# Fatigue assessment according to Eurocode 3 (steel structures)

**Fatigue Assessment According to Eurocode 3 (Steel Structures)**

Vérification à la fatigue selon l'Eurocode 3 (constructions métalliques)

Betriebsfestigkeitsnachweis für Stahlbauten nach Eurocode 3

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**SUMMARY**
The fatigue assessment of steel structures according to the limit state equation of Eurocode 3 is defined in terms of service life (number of cycles). The safety factors are derived from the "Level II" method of constant sensitivity factors.

**RESUME**
Les états limites considérés dans l'Eurocode 3 pour la vérification de la sécurité à la fatigue des constructions métalliques sont définis sur l'axe des durées de vie. Les facteurs de sécurité à utiliser pour la vérification sont déterminés selon la procédure issue des théories de "Level II" et faisant intervenir des facteurs de sensibilités constants.

**ZUSAMMENFASSUNG**
Für den Betriebssicherheitsnachweis für Stahlbauten nach Eurocode 3 werden der Grenzzustand in der Achse der Lebensdauer definiert und nach dem aus der Level II-Methode abgeleiteten Verfahren der konstanten Wichtungsfaktoren die Sicherheitsfaktoren für den Nachweis abgeleitet.

## 1. GENERAL

The calculative safety assessment for the fatigue behaviour of dynamically loaded steel structures should be carried out in a way to attain as often as possible a target reliability expressed by the safety index $\beta_T$, not falling below a minimum value min $\beta$ , if possible and also not exceeding a maximum value max $\beta$ for economic reasons.

The justification for a proposal for the determination of the safety elements is developed in the following using the procedure of global sensitivity factors which is derived from the Level II method /3/.

## 2. ASSESSMENT FOR A DETERMINED $\Delta\sigma$-LEVEL AND A DETERMINED NUMBER OF CYCLES n

Fig. 1 demonstrates a scatter-band of test results from fatigue tests with a component with a critical detail for different levels of damaging stress ranges $\Delta\sigma$, which are calculated as nominal stresses for measured test loads.

As a resistance model the S-N-lines for defined survival probabilities can be illustrated in double logarithmic scale as for instance fig. 2.

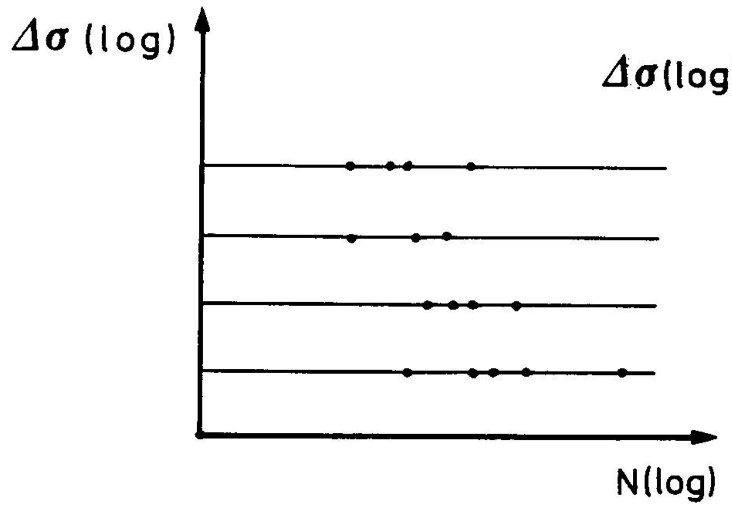


Fig. 1: Scatter-band of fatique test results

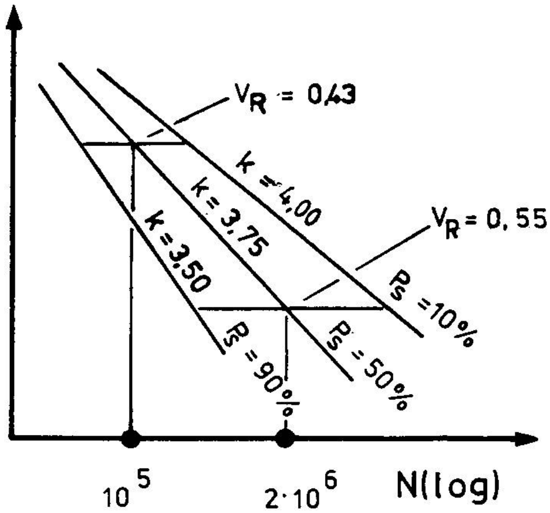


Fig. 2: S-N-lines for small test specimens

The safety assessment should be carried out for a structural component designed in the same way as the test elements.

The limit state equation for fatigue for the stress range level $\Delta\sigma_i$ is then

$$g\ (X_i) = N_i - n_i = 0 \qquad (1)$$

see fig. 3.

Here a log-normal distribution with the variation coefficient $V_R$ - for instance for small components according to fig. 4 - is assumed for the basic variable for the "resistance" $N_i$, and for the "action" $n_i$ the sum of the cycles of all measuring time intervals is extrapolated over a designed service life and defined as,fig. 5

$$n_i = \sum_{T_L} n_t \quad . \tag{2}$$

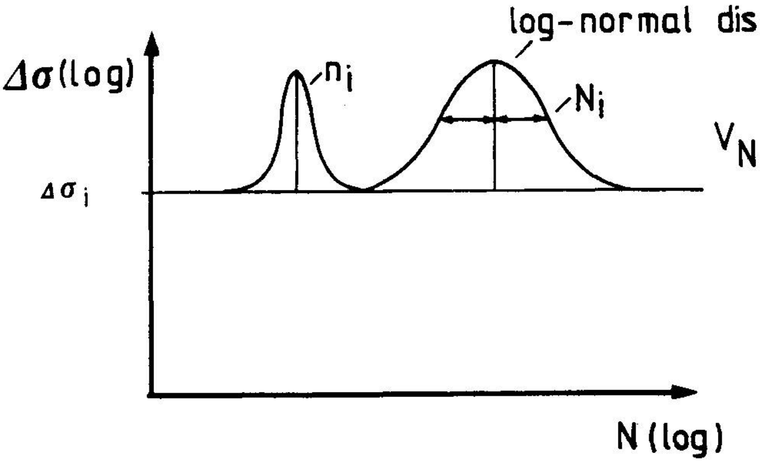



Fig. 3: Distributions of N and n

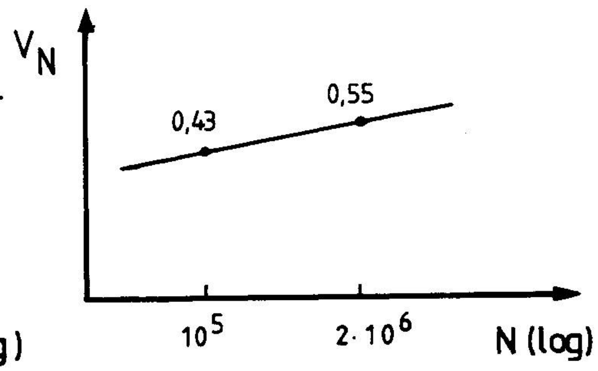



Fig. 4: Variation coefficient (N-log normal)

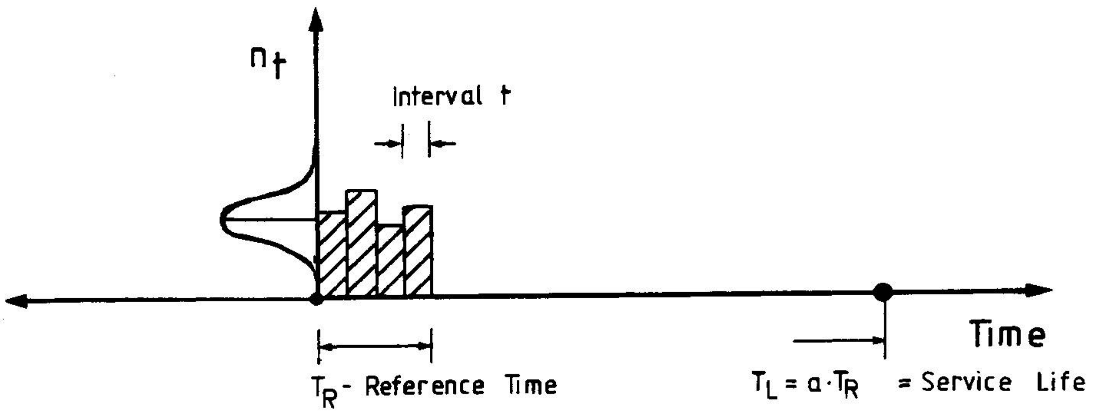



Fig. 5: Number of stress range cycles

If a symmetrical density distribution of the numbers of cycles is assumed over the measuring time intervals within service life, then the variation coefficient for $n_i$ is zero.

The design values $N_i^*$ and $n_i^*$ can be expressed according to /4/

$$\left.\begin{aligned} N_i^* &= m_N \exp\left(-\alpha_R \beta V_R - \tfrac{1}{2} V_R^2\right) && \text{with } \alpha_R = 1{,}0 \\ n_i^* &= m_n \left(1 + \alpha_S \beta V_S\right) = m_n && \text{due to } \alpha_S = 0 \end{aligned}\right\} \tag{3}$$

The design values $N_i^*$ for various stress levels $\Delta\sigma_i$ will then lie on the design S-N-curve with the slope k* which is influenced by $\beta$, the assumed variation coefficient and by the slope of the 50 %-S-N-line, see fig. 6.

For target values $\beta_T$ of the order of 2 the slope is k* = 3,0. This value is also obtained in tests with full scale structural components for which the variation coefficients are smaller and the scatter-bands are more parallel as compared with the test results obtained with small test-specimens.

The safety verification for a deterministically given stress range can now be carried out in the time scale with $\gamma_{mN}$, with reference to the characteristic values $N_k$ (for example $N_k = N_{50\,\%}$) according to fig. 7.

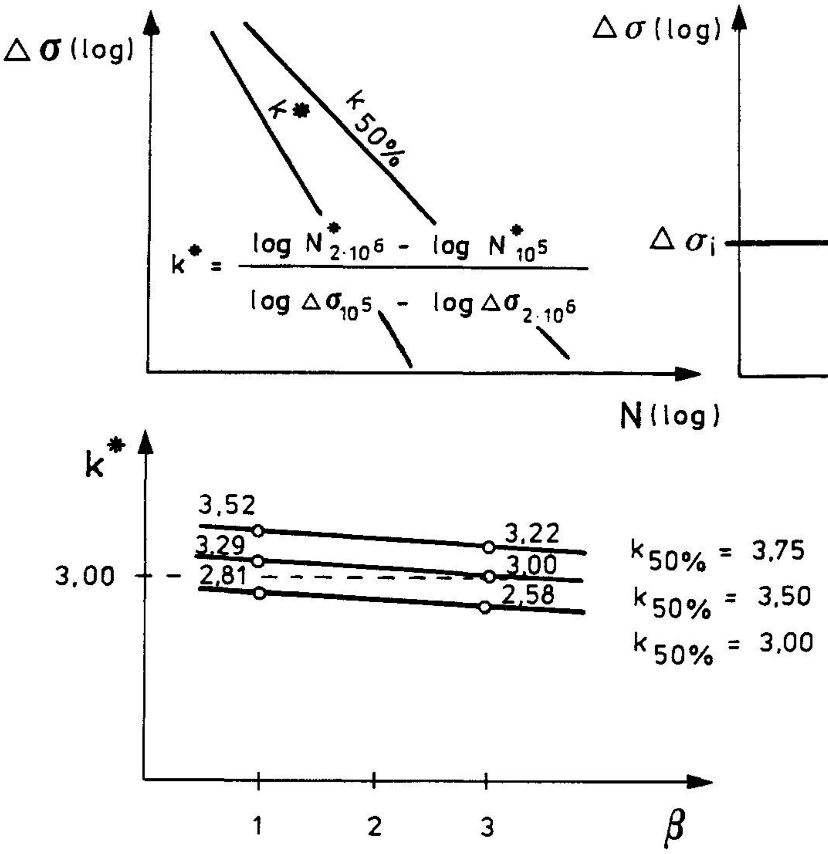
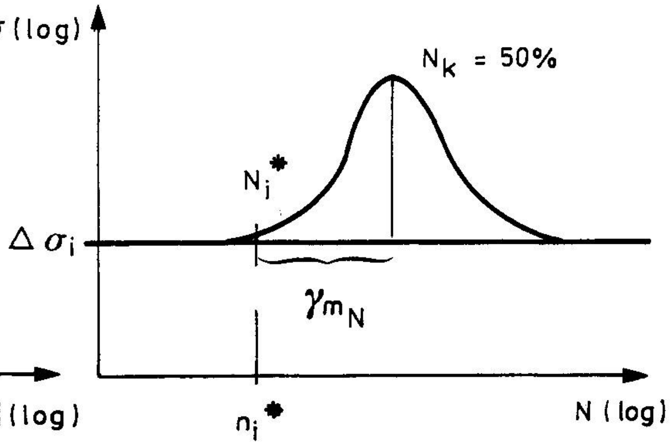


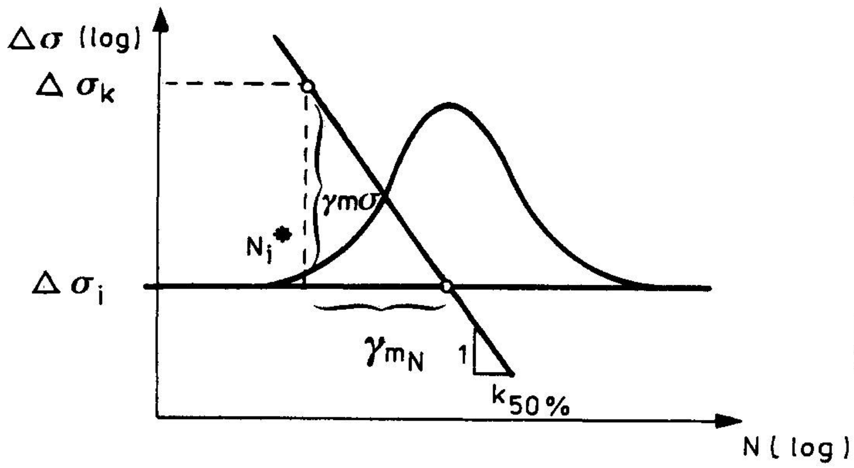

Fig. 6: Design values for the slope k*.




$$\gamma_{mN} = \frac{N_k}{N_i^*}$$

Fig. 7: Safety factor in N-scale.

An equivalent verification with respect to the stress range scale is performed with the slope k of the characteristic S-N-line according to fig. 8.




$$\gamma_{m\sigma} = \sqrt[k]{\gamma_{mN}}$$

$$\frac{\log N_k - \log N_i^*}{\log \Delta\sigma_k - \log \Delta\sigma_i} = \frac{\log \gamma_{mN}}{\log \gamma_{m\sigma}} = k$$

Fig. 8: Safety factor in $\Delta\sigma$-scale.

## 3. ASSESSMENT FOR SPECTRA WITH VARIOUS DETERMINED $\Delta\sigma_i$-LEVELS AND VARIOUS DETERMINED NUMBERS OF CYCLES $n_i$

If according to fig. 9 the stress ranges $\Delta\sigma_i$ are acting at different levels, for example $\Delta\sigma_i$ with $n_i$ and $\Delta\sigma_A$ with $n_A$, the equivalent-damage stress range cycles $n_e$ at the reference level $\Delta\sigma_A$ can be expressed using Miner's rule, fig. 10.

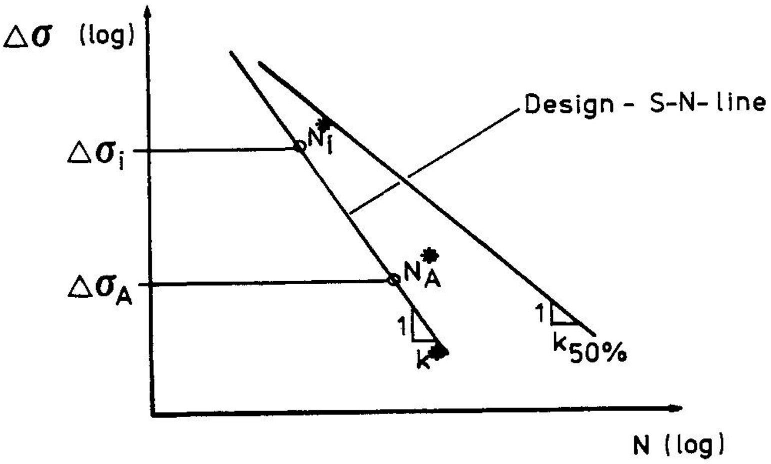



Fig. 9: Design-S-N-line.

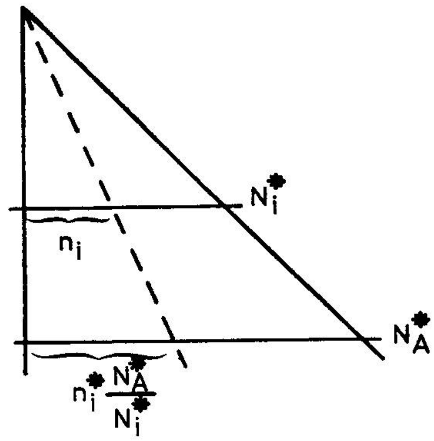



Fig. 10: Equivalent-damage stress cycles on different levels of stress range.

$$n_e^* = \sum n_i^* \frac{N_A^*}{N_i^*} \tag{4}$$

Considering the equation of the S-N*-line

$$\Delta\sigma_i^{k^*} \cdot N_i^* = \Delta\sigma_A^{k^*} \cdot N_A^* \tag{5}$$

it follows from (4)

$$n_e^* = \sum n_i^* \left(\frac{\Delta\sigma_i}{\Delta\sigma_A}\right)^{k^*} \tag{6}$$

The design-equation is then

$$g(X_i^*) = N_A^* - n_e^* = 0 \tag{7}$$

or using characteristic values and safety factors, fig. 11

$$\frac{N_{Ak}}{\gamma_{mNA}} - n_e = 0 \qquad (8)$$

By defining the reference level $\Delta\sigma_A$ as the level of the equivalent-damage stress range $\Delta\sigma_e$ for the number of load cycles $\sum n_i$, fig. 12.

$$n_e^* = \sum n_i^* \left(\frac{\Delta\sigma_i}{\Delta\sigma_e}\right)^k = \sum n_i \qquad (9)$$

the equivalent-damage stress range.

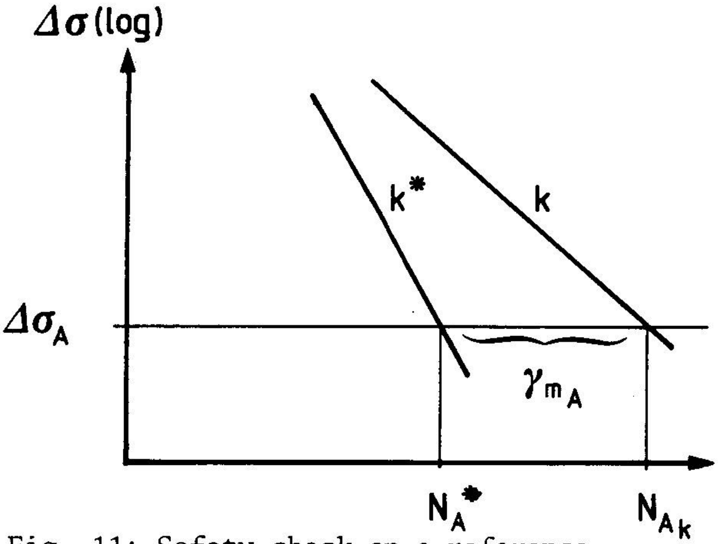



Fig. 11: Safety check on a reference stress range level $\Delta\sigma_A$.

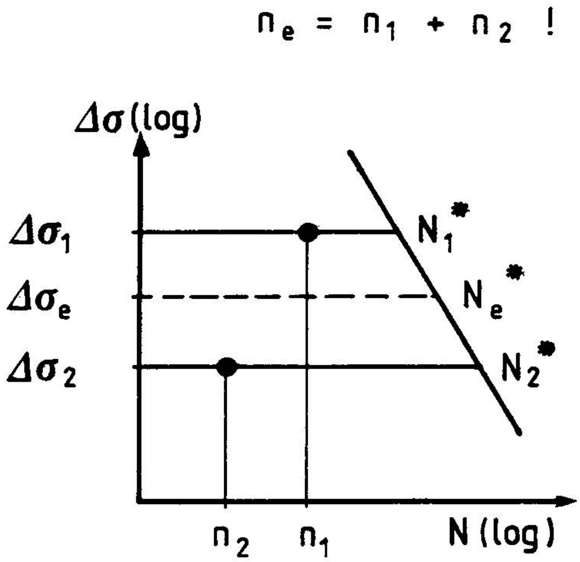



Fig. 12: Safety check on the equivalent-damage stress range level $\Delta\sigma_e$.

$$\Delta\sigma_e = \left(\frac{\sum n_i \Delta\sigma_i^{k^*}}{\sum n_i}\right)^{\frac{1}{k^*}} \qquad (10)$$

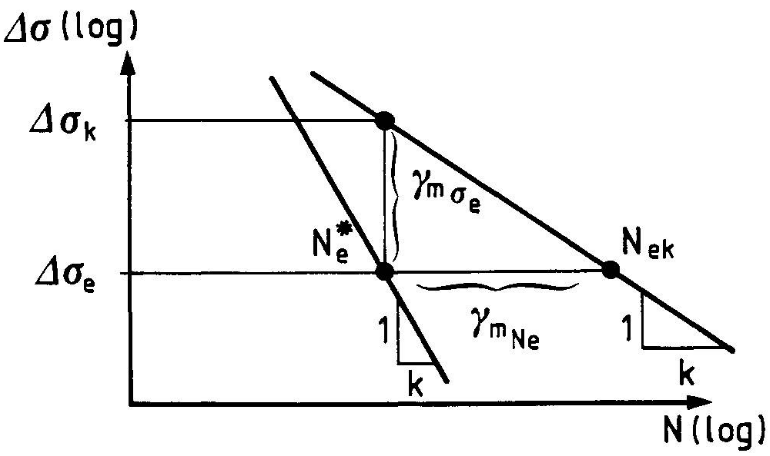
follows and the verification is performed by

$$N_e^* - \sum n_i = 0 \qquad (11)$$

The equivalent verification in terms of stresses ca be carried out using $\gamma_{m\sigma}$ according to fig. 13.

In fig. 14 $\gamma_{m_N}$ - values and $\gamma_{m\sigma}$ - values related to the 50 %-S-N-lines given as characteristic values are specified for different slopes and $\beta$-values.

For steel structures $\beta$ = 2,0 is considered to be sufficient, if the appropriate values for $\Delta\sigma_i$ and $n_i$ are correct.




$$\frac{\Delta\sigma_k}{\gamma_{m\sigma}} - \Delta\sigma_e = 0$$

Fig. 13: Safety check in $\Delta\sigma$ scale with equivalent-damage stress range $\Delta\sigma_e$.

$\gamma_{m\sigma}$ - Values

| $k_{50\%}$ \ $\beta$ | | 1,0 | 1,5 | 2,0 | 2,5 | 3,0 |
|---|---|---|---|---|---|---|
| 3,75 | $N_{10^5}$ | 1,12 | 1,19 | 1,26 | 1,33 | 1,41 |
| | $N_{2\cdot10^6}$ | 1,16 | 1,25 | 1,34 | 1,44 | 1,55 |
| 3,50 | $N_{10^5}$ | 1,13 | 1,20 | 1,28 | 1,36 | 1,45 |
| | $N_{2\cdot10^6}$ | 1,17 | 1,27 | 1,37 | 1,48 | 1,60 |
| 3,00 | $N_{10^5}$ | 1,16 | 1,24 | 1,33 | 1,43 | 1,53 |
| | $N_{2\cdot10^6}$ | 1,20 | 1,32 | 1,44 | 1,58 | 1,73 |

$\gamma_{mN}$ - Values

| $k_{50\%}$ \ $\beta$ | | 1,0 | 1,5 | 2,0 | 2,5 | 3,0 |
|---|---|---|---|---|---|---|
| 3,75 | $N_{10^5}$ | 1,54 | 1,91 | 2,36 | 2,93 | 3,63 |
| | $N_{2\cdot10^6}$ | 1,73 | 2,28 | 3,00 | 3,95 | 5,21 |
| 3,50 | $N_{10^5}$ $N_{2\cdot10^6}$ | " | " | " | " | " |
| 3,00 | $N_{10^5}$ $N_{2\cdot10^6}$ | " | " | " | " | " |

Fig. 14: $\gamma_m$-values for safety checks in N-scale or $\Delta\sigma$-scale using 50 % values as characteristic values

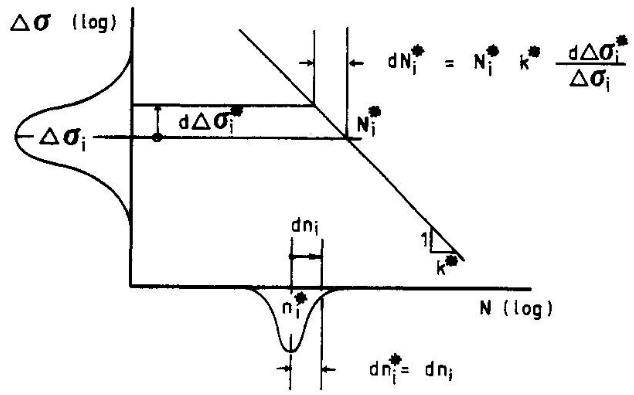
4. ASSESSMENT WITH VARYING $\Delta\sigma_i$-LEVEL AND VARYING NUMBERS OF CYCLES $n_i$

Assuming that the values of $\Delta\sigma_i$ and $n_i$ are only correct in the mean of all design cases and that they vary with $d\Delta\sigma_i$ and $dn_i$ around the correct mean value with the variation coefficients 0,10 (which corresponds to the variance of specified dead loads), the sensitivity factors $\alpha_i$ for all variables can be derived from the characteristic equation in fig. 15 using

$$\alpha_i = \frac{\frac{\partial g}{\partial x_i} S_i}{\sqrt{\sum_j \left(\frac{\partial g}{\partial x_j} \cdot S_j\right)^2}} \tag{12}$$

These sensitivity factors can be combined into $\alpha_R$ = 1,0 for the "resistance model" and $\alpha_S$ = -0,30 for the "acting model".

The $\gamma_m$-values and $\gamma_{SYS}$-values for verifications in terms of time or stresses using the 50 %-S-N-lines as characteristic values are illustrated in fig. 16.




characteristic equation :

$$N_i^* - N_i^* k^* \frac{d\Delta\sigma_i^*}{\Delta\sigma_i} - n^* - dn^* = 0$$

**Estimated datas :**

$N_{50\%} = 2 \cdot 10^6$ ; $V_N = 0{,}55$ ; $\sigma_N = 1{,}1 \cdot 10^6$

$d\Delta\sigma_{50\%} = 0$ ; $\sigma_{\Delta\sigma} = 0{,}10\ \Delta\sigma_i$

$n_{i_{50\%}} = n_i$ ; $V_n = 0$ ; $\sigma_n = 0$

$dn_{i_{50\%}} = 0$ ; $\sigma_{dn} = 0{,}10\ n_i$

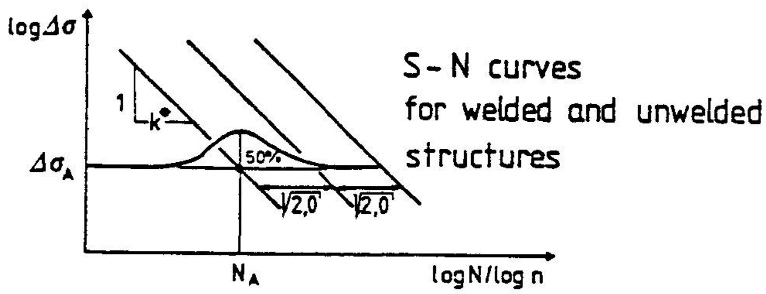
**Sensivity - factors :**

$\alpha_N \sim 1{,}0$ ; $\alpha_{\Delta\sigma} \sim -0{,}30$ ; $\alpha_{ni} \sim 0$ ; $\alpha_{dn_i} \sim -0{,}10$

**Overall Sensivity factors :**

$\alpha_R \sim 1{,}0$ ; $\alpha_S \sim -0{,}30$

Fig. 15:Consideration of a scatter for $\Delta\sigma_i$ and $n_i$.



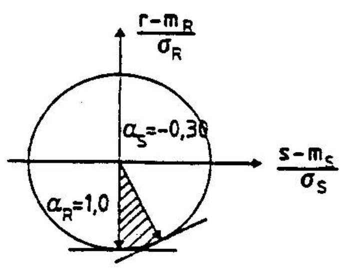

$$N_R^* \geq n_S^*$$

$$N_A \exp(-\alpha_R \beta V_R - 0{,}5\ V_R^2) \geq \sum_j n_j \left(\frac{\Delta\sigma_j\ (1 - \alpha_S \beta V_S)}{\Delta\sigma_A}\right)^{K^*}$$

$$\frac{N_A}{\gamma_{m_N} \cdot \gamma_{sys_N}} \geq n_e = \sum n_j \left(\frac{\Delta\sigma_j}{\Delta\sigma_A}\right)^{K^*}$$

For $K^* = 3{,}0$ ; $V_R = 0{,}55$ ; $V_S = 0{,}10$

| $\beta$ | 1,5 | 2,0 | 2,5 | 3,0 |
|---|---|---|---|---|
| $\gamma_{m_N}$ | 2,30 | 3,00 | 4,00 | 5,20 |
| $\gamma_{sys_N}$ | 1,15 | 1,20 | 1,25 | 1,30 |
| $\gamma_{Glob_N}$ | 2,65 | 3,60 | 5,00 | 6,80 |




$V_R \geq 0{,}50$

$V_S \leq 0{,}10$

$$\frac{\Delta\sigma_K}{\gamma_{m_\sigma} \cdot \gamma_{sys_\sigma}} \geq \Delta\sigma_e = \left[\frac{\sum n_j\ \Delta\sigma_j^{K^*}}{\sum n_j}\right]^{1/K^*}$$

| $\beta$ | 1,5 | 2,0 | 2,5 | 3,0 |
|---|---|---|---|---|
| $\gamma_{m_\sigma}$ | 1,30 | 1,45 | 1,60 | 1,75 |
| $\gamma_{sys_\sigma}$ | 1,05 | 1,06 | 1,08 | 1,09 |
| $\gamma_{Glob_\sigma}$ | 1,40 | 1,55 | 1,75 | 1,90 |

Fig. 16: Safety check for fatigue taking into account of a scatter for $\Delta\sigma_i$ and $n_i$.

In the Euorcode 3 the S-N-lines have been defined as (m - 2s)-values so that $\gamma_m$, may be defined, $\gamma_m = 1{,}0$.

## 5. MEAN STRESS INFLUENCE

The mean stress influence in Eurocode 3 is generally considered to be negligible; if the mean stress influence is significant, for instance for small welded elements or stress relieved components with minor residual stresses, the verification according to Eurocode 3 is on the safe side. In addition the mean stress influence can be considered by taking into account a bonus factor for the stress-ranges, which depends on the mean stress according to fig. 17.

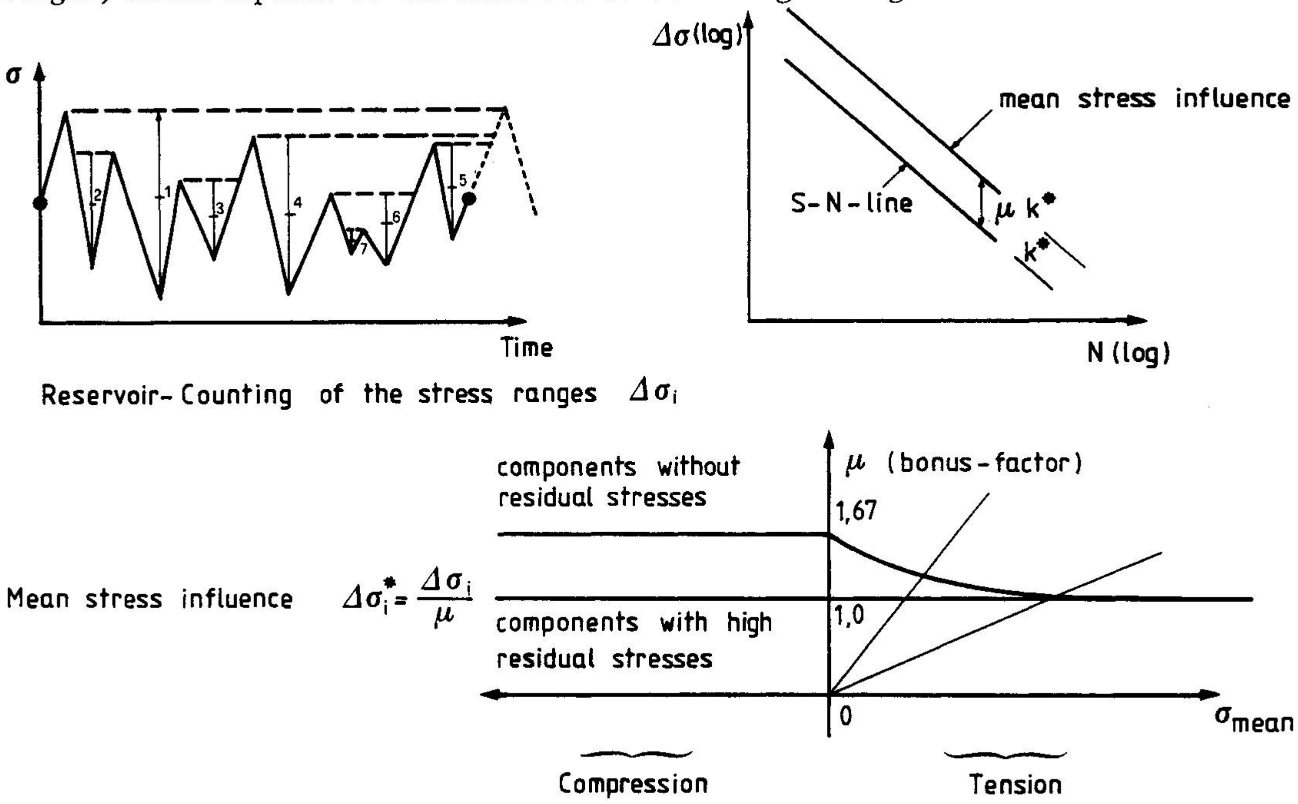



Fig. 17: Bonus factor for mean-stress-influence.

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