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# Extreme and First-Passage Time of Ship Collision Loads

## Distribution du risque maximal de collisions de navires

## Verteilung des extremen Risikos bei Schiffzusammenstößen

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**SUMMARY**
The paper outlines a general theory from which the distribution function of the extreme peak collision load encountered during a certain intended lifetime can be calculated assuming the arrival of ship collisions to be specified by a Poisson counting process.

**RÉSUMÉ**
L'article esquisse une théorie générale permettant de déterminer la fonction de distribution du risque maximal de collision dans une période donnée étant supposé que les collisions de navire interviennent selon une distribution de Poisson.

**ZUSAMMENFASSUNG**
Der Artikel stellt eine allgemeine Theorie dar, welche die Verteilungsfunktion eines extremen Kollisionsrisikos während einer bestimmten Zeit voraussieht. Es wird vorausgesetzt, daß das Auftreten einer Schiffskollision nach einer Poissonschen Verteilung geschieht.

## 1. INTRODUCTION

Clearly the impact forces from collision between greater ships and bridge piers must be considered as part of the design basis of the pier [1 - 3]. Collisions are caused by a variety of events including human and mechanical errors [2]. Since almost all decisive parameters such as impact velocity magnitude and direction of wind, waves and current are uncontrolled, stochastic modelling seems to be the only natural way of modelling. This means that the feasibility of the project is decided from the reliability of the system to withstand impact loadings below a certain design load during the intended lifetime of the structure.

In this paper the reliability problem is decoupled into two minor problems concerning the distribution of impact loads on condition that collision does take place, and estimation of the probability rate, at which larger ships collide with the pier.

The distribution function of the extreme peak collision load encountered during a certain intended lifetime can be calculated assuming the arrival of ship collisions to be specified by Poisson counting processes. The equivalent first passage time problem, i.e. the distribution function of the elapsed time until a certain design impact load is exceeded for the first time, is also indicated. The *average* collision force can be estimated with known impact energy by the well-known Minorsky formula [4 - 7]. However, the average impact load is of minor interest because the *peak* value of the impact load can amount to more than double the average value, depending on strength and water filling of the bow construction [3, 8, 9].

At the present state the probability rate at which a ship will encounter the pier can only be estimated by rather costly simulation studies. It is assumed that somewhere before the passage of the bridge a decisive event, such as fixation of the rudder, machine stop, etc., takes place with a known rate of occurrence carrying the ship out of control. For specific samples of position, velocity, course of the ship at the instant of such events, and magnitude and direction of wind, waves and current the corresponding ship path can be obtained by solving the manoeuvring differential equations. The conditional collision rate can then be estimated from the relative number of realizations at which the ship will encounter the pier. It is essential to the cost of the calculation that the number of independent stochastic variables can be reduced. The socalled »rosette method» developed by the first author (P. Thoft-Christensen) is effective in this respect, because it handles the influence from position and direction relative to the pier independently of the other stochastic variables [10]. The applicability of the »rosette method» is demonstrated by a numerical example assuming that the fatal event originates from locking of the rudder.

## 2. EXTREME PEAK COLLISION LOADS

It is assumed that the relevant ships can be grouped into M classes of equal properties according to the parameters of importance to the peak impact collision (dead weight, impact velocity, bow construction, etc.). Further, the number of ship collisions against a specific pier from ships of class $i \in \{1, \ldots, M\}$ during the time interval $]0, t[$ is specified by the counting process $\{N_i(\tau), \tau \in ]0, t]\}$ (see [11]). Let $P_{i,j}, j \in \{0, \ldots, N_i(t)\}$, $P_{i,0} = 0$, signify the $j^{th}$ load from ships in class i. All $P_{i,j}$ are assumed to be identically distributed as $P_i$ with the distribution function $F_{P_i}$.

The distribution function $F_{P_{max}}(\cdot, t)$ of the extreme impact force $P_{max}$ in $]0, t]$ among impact forces from all considered M classes can then be derived under the following assumptions:

- The collision loads are mutually independent stochastic variables.
- The counting processes are Poisson processes with the intensities $\nu_i: ]0, t] \curvearrowright R_0$ and are mutually independent.

One gets

$$F_{P_{max}}(p, t) = P(P_{max} \leqslant p) = \sum_{n_1=0}^{\infty} \cdots \sum_{n_M=0}^{\infty} P(P_{1,0} \leqslant p \wedge P_{1,1} \leqslant p \wedge \ldots \wedge P_{1,n_1} \leqslant p$$

$$\wedge \ldots \wedge P_{M,0} \leqslant p \wedge P_{M,1} \leqslant p \wedge \ldots \wedge P_{M,n_M} \leqslant p \mid N_1(t) = n_1 \wedge \ldots \wedge N_M(t) = n_M) \times$$

$$P(N_1(t)=n_1 \wedge \ldots \wedge N_M(t)=n_M) = \sum_{n_1=0}^{\infty} F_{P_1}^{n_1}(p)P(N_1(t)=n_1) \times \ldots \times \sum_{n_M=0}^{\infty} F_{P_M}^{n_M}(p)P(N_M(t)=n_M)$$

$$= \prod_{i=1}^{M} \left( \sum_{n_i=0}^{\infty} F_{P_i}^{n_i}(p)\frac{1}{n_i!}\left[\int_0^t \nu_i(\tau)d\tau\right]^{n_i} \exp\left(-\int_0^t \nu_i(\tau)\,d\tau\right)\right)$$

$$= \exp\left(-\sum_{i=1}^{M}(1-F_{P_i}(p))\int_0^t \nu_i(\tau)d\tau\right) \tag{1}$$

## 3. FIRST PASSAGE TIME OF COLLISION LOADS

Let L signify the first passage time, i.e. the elapsed time until a collision load of magnitude p is exceeded for the first time. The distribution function $F_L$ of L can then easily be determined because the event $\{L \leq t\}$ occurs if and only if the event $\{P_{max} > p\}$ occurs in the interval $]0, t]$.

$$F_L(t) = P(L \leq t) = P(P_{max} > p) = 1 - \exp\left(-\sum_{i=1}^{M}(1-F_{P_i}(p))\int_0^t \nu_i(\tau)d\tau\right) \tag{2}$$

If the Poisson processes are assumed to be homogeneous ($\nu_i$ independent of $\tau$) then (2) reduces to

$$F_L(t) = 1 - \exp\left(-\frac{t}{t_R}\right) \tag{3}$$

where

$$t_R = \left(\sum_{i=1}^{M}(1-F_{P_i}(p))\nu_i\right)^{-1} \tag{4}$$

is the return period (expected first passage time) of peak impact forces exceeding the level p.

## 4. COLLISION RISK ASSESSMENT

The risk assessment can now be made either by specifying a specific fractile in the distribution function $F_{P_{max}}$ or alternatively by specifying a sufficiently large return period $t_R$. Note that $V[L] = 1$ so that e.g. the design level p will be exceeded at least once during the period $0.1\, t_R$ with a probability as high as $1 - e^{-0.1} = 9.5\%$.

The peak impact load depends primarily on the ship magnitude measured by the dead weight D, [3, 8, 9]. When the sample space $\Omega_D = ]0, \infty[$ of D is divided into M disjoint intervals and $M \to \infty$ as the length of the subdivision passes to zero the Riemann sums in (1) and (4) converge. In the limit

$$F_{P_{max}}(p, t) = \exp\left(-\int_{\Omega_D}(1-F_P(p|x))\int_0^t d\nu(\tau, x)d\tau\right) \tag{5}$$

$$t_R = \left(\int_{\Omega_D}(1-F_P(p|x))d\nu(x)\right)^{-1} \tag{6}$$

where $F_P(\cdot|x)$ is the distribution function of impact loads on condition of $D = x$ and $d\nu(\tau, x)$ is the rate of ship collisions at time $\tau$ for ships with dead weight in the interval $]x, x + dx]$.

The last-mentioned quantity can be written

$$d\nu(\tau, x) = \nu(\tau|x)f_D(x)dx \tag{7}$$

where $\nu(\tau|x)$ is the collision rate on condition of $D = x$, and $f_D$ the frequency function of D. If it is assumed that $\nu(\tau|x)$ can be written

$$\nu(\tau|x) = g(x)\nu_0(\tau) \tag{8}$$

where

$$g(d_0) = 1 \tag{9}$$

then $\nu_0$ is the collision rate on condition of the scaling deadweight $D = d_0$ and $g(\cdot)$ weights the collision risks of ships different from $d_0$. (8) is valid, if the relative probability of collisions from ships of different magnitudes remain unchanged at all times. (5) and (6) can then be written

$$F_{P_{max}}(p, t) = \exp(-E[g(D)(1 - F_P(p|D))]\int_0^t \nu_0(\tau)d\tau) \tag{10}$$

$$t_R = (\nu_0 E[g(D)(1 - F_P(p|D))])^{-1} \tag{11}$$

The functions $g$, $F_P$ and $\nu_0$ are investigated further in the succeeding sections.

In more advanced models the peak impact load may depend on impact velocity, mass, bow construction etc. If these parameters are assembled in the vector valued quantity $\underset{\sim}{R}$ then the condition analogous to (8) is

$$\nu(\tau|\underset{\sim}{r}) = g(\underset{\sim}{r})\nu_0(\tau) \tag{12}$$

The equations (10) and (11) are unaltered if the combined stochastic variables $g(D)$ and $F_P(p|D)$ are replaced by $g(\underset{\sim}{R})$ and $F_P(p|\underset{\sim}{R})$.

## 5. RELATIVE RISK OF SHIP COLLISION

The function g weights the collision risk of ships different from the scaling deadweight $D = d_0$. Due to the fact that the beam of a ship is increasing with the deadweight d the probability of collision will increase with d. On the other hand it can be argued that larger ships will have relatively smaller probability of collision than smaller ships, because in greater ships the steering and safety systems are doubled, the crew is better trained, there will probably be a pilot on board, etc.

In lack of relevant data it is difficult to estimate the relative strength of these mutual reversed tendencies. Therefore, the best choice in this case is probably to select $g(d) \equiv 1$, and apply a scaling value $d_0$ in the vicinity of the final design load at the calculation of $\nu_0$.

## 6. CONDITIONAL DISTRIBUTION OF IMPACT FORCES

As mentioned earlier to a first approximation the peak impact load P depends only on the magnitude of the ship d. Woisin has indicated a triangular distribution of P with the following conditional expected value [3, 8, 9] (see figure 1)

$$E[P|d] \sim 0.88\sqrt{d} \tag{13}$$

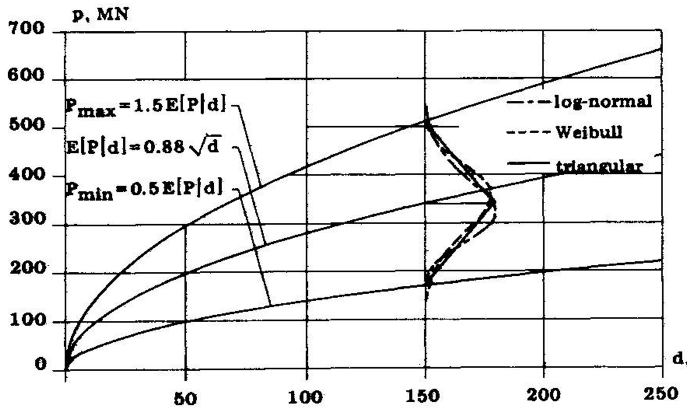


Figure 1. Conditional frequency functions of peak impact forces.

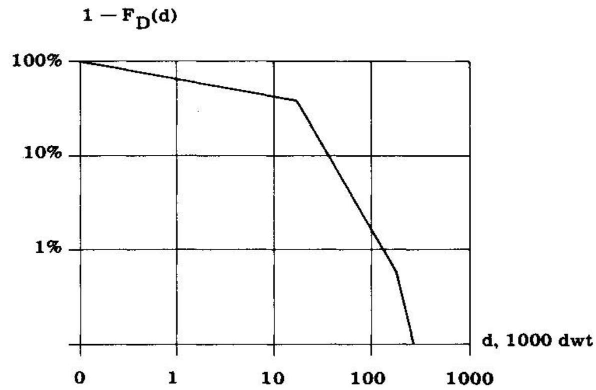


Figure 2. Complementary distribution function of magnitude of ships passing the Great Belt, Denmark, [2, 3].

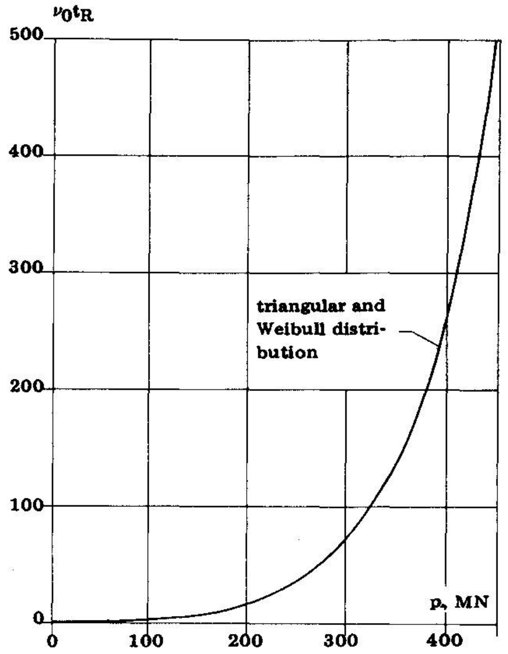


Figure 3. Non-dimensional return period as a function of design impact load.

Figure 1 also shows the density functions for the corresponding log-normal and Weibull distributions both with the same conditional expectation E[P|d] and the same conditional coefficient of variation $V[P|d] = 1/\sqrt{24}$ as the triangular distribution.

The non-dimensional return period $\nu_0 t_R$ (see (11)) can then be calculated as a function of the design impact load p if a distribution of deadweights D is chosen and by assuming $g(D) \equiv 1$. Let the distribution of deadweight D be as shown in figure 2 [2, 3], then the results corresponding to each of the three conditional distributions above are as shown in figure 3. The results are so close that only one curve is shown. The difference between the results is everywhere below 0.5%. As a consequence it can be stated that the non-dimensional return period $\nu_0 t_R$ is highly insensitive to moments in the conditional

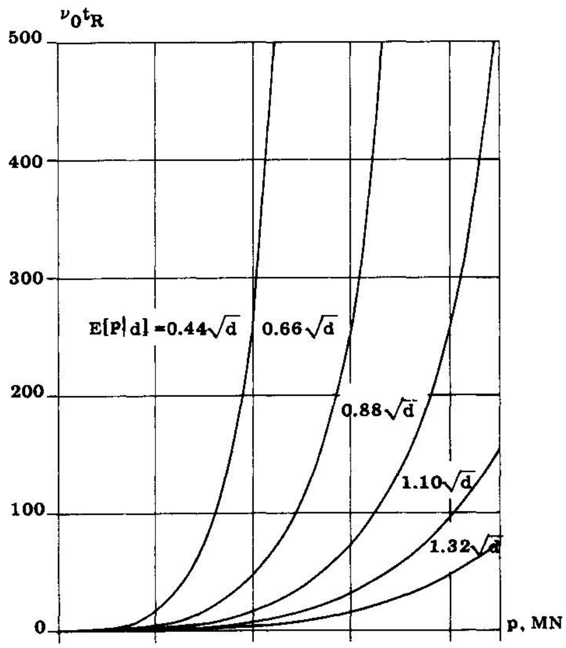


Figure 4. Non-dimensional return period. Dependence on conditional expectation of impact loads. Weibull distribution, $V[P|d] = 1/\sqrt{24}$.

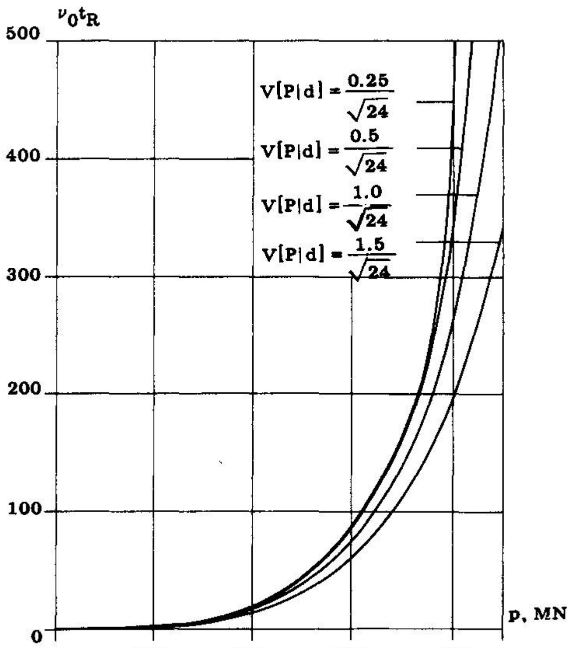


Figure 5. Non-dimensional return period. Dependence on conditional coefficient of variation. Weibull distribution, $E[P|d] = 0.88\sqrt{d}$.

distribution of P beyond the second moment properties. The dependence of $\nu_0 t_R$ on the second order moments is shown in figures 4 and 5. As expected the return period diminishes as the conditional expectation and coefficient of variation are increased. The dependence on the conditional expectation E[P|d] is considerable. Therefore, the statistical errors inherent in the estimate (13) should be recognized, when the present method is applied.

## 7. ESTIMATION OF COLLISION RATE

Let a ship be characterized by the parameter set $\underset{\sim}{R} = \underset{\sim}{r}$ and consider the set of events $E_i$ which renders the ship out of control and makes a collision to the pier possible. Let the probability rate of the event $E_i$ be $\nu_{0,i}$ and let the event that ships with $\underset{\sim}{R} = \underset{\sim}{r}$ encounter the pier be denoted C. Then the collision rate $\nu_0(t)$ is

$$\nu_0(t) = \sum_i P(C|E_i)\nu_{0,i}(t) \tag{14}$$

where the conditional probabilities $P(C|E_i)$ are assumed to be independent of time. The failure rates $\nu_{0,i}$ must be estimated at best from available data.

On condition of the event $E_i$, the event C, i.e. collision with the pier takes place, is determined by a finite set of stochastic variables $\underset{\sim}{X}$, such as position of the ship, its speed and heading angle, speed and direction of wind and current, direction and height of waves, etc. at the instant of failure. The collision probabilities $P(C|E_i)$ can hardly be determined analytically. It is therefore necessary to estimate them by numerical simulation. Here a simulation method developed by Thoft-Christensen [10] will be demonstrated, when $E_i$ signifies locking of the rudder at a random angle $\delta$. Such a locking may occur during the continuous rudder manoeuvre due to the fact that unstable and neutrally stable ships cannot maintain a straight course without continuous rudder control.

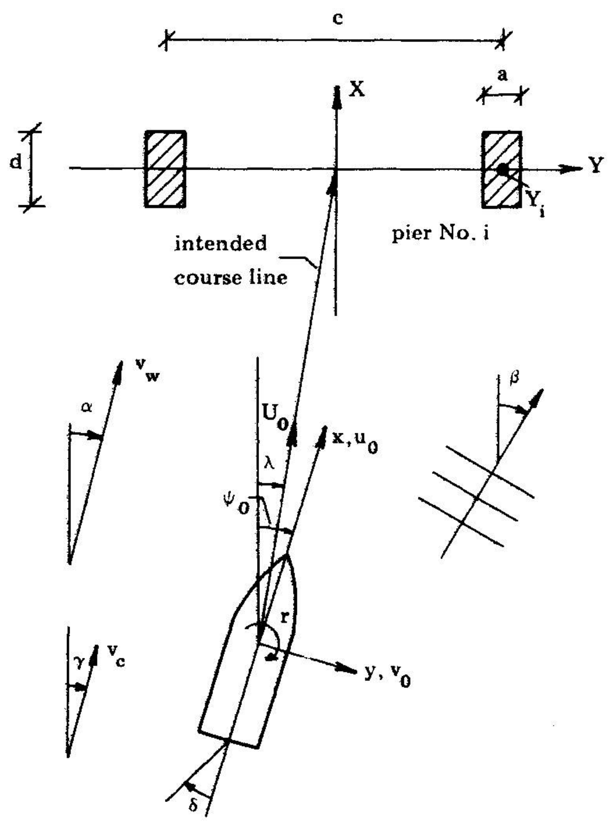


Figure 6. Decisive parameters of ship collision problem.

Ship positions are specified in relation to an inertial XYZ-coordinate system with origo at the centre of the free bridge span (see figure 6). In the same figure a body-fixed xyz-coordinate system is defined with origo somewhere in the symmetry plane and axes parallel to the principal axes of the ship.

The above-mentioned parameter set $\underset{\sim}{X}$ at the instant of failure contains at least the following stochastic variables:

| | |
|---|---|
| $X_0, Y_0, \psi_0$: | ship position and heading angle at failure |
| $u_0, v_0, r_0$: | horizontal ship velocity components and yaw rate |
| $\delta$: | locked rudder angle at failure |
| $v_w, \alpha$: | wind speed and direction |
| $h_s, \beta$: | significant wave height and direction |
| $v_c, \gamma$: | current speed and direction |

For the sake of simplicity it is assumed that all ships intend to approach the bridge in a straight course line towards the centre of the free span with speed $U_0$. Then

$$\left.\begin{aligned} X_0 &= -s\cos\lambda \\ Y_0 &= -s\sin\lambda \\ \dot{X}_0 &= U_0\cos\lambda \\ \dot{Y}_0 &= U_0\sin\lambda \\ r_0 &= 0 \end{aligned}\right\} \qquad (15)$$

where s is the distance to the centre of the free span and $\lambda$ specifies the intended course line.

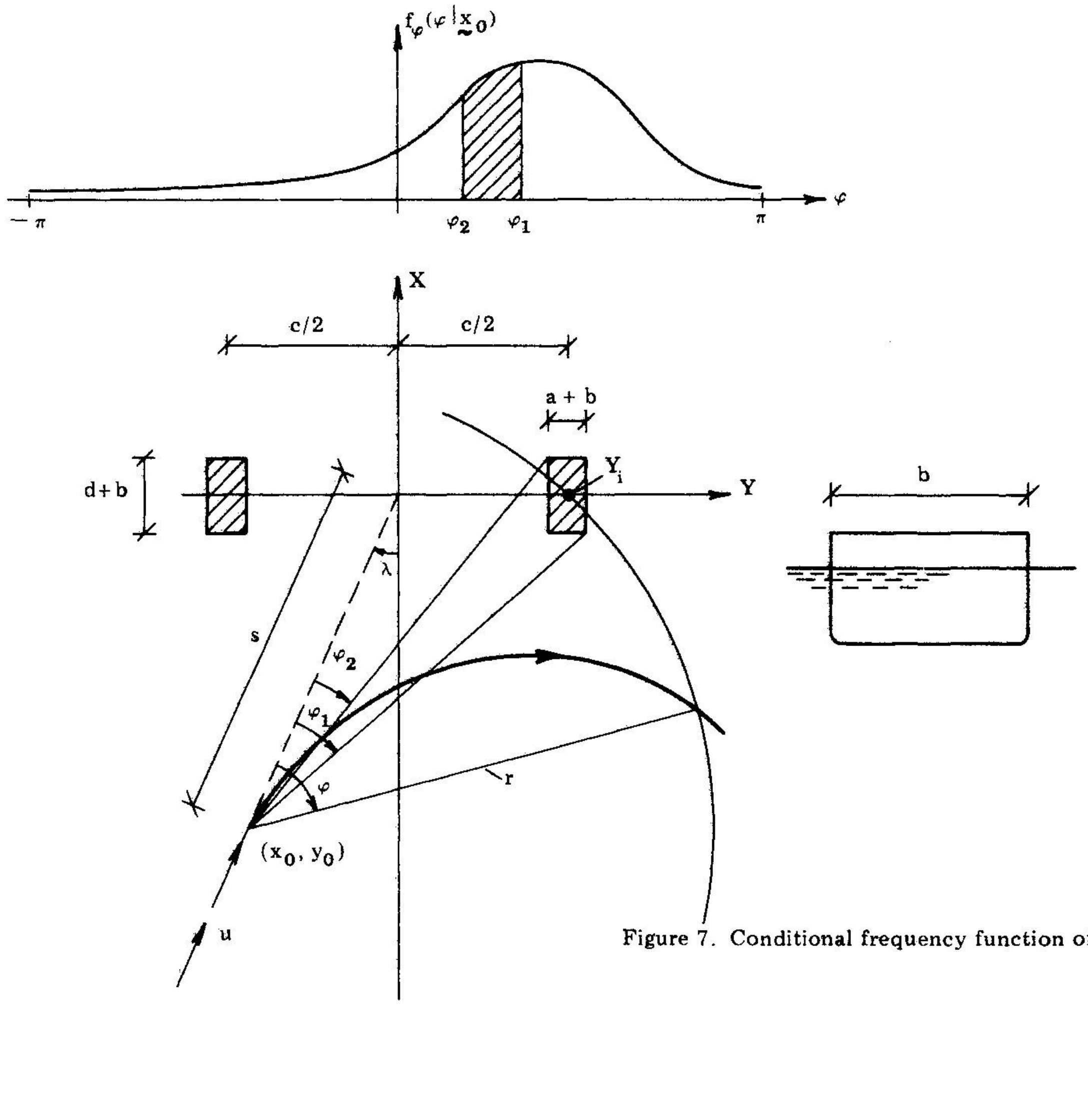


Figure 7. Conditional frequency function of collision angles.

Let the independent variables be grouped into the following two subsets $\underset{\sim}{X}_0 = (s, U_0, \lambda)$ and $\underset{\sim}{X}_1 = (\delta, v_w, \alpha, h_s, \beta, v_c, \gamma)$. The horizontal ship velocity components $u_0$, $v_0$ and the equilibrium rudder deflection depend on the parameters $\underset{\sim}{X}_0$ and $\underset{\sim}{X}_1$, and can be calculated from equilibrium conditions of the loads on the ship at straight course line.

The distance r to the considered pier at the instant of failure (locking of the rudder) depends only on the distance s and the angle $\lambda$ (see figure 7). The trajectory of the ship will cross the circle with radius r at a certain angle $\varphi$, defined as shown in figure 7. This angle $\varphi$ depends (on condition of the sample $\underset{\sim}{X}_0 = \underset{\sim}{x}_0$) merely on the stochastic vector $\underset{\sim}{X}_1$, i.e.

$$\varphi = f(\underset{\sim}{X}_1, \underset{\sim}{x}_0) \tag{16}$$

The ship will collide with the pier for $\varphi \in \,]\,\varphi_1, \varphi_2\,[$ (see figure 7). Therefore, the conditional probability of collision $P(C \mid E_i)$, is given by

$$P(C \mid E_i) = \int_{\Omega_{\underset{\sim}{X}_0}} (F_\varphi(\varphi_1 \mid \underset{\sim}{x}_0) - F_\varphi(\varphi_2 \mid \underset{\sim}{x}_0))\, f_{\underset{\sim}{X}_0}(\underset{\sim}{x}_0)\, d\underset{\sim}{x}_0 \tag{17}$$

where $F_\varphi(\cdot \mid \underset{\sim}{x}_0)$ is the distribution function of $\varphi$ on condition of $\underset{\sim}{X}_0 = \underset{\sim}{x}_0$ and where $f_{\underset{\sim}{X}_0}$ and $\Omega_{\underset{\sim}{X}_0}$ indicate the joint frequency function and sample space of $\underset{\sim}{X}_0$.

Clearly $\varphi_1$ and $\varphi_2$ depend on $\underset{\sim}{x}_0$ as well as the beam b of the ship and the dimensions of the pier. This fact can be taken into consideration by assigning equivalent dimensions a + b and b + d to the pier (see figure 7). Then the ship can be considered as a particle.

For sample values of the ship velocity $U_0$ and the angle $\lambda$ a number of samples of $\underset{\sim}{X}_1$ are generated numerically. For each of these samples a ship trajectory is obtained from the manoeuvring equations governing the ship motion, and the crossing angles $\varphi_1, \varphi_2, \ldots$ at a number of concentric circles with preselected radii $r_1, r_2, \ldots$, are registered (see figure 8). From these sample values the conditional distribution functions $F_\varphi(\cdot \mid r_i, u, \lambda)$, $i = 1, 2, \ldots$, can be estimated. Actually the power of this so-called rosette method [10] originates from the fact that information is obtained for a great number of conditional distribution functions for each ship path realization.

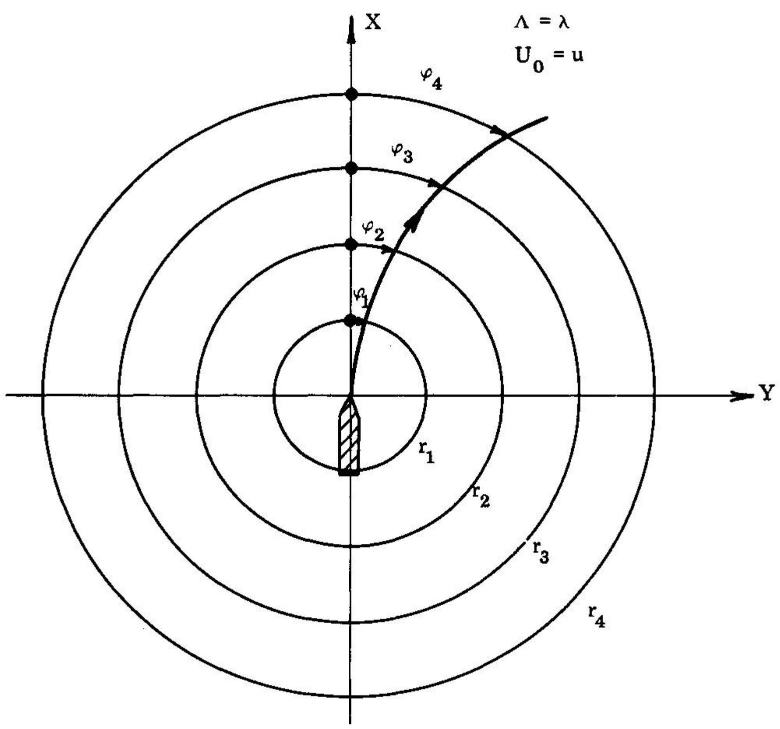


Figure 8. Collection of sampling values.

## 8. NUMERICAL EXAMPLE

In this section the method outlined above will be demonstrated with the following assumptions:

- $\lambda \equiv 0$
- Wave loads are ignored
- Wind direction is parallel to the Y-axis
- Current direction is parallel to the X-axis

Further it is assumed that the speed of wind and current are distributed according to the following Weibull distributions (v in m/s):

$$F_{V_w}(v) = 1 - \exp(-(\frac{v}{5.84})^{2.817}) \tag{18}$$

$$F_{V_c}(v) = 1 - \exp(-(\frac{v}{2.19})^{4.542}) \tag{19}$$

For a period of 75% of the time the wind direction is assumed to be in the positive Y-direction, whereas the current is directed in the positive and negative X-direction with equal probability. In both cases the directions are assumed to be independent of the corresponding speeds.

The density function $f_\Delta$ of the rudder angle $\delta$ is shown in figure 9. In the interval $[\delta_1 - 10°, \delta_1 + 10°]$, where $\delta_1$ is the equilibrium value to maintain a straight course line, the density function follows a normal distribution with standard deviation $\sigma = 5°$. In some failure situations the rudder locks in the maximum rudder angle $\pm\delta_2$ with a probability of $\frac{1}{2}(1-p)$, where p is the probability of the distributed part of the sample space. In this example $p = 0.75$ and $\delta_2 = 35°$.

The distance s at failure between the ship and the bridge is assumed to be uniformly distributed in the interval [0, 4000 m]. Failures outside this interval are not considered to imply any risk of collision either because the ship can be stopped or collision can be prevented in other ways.

The ship velocity at failure $U_0$ is assumed to be Weibull distributed with expectation $E[U_0] = 8$ kn and standard deviation $\sigma_{U_0} = 2$ kn.

The hydrodynamic forces in the manoeuvring equations can be modelled either by the linear model of Abkowitz [12] or the non-linear model suggested by Norrbin [13]. In the present study the latter method has been applied with hydrodynamic coefficients taken from [14]. The wind coefficients of the ship are specified according to [15] and the manoeuvring equations are solved numerically by means of a 4th order Runge-Kutta integration scheme.

The conditional probability of collision is then calculated as a function of the width of the free span c and the result is shown in figure 10.

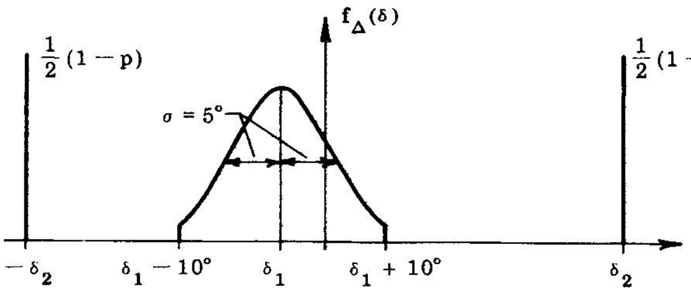


Figure 9. Density function of rudder angle at failure [10].

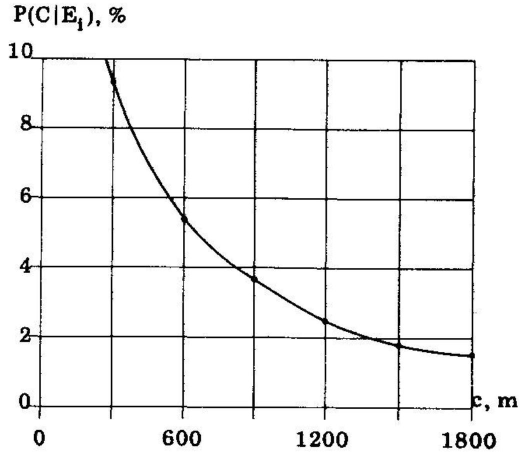


Figure 10. Probability of collision as a function of the free span c.

## 9. CONCLUDING REMARKS

A general method has been developed from which the distribution of extreme impact forces can be calculated. It is demonstrated how the reliability problem can be decoupled into two minor problems concerning the distribution of impact loads on condition that collision does take place and calculation of the probability rate at which larger ships will encounter the pier. A computer program has been developed from which the latter quantity can be calculated when rudder locking is the main cause of ship collision. Extension of the method to other failure sources is straightforward.

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