A Calculation of Reinforced Concrete Beams Under Bending and Torsion Using Three-Dimensional Finite Elements

Le calcul des poutres en béton armé soumises à la flexion et à la torsion, utilisant les éléments finis tridimensionnels

Berechnung von Stahlbetonbalken unter Biegung und Torsion mit dreidimensionalen Finiten Elementen

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SUMMARY
The problem of solid prestressed or reinforced concrete members under pure torsion or under combined loading is pointed out. The solution with the help of three-dimensional finite elements and a coordinating 3d-concrete-material model is shown. A comparison of calculation with a test result is given.

RÉSUMÉ
L'auteur a examiné le problème d'éléments massifs en béton précontraint ou béton armé, soumis à la torsion pure ou à un chargement combiné. Il montre la solution utilisant des éléments finis tridimensionnels et le modèle polyaxiale pour le matériau béton. Il donne une comparaison de calcul avec le résultat d'un essai.

ZUSAMMENFASSUNG
Die Problemstellung von massiven Bauteilen aus Stahlbeton oder Spannbeton unter reiner Torsion oder kombinierter Beanspruchung mit Torsion sowie die Lösungsmöglichkeit mittels räumlichter finiter Elemente werden aufgezeigt. Ein hierzu erforderliches Materialmodell auf der Grundlage schrittweiser elastischer Ansätze wird vorgestellt. Die Vergleichsrechnung eines Testbeispiels wird ebenfalls gezeigt.

1. A SURVEY OF PREVIOUS WORK DONE AT THE TECHNISCHE HOCHSCHULE DARMSTADT CONCERNING PROBLEMS OF BENDING AND TORSION ON REINFORCED CONCRETE BEAMS

In his thesis [1] the first named author of this paper examined the influence of material behavior on the lateral buckling of reinforced and prestressed concrete beams. With regard to concrete pressure zones, the torsional stiffness was determined with the help of the Boundary Element Method (BEM). The influence of warping torsion on reinforced concrete was referred to, and for the I-cross-section, a simple method for determining the warping stiffness was given. It was also shown, however, that for the combined stresses of bending, shear and torsion on a reinforced concrete beam, with a arbitrary cross-section, no suitable calculating method was available. For this reason, researchers of the Institut für Massivbau at the Technische Hochschule Darmstadt have been working on this problem for the past ten years.

Bertram [2] following the tests of Lampert and Thürlimann [3] , which used reinforced concrete beams with rectangular cross-sections and T-sections, developed a method for calculating the ultimate moment and the interaction between the ultimate bending moment and torsional moments. Information was also given for determining the torsional stiffness. However, the procedure proved to be too cumbersome and not general enough. (For example, determining torsional stiffness, influenced by the position of the neutral axis, using only the bending moment).

In his thesis [4] , Rützel developed a procedure for thin-walled reinforced concrete beams, with and without prestressing, with which one could ascertain the stresses and deformations caused by bending, normal force, shearing force and torsion in relation to material non-linearity. He showed that for thin-walled beams, especially in State II, the portion of the warping torsion at the torsion carrying behavior should not be neglected. Rützel took into consideration the alteration of the torsion resistance after cracking by changing the position of the neutral axis. Since the strains from the various loads were determined separately - and after the superposition - there had to follow in each case a reconsideration of the assumptions, which resulted in subsequent revisions and improvements in them. Because of this, the procedure is also relatively cumbersome.

Bertram's [2] and Rützel's [4] methods, in addition, are not well suited for implementation into a general computer code, because the numerical processing is too costly and the prior assumptions used are too specific. Therefore, a new way had to be sought out, and the Finite Elemement Method (FEM) proved to be especially suitable for this purpose.

With the help of the FEM, Maurer [5] developed a computational procedure for box girders under random loads. He used layered plate elements. With it, the stresses of the cross-section could be determined; distinct not only from the overall loads, but also from the cross-sectional deformation or the transverse bending. The FEM-program has manifold applications: it is able to take into account the non-linear stress-strain relationship of the concrete in compression, the cracking in tension, the influence of prestressing, as well as the yielding of the reinforcement. In his thesis, Maurer showed that with his program, there was good conformity between his calculation results and test results.

Subsequently, following the course set by [1], Sauer [6] developed the theoretical fundamentals for applying the BEM to the solution of shear and torsion problems for elastic beams, and wrote a computer code for determining St. Venant's torsional stiffness, warping resistance and position of the shear center as well as the shear stresses, that could be used for beams with arbitrary, polygonal-sided cross-sections.

Röder's thesis [7] investigated the lateral buckling of reinforced and prestressed concrete beams in consideration of non-linear material behavior: non-linear stress-strain behavior of the concrete in compression, cracking in tension and the yielding of the reinforcement. In contrast to [1], where the bifurcation problem was dealt with, Röder treated the investigation of stability as a problem in accordance with "Sedond Order Theory", but to do so, he had to presume there were imperfections in the beam or excentricity of the loading. For the given loads, the state of strain was determined for various points on the beam and verified with the conditions of equilibrium. From the state of strain, the stiffness was figured, and the calculations were performed repeatedly. This iterated method of calculation was terminated if the deformation did not differ essentially from the preceding step. The torsion stiffness was determined by the procedure for the concrete area under compression, in accordance with Sauer [6], and for the cracked area, he used a truss model. The procedure used various prior assumptions about the cracked area, and because of this it was not suitable for making calculations concerning beams primarily loaded by torsion.

Finally, it should be mentioned that a combination of the FEM and the BEM promises distinct advantages, especially with regard to computation time.

2. CALCULATIONS WITH THREE-DIMENSIONAL FINITE ELEMENTS

As the preceding discussion illustrates, the calculation of torsion carrying behavior has led those of us at the Technische Hochschule Darmstadt along a path of steady progression from simple "beam statics", through a consideration of geometric and material non-linearity in the beams toward spatial models of box girders, with plate elements considering realistic material behavior. The last cited procedure, however, provides no method for analyzing solid beams. Since under pure torsion the carrying mechanism is fairly clear (and assuming "hollow cross-sections, with presumed wall thicknesses, useful results are derived not only relative to the ultimate load, but also to deformation behavior for State II), one could say the research is extensively finished. However, under combined stress (torsion, bending, shear, normal force), especially as related to deformation behavior, there is still a gap to be closed. Because the aforementioned problem generally concerned with conditions of three-dimensional stress and deformation, three-dimensional (spatial) finite elements should be used.

After test computations, we decided upon using hexahedral elements, with a variable number of nodes (8 to 21 nodes) and isoparametric displacement function.

For calculations with non-linear material specifications, however, relatively simple elements appeared to us to be the most suitable (we use the element-type mainly with eight nodes (corner nodes)), and thereby we reached a linear displacement function.

In essence, the questions to be asked, subsequent to the idealization of the reinforcement, are about the bond behavior and non-linear stress-strain relationship of the concrete, here too, cracking must be taken into consideration. To consider a practical and realistic comprehension of beam behavior for reinforced concrete structural members under torsion there are still several detailed problems to be examined:

a. Questions of load carrying in.
b. Local tension problems at the stirrup corners.
c. Splitting of the concrete layers outside the reinforcement.

2.1 Idealization of the Reinforcement

For calculating the beam segments, the reinforcement rods were individually idealized using truss elements with linear displacement function, only normal force are considered. The possible yielding of the reinforcement was taken into account by considering a bi-linear stress-strain relationship.

2.2 Idealization of the Bond

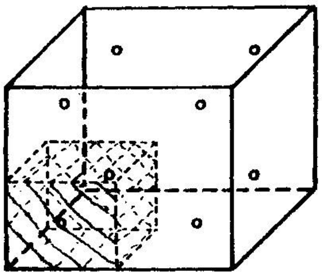
To calculate the bond behavior, a bond element was developed [9] for the computer code ADINA. The theoretical work and experimental certification was performed largely by Dörr [10] and [11]. The element is compatible with the concrete and steel elements used. It possesses no geometrical dimensions, but the element connects the concrete and steel, so that the forces parallel and perpendicular to the reinforcement can be transmitted.

A bi-linear relationship between the relative displacement (concrete-reinforcement) Δ and the bond stress τ was supposed. This procedure represented an approximation and neglected the dependence of the bond stress upon the transverse pressure Q. (Fig. 2).

2.3 Idealization of the Concrete

2.3.1 Elements

Three-dimensional continuum elements were used in cubic form (hexahedron), with eight nodes and three degrees of freedom per node. The element in this form is based on the linear displacement function.



Test calculations with elastic material behavior and with cross-sectional types frequently found in real structures, gave us information about the necessary refined grid for the required precision. Figure 1 shows a few of the F.E. idealizations utilized. (The comparative precision was calculated according to Sauer [6]).

Since we want to confine ourselves, relatively short parts of real structures are to be modelled and analyzed, and the required computer time remains, even with grid refinement, justifiable. Using short structural members, in which the loadings along the beam axis is constant, we want to analyze the influence of torsion carrying behavior under normal force and bending moments. The calculations are done using the computer code ADINA [8].

For our presumptions to be valid, and compatible with the previous test results, we have to use realistic material models.

The material models used were selected and developed not only with regard to the problem to be analyzed, but also relative to F.E. idealization.

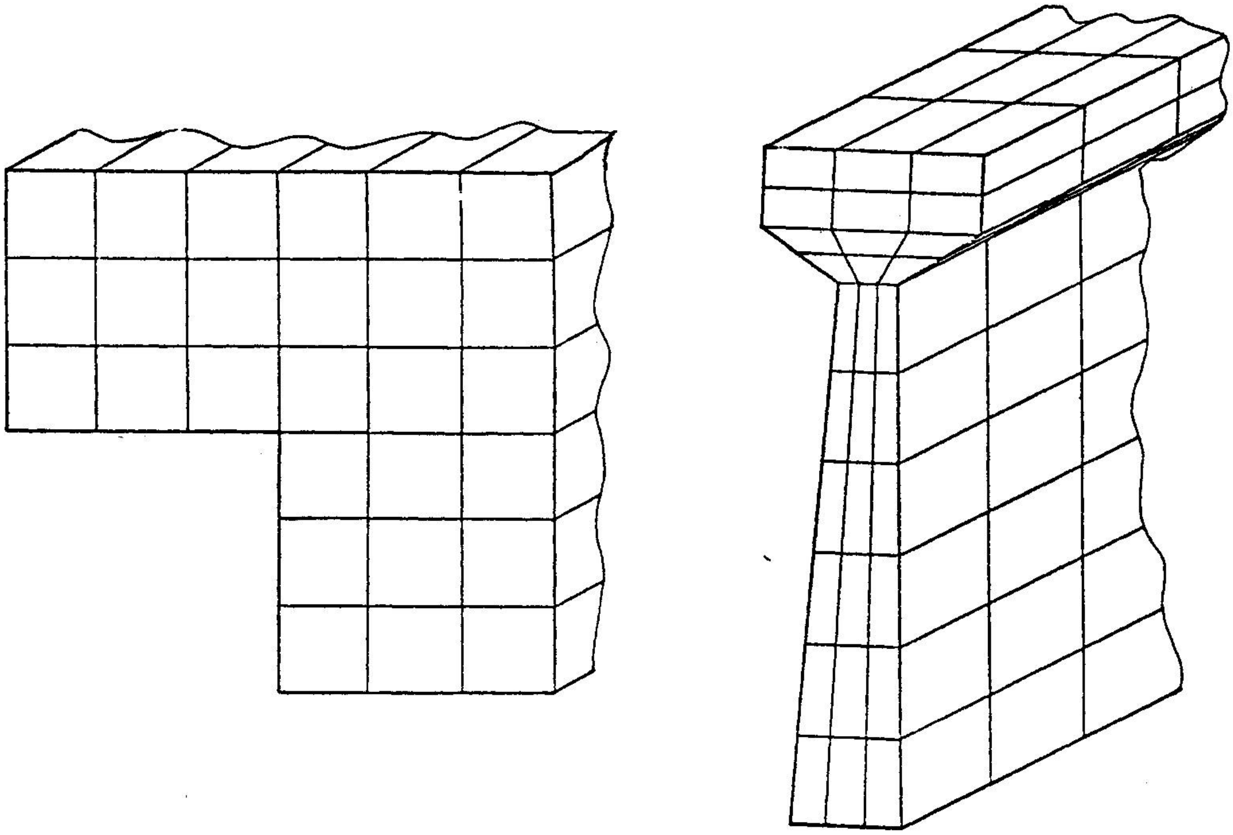


Fig. 1 F.E. Mesh for solid beams

The stiffness of the elements was formed of at least eight integration points. On these integration points, the state of stress and strain was also determined. In the event of cracking, the cracks were not supposed to be discrete, but rather "smeared" over the integration area. In this way, the conditions of stress or cracking at the integration point were determined, and the formation of element stiffness ascertained by using the integration points. Such a methodology can produce a great amount of information about the non-linearity of material.

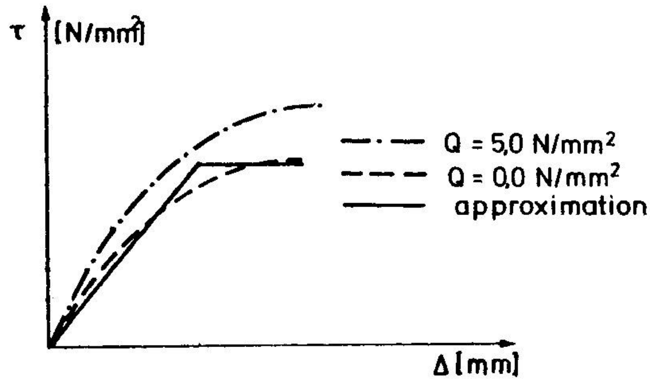


Fig. 2 Bond-Behavior

2.3.2 Multi-Axial Strength

The dependence of the concrete's strength on the state of stress was taken into consideration. We used a principle-stress-related representation of multi-axial strength. Figure 3 shows the multi-axial compression strength, which is normalized by the uni-axial compression strength (σ_c).

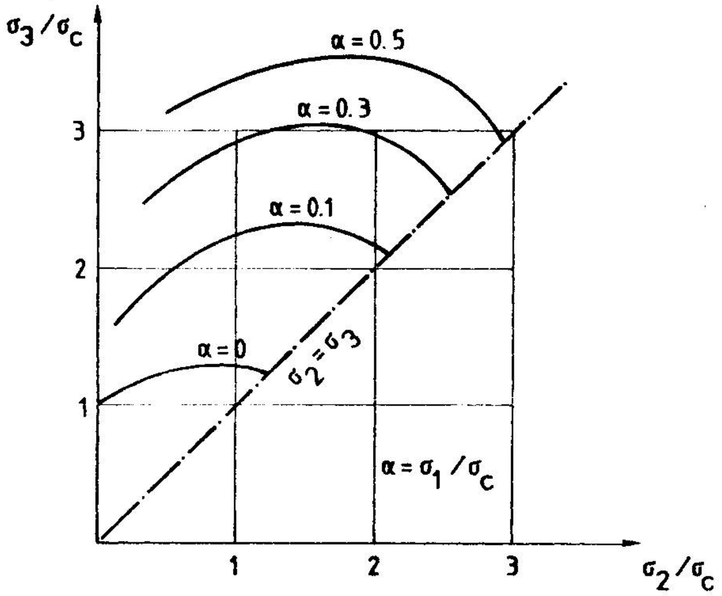


Figure 3 Concrete strength under multiaxial compression

The sets of curves, whose lowest ($\sigma_1 = 0$) represents the bi-axial strength, shows quite visibly the increase of the compression strength with increasing transverse pressure (the smallest principle compressive stress). In the program, the sets of curves were replaced by six polygons, between which we interpolate in the linear form. Current values for the multiaxial compressive strength were determined by the results of the international comparative test program, for example [12], and already published in [13]. For a specific condition of stress (σ_2, σ_1), a value γ was determined, that gave us the greatest bearable compressive strength σ_3 for these conditions of stress, in comparison to a uni-axial compressive strength: min $\sigma_3 = \sigma_c \cdot \gamma$.

For the tensile strength with regard to the conditions of stress, a simple prior assumption, corresponding to Figure 4, was used. It means: in tri-axial tension, everywhere the uni-axial tensile strength is valid; in the compression-tension regions, dependent upon σ_2 and σ_3, the value for the tensile strength was determined through linear interpolation. If the greatest compressive stress approaches the compressive strength, then the tensile strength is set to zero.

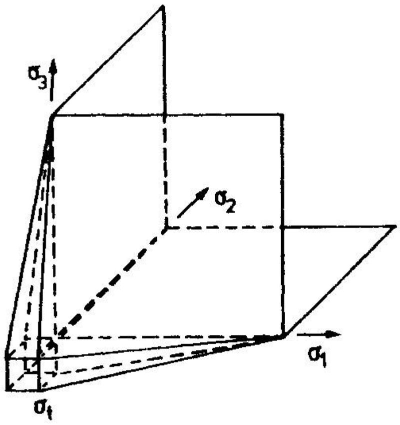


Figure 4 Tensile strength under multiaxial conditions

2.3.3 Multi-Axial Stress-Strain Behavior

The material model is compiled in an incremental form; that means: we considered the non-linear material relationship during a load step as a linear-elastic material relationship. However, it was ensured through a process of iteration, that for the respective states of strain, the corresponding tangents and secant stiffness were used.

We employed an orthotropic, linear-elastic material model, by which the material characteristics varies correspondingly to the strains. The directions of the orthotropic axes are identical with those of the principle stress axes. The values of the tangents or secant moduli of the orthotropic axes were determined from the distorted uni-axial stress-strain relationship. For the uni-axial case, we used the simple relationship:

$$\sigma_{(\varepsilon)} = \tilde{\sigma}_c \left[1 - \left(1 - \frac{\varepsilon}{\tilde{\varepsilon}_c} \right)^{\alpha} \right]$$

with

$$\alpha = E_o \cdot \tilde{\varepsilon}_c / \tilde{\sigma}_c$$

$$\curvearrowright \quad E_{(\varepsilon)} = E_o \cdot \left(1 - \frac{\varepsilon}{\tilde{\varepsilon}_c} \right)^{\alpha - 1}$$

For the multi-axial region, the curve was distorted for all principle stress axes, with the factor γ defined by using the multi-axial strength. Its value $\sigma_c = \gamma \cdot \tilde{\sigma}_c$, $\varepsilon_c = \gamma \cdot \tilde{\varepsilon}_c$ etc.

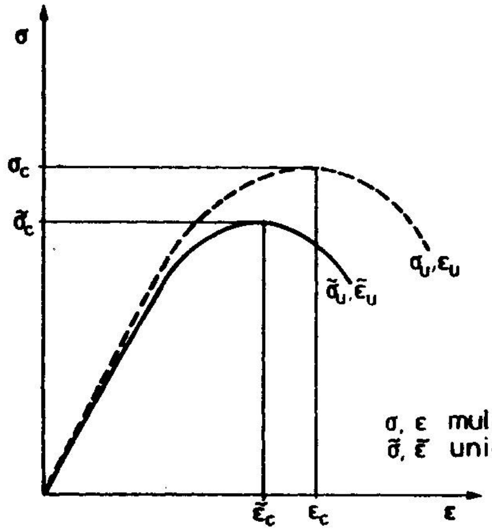


Figure 5 Uniaxial and multiaxial stress-strain-relation

2.3.4 Post-Cracking Behavior

From a reinforced concrete test, the total stiffness, after the development of the first cracks, is still clearly greater than the stiffness of the reinforcement alone, meaning: the concrete between the cracks is still carrying a load. If the carrying effect of the concrete after cracking is not considered in a computational model, then one mathematically obtains, after the appearance of the first cracks, incontinuity in the deformation plot - although in reality this is not clearly observed. In order to avoid this after cracking we set the remaining concrete tension perpendicular to the crack not to zero, but reduced the stresses step-wise, as indicated in Figure 6, with regard to the unit elongation perpendicular to the crack.

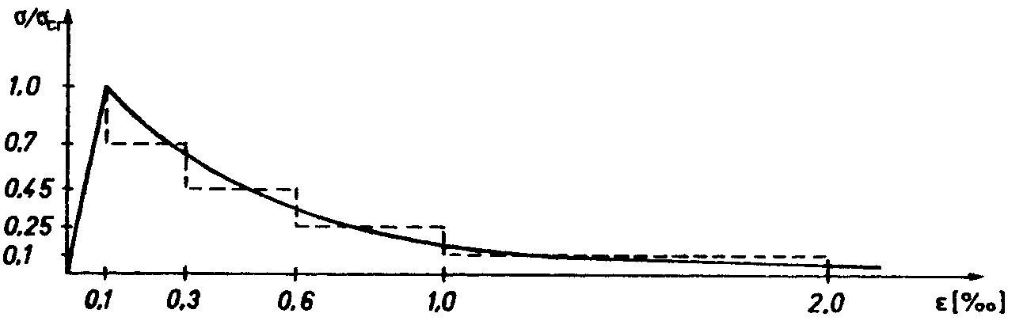


Figure 6 Stresses in the concrete after cracking

At the beginning of cracking, the stresses parallel to the crack are zero, because the crack was defined as perpendicular to the principle tensile stress. Through load re-arrangement and alteration, however, deformations parallel to the crack can occur. As a consequence, the crack inter-lock can, dependent upon the width, still transfer shear forces. Defining the resistance against the displacement parallel to the crack (corresponding to the elastic shear modulus (G_o)) as G_r, and plotting the relationship G_r/G_o, depending upon the width of the crack, results in a plot like that shown in Figure 7. The dependence upon the width of the crack was approximated in the computer code in a step-wise manner.

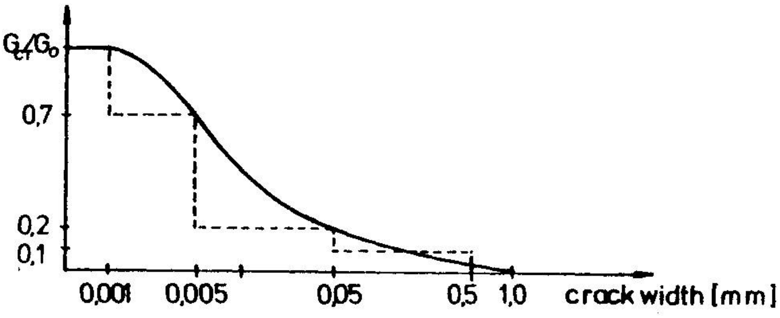


Figure 7 Shear-stiffness of cracked concrete

2.3.5 Loading - Unloading

The stress-strain behavior described in 2.3.3 was only used for the virginal load (1). For unloading (2) and re-loading (3) - as long as the structure still showed no cracks - the original stiffness was used (see Fig. 8). If re-loading reached a point over the first load (4), then the virginal loading curve again was taken.

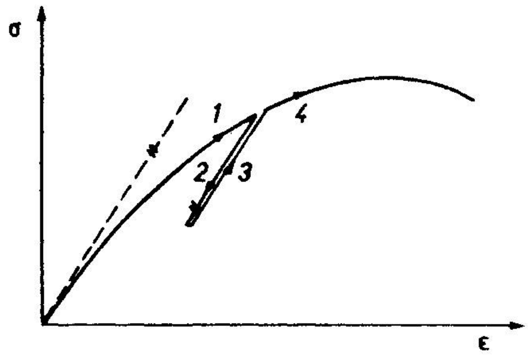


Figure 8 Loading - Unloading

2.4 Special Problems

With regard to local stress concentration at the corner of the stirrups, and the problem of splitting of the concrete layer outside of the reinforcement, we are now working on a detailed, computational analysis, also with a 3-D-model. With such an analysis, the corner of a stirrup, the longitudinal reinforcement truss and the surrounding concrete can be represented. We expect from it information about the carrying behavior of this area. Furthermore we would like to mention the results of the analysis in the overall computational models.

2.5 Example

With the aforementioned procedure, the various cross-sections and varying loads should be able to be analyzed. At the present time, we have the first results of the calculation for the tests [14].

Fig. 9 illustrates an F.E. idealization with 3 x 4 x 5 elements for concrete. The reinforcement is modelled by means of truss elements at the beam surface.

The comparison of the distortion and the stirrup stress from calculations and a test is shown in Fig. 9.

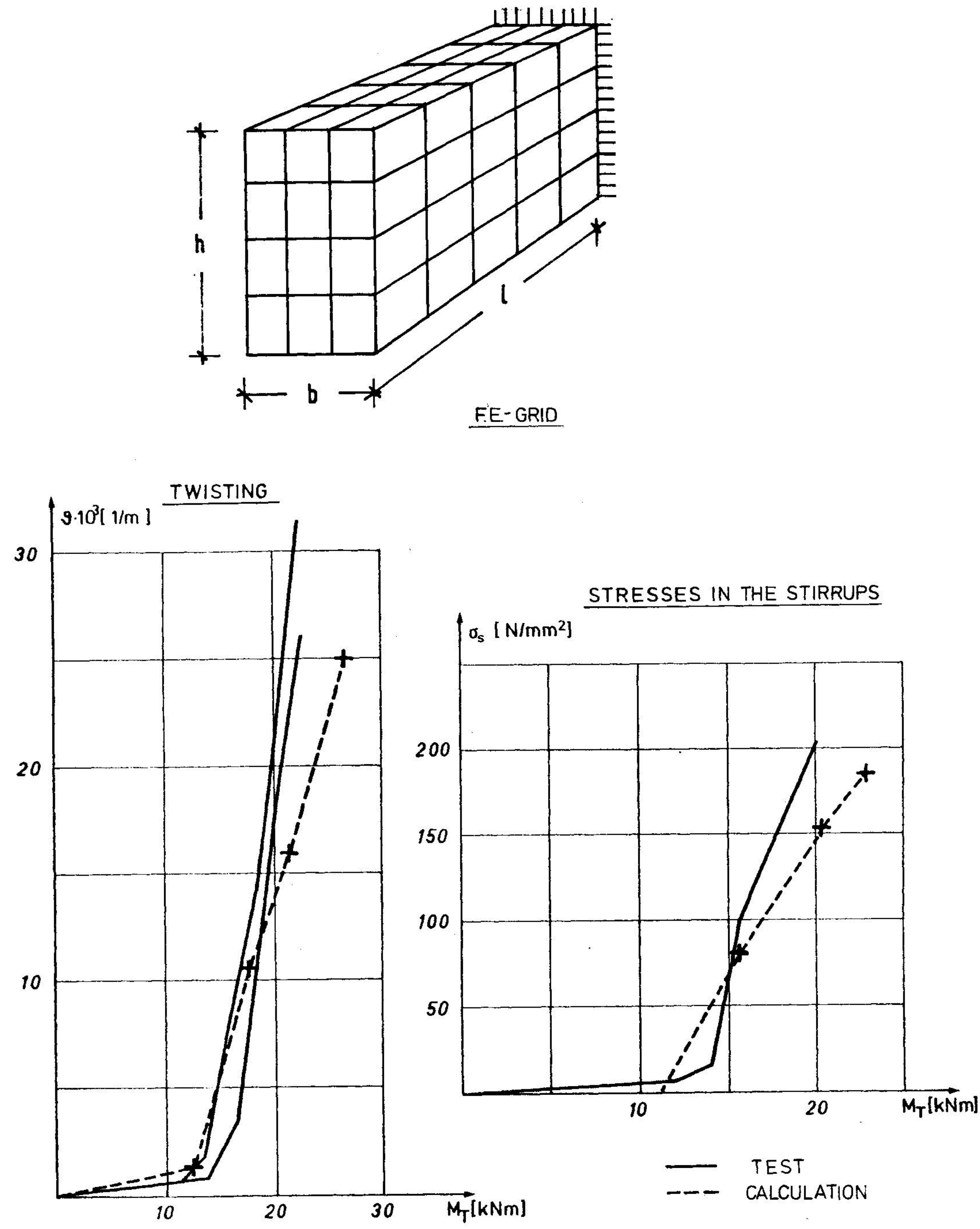


Figure 9 F.E.-Model and a comparison of calculation and test results

3. OUTLOOK

Through parametric studies of concrete members, under combined loadings, using the described computational models, the interaction between the individual types of loading should be able to be determined and demonstrated.

We believe, however, that the computational model can also be a useful tool for analyzing a series of wider problems concerning reinforced concrete structures - especially for the researcher.

ACKNOWLEDGEMENT

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