Determination of fatigue life of bicyclic loaded metal structures

Autor(en):   **Trufiakov, V.I. / Kovalchuk, V.S.**

Determination of Fatigue Life of Bicyclic Loaded Metal Structures

Détermination de la durée de vie de structures métalliques soumises à des charges alternées bicycliques

Bestimmung der Lebensdauer von Metallkonstruktionen, die durch Doppelfrequenzbelastung beansprucht werden

V.I. TRUFIAKOV
Professor
E.O. Paton Electric Welding Institute
Kiev, USSR

V.S. KOVALCHUK
Dr. Ing.
E.O. Paton Electric Welding Institute
Kiev, USSR

SUMMARY
It was established that at a fixed frequency ratio a linear dependence is observed between the amplitude ratio of two-frequency cycle and the decimal logarithm of fatigue life ratio. It is invariant to the variation of the main factors determining the fatigue strength of materials and joints. The given relationship permits to determine the fatigue life at bicyclic loading from Wöhler curves. Determination procedure is given.

RESUME
Il a été établi que, pour un rapport fixé des fréquences bicycliques, on peut observer une relation linéaire entre le rapport des amplitudes et le logarithme décimal du rapport des durées de vie. Cette relation est indépendante de la variation des principaux paramètres caractérisant la résistance à la fatigue des matériaux et des assemblages. La relation proposée permet de déterminer à partir des courbes de Wöhler la durée de vie sous un chargement bicyclique. On donne le processus de cette détermination.

ZUSAMMENFASSUNG
Es hat sich gezeigt, dass bei konstantem Verhältnis der zwei Frequenzen ein linearer Zusammenhang zwischen dem Spannungsamplitudenverhältnis und dem dekadischen Logarithmus des Lebensdauerverhältnisses besteht. Dieser Zusammenhang ist invariant zu den Grundfaktoren, die die Dauerfestigkeit der Werkstoffe und Verbindungen bestimmen. Diese Gesetzmässigkeit erlaubt es, die Lebensdauer bei der Doppelfrequenzbelastung mit den Wöhlerkurven zu bestimmen. Das Verfahren dazu wird angegeben.

Bicyclic loading (Fig.1), at which smaller value but higher-fre-
quency components generated by all kinds of vibrations are super-
imposed on the main low-frequency cycles, is characteristic not
only of aircraft, ships, reactor bodies, gas-turbine engines, but
also of bridges [1], crane girders [2], mast-aerial constructions
[3] and other structures (Fig.2). The data in the Table permit to
judge   frequency-amplitude ratios of loading components typical
for the elements of these structures.

The detrimental effect of vibrations is usually not taken into
account when calculating the
cyclic fatigue life of structu-
res or it is only taken into
account by increasing the am-
plitude of the main alternating
loading by a value correspon-
ding to the high-cyclic compo-
nent. However, as shown by the
research, the results of which
have been generalized in the
previously compiled reviews and
separate publications[4,5,6,9,
et al.] the fatigue strength un-
der bicyclic loading decreases
to a greater extent, than at
one-cyclic loading with a ma-
ximum amplitude $\sigma_{a.s}$, equal to
the total value of the amplitu-
des of both cyclic components
($\sigma_{a.l}$ and $\sigma_{a.h}$). Here, the fati-
gue life variation can be affec-
ted not only by the amplitude
ratio $\sigma_{a.h}/\sigma_{a.l}$ (Fig.3), but
also by the frequency ratio
f_h/f_l [7,8]. With the increase
of f_h/f_l , the fatigue life mo-
notonically decreases. At the
same time, in case of small
frequency ratios (at bending,
axial loading and twist-ben-
ding) or when the rotation
frequency is higher than that
of bending (at testing for ro-
tation bending),not the de-
crease but the increase of fa-
tigue life was observed in
some studies under bicyclic
loading (Fig.3).

Fig.1.Bicyclic loading (scheme)

It should be noted that under
rotation bending conditions,
the bicyclic loading is only
ensured when the stress ampli-
tude ratio is not high, and
the bending frequency exceeds
the rotation frequency. With
the amplitude ratio increase
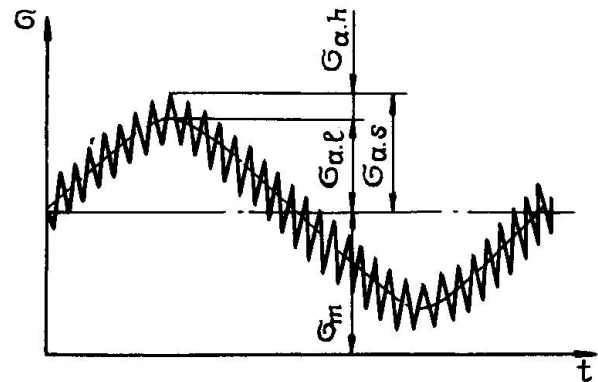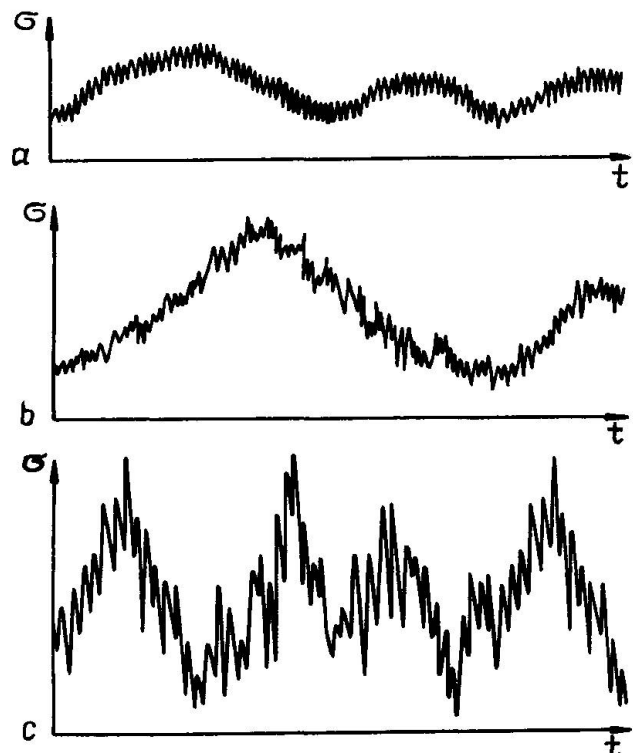and at excess of rotation fre-

Fig.2.Change of stresses determi-
ned by oscillography:a)main beam
of a bridge span l =23m; b) in a
brace of a bridge lattice girder
l =77m; when a train is passing
at 83 km/h speed; c)in the mast
guy-ropes.

Table. Amplitude and frequency ratios of load components

Elements of structures and workpieces	Possible ratios of	
	amplitudes $\sigma_{a.h}/\sigma_{a.s}$	frequencies f_h/f_l
1. Main beams and braces of railway bridge girders	0.05...0.25*	30...100*
2. Metal structures of radio- and telemasts	0.1...0.5*	1.5...150*
3. Crane girders	0.01...0.25	10...1000
4. Power plant elements	0.03...0.5	100...1000
5. Hydraulic turbines	0.1...0.5	2.5...150
6. Spindles of blooming and rolling mills	0.01...0.5	15...30
7. Cutting chains of coal cutters	0.1...0.5	15...20
8. Fuselages, vane suspensions, ailerons and stabilizers of passanger planes	0.03...0.5	1.5...5000

*According to the data of measurements in several installations. It is possible that the amplitude and frequency range can be even greater for bridges and masts.

quency over the bending one, the nature of stress change noticeably differs from the bicyclic case. Here, the fatigue testing results of a different quality are obtained. Investigations of aluminium alloys at similar levels of stresses and frequency ratios showed that in case of rotation bending the superimposition of a high-frequency component increases the fatigue life, and under bending in one plane it considerably decreases. Furtheron, testing results only for axial loading and plane bending are given.

Alongside the experimental evaluation, various authors have made suggestions also for fatigue life calculation under bicyclic loading. They are based on linear cumulative damage hypothesis [3,10, et al.], the hypothesis of spectral summation [11], energy concepts [8], and empirical relationships [6,12,13, et al]. These suggestions mostly referred to specific workpieces, loading conditions and materials. Thus, evidently, their validity, as shown by the comparison of calculated values and experimental data of other authors, is preserved in certain rather narrow ranges of amplitude and frequency ratio variations [14] It is to be noted here that, apparently, Miner's hypothesis
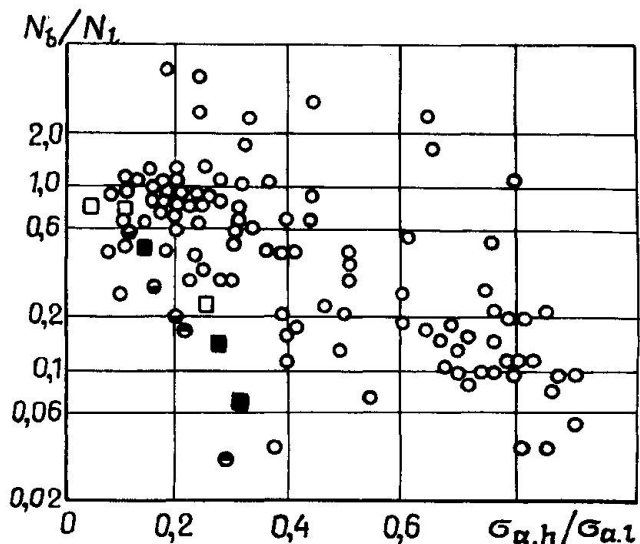

Fig.3.The effect of amplitude ratio on fatigue life change at bicyclic loading. o - [5], □ - [9], ■ - [6], ◐ - [4], ◑ - [4].

cannot be used to describe the fatigue damage accumulation. Possibly, it may lead to satisfactory results only in low-cycle region at relatively low f_h/f_l and for conditions not associated with significant temporal processes [15]. The unacceptability of Miner's hypothesis in multicycle area is demonstrated, in particular, by data in the work [16]. Specimen testing at bicyclic loading and at onecyclic loading with an equal value of maximum stresses showed that at bicyclic loading the fatigue life is noticeably decreased even when the average stress value is equal to the maximum average value of low cyclic component (Fig.5).

The suggestions made do not permit to conclude that at present a new quite well-based procedure for bicyclic loading fatigue life evaluation is being established or contemplated. It should be based on the premises or relationships, expressing regular connection between the parameters of bicyclic loading and fatigue strength valid in a wide range of amplitude and frequency ratio variation. The approach to the bicyclic loading parameter selection to obtain the simplest estimation dependences has not yet been sufficiently well grounded, either. In the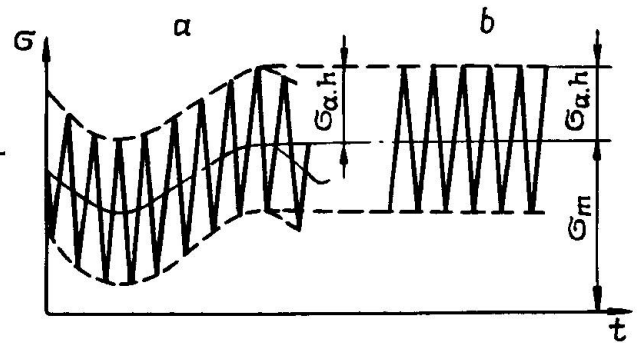 investigations performed the total stress $\sigma_{a.s} = \sigma_{a.l} + \sigma_{a.h}$ was used to estimate the bicyclic loading fatigue life, and the relationship was established between $\sigma_{a.h}/\sigma_{a.s}$ and $N_b/N_{l.s}$, where N_b - bicyclic loading fatigue life and $N_{l.s}$ - fatigue life corresponding to the summary amplitude $\sigma_{a.s}$. The usage of $\sigma_{a.h}/\sigma_{a.s}$ ratio excludes the possibility of obtaining a linear connection between bicyclic loading parameters and fatigue life N_b since at linear change of $\sigma_{a.l}$ and $\sigma_{a.h}$ amplitudes the ratio $\sigma_{a.h}/\sigma_{a.s}$ varies non-linearly . As will be shown later, it is also expedient to evaluate the relative fatigue life by N_b/N_l ratio, and not $N_b/N_{l.s}$ i.e. to determine the effect of high cyclic component on the decrease of initial fatigue life N_l , corresponding to loading with $\sigma_{a.l}$ amplitude and not to the conditional fatigue life $N_{l.s}$ at hypothetical amplitude $\sigma_{a.s}$.

The advantages of using these ratios are shown in the investigation [14]. Fig.5.in the coordinates $\sigma_{a.h}/\sigma_{a.l}$, $lg(N_l/N_b)$ shows the results of all tests made by different authors with an average range of frequency ratio

Fig.4.Loadings at pure bending compared in the work [16].
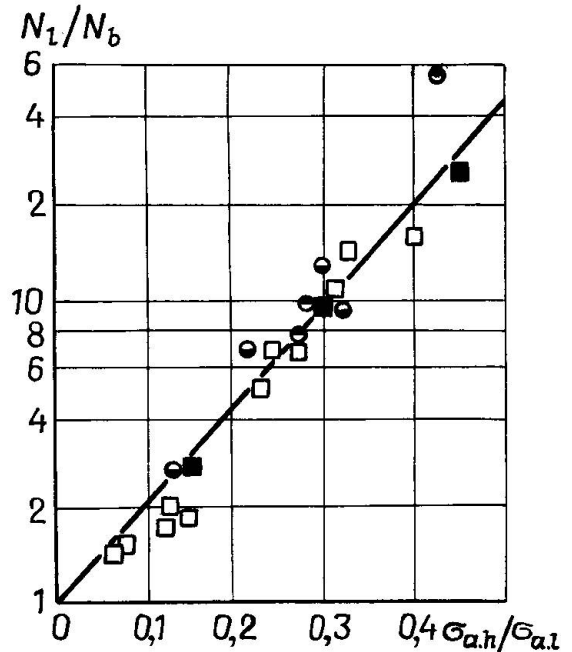

Fig.5.The generalization of testing results at two-frequency loading: □ - D16AT alloy,tension [9] ,■ - D16T alloy,tension [6] , ⊖ -OX12H steel,bending [4] , ⦿ - 45 steel,bending [4] .

(mainly f_h/f_l =100...600), which could be converted to the given
system. There is an agreement between the test results, despite
the fact that the specimens differed in shape and sizes and were
fabricated of different materials. In the given coordinate sys-
tem a linear dependence was observed in a rather wide range be-
tween $\sigma_{a.h}/\sigma_{a.l}$ and N_l/N_b .

If we proceed from the existence of such a dependence, then the
bicyclic loading frequency can be determined from the fatigue
curves of onecyclic loading:

$$N_b = \frac{N_l}{\mathscr{æ}}$$ (1)

where $\mathscr{æ}$ - factor dependent on $\sigma_{a.h}/\sigma_{a.l}$ amplitude ratio and f_h/f_l
frequency ratio.

To confirm the invariance of $\mathscr{æ}$ factor at the change of the main
factors, affecting the fatigue strength of materials and joints,
and also to obtain an appropriate expression for its determina-
tion, the large-scale specimens (Fig.6) were tested at one- and
bicyclic loading at various stresses - from the level slightly
above the endurance limit up to the yield strength. Here, the
$\sigma_{a.h}/\sigma_{a.l}$ ratio varies from 0.05 to 0.9,

155x20 mm section flat specimens (Fig.6a) were made of low-carbon
steel (σ_T =300 MPa, σ_B =510 MPa) and 12X18H10T chrome-nickel steel
(σ_T =350 MPa, σ_B =650 MPa). The concave recesses in the sides crea-
ted in various samples a stress concentration α_σ , equal to 1.5;
2.5 and 3.5. Specimens of high-strength steel (σ_T =1070 MPa, σ_B =
1120 MPa) had a 80x18 mm section and a stress raiser in the form
of a whole in the thinned part of the plate (Fig. 1,c). Taking
into account a possible effect of residual stresses [17]   on
the fatigue strength of welded
structures, the role of this factor
in $\mathscr{æ}$ change was evaluated on speci-
mens with beads (Fig.1,b), and on an
actual joint (Fig.1,d). In the stress
raiser zone the beads created initial
residual stresses close to the base
metal yield strength. The tests were
performed both at axial loading and
at bending. As for the low-cyclic
loading, the tests were performed at
a symmetrical, zero-to-tension and
asymmetrical cycles (r=+0.5). To eli-
minate the errors associated with a
possible effect of frequencies at
one- and bicyclic loading, f_l fre-
fuency in a multicycle region did
not exceed 20 min^{-1} . f_h frequency
was 300 min^{-1} under axial loading
and 1800 min^{-1} at bending. The re-
quired ratio f_h/f_l with 10^2-5.10^3
range was ensured by varying f_l . An
initial stage of crack development
was used as a fatigue testing com-
pletion criterium. The testing was
stopped when crack depth reached 2-
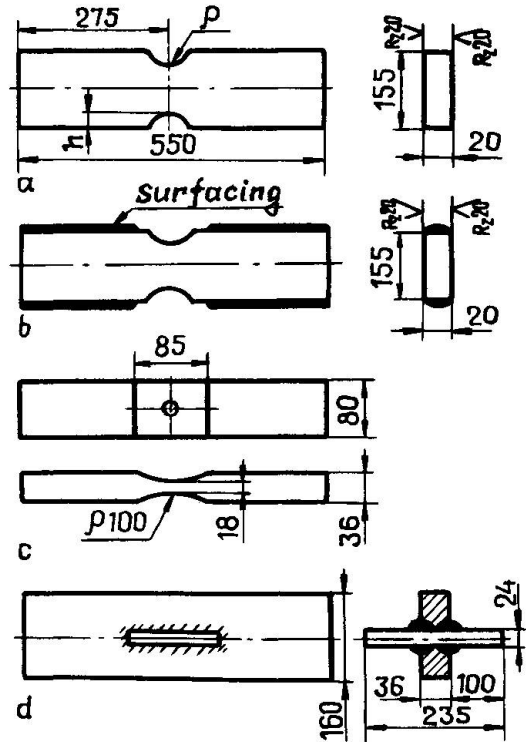3 mm. Part of the specimens were

Fig.6.Specimens for testing
at one- and bicyclic loa-
ding.

tested till they completely broke down.

Each factor studied significantly affected the fatigue strength of samples at one- and bicyclic loading. However, the studies performed have proved that $\mathscr{x}$ coefficient remains invariant to the changes associated with the degree of stress concentration, residual stresses, stress ratio, type and level of loading, temperature and fatigue testing completion criterium. In this case all the relationships between $lg\,\mathscr{x}$ and $\sigma_{a.h}/\sigma_{a.l}$ are of a stable linear type. The main dependences are shown in Fig. 7. At the same time, the change of $\mathscr{x}$ is drastically affected by the amplitude (Fig.7) and frequency (Fig.8) ratios. With the increase of these ratios $\mathscr{x}$ becomes considerably greater. These two factors mostly determine $\mathscr{x}$ factor value. It is to a lesser degree that $\mathscr{x}$ is sensitive to steel properties (Fig.8,b). With the change of frequency ratios and strength properties of material a linear relationship is maintained between $lg\,\mathscr{x}$ and $\sigma_{a.h}/\sigma_{a.l}$

The established dependences give the following expression for determining the factor of bicyclic loading fatigue life decrease.

$$\mathscr{x} = \left(\frac{f_h}{f_l}\right)^{\vartheta\,\frac{\sigma_{a.h}}{\sigma_{a.l}}} \qquad (2)$$

where ϑ - factor, reflecting the material effect. It may change within 1.3...1.8 for steels. $\mathscr{x}$ value may also be determined according to the nomogram, given in Fig.9.

When $\mathscr{x}$ factor in the initial equation (1) is replaced by its value (2), we obtain a final formula for bicyclic loading fatigue life determination:

$$N_b = \frac{N_l}{\mathscr{x}} = \frac{N_l}{\left(\frac{f_h}{f_l}\right)^{\vartheta\,\frac{\sigma_{a.h}}{\sigma_{a.l}}}} = N_l\left(\frac{f_l}{f_h}\right)^{\vartheta\,\frac{\sigma_{a.h}}{\sigma_{a.l}}} \qquad (3)$$

In this formula, N_l - is the fatigue life of material, joint, element, etc., which, in the assigned conditions corresponds to low cyclic loading with $\sigma_{a.l}$ amplitude. According to formula (3) the bicyclic loading fatigue life N_b is always less than N_l . At the same time, as noted above
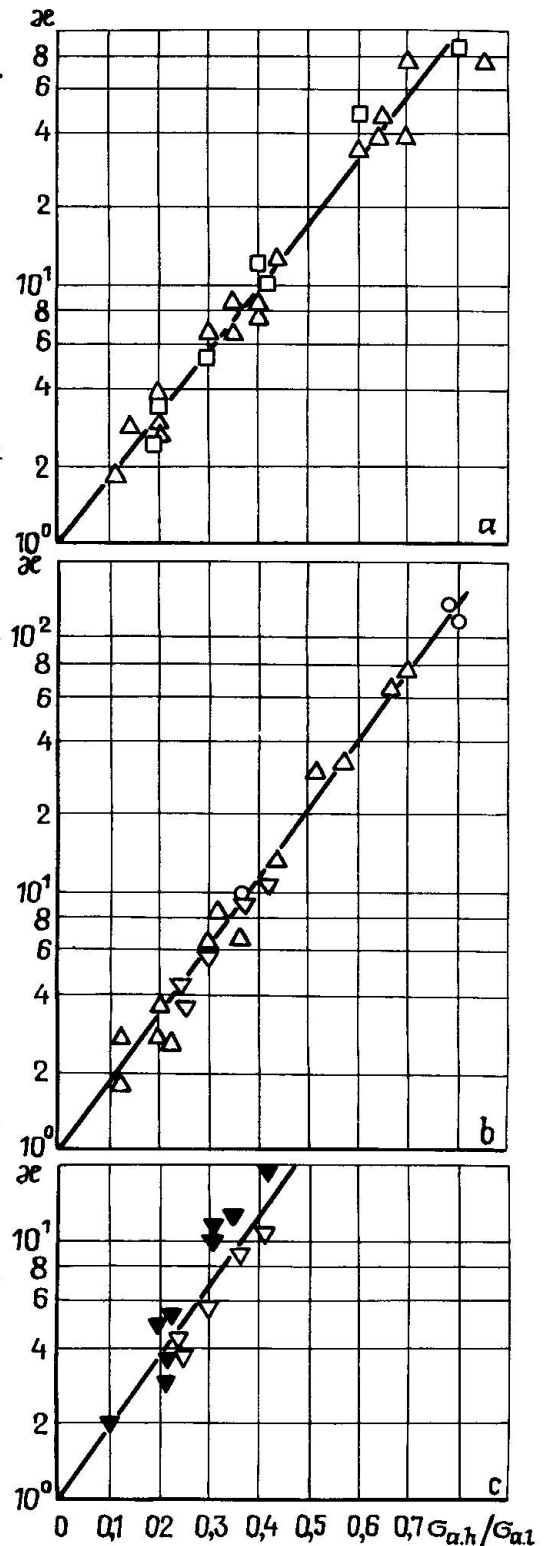

Fig.7.Coefficient $\mathscr{x}$ invariance to the change of: a)stress concentration: □ - α_6=1.5; ∆ -α_6=2.5; ∆ -α_6=3.5; b)cycle asymmetry: o -r=0.5; ∇ -r =0; ∇ -r =-1; c)resi- dual stressed state: ∇ - Ω =0; ∇ - Ω = σ_τ .

at small frequency ratios
an increase of bicyclic lo-
ading fatigue life was ob-
served in certain investiga-
tions. In this connection
it becomes necessary to de-
termine the boundaries of
formula (3) applicability.
In case of small values of
f_h/f_l the change of $\sigma_{a.s}$ am-
plitude depends not only on
$\sigma_{a.h}/\sigma_{a.l}$, but also on phase
shift between the componen-
ts. In a certain interval
of φ phase shift, the $\sigma_{a.s}$
summary amplitude can de-
crease to values lower than
$\sigma_{a.l}$ [14]. If we take into
consideration the fact that
$\sigma_{a.h}/\sigma_{a.l} < 0.3$ and the re-
sulting load cycle form $\sigma_{a.s}$
is distorted without the
appearance of additional
extremums, then the incre-
ase of fatigue life in the
given conditions under the
effect of bicyclic loading
is readily explained.
When $\sigma_{a.h}/\sigma_{a.l} < 0.5$ the sum-
mary amplitude depending
upon φ angle, can be both
lower and higher than the
low-frequency component am-
plitude. Accordingly, N_b can
be increased or decreased
compared to N_l . At $\sigma_{a.h}/\sigma_{a.l} >$
> 0.5 the summary amplitu-
de is higher than the low-
cyclic component amplitude.
The fatigue life will de-
crease under the effect of
this factor, and, possibly,
due to the appearance of
additional extremums.

The situation becomes dif-
ferent at $f_h/f_l > 10$. Irres-
pective of amplitudes and
initial phases of loading
components $\sigma_{a.s} > \sigma_{a.l}$. The

Fig.8.Relationship between æ and
frequency ratio(a) and steel type(b):
1- $f_h/f_l =10^2$; 2- $f_h/f_l =10^3$; 3- $f_h/f_l =$
5.10³; 4-low-carbon steel; 5-chrome-
nickel steel; 6-high-strength steel.

Fig.9.Nomogram for æ determination.

analysis performed showed that $\sigma_{a.s}$ deviation from the maximum
possible value at a synphase summation of harmonic loads did
not exceed 5% within the whole range of amplitude ratios. The
damage accumulation is also favoured by individual stress cycles,
changing with f_h frequency. In such conditions, N_b will al-
ways be much lower than N_l . Thus, at $f_h/f_l \geqslant 10$, the formula
(3) can be used without restrictions. When $f_h/f_l < 10$, it is
valid for a fixed value of phase shift, meeting maximum value of

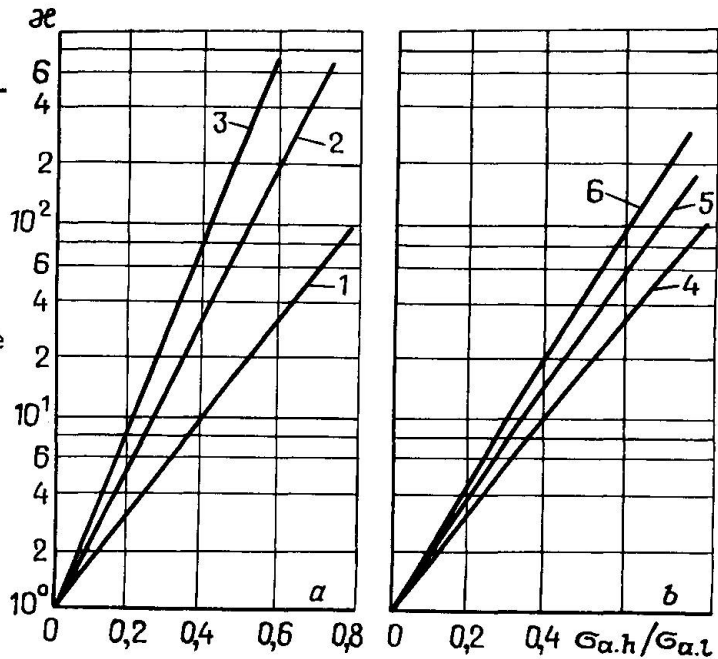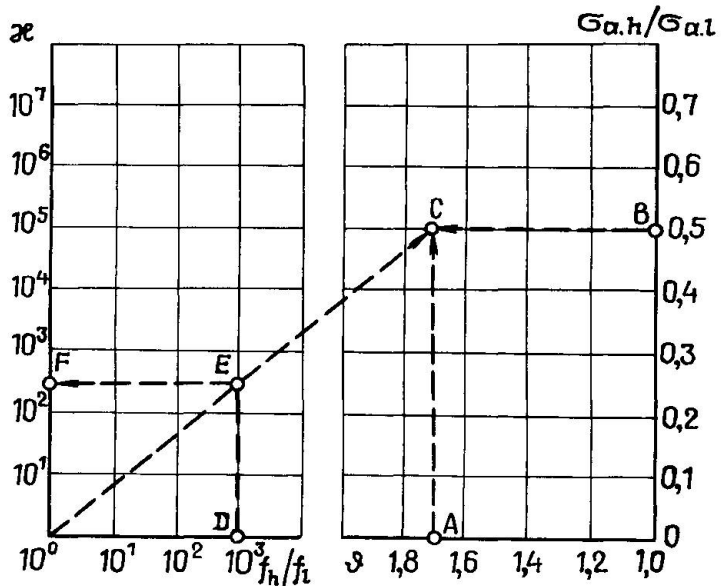resulting amplitude $\sigma_{a.s}$ .For other values of phase shift the formula (3) can give only under-estimated values of N_b .

The validity of formula (3) was also verified by comparing the experimental data obtained earlier by other authors with the calculated data. In most cases the ratio between calculated data and the experimental ones was close to a unit, while individual deviations lay within the scattering range of 0.75-1.5.

It appears that there is every reason to believe, that the proposed method of bicyclic loading fatigue life estimation is quite suitable for engineering calculations and can be used within a wide range of amplitude and frequency ratio change.

REFERENCES

1.SERGIEVSKY,A.:Fatigue strength of railway bridge spans in service , Transzheldorizdat, 1963.
2.GASSNER,E., SVENSON O.: Einfluss von Störschwingungen auf die Ermüdungsfestigkeit, Stahl und Eisen, 1965, N5.
3.GUSHCHA,O., LEBEDEV,V., ROYTSTEIN,M.:Masts with guy-ropes in service, Promyshlennoje Stroitelstvo, 1970, N4.
4.ZAITSEV,G., PARADZHOV,R.M.: Strength of steels used for hydraulic turbines, NIIinformtyazhmash, 1970.
5.BUGLOV,E., FILATOV,M., KOLIKOV,E.: Material resistance at bicyclic loading, Problemi prochnosti, 1973, N5.
6.DVORKIN,Ya.: The effect of bicyclic loading parameters on D16T alloy samples fatigue life, Zavodskaya laboratorija, 1973, N4.
7.BUGLOV,E., KOLIKOV,E., FILATOV,M.: Studies of fatigue at biharmonic loading, Problems of strength, 1970, N1.
8.ZAITSEV,G., ARONSON, A.: Fatigue strength of hydraulic turbines, Mashinostrojenije, 1975.
9.BOGDANOV,B.: Strength and fatigue life of aircraft structures, Kiev, CVF, 1965, p.124-127.
10.STARKEY,W. and MARCO,S.: Effects of complex stress-time cycles on the fatigue properties of metals, Transactions of the ASME, 1957, vol.79, N6, p.1329-1336.
11.REICHER,V. A hypothesis of spectral summation and its application to fatigue strength determination at random loads.-In:Problems of strength in structural mechanics, 1968, p.267-274.
12.AOKI,J., HATSUKO,K., NAKAMURA,H., KUNIO,T.: The study on the fatigue strength by varying the stress amplitude with the superimposed the based sinusoidal stress wave, Transactions of JSME, 1965, vol.31, N222.
13.YAMADA,T., and KITAGAWA;S.:Investigation of fatigue strength of metals under actual service loads (with two superimposed cyclic loadings), Bulletin of JSME, 1967, N38, p.245-252.
14.TRUFIAKOV,V., KOVALCHUK,V.: Change of bicyclic loading fatigue strength, In:Strength of welded joints and structures, 1974.
15.BUGLOV,E.: Low-cyclic fatigue and some properties of structural material hysteresis curve at bicyclic loading, In: Strength of materials and structures, Naukova dumka, 1975.
16.NISHIHARA,T., and YAMADA,T.: Fatigue life of metals under varying repeated stresses, Proceedings of Sixth Japan National Congress for Applied Mechanics, 1956.
17.TRUFIAKOV,V.: Welded joint fatigue, Kiev, Naukova Dumka, 1973.