Fatigue life estimation using fracture mechanics

Fatigue Life Estimation Using Fracture Mechanics

Estimation de la durée de vie à l'aide de la mécanique de la rupture

Lebensdauerberechnung mit den Methoden der Bruchmechanik

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SUMMARY
The main portion of the fatigue life of welded details with weld defects or high stress concentrations is taken up by fatigue crack propagation. It can be conveniently analyzed using fracture mechanics. Fatigue crack propagation behaviour of tensile plates with fillet welded attachments (gusset specimens) is investigated experimentally and analytically. The results are extended to estimate the fatigue crack propagation life of beams with attachments welded to the flange tip or the web. The analytical results are generally in good agreement with test results.

RESUME
La plus grande partie de la durée de vie est absorbée par le temps nécessaire à la propagation des fissures de fatigue dans les détails constructifs soudés comportant des défauts de soudure ou des concentrations de contraintes importantes. Cette propagation des fissures peut être décrite analytiquement par les méthodes de la mécanique de la rupture. Cette analyse a été appliquée à l'étude du comportement à la fatigue d'éprouvettes avec goussets soudés et vérifiés expérimentalement. Les résultats ont ensuite été étendus aux poutres munies de goussets soudés à l'aile ou à l'âme. Les résultats de l'analyse théorique correspondent généralement bien aux résultats d'essais.

ZUSAMMENFASSUNG
Die Lebensdauer von geschweissten Konstruktionen, die Schweissfehler oder hohe Spannungskonzentrationen enthalten, wird im wesentlichen durch das Wachstum der Ermüdungsrisse bestimmt; dieses kann mit den Methoden der Bruchmechanik analytisch beschrieben werden. Das Ermüdungsverhalten von Zugproben mit aufgeschweissten Knotenblechen wird experimentell und analytisch untersucht. Die Resultate werden auf Träger übertragen, welche mit am Flansch oder am Steg angeschweissten Knotenblechen versehen sind. Die rechnerischen Werte stimmen gut mit den Versuchsresultaten überein.

1. INTRODUCTION

The fatigue life of structural details containing weld defects or high stress concentration can be analyzed using fracture mechanics methods [1] [2]. In order to use this extensively, knowledge of the fatigue crack behavior at an early stage of fatigue crack propagation is important. In fact, during this stage a large portion of the fatigue life elapses.

The fatigue crack propagation behavior in the early stages of fatigue is especially monitored by a dye-marking technique on specimens with longitudinal attachments. The test results are used to verify the analytical procedure. Based on these experiences, the fatigue crack propagation life of attachments (gussets) welded to the flange tip or to the web of beams are computed and compared with test results.

2. FATIGUE CRACK PROPAGATION ANALYSIS USING FRACTURE MECHANICS

The fracture mechanics analysis of the fatigue crack propagation consists of the two basic equations (1) and (2). The fatigue crack growth rate, da/dN in mm/cycle, can be empirically expressed as a function of the stress intensity factor range, ΔK in $N/mm^2 \cdot \sqrt{mm}$, as follows :

$$\frac{da}{dN} = C\,\Delta K^m \,, \tag{1}$$

where C and m are material dependent constants. Throughout the numerical analyses described later in this paper, fixed values of $C = 1.52 \cdot 10^{-13}$ and $m = 3$ are selected ($C = 48 \cdot 10^{-13}$ for m/cycle and $MPa \cdot \sqrt{m}$).

The stress intensity factor range is expressed as follows :

$$\Delta K = F(a)\,\Delta\sigma\,\sqrt{\pi a} \,, \tag{2}$$

where $\Delta\sigma$ is the nominal stress range, a is the governing crack size, and F(a) is a correction factor. For a crack emanating from a fillet weld toe, F(a) can be conveniently expressed by the product of different correction factors [3] :

$$F(a) = F_S\,F_E\,F_G\,F_W \,, \tag{3}$$

where F_S accounts for the effect of the free front surface, F_E for the elliptical crack shape, F_W for finite plate width (free back surface), and F_G for non-uniform stresses due to the stress concentration effect. Obviously, Eq. (3) contains some approximation due to interaction effects between the individual correction factors which are not considered when used in the combination. This expression is, however, very useful in practice, because it is simple and easy to use.

The fatigue crack propagation life N_p can be computed by rewriting Eq. (1) and integrating between two crack sizes :

$$N_p = \frac{1}{C}\int_{a_i}^{a_f} \frac{1}{\Delta K^m}\,da \,. \tag{4}$$

N_p is the number of cycles necessary to propagate a crack from a given (initial) crack size, a_i, to a final crack size, a_f, under constant stress range. Substituting Eq. (2) into Eq. (4), N_p can be obtained by numerical integration, for example using Simpson's rule.

It has been reported that, below a certain threshold value, ΔK_{th}, existing fatigue cracks do not propagate [2]. Substituting the ΔK_{th} values into ΔK of Eq. (2), one can estimate the fatigue limit for a given initial crack size. This gives a threshold value of the stress range below which existing initial cracks of a given size do not propagate.

3. FATIGUE TESTS OF TENSILE SPECIMENS WITH GUSSET PLATES

A tensile plate with two longitudinally welded gusset attachments, as shown in Figure 1 is often used for fatigue tests of attachments. Fatigue cracks always initiate and propagate from the fillet weld toe at the end of the attachment. In order to verify the fatigue crack propagation behavior at an early stage of fatigue, dye-penetrant was applied to the fillet weld toe of the gusset specimens during fatigue tests [4]. The dye left clear markings of the geometry of the crack on the fatigue fracture surface. These dye-marked cracks were always of semi-elliptical shape, and were present at as early as 20 percent of the total fatigue life of the specimen, N, as shown in Figure 1.

The depths of these early cracks were 0.27 to 0.42 mm. In other words, at least 80 percent of the total fatigue life consists of fatigue crack propagation. The stress intensity factor range of these semi-elliptical surface cracks, emanating from the weld toe at the gusset end, can be conveniently expressed by Eq. (3). The correction factors used are as follows :

$$F_S = 1.12 - 0.12 \frac{a}{b}, \tag{5}$$

$$F_E = \frac{1}{E_k} = \frac{1}{\int_0^{\pi/2} (1 - k^2 \sin^2 \varphi)^{1/2} \, d\varphi}. \tag{6}$$

E_k is the complete elliptical integral of the second kind, where $k^2 = 1 - a^2/b^2$, and the crack shape is expressed by its depth, a, and width, 2 b. Normally, a wide variation of the crack shape was observed at the weld toe due to the weld toe profile, weld toe irregularity, and coalescence of multiple minute cracks.

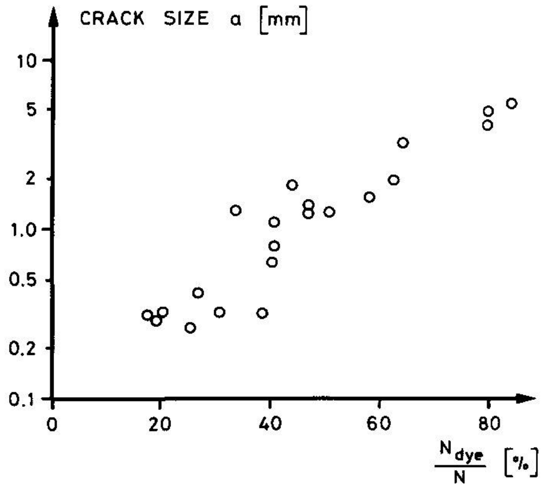

Figure 1 : Dye-marked crack depth at a given percentage of the total fatigue life.

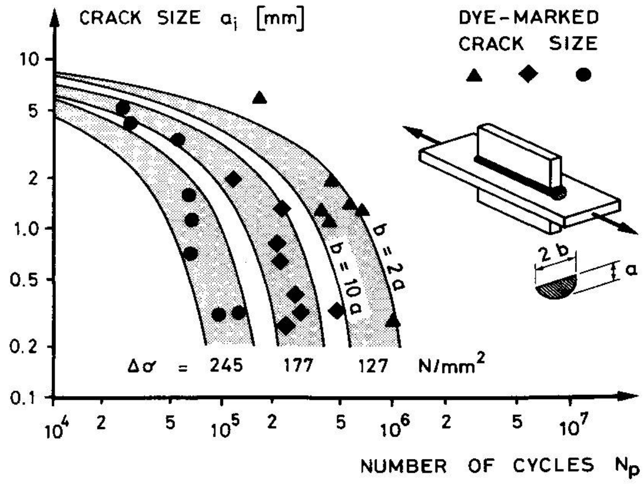

Figure 2 : Computed crack propagation life, compared with test data of gusset specimens.

The inclusion of all these factors in the analysis would yield considerable and unnecessary complexity. Therefore, only an upper and a lower bound of the crack geometry aspect ratio, a/b, are selected to represent all crack shapes [4]. The upper bound is selected as b = 2 a; the lower bound is expressed by b = 10 a, if $a \leq 1$ mm, and $b = 10\ a^{0.3}$, if $a > 1$ mm. This crack shape is substituted into Eq. (6). The correction factor for a finite plate width (with free back surface), is defined as follows :

$$F_W = \sqrt{\frac{2\,t}{\pi\,a} \tan\left(\frac{\pi\,a}{2\,t}\right)}, \tag{7}$$

where t is the plate thickness.

The geometry correction factor F_G is determined from the stress distribution normal to the line where the fatigue crack is to propagate [3]. At the attachment end, the stress is raised once due to the fillet weld and again by the presence of the gusset plate. The stress concentration due to the fillet weld was computed by finite element analysis [5]. The stress concentration due to the gusset plates is estimated from stress measurements using strain gages mounted 3 mm away from the fillet weld toe. This stress concentration is about 1.5.

The computed fatigue crack propagation life N_p, of the gusset specimens, is plotted against the initial crack size a_i, as shown in Figure 2. The three stress ranges indicated correspond to the applied stress ranges used in the fatigue tests. The shape of the semi-elliptical surface crack is one of the dominating parameters affecting the computed N_p. The two bounds of the observed crack shapes result in a band of computed N_p for each stress range. The observed dye-marked crack depth and the corresponding number of cycles that are needed to propagate the crack from this size to failure are also plotted in Figure 2 for all test specimens. These results are directly comparable to the computed N_p. In spite of the fact that a simple expression of ΔK was used for the analysis, the computed results are in good agreement with the test results.

When a given initial crack size is selected, the computed N_p can then be expressed in the same way as a regular S-N diagram. The computed N_p is plotted

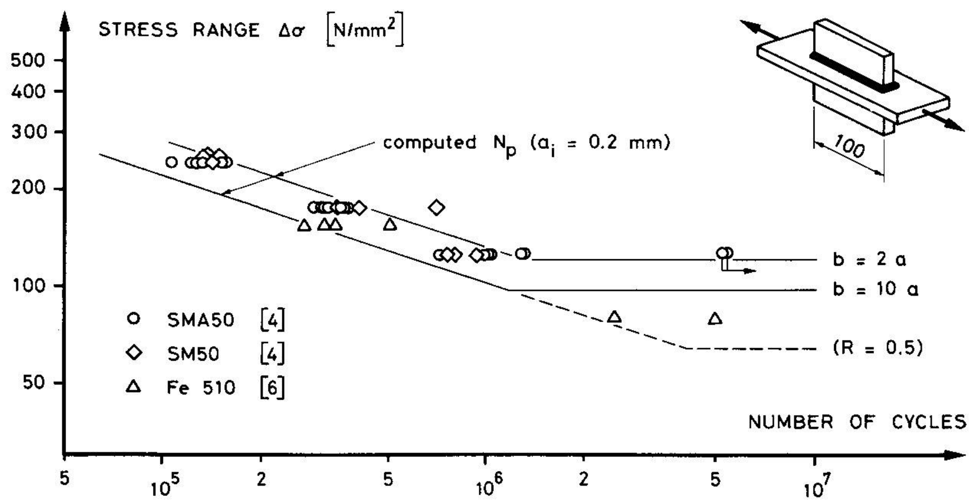

Figure 3 : Comparison between fatigue test results and the computed crack propagation life N_p for a_i = 0.2 mm.

and compared with the fatigue test data, as shown in Figure 3. Assuming the initial crack size to be a_i = 0.2 mm, almost the entire fatigue life is spent in propagating the cracks. As shown before in Figure 1, fatigue cracks of 0.27 to 0.42 mm deep already existed at the fillet weld toe at about 20 percent of the life.

The theoretical fatigue limits, computed with ΔK_{th} = 190 N/mm^2 • $\sqrt{mm}$ (R = 0.1), for the two limiting crack shapes, are also shown in Figure 3. In fact, two specimens tested at $\Delta\sigma$ = 127 N/mm^2 had not failed after 5 million stress cycles. A favorable fillet weld toe profile may reduce the initial stress condition and hence increase the fatigue limit of the specimen. Nine other specimens failed between 7 • 10^5 cycles and 2 million cycles. Note that two specimens tested at 80 N/mm^2 also showed finite lives [6].

If high tensile residual stresses exist at the fillet weld toe, a higher stress ratio exists which consequently reduces the ΔK_{th} value [2]. Considering this effect and therefore introducing ΔK_{th} = 125 N/mm^2 • $\sqrt{mm}$ (R = 0.5), the fatigue limit computed for the crack shape of b = 10 a is then below the stress range at which the two specimens were tested.

4. FATIGUE ANALYSIS OF GUSSET PLATES WELDED TO THE FLANGE TIP

4.1. Rectangular gusset plates

When an attachment is welded directly to the flange tip of a beam, a part of the stress is transferred to the attachment, which causes high stress concentration at the weld toe at the attachment end. Therefore, the crack normally initiates at the weld toe and propagates perpendicular to the principal stress. The fatigue crack propagation behavior may be modelled in three ways, depending on the exact location of the crack initiation, as shown in Figure 4.

For the edge crack, F_S = 1.12 is used. In the second model, F_S = 1.38 and F_E = $2/\pi$ are used for a quarter circular corner crack until it reaches the plate thickness of the flange. Then the crack propagates as an edge crack. The computed propagation life is the sum of these two steps. In the third model for a semi-elliptical

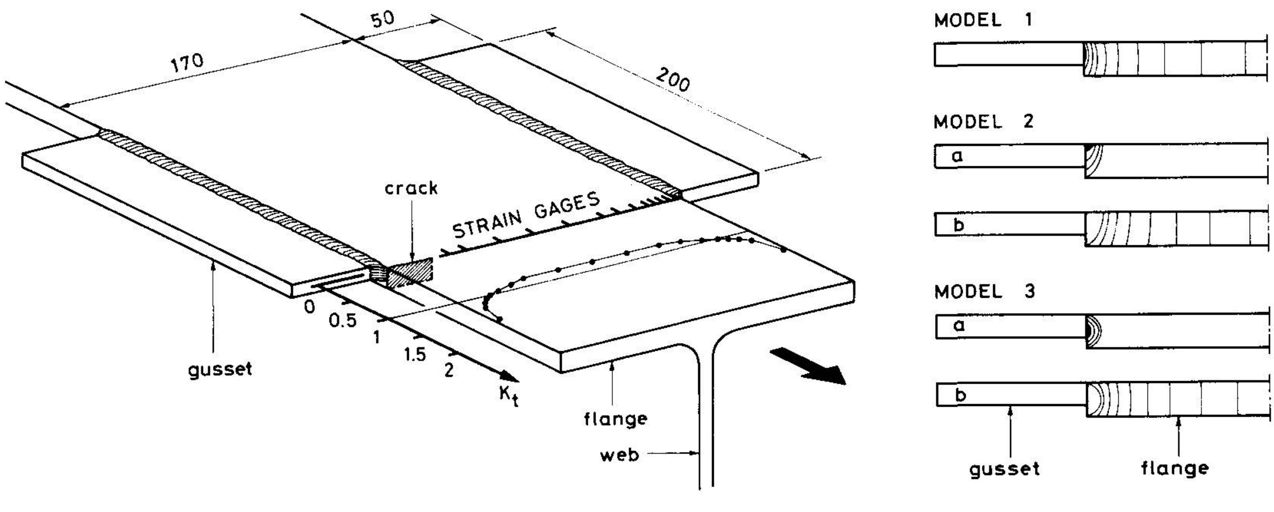

Figure 4 : Stress concentration measurement and models of fatigue cracks emanating from the weld toe at the end of gusset plates welded to the flange tip.

surface crack, F_S = 1.12 - 0.12 a/b and F_E = $1/E_k$. Again the second stage of growth is that of an edge crack. F_W is assumed to be unity for all models since the crack size is generally small compared to the width of the flange. Thus, all correction factors are defined, except F_G.

It is proposed to divide F_G further into F_{G1} and F_{G2}. F_{G1} accounts for the local stress concentration due to the fillet weld at the end of the gusset, and F_{G2} for the global stress concentration due to the gusset. In the following analysis, the F_G value for a transverse fillet weld computed for a stiffener specimen [5] is used to define F_{G1}, provided that the fillet weld profiles are the same. The F_{G2} value is computed from the stress distribution measured by strain gages mounted across the flange plate, as shown in Figure 4. Note that a stress concentration of about 1.5 was measured, 4 mm away from the fillet weld toe. Since the stress concentration effect of the fillet weld is very localized, this value measured by the strain gages is mainly due to the gusset. The local effect due to F_{G1} is large, but decays quickly to unity after about 3 mm crack size.

The fatigue crack propagation life for a crack with inital size a_i = 0.2 mm, growing to a size a_f = 50 mm, is computed and plotted in Figure 5. Usually, a large difference in the computed N_p is expected depending on the assumed crack shape, as was previously shown in Figure 3. However, for the gusset plate attached to the flange tip, only the first stage of fatigue crack propagation is affected by the crack shape. It results in a relatively small difference of the computed N_p for the different crack shape models. The fatigue test data for 200 mm long gussets welded to the flange tip [7] are in good agreement with the test results.

4.2. Fatigue strength improvement by grinding the weld toes

One way to improve the fatigue life of a beam with gussets welded to the flange tip is to reduce the stress concentration at the end of the gusset. This can be achieved by grinding the fillet weld. A further reduction of the stress concentration can be expected when a smooth transition from the gusset to the flange is created. If the weld toe is ground completely smooth, the correction factor F_{G1} can be set equal to unity. Assuming all other factors to be the same as those previously used to compute N_p in the as-welded condition, the propagation life for the gusset with ground weld toes can then be computed; the results are plotted in Figure 6. More than a 50 percent improvement of the fatigue life can be expected by grinding completely the weld toe. The fatigue limit computed with ΔK_{th} = 190 $N/mm^2 \cdot \sqrt{mm}$ (R = 0.1) is almost doubled.

Fatigue test data of beams with welded gussets of various end shapes [7] indicate a significant improvement of fatigue life, compared with the as-welded gusset end. The fatigue tests were carried out for three radii of circular transition from flange to gusset : r = 10, 50 and 70 mm. Note that a large circular transition radius does not consistently improve fatigue life. This may be attributed to the imperfect grinding especially at the fillet weld toe.

The analysis shows that grinding the gusset, so as to obtain a smooth transition from gusset to flange plate, significantly improves the fatigue life of this detail. It should be noted that care must be taken when grinding the weld toe. Small portions of the weld toe missed by the grinding operation retain their local stress concentration. Moreover, grinding marks left on the ground surface, perpendicular to the stress direction, may well be the initiation point of fatigue cracks. In addition, the fatigue tests were carried out at stress ranges close to the computed fatigue limit. When this is the case, the fatigue life

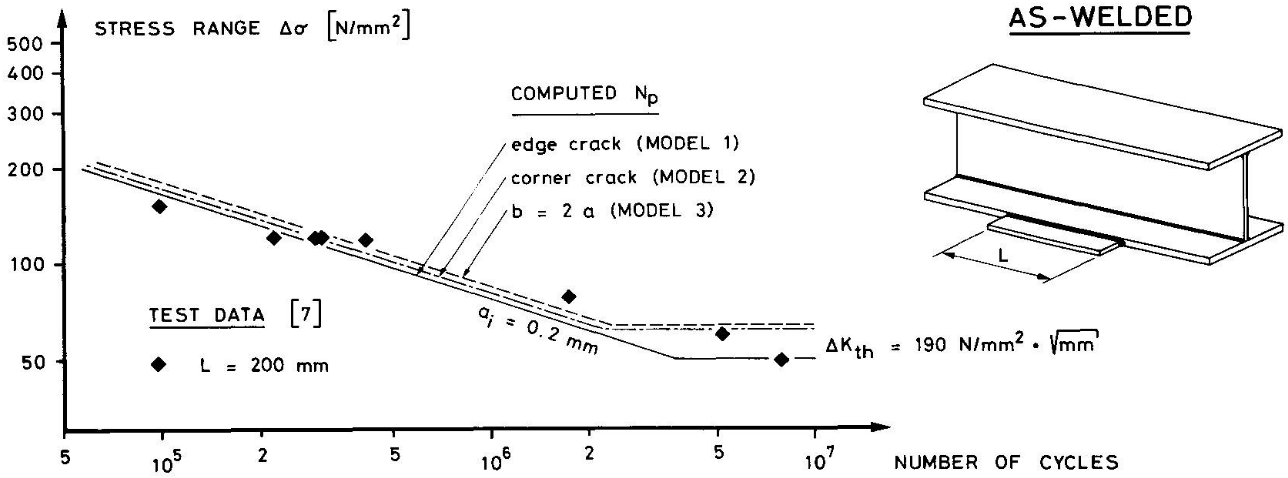

Figure 5 : *Comparison between crack propagation life and observed fatigue life for rectangular gussets attached to the flange tip (as-welded).*

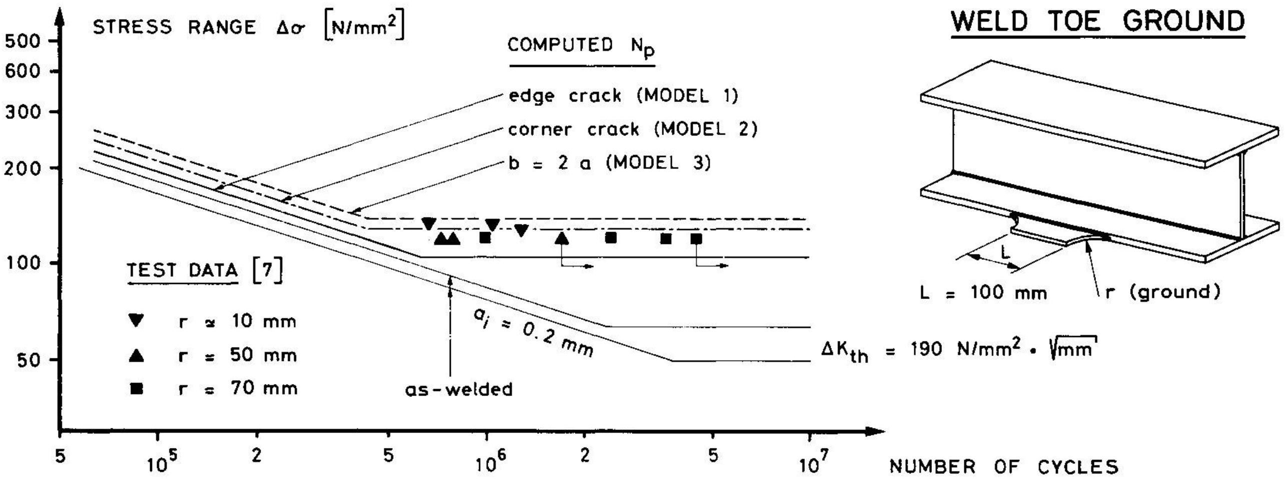

Figure 6 : *Computed crack propagation life for gussets with various end radii, and comparison with test results.*

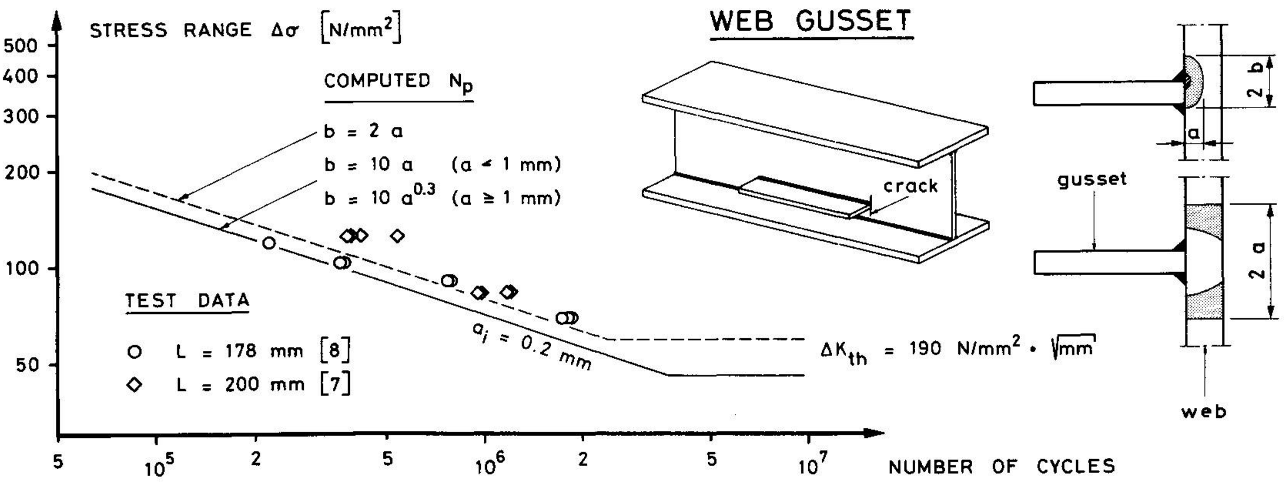

Figure 7 : *Comparison of analytical prediction with test results for gussets attached to the web.*

depends very much on the surface condition. In other words, a small imperfection may cause fatigue crack initiation, whilst a perfectly smooth surface gives longer life or even run-out. In fact, a large scatter of the fatigue data is observed in these types of tests.

5. FATIGUE ANALYSIS OF GUSSET PLATES WELDED TO THE WEB

When a gusset plate is horizontally attached to a beam web, fatigue cracks initiate and propagate from the toe of the fillet weld, at the end of the attachment [7] [8]. The fatigue crack propagation behavior is similar to that observed for the gussets welded to the tensile specimens previously described. Once the crack has completely penetrated the web, the crack becomes a through-thickness crack, propagating vertically upwards and downwards in the web, as schematically shown in Figure 7. The first stage of the fatigue crack propagation behavior can be modelled in the same manner as for the gusset specimens. For the geometry correction factor, however, two factors, F_{G1} and F_{G2}, are used in the same manner as for the gusset welded to the flange tip. Once the crack has completely penetrated the web plate, only F_{G2} is used to account for the global effect due to the gusset.

The computed crack propagation life for an initial crack size of 0.2 mm is plotted in Figure 7; the final crack size is again assumed as a_f = 50 mm. Fatigue test data of the attachments welded to the web are also plotted for comparison [7] [8]. Although a more accurate stress analysis might be needed for this welded detail, the computed crack propagation lives N_p are in relatively good agreement and show the trend of the fatigue test data. What must be done in future fatigue tests of this detail is to check whether the fatigue crack initiates at an early stage. Also, a more precise stress analysis or stress measurements must be carried out.

6. SUMMARY

The fatigue crack propagation behavior of gusset plates welded to tensile specimens is used to demonstrate how fatigue cracks propagate and how fracture mechanics analysis can be used to predict the crack propagation behavior. The results are then applied to similar structural details, such as attachments welded to the flange or web of beams. The computed fatigue propagation life of these structural details is generally in good agreement with test data. It is speculated that the majority of the fatigue life of these details is spent in propagation of cracks from minute sizes, for example a_i = 0.2 mm, and that the crack initiation period is very short or even non-existant.

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