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Analysis and Design of Plane Steel Structures

Conception basée sur la sécurité appliquée aux structures planes en acier

Berechnung und Entwurf ebener Stahlkonstruktionen

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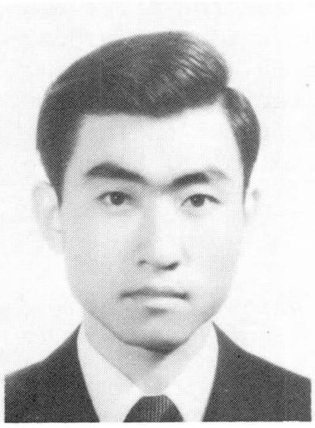

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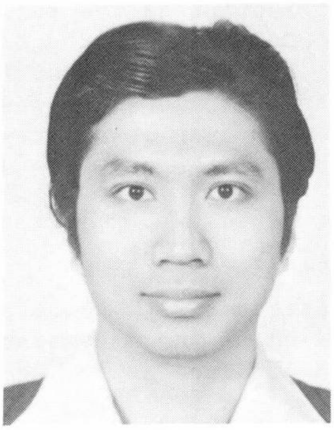

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SUMMARY
A computer program which can be used to find the probability of failure of the structure under consideration has been set up. The weakest link model is assumed and is applied to either statically determinate or statically indeterminate structures for the failure stage. Lognormally distributed loadings consisting of dead load, live load and impact effects are assumed. A series expansion technique is used in order to avoid the table checking work. The probability of failure of each member as well as the whole structure can be found.

RESUME
L'article traite d'un programme destiné au calcul de la probabilité d'observer une défaillance au niveau d'une structure. Le principe de l'étude du maillon le plus faible de la chaîne est appliqué aux structures statiquement déterminées ou indéterminées, au stade de la ruine. Les charges distribuées selon une loi lognormale sont: le poids propre, les surcharges et l'effet d'impacts divers. Une technique d'expansion de séries est utilisée afin d'éviter le parcours de table de vérification. Les probabilités de ruine de chaque élément ainsi que de l'ensemble de la construction sont déterminées.

ZUSAMMENFASSUNG
Ein Computerprogramm wird vorgestellt, das die Versagenswahrscheinlichkeit eines Bauwerkes berechnet. Dabei wird das Modell des schwächsten Gliedes in der Kette sowohl für statisch bestimmte als auch für statisch unbestimmte Bauwerke für den Zustand des Versagens angewendet. Lognormal verteilte Last, die aus Eigengewicht, Nutzlast und Aufpralleffekten besteht, wird angenommen. Die Versagenswahrscheinlichkeit kann für jedes einzelne Glied sowie für das ganze Bauwerk gefunden werden.

1. INTRODUCTION:

For a long period, many contributors devote their efforts in the field of optimum design of structures. Criterions they used generally are weighting consideration, i.e., the optimum design of a structure is the design which need minimum weight of the materials while provide equal function of the structure required. Recently, there is another argument, followed by the rapid development of reliability design concept, that the optimum design should be the design which possesses the minimum probability of failure in different designs which have equal functions. Furthermore, the argument states that the best design is a design that individual members of the structure possesses equal probability of failure. So as to give a rational and economical design which assure equal safety of all parts of the structure.

A simple flowchart of the reliability design is shown in Fig.1. From the figure we see that one contribution of the reliability design is to check the different preliminary designs and calculate the probability of failure both for individual elements and for the whole structures. It is obviously that there are so many choice when we are in the process of design. "Which one is better and which one is the best?" is the question we are trying to answer. Probability of failure calculated iteratively in the computer should be a method we can follow to solve the above question.

2. APPLICATION OF SAFETY MARGINS:

Fig.2 shows a prismatic bar with resistance R and subject a tension force S. R and S are considered to be random variable. Thus the probability of failure of the member can be obtained as

$$P_f = P\ (\ M.S. \leqq 0\ ) = \int_o^{\infty} f_S(s)\ F_R(s)\ ds \qquad (2\text{-}1)$$

or

$$p_f = \int_o^{\infty} (\ 1 - F_S(r))\ f_R(r)\ dr \qquad (2\text{-}2)$$

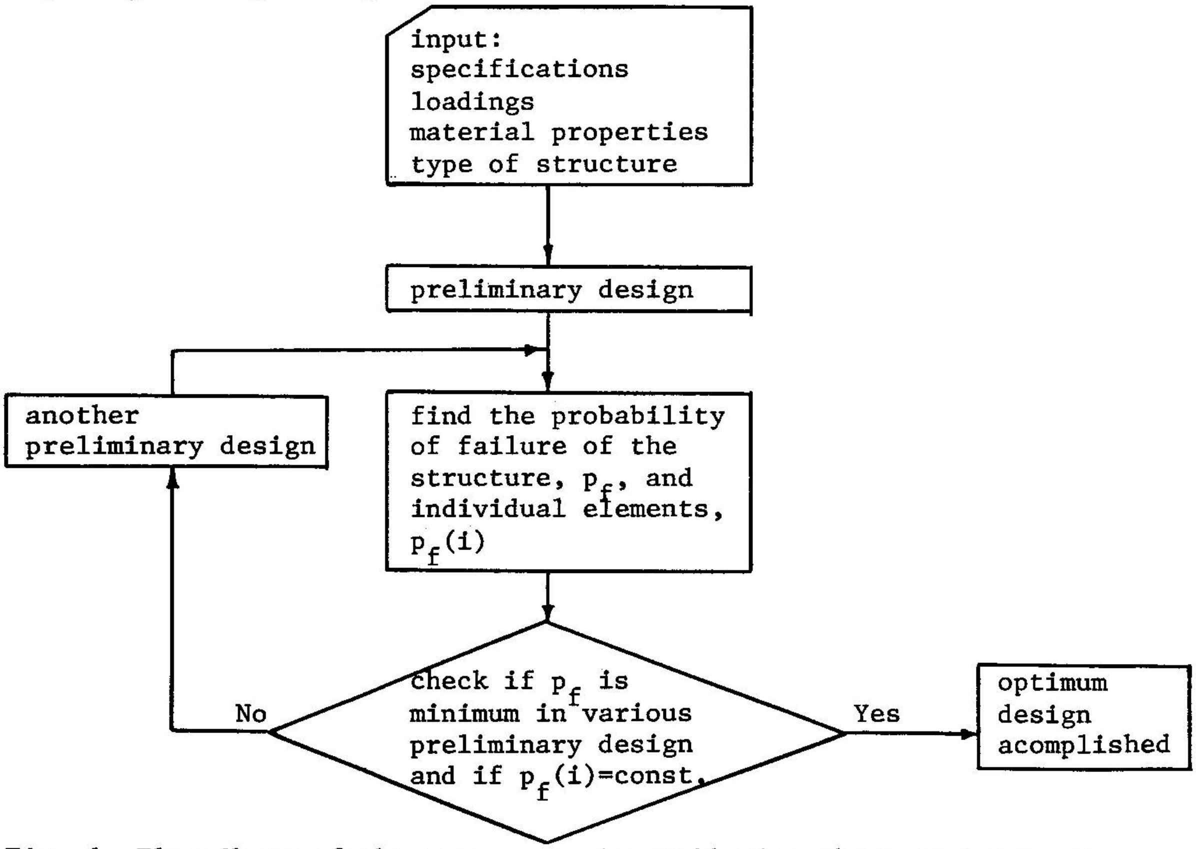


Fig. 1 Flow Chart of the Optimum Design Following the Reliability Concept

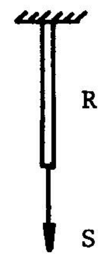


Fig. 2 Prismatic Member with Resistance R and Load S.

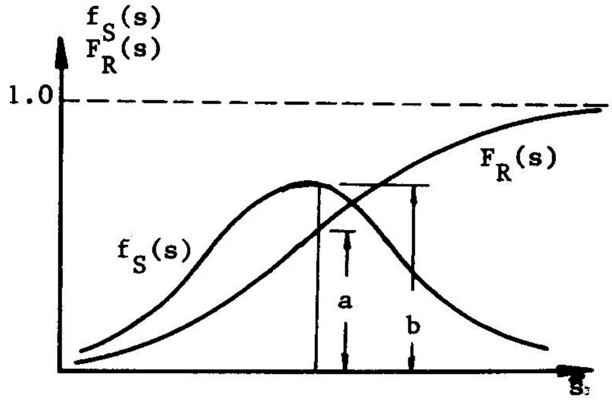


Fig. 3 Relationship Between Load and Resistance

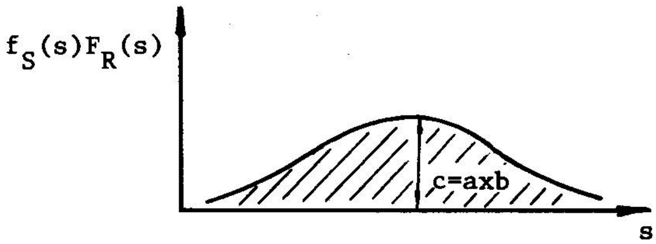


Fig. 4 Probability of Failure with R and S in Fig. 3

where P(.) means the probability of .; f_S and f_R are the probability density function of S and R, respectively; F_S and F_R are the cumulative distribution function of S and R, respectively. The relationship of S and R and the meaning of probability of failure are showed in the Fig.3 and Fig.4.

In case of S and R are all normally distributed, p_f can be obtained by substituting $\mu_{M.S.}$ (mean value of the safety margin) and $\sigma^2_{M.S.}$ (Variance of the safety margin) into the normal function

$$p_f = P\ (\ M.S. \leqq 0\ ) = P(\ R - S \leqq 0\ ) = \Phi\left(\frac{0-\mu_{M.S.}}{\sigma_{M.S.}}\right)$$

$$= 1 - \Phi\left(\frac{\mu_{M.S.}}{\sigma_{M.S.}}\right) = 1 - \Phi\left(\frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) \qquad (2-3)$$

where μ_X and σ_X^2 are the mean value and the variance of the random variable X, respectively. If loading are the combination of the dead load(S_D) and live load(S_L), and they are assumed to be statistically independent random variables, then we have

$$\sigma_S^2 = Var(S) = Var(\ S_D + S_L\ ) = Var(\ S_D\ ) + Var(S_L) + 2Cov(S_D,\ S_L)$$

$$= \sigma^2_{SD} + \sigma^2_{SL}$$

Thus equ(2-3) will be

$$p_f = 1 - \Phi(\ \mu_{M.S.} / \sigma_{M.S.}\ )$$

$$= 1 - \Phi\left((\ \mu_R - \mu_S\ ) / \sqrt{\sigma_R^2 + \sigma_{SD}^2 + \sigma_{SL}^2}\right) \qquad (2-4)$$

or

$$p_f = 1 - \Phi(\beta) \qquad (2-5)$$

where $\beta = \dfrac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_{SD}^2 + \sigma_{SL}^2}}$ is named as safety index by Ang(1).

More rational expression is considered that R and S are following lognormal distribution. In this case, safety margin is R/S instead of R - S in the equation (2-3) and

$$p_f = 1 - \Phi\left(\frac{\ln\left(\frac{\mu_R}{\mu_S}\right)\left(\frac{1+V_S^2}{1+V_R^2}\right)^{1/2}}{\sqrt{\ln(1+V_R^2)\,(1+V_S^2)}}\right) \qquad (2-6)$$

Value of $\Phi(.)$ can be obtained from the normal distribution table. $\Phi(.)$ is expanded into series herein thus the value of $\Phi(.)$ can be easily obtained in computer.

3. WEAKEST LINK MODEL:

For statically determinate structure, failure of any member will cause the failure of the whole structure. For example, collapse of a statically determinate truss occurs simultaneously when any member in the truss failed. The terminology "failure" herein means totally plastic reached for a section, thus the member will deform continuously without any increasing of load. The probability of failure of this type of structure can be obtained by using weakest link model. And the formula of the weakest link model is

$$p_f = \int_0^\infty F_R(x)\, f_S(x)dx = \int_0^\infty 1 - \prod_{i=1}^{n} [1 - F_{R,i}(x)dx]\;\; f_S(x)\, dx$$

$$= 1 - \prod_{i=1}^{n} (1 - p_f(i)) \qquad (3-1)$$

For statically indeterminate structure, there are many modes and the failure paths depends on the degree of indetermenacy. Yao and Yeh(2) shows even for satically indeterminate structure, weakest link model can still be properly adopt and leads very small error. So equation (3-1) is used herein for calculating the probability of failure both for statically determinate and statically indeterminate structures.

4. FAILURE CRITERION:

Galambos and Rarindra(3,4) suggest limit equations of bar element in plane structures are

$$1.\ \frac{R}{Ru} + \frac{C_m M_o}{M_p(1-(R/R_E))} \leqq 1.0 \qquad (4-1)$$

$$2.\ \frac{Mo}{1.18M_p} + \frac{R}{Ry} \leqq 1.0 \qquad (4-2)$$

$$3.\ \frac{R}{R_u} \leqq 1.0 \qquad (4-3)$$

where

$$R_u = \begin{cases} AF_y\,(1 - 0.25\,\lambda^2) & \text{when } \lambda \leqq 2 \\ AF_y\,/\,\lambda^2 & \lambda \geqq 2 \end{cases}$$

$$R_E = \frac{A\pi^2 E}{(\frac{KL}{r})^2}$$

$$\lambda = \frac{KL}{r}\left(\frac{1}{\pi}\right)\sqrt{F_y / E}$$

$$M_p = ZF_y$$

$$C_m = 0.4 + 0.6\, n$$

M_1, M_2 are member end moments.

M_o = the larger one of M_1 and M_2

$$n = \text{moment ratio} = \frac{M_o}{\text{moment at the other end}}$$

$$R_y = AF_y$$

5. DETERMINATION OF THE RESISTANCE FACTOR, ϕ:

There are certain relationship between the expected resistance R_m and nominal resistance R_n. For instance, for A36 steel, its nominal resistance is 36 ksi. Generally, the expected resistance is greater than the nominal resistance. Galambos and Raindra(3,4) suggest their relation be

$$R_m = \left(\frac{\text{Test strength}}{\text{Prediction by theory}}\right)_m$$

$$\times \left(\frac{\text{Prediction by theory}}{\text{Prediction by interaction equation}}\right)_m$$

$$\times \left(\frac{\text{Prediction by interaction equation}}{R_n}\right)_m \times R_n$$

$$= B_{EX}\, B_{TH}\, B_{MAT}\, R_n \qquad (5-1)$$

where the interaction equation are the governing equations (4-1), (4-2), and (4-3). B_{EX}, B_{TH}, B_{MAT} represent the "bias" of "experiment", "theory", and "material", respectively. Galambos suggests that the mean value of B_{EX}, B_{TH}, B_{MAT} be 1.005, 1.01, 1.05, respectively, and the coefficient of variation of B_{EX}, B_{TH}, B_{MAT} be 0.093, 0.04, 0.10, respectively. R_n in the equation (5-1) can be solved from equations (4-1), (4-2), and (4-3).

The criteria of L.R.F.D. (Load and Resistance Factor Design) is

$$\phi R_n = \sum_{k=1}^{n} r_k\, S_{nk} \qquad (5-2)$$

when dead load and live load are considered individually, equation (5-2) will be

$$\phi R_n = \sum_{k=1}^{2} r_k\, S_{nk} = r_E\,(r_D S_{DL} + r_L S_{LL}) \qquad (5-3)$$

where ϕ is the resistance factor, r_D, r_L, r_E are load factors related to dead load, live load and uncertainties including impact effect. Galambos (4) suggests the resistance factor ϕ can be determined by

$$\phi = \left(\frac{R_m}{R_n}\right) \exp\left(-\alpha\beta V_R\right) \qquad (5-4)$$

where

$$V_R = \sqrt{V_{EX}^2 + V_{TH}^2 + V_{MAT}^2 + V_F^2}$$

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_{SD}^2 + \sigma_{SL}^2}}$$

$$\alpha = \frac{\sqrt{\sigma_{SD}^2 + \sigma_{SL}^2}}{\sigma_{SD} + \sigma_{SL}}$$

V_F = Variance of other factor with out considered clearly

6. SERIES EXPANSION OF $\Phi(.)$:

For a normal distribution random variable X, the cumulative distribution function of X can be written as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}\xi^2\right) d\xi \qquad (6-1)$$

Equation (6-1) can be expanded by Taylor's series as

1. when $\beta \geqq 4$:

$$p_f = 1 - \Phi(\beta) = \frac{1}{2} - \frac{1}{\sqrt{\pi}}\left(\frac{\beta}{\sqrt{2}} - \frac{\beta^3}{1!\cdot 3\cdot(\sqrt{2})^3} + \frac{\beta^5}{2!\cdot 5\cdot(\sqrt{2})^5} - \frac{\beta^7}{3!\cdot 7\cdot(\sqrt{2})^7} + \ldots\ldots\right) \qquad (6-2)$$

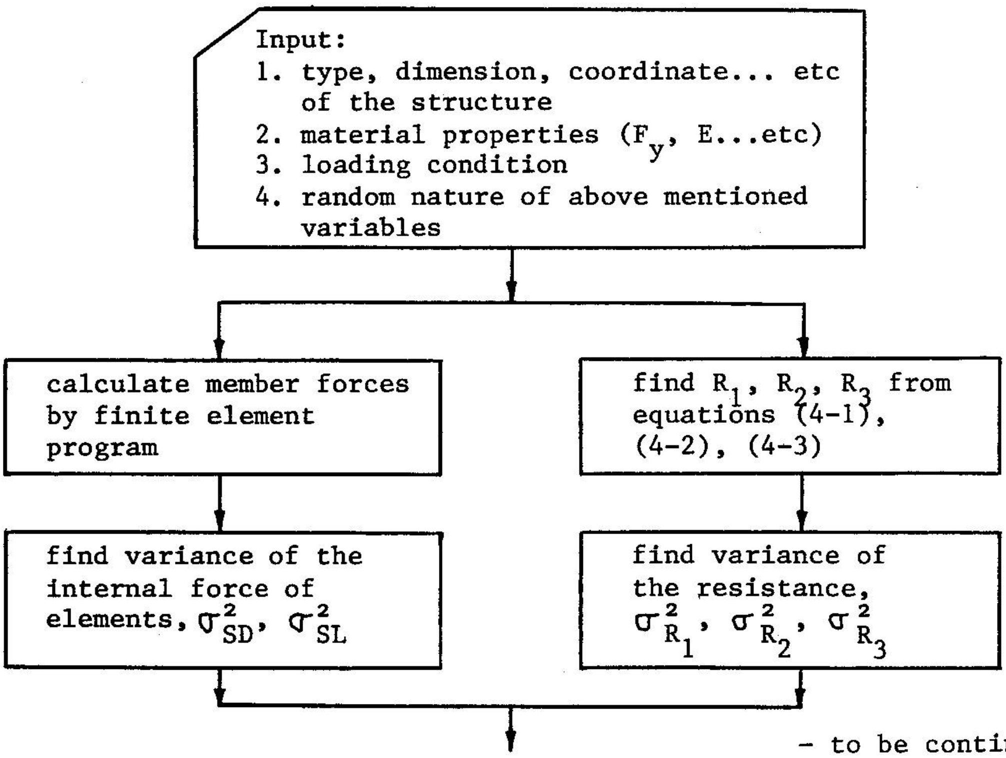
2. when $\beta > 4$:

$$p_f = 1 - \Phi(\beta)$$

$$= \frac{1}{2}\frac{1}{\sqrt{\pi}}\exp\left(-\left(\frac{\beta}{\sqrt{2}}\right)^2\right)\left(\frac{2}{\beta} - \frac{(\sqrt{2})^3}{2\cdot\beta^3} + \frac{1\cdot 3\cdot(\sqrt{2})^5}{2^2\cdot\beta^5} - \frac{1\cdot 3\cdot 5\cdot(\sqrt{2})^7}{2^3\cdot\beta^7} + \frac{1\cdot 3\cdot 5\cdot 7(\sqrt{2})^9}{2^4\cdot\beta^9} - \frac{1\cdot 3\cdot 5\cdot 7\cdot 9(\sqrt{2})^{11}}{2^5\cdot\beta^{11}} + \frac{1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11\cdot(\sqrt{2})^{13}}{2^6\cdot\beta^{13}} - \frac{1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11\cdot 13\cdot(\sqrt{2})^{15}}{2^7\cdot\beta^{15}} + \ldots\ldots\right) \qquad (6-3)$$

7. FLOW CHART OF THE PROGRAM:

Two flow charts will be introduced following, one is for the calculation of the probability of failure both for individual element and for the whole structure, and the other is for the goal of optimum design. They are shown in Fig.5 and Fig.6.



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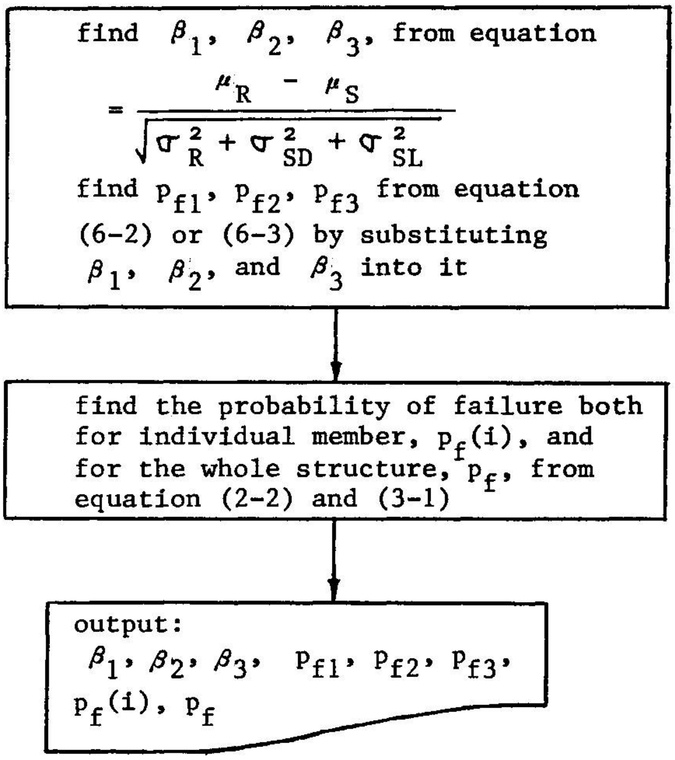

Fig. 5 Flow Chart of the Program for Calculation of the Probability of failure

8. CONCLUSIONS:

The probability of faiture for each member can controlled in expected ranges, say, 10E-6 to 10E-5. It is the goal of an optimum design. Variance of the variables can be properly considered in reliability design. So the reliability design is actually more rational than the conventional design which treat all variables to be constant. Well developed computer program is necessary for future design work in different types of the structures especially in the field of reliability design.

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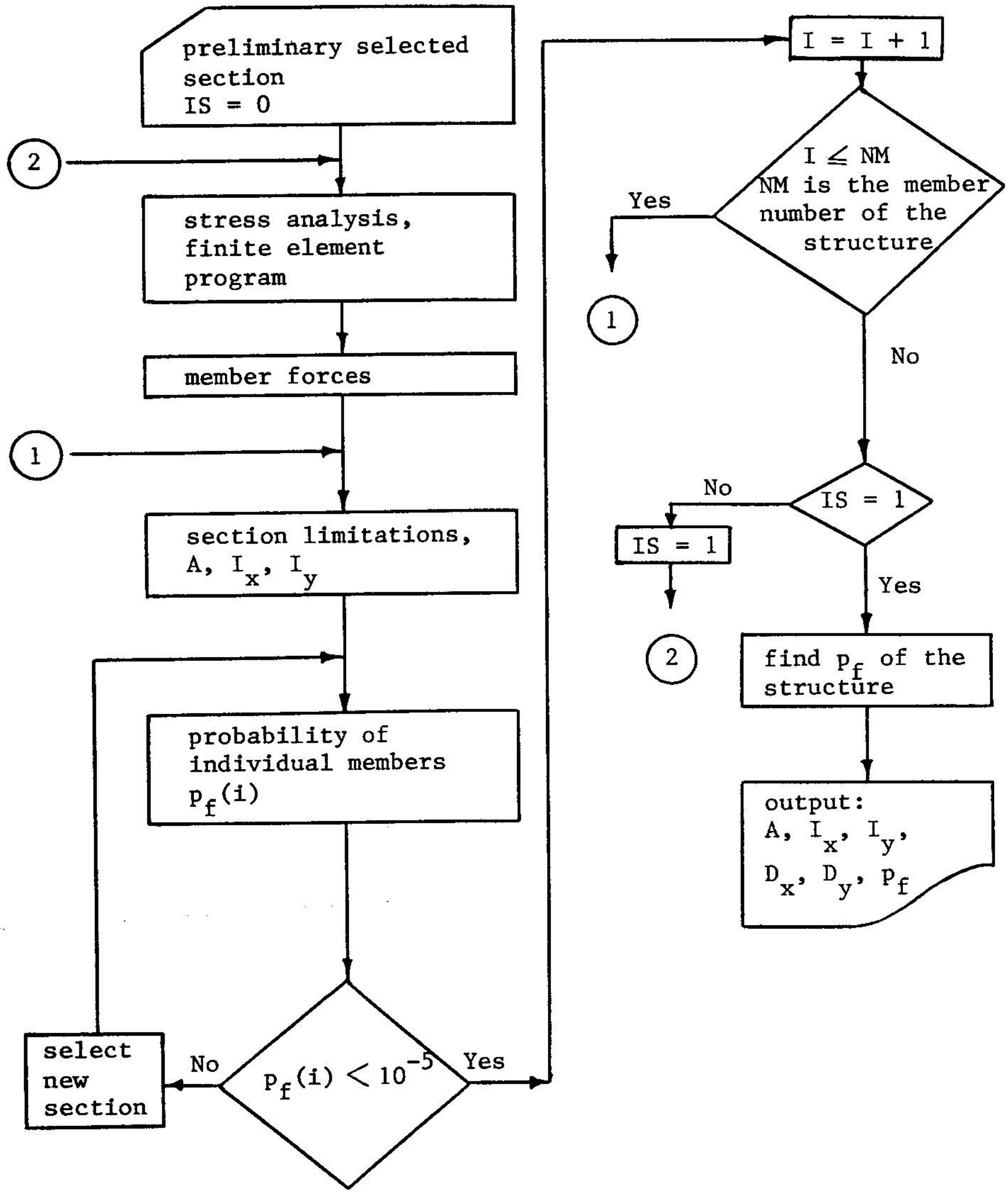

Fig. 6 Flow Chart of the Program for Optimum Design to Select Proper Sections