**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte

**Band:** 37 (1982)

**Artikel:** Fatigue crack propagation and reliability of structural elements

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**DOI:** https://doi.org/10.5169/seals-28909

**Fatigue Crack Propagation and Reliability of Structural Elements**

Propagation des fissures de fatigue et fiabilité des éléments de construction

Ermüdungsrissausbreitung und Betriebszuverlässigkeit von Konstruktionsteilen

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**SUMMARY**
Reliability theory and fracture mechanics principles were combined to indicate the time-dependent failure rate and survival probability of structural elements subjected to random loading. The analysis is illustrated for random loading of the Gaussian, stationary, narrow-band form.

**RESUME**
La théorie de la fiabilité et les principes de la mécanique de la rupture sont considérés conjointement, en mettant en évidence la dépendance de la vitesse de défaillance en fonction du temps et la probabilité de survivance des éléments de construction soumis à des charges aléatoires. L'analyse est illustrée pour une charge aléatoire du type Gaussien stationnaire à bande étroite.

**ZUSAMMENFASSUNG**
Die Zuverlässigkeitstheorie und die Methoden der Bruchmechanik wurden angewendet, um die zeitabhängige Ausfallsrate sowie die Überlebenswahrscheinlichkeit von Bauteilen unter variablen Lasten zu bestimmen. Die Betrachtungen werden für einen stationären Gauss'schen Schmalband-Belastungsprozess veranschaulicht.

## 1. INTRODUCTION

The materials fatigue damage occurs in two stages :
a) the damage nucleation, a process which develops at microstructural levels ending with fatigue cracks initiation, b) crack propagation until the overall load carrying capacity is lost. The nucleation stage can be extended over 80 - 90 % of the fatigue life, in the case of smooth elements loaded in neutral environ - ment, or can be suppressed in the presence of accute stress concentrators, preexistent defects or corrosive environment.In welded metallic structures stress concentrators arise from particular joint configuration while defects such as undercuts, lack of penetration or even cracks can be induced during the welding process. Undere these circumstance the damage nucleation period is short and the reliability of a structural element or of the whole structure is governed by the random interaction of the load, defects size and the material respose to the fatigue crack propagation.

In the following a procedure will be presented, in order to estimate the reliability of metallic elements containing accute defects which propagate under random loadings.

## 2. RELIABILITY ANALYSIS

In a metallic element which contains a crack of semilength a ( figure 1 ), the descrease of residual strength $R_r$ as against the number of loading cycles is expressed ba the general relationship :

$$\frac{dR_r}{dn} = \frac{dR_r}{da} \cdot \frac{da}{dn} \qquad (1)$$

The residual strength $R_r$ of an element containing a fatigue crack can be evaluated by resorting to the basic relationships of fracture mechanics :

$$R_r = \frac{K_{IC}}{\sqrt{\pi \beta a}} \qquad (2)$$

where $K_{IC}$ is the fracture toughness ( under plain strain conditions ) of the material and $\beta$ is a correction for the finite dimensions of the element [1],[2] . The rate of fatigue crack propagation da/dn can be evaluated by a Paris type relationship [3] :

$$\frac{da}{dn} = C(\sigma_a \sqrt{\pi \beta a})^k \qquad (3)$$

where $\sigma_a$ is the uniform stress amplitude applied sufficiently re-

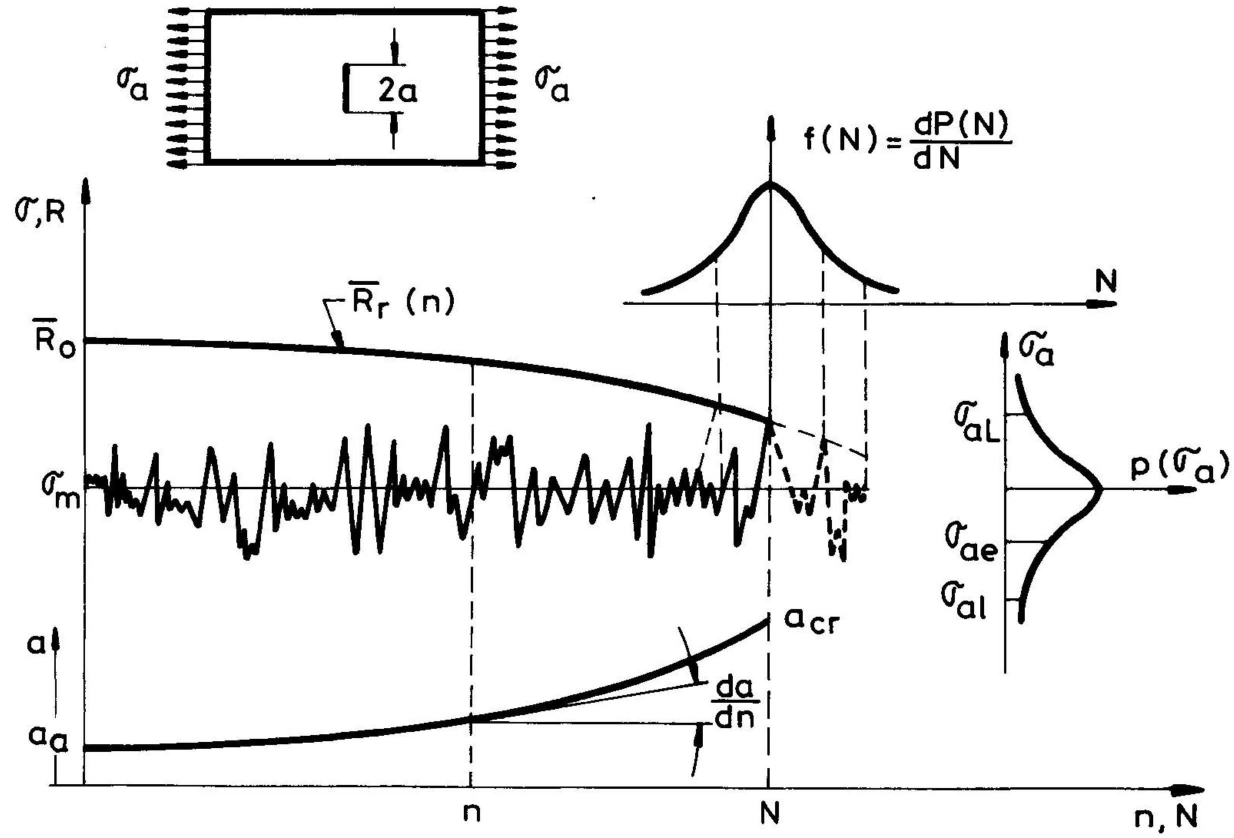



Figure 1.

moted from the crack tip region ( figure 1 ) ; C and k are material constants.

If the random loading is stationary and centred arround the mean stress $\sigma_m$ with the probability density $p(\sigma_a)$ defined on the domain $(\sigma_{al};\sigma_{aL})$ as is illustrated in figure 1 , then the mean rate of crack propagation $\overline{da/dn}$ derived by a mean type operation in the relationship (3) is:

$$\frac{\overline{da}}{dn} = \frac{C}{\Lambda_0}(\sqrt{\pi\beta a})^k \int_{\sigma_{ae}}^{\sigma_{aL}} \sigma_a^k p(\sigma_a) d\sigma_a \qquad (4)$$

where $\sigma_{ae}$ is the inferior threshold limit of the stress amplitude under which the fatigue cracks do not propagates any more 1

If $\sigma_{ae} < \sigma_{al}$ , then in eq. (4) the inferior limit of integration is $\sigma_{al}$, while if $\sigma_{ae} > \sigma_{al}$ the correction for the load distribution truncation must be introduced :

$$\Lambda_0 = \int_{\sigma_{ae}}^{\sigma_{aL}} p(\sigma_a) d\sigma_a \qquad (5)$$

Taking the derivative in the relationship (2) one results from the conjunction with the relationship (1) and (4) the rate of the mean residual strength decreasing as against the applied loading cycles :

$$\frac{\overline{dR_r}}{dn} = -\frac{1}{2} \cdot \frac{\pi\beta C (K_c)^{k-2}}{\Lambda_0 \overline{R}_r^{k-3}} \int_{\sigma_{ae}}^{\sigma_{aL}} \sigma_a^k p(\sigma_a) d\sigma_a \qquad (6)$$

By integrating in this equation and considering the relationship (2) for the initial undamaged state of the element with the mean strength $\overline{R}_0$ and a preexisting crack of semi-length $a_0$(i.e. $\overline{R}_0 = K_{IC}/\sqrt{\pi\beta a_0}$) one results the mean residual strength $R_r$ after n - loading cycles :

$$\overline{R}_r = \overline{R}_0 \left[1 - n\left(\frac{k}{2} - 1\right)\frac{\pi\beta C}{\Lambda_0}\left(\frac{K_{IC}}{\overline{R}_0}\right)^{k-2} \int_{\sigma_{ae}}^{\sigma_{aL}} \sigma_a^k p(\sigma_a) d\sigma_a\right]^{1/(k-2)} \qquad (7)$$

In this stage of analysis it is pertinent to outline that the mean residual strength after n-loading cycles can be computed as function of :

- the characteristic parameters of the loading distribution $p(\sigma_a), \sigma_{al}; \sigma_{aL}$;
- the parameters associated with the material strength to fatigue crack propagation : k , C and $\sigma_{ae}$ ;
- a geometric parameter $\beta$ which depends on the shape and dimensions of the metallic element;

- initial mean static strength $\overline{R}_0$ and fracture toughness $K_{IC}$ .

It is to be remarked in the eq. (7) that the ratio $(K_{IC}/\overline{R}_0)^2 = \overline{J}\Pi\beta a_0$ is proportional with the initial crack length.

When the application of $K_{IC}$ concept is not appropriate due to crack - tip plasticity effects one can resort to the critical crack opening displacement ( COD ) concept [1] for making explicite the residual static strength.

Knowing the expression of the residual strength $\overline{R}_0$ as function of applied loading cycles one can determine the failure risk $\lambda(N)$, or the probability of failure in the (N+1) cycle if the involved metallic element has survived N cycles. This probability is determined by the condition that the maximum loading $\sigma_{max}$ (i.e. $\sigma_{max} = \sigma_a + \sigma_m$ ) applied in the N-th cycle be grater than the residual strength under singular loading [4] :

$$\lambda(N) = \text{Prob.}(\sigma_{max} > \overline{R}_r) = \int_{\overline{R}_r - \sigma_m}^{\sigma_{aL}} p(\sigma_a)\, d\sigma_a \tag{8}$$

In the reliability theory one defines the reliability function Q(N) which quantifies for an element or a structure the cumulative probability for attaining ( survival ) an endurance of N cycles. The complementary function $P(N) = 1 - Q(N)$ defines the cumulative probability that fracture occurs at a certain endurance N. The following relationship between these functions exists [4],[5]:

$$Q(N) = 1 - P(N) = \exp\left[ -\int_0^N \lambda(N)\, dN \right] \tag{9}$$

From eqs. (8) and (9) one can estimate the reliability parameters of a structural element which instantaneous residual strength (eq.7) is governed by the fatigue crack propagation under random loadings.

If the loading pattern ( applied in a region remote from the crack influence ) is defined by the probability density of loading maxima $p(\sigma_{max})$ then eq.(7) remains valid in its functional form, but with the particularization $\sigma_a \rightarrow \sigma_{max}$ and appropriate experimental constants C and k.

3. THE CASE OF NORMAL NARROW - BAND RANDOM LOADING

As an exemplification, at the application of the general theory the case of a normal or Gaussian narrow - band random stationary loading with a zero mean will be considered. For such a loading case the loading maxima repartition coincides with amplitudes repartition, being of the Rayleigh type [6] :

$$p(\sigma_a)=\frac{\sigma_a}{d_\sigma^2}\exp\left(-\frac{\sigma_a^2}{2d_\sigma^2}\right) \qquad (10)$$

which in conjunction with the relationship (7) one determines the expression of the mean residual stress $\overline{R}_r$ after n-loading cycles :

$$\overline{R}_r=\overline{R}_0(1-A_n)^{1/(k-2)} \qquad (11)$$

where :

$$A=\left(\frac{k}{2}-1\right)\pi\beta C(\sqrt{2}\,d_\sigma)^k\ \frac{\Gamma\left(\frac{\sigma_{aL}^2}{2d_\sigma^2};1+\frac{k}{2}\right)-\Gamma\left(\frac{\sigma_{ae}^2}{2d_\sigma^2};1+\frac{k}{2}\right)}{1-\exp(-\sigma_{aL}^2/2d_\sigma^2)}\left(\frac{K_{IC}}{\overline{R}_0}\right)^{k-2} \qquad (12)$$

Since the Rayleigh distribution was truncated at the superior limit $\sigma_{aL}$ , the fracture cannot occur as long as the residual strength $\overline{R}_r$ was not decreased by the fatigue damage down to the superior limit of the loading distribution $\sigma_{aL}$ , when the corresponding number of cycles $n^*$ are :

$$n^*=\frac{1}{A}\left[1-\left(\frac{\sigma_{aL}}{\overline{R}_0}\right)^{k-2}\right] \qquad (13)$$

The failure risk is zero for $0\leqslant n\leqslant n^*$ and increases for $n>n^*$ according to the relationship (8) which yields :

$$\lambda(N)=\left\{\exp\left[-\frac{\sigma_{aL}^2}{2d_\sigma^2}\left(\frac{1-A_n}{1-A_{n^*}}\right)^{\frac{2}{k-2}}\right]-\exp\left(-\frac{\sigma_{aL}^2}{2d_\sigma^2}\right)\right\}\left[1-\exp\left(-\frac{\sigma_{aL}^2}{2d_\sigma^2}\right)\right] \qquad (14)$$

Now, the reliability function Q(N) and the cumulative probability P that the fatigue crack propagation till to the complete fracture be accomplished in N loading cycles results from the conjunction of eq.(9) and (14).

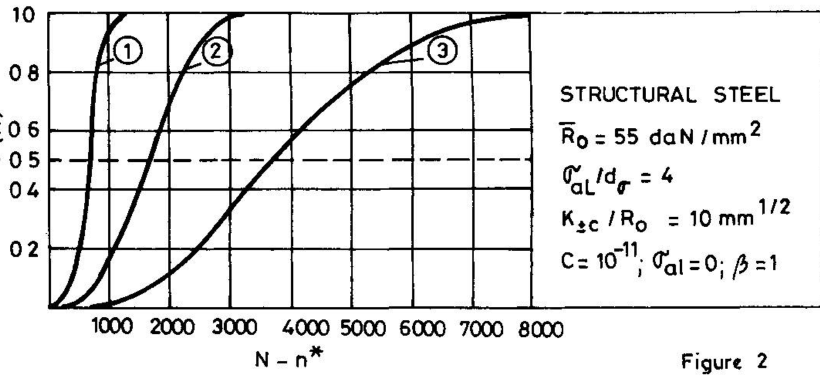
$$P(N)=1-\exp\left\{\frac{(N-n^{*})\exp\left(-\frac{\sigma_{aL}^{2}}{2d_{\sigma}^{2}}\right)+\frac{k-2}{2}\cdot\frac{1-An^{*}}{A}\left(\frac{2d_{\sigma}^{2}}{\sigma_{aL}^{2}}\right)^{\frac{k-2}{2}}\left\{\Gamma\left[\frac{\sigma_{aL}^{2}}{2d_{\sigma}^{2}}\left(\frac{1-AN}{1-An^{*}}\right)^{\frac{2}{k-2}};\right.\right.}{1-\exp\left(-\frac{\sigma_{aL}^{2}}{2d_{\sigma}^{2}}\right)}\right. \longrightarrow$$

$$\longrightarrow \left.\frac{\left.\left.\frac{k}{2}-1\right]-\Gamma\left(\frac{\sigma_{aL}^{2}}{2d_{\sigma}^{2}};\frac{k}{2}-1\right)\right\}}{1-\exp\left(-\frac{\sigma_{aL}^{2}}{2d_{\sigma}^{2}}\right)}\right\} \qquad (15)$$

In the relationships (12) and (15) $\Gamma(t,\vartheta)$ is the incomplete Gamma function whose integral expression is :

$$\Gamma(t,\vartheta)=\int_{0}^{t} t^{\vartheta-1}\, e^{-t}\, dt \qquad (16)$$

In figure 2 it is illustrated the cumulative failure probability P(N) as against the number of cycles $(N-n^{*})$ for a 50 class steel element with $\bar{R}_{0} \doteq 55$ dN/mm$^{2}$ $K_{IC}/\bar{R}_{0}=10$ ; $\beta=1$ and $\sigma_{ac}=0$ . The fa -




| | $\sigma_{aL}/R_0$ | $n^*$ (cycles) |
|---|---|---|
| ① | 0.500 | $1.61.10^{4}$ |
| ② | 0.250 | $1.36.10^{5}$ |
| ③ | 0.125 | $2.24.10^{6}$ |

Figure 2

tigue crack propagation parameters $k=4$ and $C=10^{-11}$ are typical for this type of steel [1] . The considered random loading is of Rayleigh type with $\sigma_{aL}/d_{\sigma}=4$ , the different pattern of loading intensity being specified by the ratio $\sigma_{aL}/\bar{R}_{0}=0{,}125; 0{,}250; 0{,}500$.
It is to be remarked from figure 2 that the statistical variability increases as the random loading intensity decreases.

The proposed analysis is pertinent for the reliability evaluation of a structural element in which a fatigue crack is propagating. By applying the theorems of probability composition [4] [5] one can further estimate the overall reliability of a structure with elements arrangend in series (static determinate) parallel ( redundant ) or a combination of both pattern.

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