associated problem $\tilde{\Phi}$ [4, 5] /tildas denote the Laplace transform/ over a finite space of functions $V_N \subset V$, where V is the infinite dimensional space. V will be the subspace of a Sobolev space $W_2^{(2)}(\Omega)$ generated by all functions from $W_2^{(2)}(\Omega)$, which satisfy the main /also stable/ boundary conditions for the associate elastic problem. Similar conditions are also for the Finite Element Method for viscoelastic structures [6, 7].

Necessary condition for the existence of the minimum of functional $\tilde{\Phi}$ of the given associated problem is

$$\frac{\partial \tilde{\Phi}}{\partial \tilde{\delta}} = 0 \qquad \tilde{\delta} = (\tilde{\delta}_1, \tilde{\delta}_2, \dots, \tilde{\delta}_N)^T \tag{1}$$

where

$$\tilde{\Phi} = \frac{1}{2}\tilde{\delta}^T \tilde{K} \tilde{\delta} - \tilde{\delta}^T \tilde{F} = \tilde{W} + \tilde{U} \tag{2}$$

and

$\tilde{\Phi}$ - total potential energy
$\tilde{K}$ - stiffness matrix
$\tilde{F}$ - load vector
$\tilde{W}$ - potential energy due to external forces
$\tilde{U}$ - strain energy

Suitable set of displacement functions satisfy the given boundary conditions and also linear changes of displacements v and w across the thickness of the particular layers we will choose in the form [3]

$$\begin{Bmatrix} \tilde{v}_i \\ \tilde{w}_i \end{Bmatrix} = \sum_{n=1}^{s} \begin{bmatrix} 1-\bar{x}_i, \bar{x}_i, 0, 0 \\ 0, 0, 1-\bar{x}_i, x_i \end{bmatrix} \begin{Bmatrix} \tilde{v}_{1i} \\ \tilde{v}_{2i} \\ \tilde{w}_{1i} \\ \tilde{w}_{2i} \end{Bmatrix} \times \begin{matrix} [Y_n(y)]' \\ [Y_n(y)] \end{matrix} \tag{3}$$

where

$\bar{x}_i = x/b_i$

b_i - is a width of the i - strip /layer/

s - is a number of members of trig. series

and $\tilde{v}_{1j}$, $\tilde{v}_{2j}$, $\tilde{w}_{1j}$, $\tilde{w}_{2j}$ are displacements parameters, location of which is indicated od Fig. 1.

$Y_n(y)$, $Y_n'(y)$, / $n = 1, 2, \dots, s$ /are the basic functions and their first derivatives, and must a priori satisfy given boundary conditions for $y = 0$ and $y = l$. In our case

$$Y_n(y) = \sin(n\pi y/l).$$

The system of linear algebraic equations, derived on the above mentioned basic assumptions, solution of which is the set of vector functions $\{\tilde{\delta}\}$ ha formaly the same form for Finite Elements Method /FEM/ and also for Layer Finite Strip Method /LFSM/

$$[\tilde{K}]\{\tilde{\delta}\} = \{\tilde{F}\} \tag{4}$$

but stiffness matrix for FSM has the form

$$[\tilde{K}] = \int_{\Omega} [B]^* [\tilde{D}][B] \, d\Omega \tag{5}$$

and for LFSM

$$[\tilde{K}] = \sum_i \sum_k [\tilde{K}_{i(k)}] = \sum_i \sum_k \left(\sum_n [\tilde{K}_{i(k)n}]\right) \tag{6}$$

where

$$[\tilde{K}_{i \textregistered n}] = \int^{l}\int^{b_i}\int^{h_k} [B_{i \textregistered n}]^* [\tilde{D}_{i \textregistered n}][B_{i \textregistered n}]\,dx\,dy\,dz \tag{7}$$

and $[B]$ is strain matrix.

In both cases the determination of property matrix is very important and requires adequate attention.

3, DERIVATION OF PROPERTY MATRIX

As a spatial case we consider standard linear viscoelastic material where

$E^{ijkl}_{(\alpha)}$ is a tensor of moduli of elasticity

$\eta^{ijkl}_{(\alpha)}$ is a tensor of moduli of viscosity

We assume, that the Maxwell element /of linear viscoelastic material, or called also Zener's material/ has a homogeneus relaxation spectrum and hence it holds

$$\eta^{ijkl}_{(\alpha)} = K\,E^{ijkl}_{(\alpha)} \tag{8}$$

where K is the inverse value of relaxation time of the material of the structure. According to [8], this model is appropriate, under some simplifications, for the expression of concrete.

The differential equation describing this model is

$$K(E^{\alpha\beta\gamma\delta}_{(1)} + E^{\alpha\beta\gamma\delta}_{(2)})\dot{\varepsilon}_{\gamma\delta} + E^{\alpha\beta\gamma\delta}_{(1)}\varepsilon_{\gamma\delta} = K\dot{\sigma}^{\alpha\beta} + \sigma^{\alpha\beta} \tag{9}$$

After Laplace transform of equation (9) where $p_i = 1/\tau_i$ is parameter of Laplace transform and tildas denote this transformation we obtain

$$(p + \frac{1}{K})\tilde{\sigma}^{ij} = [p(E^{ijkl}_{(1)} + E^{ijkl}_{(2)}) + \frac{1}{K}E^{ijkl}_{(1)}]\tilde{\varepsilon}_{kl} \tag{10}$$

In contracted form, when we denote

$$E^{ij} = E^{ij}_{(1)} + E^{ij}_{(2)}$$

we obtain

$$(p + \frac{1}{K})\tilde{\sigma}^{11} = (pE^{11} + \frac{1}{K}E^{11}_{(1)})\tilde{\varepsilon}_{11} + (pE^{12} + \frac{1}{K}E^{12}_{(1)})\tilde{\varepsilon}_{22}$$

$$(p + \frac{1}{K})\tilde{\sigma}^{22} = (pE^{12} + \frac{1}{K}E^{12}_{(1)})\tilde{\varepsilon}_{11} + (pE^{22} + \frac{1}{K}E^{22}_{(1)})\tilde{\varepsilon}_{22} \tag{11}$$

$$(p + \frac{1}{K})\tilde{\tau}_{12} = (pE^{66} + \frac{1}{K}E^{66}_{(1)})\tilde{\gamma}_{12}$$

In the case of orthotropy and introducing a constracted form of E^{ij} we can rewrite equation (10)

$$[\tilde{\sigma}^{ij}] = (p + 1/K)^{-1} [d^{ij}][\tilde{\varepsilon}_{kl}] \qquad (12)$$

where the matrix of viscoelastic coefficients is

$$d^{ij} = \begin{bmatrix} pE^{11} + \frac{1}{K}E^{11}_{(1)} & pE^{12} + \frac{1}{K}E^{12}_{(1)} & 0 \\ pE^{12} + \frac{1}{K}E^{12}_{(1)} & pE^{22} + \frac{1}{K}E^{22}_{(1)} & 0 \\ 0 & 0 & pE^{66} + \frac{1}{K}E^{66}_{(1)} \end{bmatrix} \qquad (13)$$

The stresses in the elastic orthotropic two-dimensional structure we obtain from the known equation

$$[\sigma^{ij}] = [c^{ij}][\varepsilon_{kl}] \qquad (14)$$

where the matrix $[c^{ij}]$ depends on the arrangement of reinforcing bars, or fibres. In the case, when the reinforced bars are arranged in two perpendicular directions, parallel with the directions of coordinate axes y and z , then the matrix $[c^{ij}]$ is in form (15). If we later suppose, that the standard linear viscoelastic model has isotropic material coefficients $E^{ijkl}_{(2)}$ and $\eta^{ijkl}_{(2)}$, /Maxwell element/ and coefficients $E^{ijkl}_{(1)}$ have orthotropic properties /elastic spring element/.

$$c^{ij} = \begin{bmatrix} \bar{E}^{11}A^{-1} & \bar{E}^{11}\mu_{21}A^{-1} & 0 \\ \bar{E}^{22}\mu_{21}A^{-1} & \bar{E}^{22}A^{-1} & 0 \\ 0 & 0 & G^{1}_{12} \end{bmatrix} ; \quad A = 1 - \mu_{12}\mu_{21} \qquad (15)$$

If we introduce later

$$E^{ij}_{(1)} = \bar{E}^{ij}_{(1)} / (1 - \mu_{12}\mu_{21}) \qquad (16)$$

where $\bar{E}^{ij}_{(1)}$ are moduli of elasticity calculated according to the theory of reinforcing [2] for the ideal structure made from the composite material, then we can easily determine the matrix $[d^{ij}]$, which fully describes the viscoelastic properties of the material considered.

4. NUMERICAL EXAMPLE

We investigated reinforcered concrete beams, simply supported and uniformly loaded with intensity $q = 3{,}43$ kN/m^2. The span of the beam was 3,60 m. Cross section 12 cm x 120 cm was divided

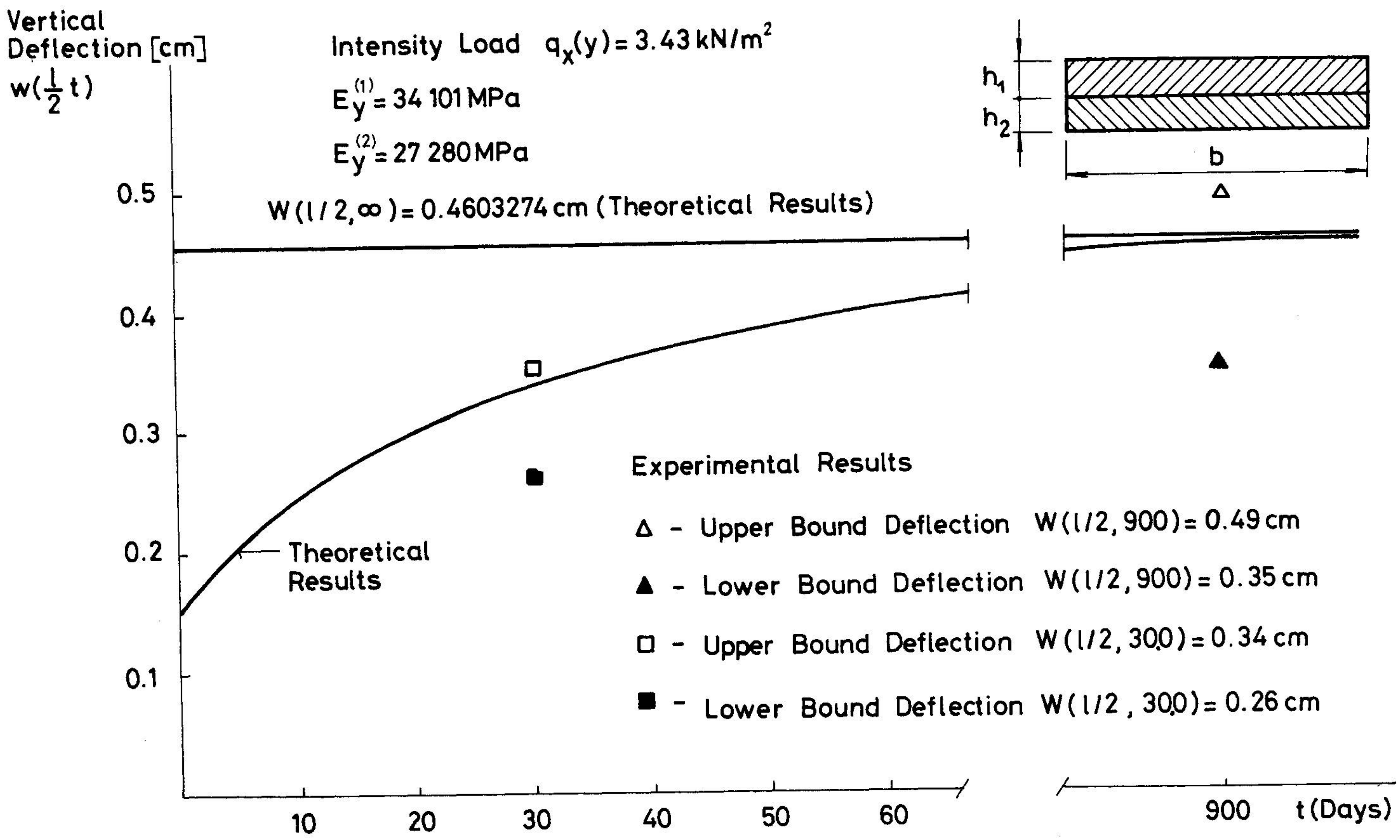

Fig.2. Vertical Time-dependend Deflection of Reinforced Concrete Beam

into two layers /Fig. 2/, where h_1 = 6,20 cm and h_2 = 5,80 cm.

The tension zone with depth h_2 bellow the neutral axes we considered weaken by cracks and microcracks. The material matrix was derived according to the method shown as above in part 3. Starting from the associated elastic problem to the viscoelastic one we used later in the inverse Laplace transform the displacement $w(x,y,t)$ from the experimental results [9]. Thus we could identify the modified stiffness of the layer in tension with cracks. From the Fig. 2 we can see that the theoretical solution of the deflection fits between upper and lower bound of experimental results for 30-days period, as well for 900-days period.

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