Fatigue Cracks in Bolt Threads

Fissures dues à la fatigue dans le fond de filet des boulons

Riss im Gewindegrund eines Schrankenbolzens — Lebensdauerermittlung

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SUMMARY
A method is proposed which allows the prediction of the fatigue life of a notched component. A knowledge of the state of the stress field in the vicinity of the notch in an uncracked component (the stress concentration) permits the study of fatigue behaviour by integration of the crack growth law. The method is demonstated by application to the development of a crack in a bolt thread. Good agreement is achieved with experimental results.

RESUME
Une méthode est proposée qui permet de prédire la durée de vie d'une pièce entaillée. Il suffit de connaître la concentration de contrainte, calculée sur la pièce entaillée, sans fissure, pour pouvoir évaluer, au moyen de l'intégration de la loi de propagation des fissures, le comportement à la fatigue. La méthode est appliquée à l'exemple de la fissure au fond d'un filet de boulon. La durée de vie calculée concorde bien avec les résultats expérimentaux.

ZUSAMMENFASSUNG
Es wird ein Verfahren angegeben, mit dessen Hilfe die Lebensdauer gekerbter Bauteile vorausberechnet werden kann. Die Kenntnis der Spannungskonzentration, berechnet am ungerissenen, gekerbten Bauteil, reicht aus, um mittels Integration des Rissausbreitungsgesetzes das Ermüdungsverhalten zu studieren. Das Verfahren wird am Beispiel des Risses im Kerbgrund eines Schraubengewindes demonstriert. Die vorausberechnete Lebensdauer der Schraubenverbindung zeigt gute Übereinstimmung mit experimentellen Ergebnissen.

1. INTRODUCTION

Highly strengthened threaded joints are sometimes sore points in machines or steel constructions, especially in the case of cyclic loading.
In order to estimate the life time of threaded connections the behaviour of a crack in the most strengthened region of the bolt is investigated using the finite element technique.
As a prerequisite for the analysis of the local stress field the load transfer in the bolt-nut connection has to be calculated. This analysis shows the well known fact that the load is not distributed evenly along the threads. The thread at the base of the nut (the first thread) carries more load than all the subsequent threads. Furthermore, a significant stress concentration arises at the notch forming the bottom of the screw profile. It is essential to allow slip motion between the thread of the nut and the thread of the bolt in the model of analysis.

With this information the local behaviour of a crack in the bolt can be analysed. Singular crack tip elements are used to take account for the stress singularity. Because of the high stress gradient at the notch the LEFM (Linear Elastic Fracture Mechanics) geometric compliance factor, Y, [1] in the commonly used relation for the stress intensity factor is strongly dependent on the crack length, a. The crack propagates into areas with decreasing stress level.

To predict the fatigue life of structures many authors integrate the crack growth formulas (as e. g. that of Paris or Forman [1]) under simplifying assumptions. One of these assumptions is the neglect of the variation of Y(a) during crack growth. In the present paper this assumption, leading to a too short fatigue life, was not made. Y(a) is obtained calculating the Y values for several crack lengths. A correlation between the variation Y(a) and the stress distribution in the uncracked notch is recognized. Paris' law is integrated in a modified form for an estimation of the fatigue life of the threaded connection. Well agreement with experiments is found.

2. CALCULATION OF THE LOAD TRANSFER IN A THREADED CONNECTION

For the threaded connection M1∅, as shown in Figure 1, a stress analysis is performed using the Finite Element Method (FEM).

2.1 The FE-Model

The axisymmetric FE-model consists of substructure elements, see Figures 1 and 2. The degrees of freedom of the nodes within the boundary of those elements are condensed out [2]. Therefore the degrees of freedom of the whole structure are only those of the boundary nodes of the substructure elements.

Load-transfering nodal points between nut thread and bolt thread are allowed to slip in the direction of their surfaces, friction is neglected.
The applied reference nominal stress, σ_n, is assumed to be 100 N/mm^2. The boundary conditions are shown in Figure 1, the substructure elements in Figure 2.

2.2 Results of the Stress Analysis

The variation of the maximum principal stresses, σ_I, is displayed in Figure 3. The stress concentrations at the notches can be recognized by an increasing isostress line density. From notch to notch both the stress levels and the stress line density become less. This implies that the stress concentration and the part of the whole load transferred from bolt to nut decrease from thread to thread.

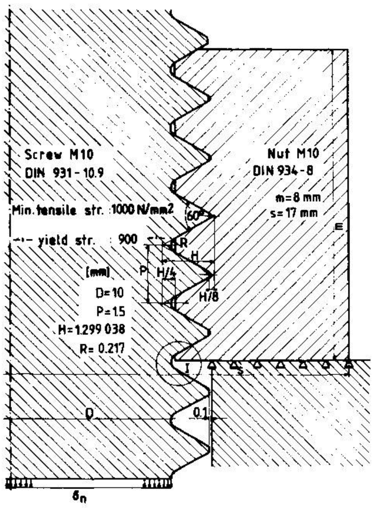

Fig. 1 Threaded conncection M10

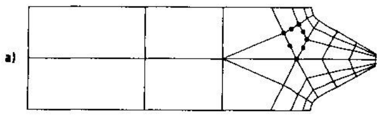

Fig. 2a Substructure element of a bolt thread

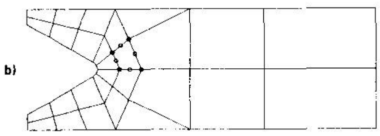

Fig. 2b Substructure element of a nut thread

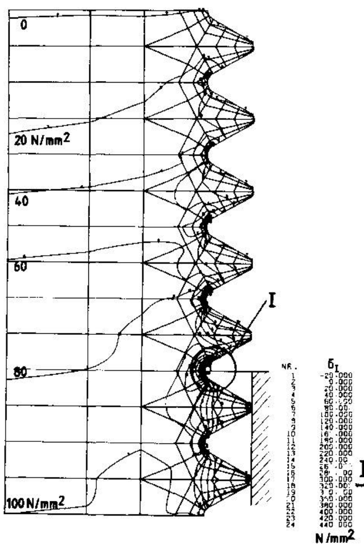

Fig. 3 Variation of principal tensile stresses in the bolt

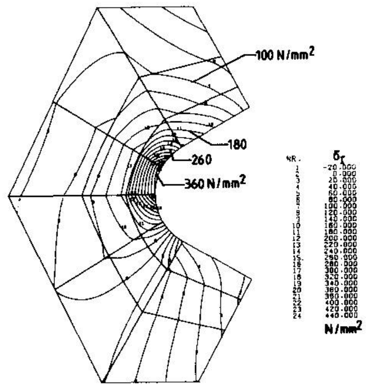

Fig. 4 Stress concentration at the notch root of the first and most stressed bolt thread (I in Fig. 1 and 3)

Several other investigations about stress analysis of threaded connections are listed in references [3-7].

3. A CRACK IN THE THREAD OF THE BOLT

The stress distribution of the first load transmitting bolt thread is presented in detail in Figure 4.
The elastic stress concentration factor, α_K, which is defined as

$$\alpha_K = \frac{\sigma_{Imax}}{\sigma_n}, \tag{1}$$

approaches the value of 4.0 at the notch root. σ_{Imax} is the maximum principal tensile stress.
In these domains of high stress concentration cracks may originate under cyclic loads.

3.1 Stress Intensity Factors

Investigations [8,9] have shown that failure of threaded connections is usually caused by tearing of the threaded bolts.
A number of local axisymmetric analyses of the bolt are now performed with several cracks assumed. The cracks are - in accordance with [1] - assumed to be normal to the maximum principal tensile stresses, i.e. normal to the notch surface. FE meshes for two sample crack lengths are shown in Figure 5. Singular elements

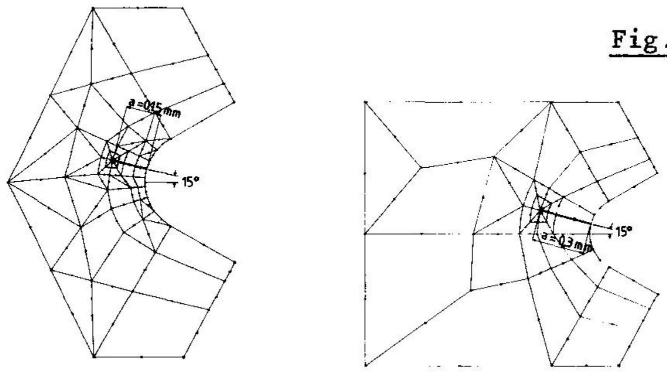

Fig. 5 Local FE models of the most stressed notch of the cracked bolt thread

[10,11] and transition elements according to Lynn [12] are used in the crack tip region.

The angle of 15^o between crack and the horizontal line as shown in Figure 5 is in good agreement with experimental results [13]. The loading condition for the local FE model is given by the displacements of the boundary nodal points which were determined in the global stress analysis.
The stress intensity factors, K_I and K_{II}, are calculated making use of the method described in [14]. The geometric compliance factor, Y(a), according to

$$K_I = Y(a)\ \sigma_n \sqrt{\pi a} \qquad (2)$$

is shown in Figure 6 in relation to the crack length, a.
The K_{II} values are very small even for greater crack lengths. Hence the assumption of a crack propagation in the initial crack direction is justified.
Checking the size of the plastic zone at the crack tip by Rice's formula [12]

$$\omega = \frac{1}{3\pi} \left(\frac{K_I}{\sigma_y}\right)^2 \qquad (3)$$

justifies the application of the linear elastic fracture mechanics concepts in this analysis.
ω is the plastic zone size, σ_y is the yield stress. Plane strain condition is assumed.

3.2 Crack Initiation

Differing from the procedure outlined in Ref. [14] the definition of the stress intensity factor range as proposed by El Haddad et al [16] is used:

$$\Delta K = Y(a)\ \Delta\sigma_n \sqrt{\pi(a+\ell_o)}\,. \qquad (4)$$

$\Delta\sigma_n$ is the applied nominal stress range. ℓ_o is a constant, characteristic for the material and surface conditions. It takes account for micro effects which could not be covered by continuum mechanics. In Ref. [16] the excellent ability of ℓ_o for eliminating discrepances between short and long crack results is demonstrated. The stress level at which a crack in acyclic loaded specimen

will not just yet propagate is defined as the threshold stress. This threshold stress at a very short crack length is shown [16] to approach the fatigue limit stress of the material, $\Delta\sigma_e$.
ℓ_o is obtained from equation (4) by:

$$\ell_o = \left(\frac{\Delta K_{th}}{Y_o \; \Delta\sigma_e}\right)^2 \frac{1}{\pi}. \qquad (5)$$

ΔK_{th} is the threshold stress intensity factor, and Y_o is the geometric compliance factor for a very short crack. The crack length, a_o, at which a crack will propagate at a certain stress level, $\Delta\sigma_n$, is obtained by (4) and (5) as:

$$a_o = \left(\frac{\Delta K_{th}}{Y_o}\right)^2 \left[\left(\frac{1}{\Delta\sigma_n}\right)^2 - \left(\frac{1}{\Delta\sigma_e}\right)^2\right] \frac{1}{\pi} \qquad (6)$$

with the simplifying assumption: $Y(a_o) \approx Y_o$.

When the applied nominal stress range, $\Delta\sigma_n$, exceeds $\Delta\sigma_e$ cracks will initiate and propagate, because a_o would become negative, formally calculated by equ. (6). In other words, if $\Delta\sigma_n > \Delta\sigma_e$ cracks of very small size will propagate

Schwalbe [17] suggests ΔK_{th} being dependent on the Young's Modulus, E, and the stress ratio R_e, as described by:

$$\Delta K_{th} = E \times (2.75 \pm 0.75) \times 10^{-5} \times (1-R_e)^{0.31}. \qquad (7)$$

R_e is the ratio of minimum to maximum stress at fatigue limit load. The dimensions are $[MN/m^{\frac{3}{2}}]$ and $[MN/m^2]$ for ΔK_{th} and E, respectively.

In order to compare computed results with results of Illgner's fatigue tests [8] two examples are chosen:

$$(1)\ \Delta\sigma_n = 180\ N/mm^2 \text{ and } (2)\ \Delta\sigma_n = 240\ N/mm^2$$

as load in addition to the prestress $\sigma_V = 140\ N/mm^2$, of the bolt. The fatigue limit stress taken from Illgner's results is $\Delta\sigma_e = 140\ N/mm^2$ and the value of Young's modulus is $E = 2.06 \times 10^5\ N/mm^2$.
Using equ. (7) in combination with equ. (5) ΔK_{th} and the characteristic length, ℓ_o, are calculated:

$$\Delta K_{th} = 144.5\ N/mm^{\frac{3}{2}} \text{ and } \ell_o = 0.0212\ mm.$$

4. LIFE TIME ESTIMATION OF THE THREADED CONNECTION

Based on tests with cyclic loaded, cracked specimens various crack growth laws have been established [18,19]. The fatigue crack growth rate is controlled by loading, geometry of the specimen (notches etc.), mean stress, stress frequence, temperature and fracture mechanics material constants.

Here Paris' law [20] will be applied. It is formulated by:

$$\frac{da}{dN} = C(\Delta K)^m. \qquad (8)$$

$\frac{da}{dN}$ is the fatigue crack growth rate, ΔK is the stress intensity factor range and C and m are material constants.
By integration of equation (8) the number of endurable cycles of the specimen may be estimated.
ΔK is used as defined in equ. (4). At a certain crack length, a_c, failure of the component will occur if either

$$K_{Imax} = Y(a_c)\,\sigma_{nmax}\sqrt{\pi\, a_c} = K_{Ic} \qquad (9)$$

or the nominal stress in the remaining net section approaches the tensile strength. K_{Imax} is the stress intensity factor at the maximum applied nominal stress σ_{nmax}. K_{Ic} is the critical stress intensity factor.
The variation of the geometric compliance factor, Y(a), especially for notched specimens like the threaded bolt under consideration, must not be neglected as will be seen later.

Figure 6 contains the variation of Y with crack length, a. Y decreases to unity at the end of the notch stress field. A distinct correlation of Y with the stress variation at the notch can be observed.

For cracks longer than the notch domain fatigue crack growth is governed by the bulk stress field only.
Hence, for cracks with $\frac{a}{r_K} > 0.02$ (with r_K being the half minor diameter of the bolt thread), Y(a) may be obtained from the geometric compliance factor of the unnotched specimen, Y_G.
According to Rooke and Cartwright [21] Y_G is given by

$$Y_G = \frac{\frac{a}{r_K}}{(1-\frac{a}{r_K})^2}\;\frac{1}{\sqrt{0.8 + \frac{4\,\frac{a}{r_K}}{1-\frac{a}{r_K}}}} \qquad (10)$$

r_K is the radius of the unnotched specimen and a is the crack length.
Richard [22] describes the relation of Y_G in the following way:

$$Y_G = \left(\frac{r_K}{r_K - a}\right)^2 \frac{1}{1-\frac{a}{r_K}} \sqrt{\frac{A + B\,\frac{a}{r_K - a}}{1 + C\,\frac{a}{r_K-a} + D\left(\frac{a}{r_K-a}\right)^2}} \cdot \qquad (11)$$

A = 1.26; B = -0.24; C = 5.35; D = 11.6.
Two important features will be recalled. First, there exists a good correlation between the stress variation of the uncracked threaded bolt and the geometric compliance factor of the cracked threaded bolt in the notch stress field. Second, the influence of the notch may be neglected for long cracks.
Therefore, Y(a) for the threaded bolt may be well approximated by

$$Y(a) = Y_G\,\frac{\sigma_I(\ell)}{\sigma_n}\,. \qquad (12)$$

Y_G is the geometric compliance factor for the unnotched specimen, $\sigma_I(\ell)$ is the principal tensile stress of the uncracked threaded bolt at $\ell = a$ (see Figure 6) and σ_n is the applied nominal stress. Similar formulas for Y(a) may also be found in Refs. [16,23].
Fig. 7 shows the variation of Y and K_{Imax} for both the unnotched specimen and the threaded bolt with crack length a.
Since Y is highly nonlinearly dependent on a, Paris' law is integrated numerically [24].

From equs. (4) and (8) the number of cycles corresponding to a certain crack length, a, is obtained by:

$$N(a) = \int_{0}^{a} \frac{da}{C\cdot(Y(a)\ \Delta\sigma\sqrt{\pi(a+\ell_o)})^m} \tag{13}$$

To get the number of cycles to failure, N_f, a_c must be taken as upper integration limit.

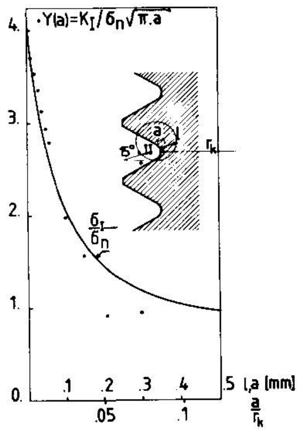

Fig. 6 Stress concentration at the notch of the uncracked bolt. Geometric compliance factors for various crack lengths.

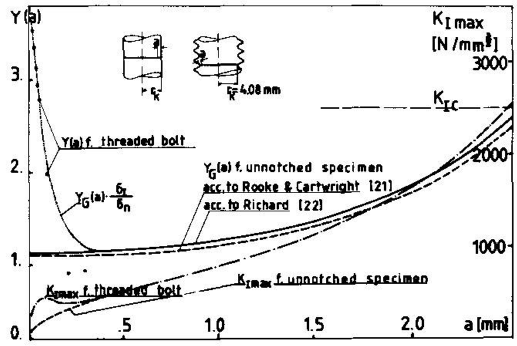

Fig. 7 Variation of the geometric compliance factor. Maximum stress intensity for the threaded bolt and the unnotched specimen ($\Delta\sigma_n$ = 180 N/mm²)

Now two sets of material constants are taken from limiting curves of the deviation band of experimental data (see Figure 251 of Ref. [17]).

$C_1 = 8.5704\times10^{-9}$; $m_1 = 3.16$. $C_2 = 4.1706\times10^{-9}$; $m_2 = 2.94$.

Results of numerical integration of (13) for the two examples described in 3.2 are shown in Figure 8.

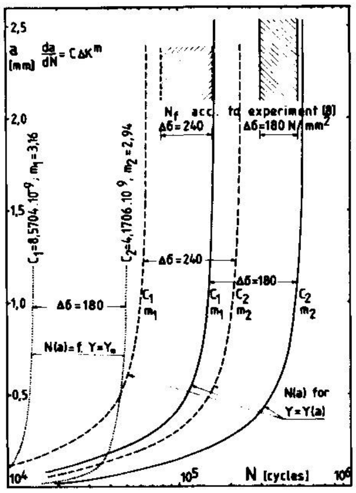

Fig. 8 Load cycles N, dependent on the actual crack length a for the threaded bolt.

Integration of the fatigue crack growth law with a constant geometric compliance factor, $Y(a) \equiv Y_o$, would render to a significant underestimation of the final fatigue life of the threaded bolt.

Taking into account the true variation of Y(a) of the cracked threaded bolt the integration of equ. (13) yields resonable agreement of the fatigue life of the threaded connection compared with results of Illgner's experiments, as shown in Figure 8.

CONCLUSIONS

The variation of the geometric compliance factor for a threaded bolt is given by equ. (12).

$\sigma_I(\ell)$ is calculated by a stress analysis of the uncracked threaded connection. Y_G is the geometric compliance factor of the unnotched specimen. By numerical integration of the appropriate crack growth law the fatigue life of a threaded connection may be predicted.

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