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Autor(en): **Di Pasquale, S.**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **46 (1983)**

PDF erstellt am: **29.06.2024**

Persistenter Link: https://doi.org/10.5169/seals-35846

Structural Analysis of the Dome in Florence

Analyse structurelle de la coupole du dôme à Florence

Analyse des Tragsystems der Domkuppel in Florenz

S. DI PASQUALE
Professor
Univ. of Florence
Florence, Italy

S. Di Pasquale, born 1931, received his architect degree at the University of Naples, Italy, and worked in the field of Space Structures. From 1976 he was involved in structural masonry problems.

SUMMARY

The results of research carried out on the dome of the Duomo in Florence, since 1976 are presented. Details are given of the characteristics of the non-linear problem due to the material properties, and about the theory proposed for its solution. Finally, a solution for non-consolidation is determined.

RESUME

Le rapport présente les résultats de recherches effectuées pour le Dôme à Florence dès 1976. La nature du problème non-linéaire est dû aux propriétés du matériau; une théorie est utilisée pour sa résolution. Une solution de non-consolidation est enfin déterminée.

ZUSAMMENFASSUNG

Es werden die Forschungsergebnisse über die Untersuchungen der Domkuppel in Florenz, die seit 1976 unternommen wurden, vorgestellt. Besonders wird über die durch das Materialverhalten bedingte Nichtlinearität des Problems und die theoretischen Annahmen für seine Lösung hingewiesen. Ferner wird eine Lösungsmöglichkeit ohne Verstärkung aufgezeigt.

1. INTRODUCTION

Paradoxically, could Filippo Brunelleschi have known the famous "π theorem", many problems arising today from the stability of his dome should appear as feigned problems. Built in the years between 1420 and 1436, the florentine S. Maria del Fiore dome is probably the geatest masonry dome all over the world. It shows an eight-edged plan, 52 mt from the ground level, inscribed within a circle with a 45 mt inner and 54 mt outer diameter. There are also an inner 2.20 mt average thickness dome and an outer one 0.80 mt thick, bound together by eight corner arches and by other sixteen ribs. At the top, about 90 mt from ground level, there is an about 800 tons heavy lantern.
Over the dome, being of fair weight of 25,000 tons, there is a wide-spreading fracture pattern, common to all wide-span masonry domes. Four major fractures bypass both the inner and outer domes and are spreading from the basis of the drum to about two thirds of the meridian length.
There are also capillary fractures, sometimes even well appreciable at a glance, actually spreading all over the structure. The problem of these fractures, and of the strengthening project, is not recent. By the end of the 17th century the dome already raised some tronbles. Many were concerned with the problem in turn; V. Viviani, the famous pupil-biograph of Galileo, other less renowned architects in the 18th century, P. L. Nervi since 1940, W.B. Parsons in the same years and, more recently, R. Mainstone.
The structural analysis of this last author is particulary interesting. Though performed assuming a membrane behaviour and a fictitious rotational dome within the real one (following some hints already expressed in nuce by L.B. Alberti). In such an analysis, the problem of the static behaviour of the dome during the years of its construction is taken into consideration for the first time. The results obtained by Mainstone, although affected by rough approximations, may be confirmed today by the more sophisticated analysis recently performed. All the work done by us about this problem since 1977 cannot be conveniently summarized in the available space. Together with researches about the mechanical problem - described in what follows - other investigations have been made about worksite management, the load-lifting devices used for the construction, the history of mechanics in Renaissance. These last researches gave a confirmation of the results obtained by all the structural analyses : the fracture phenomenon had its origins before the dates stated by known documents, maybe already at the time of its construction.
The dome structure has been studied with several methods, with the aim of obtaining results which could be compared.
Before explaining such results, some basic considerations will be very useful, mainly about the contribution given by technological and scientific research to the problem of restoration of monuments. The clause I of the Venice Chart - ruling the restoration projects - raised several perplexities among the specialists of restoration, mainly about the several conclusions and the use of not-ever justifiable strengthening techniques.
The lacking of a specific theory of masonry mechanics in many cases produced structural analyses where masonry is a no-tension material; hence, fractures take place where tensile stresses show the greatest values. The theoretical mistake affecting the so-obtained results is quite self-explanatory and needs no remark.
Obviously, where a restoration project need be quickly and pressingly worked out, such procedures may be used and cautiously accepted. From our point of view, however, the not so-pressing problem of the SMF dome quite deserves a more rigorous formulation.

To this aim a mechanical theory has been proposed, viewing masonry as no-tension materials. Since this theory is complete up to date, as for the evaluation of stresses and fractures, it needs only existence and uniqueness theorems relating to particular compliance conditions of loads and constraints. Quite general results have been already obtained for plane problems, stating the theorems governing the fracturing and neglecting body forces.
In the already tested computational techniques, the no-tension behaviour has been simulated by suitable distortions or fractures. Convergence is achieved by setting zero the constrained energy due to fracture simulating distortions. Such numerical procedures followed mechanical analyses on masonry samples in order to obtain the values of elastic coefficients.

2. Mechanical properties of materials

This section of the research is aimed to the evaluation and knowledge of the materials of the Dome, including the materials of the foundations and the analysis of sub-soil.
The historical research has been done mainly on the documents of the "Opera del Duomo"; it has permitted the characterization of the materials by their source of origin. Destructive tests have been done on mortar, bricks and stones, using samples obtained from cores with a diameter 7 cm. and length which varies with the thickness of the vault. Mineralogical-petrographical, physical and spectrographical tests were done with the aid of Manganelli Dal Fà and Franchi at "Centro di studio sulle cause di deperimento e metodi di conservazione delle opere d'arte"- - CNR - University of Florence all of this to determine the components of such materials for an ultimate control.
The availability of wood kilns for the firing of pozzolans and clays, together with aging chambers, lets us hope for good results.
In the meantime, tests have been done on the individual components and on contemporary masonry, so as to be able to compare the old and the new. This will allow an ample experimental research that uses components available today, avoiding thus further sampling. Here under we list the results of the tests done up to today, neglecting all technical information acquired by the readings of archival documents.
A relationship between a stratigraphical model, obtained from drillings performed in the surroundings of Santa Maria del Fiore, and the units that constitute the florentine sub-soil, have been deduced. Of such units have been determined the granulometric characteristics and the mechanical properties.
In a similar way we shall summarize the results of tests done on old and new materials is the following table:

	A	γ_d	γ_w	σ_c	σ_t	E_c	ν
		Ncm^{-3}	Ncm^{-3}	Ncm^{-2}	Ncm^{-2}	Ncm^{-2}	
antique brick	18,14	16,18	19,22	2772	260	1108530	0,18
recent brick	18,63	16,87	20,01	2943	256	961380	0,14
antique mortar	17,67	16,57	19,52	1962	387	784800	0,27
recent mortar	-	-	-	2118	465	794610	0,22

Where

A = coefficient of imbibition

γ_d = dry density

γ_w = wet density

σ_c = ultimate compressive strenght

σ_t = ultimate tensile strenght

E_c = Young's modulus ν = Poisson's ratio

3. Structural static and dynamic analysis

3A) Static analysis was performed as stated in what follows:

i) The order of magnitude of tensile stresses has been obtained by membrane and membrane & bending mechanical models, assuming perfectly linear elastic behaviour of materials.

ii) A mechanical model has been considered, where the fractures already spread up to the basis of the lantern, in order to evaluate the structural stability in such configuration.

iii) A particular no-tension finite-element has been implemented for accounting for the actual behaviour of masonry. Thus a dead load condition alone and combined with thermal loads have been considered.

A short survey of the massive results obtained leads to the following remarks. Type A-i simulations may all be compared to each other, but they do not exactly comply with the fracture pattern. This is due to the assumptions about material behaviour, which is actually no-tension.
Type A-ii simulation and its results are a basic reference for the analysis of the future fracture progress. In such a model the greatest values of stresses are found at the basis of the lantern: 10, Ncm-2 in tension and 75. Ncm-2 in compression.
We ought to remark that in this model the thrust-line holds always within the sections of the structure, except than at the basis of the lantern.
Type A-iii simulation has been performed on a 171 degrees-of-freedom model, corresponding to 1/16th of the whole structure, accounting also for the great openings at the levels of two internal platforms: these are the two weakest--links within the structure. These are very interesting results as far as they give the loading factor corresponding to a possible failure, due to yield compression stresses. Furthermore, we should remark that no-tension structural models lead in general to under-determined configurations, i.e. in unstable equilibrium under prescribed external loads: thus it may be viewed as a limit-model In the fractured wones the material behaves as a semi-fluid, although satisfying the Drucker stability postulates.

3B) Dynamic linear analysis of the SMF Dome

In what follows a finite-element numerical model will be described, used to investigate the structural behaviour of the S. Maria del Fiore Dome subjected to road-traffic excitations: such dynamic analysis aimed to be compared with experimentally evaluated data.
These are several available approaches to perform a linear elastic analysis of structures subjected to any dynamic loads; these are described by several authors, but the main topics may be summarized by K.J. Bathe and E.L. Wilson ("Solution

methods for EigenValue problems in Structural Mechanics", Int. Jour. Num. Meth. in Eng., 6, n. 2, 1973) and by R.W. Clough and J. Penzien ("Dynamics of Structures", McGraw, 1975).

As a matter of fact, the structural analysis of the SMF Dome needed to account also for pier and butresses down to the ground level, where constrained nodes are subjected to dynamic loads. These are defined as input acceleration diagrams (g'/g versus time), obtained by evaluation of amplitudes and frequencies of vibrations corresponding to several traffic conditions. Obviously, locally-acting dynamic loads cannot be considered, but a global ground-level accelerogram may be properly defined in the different loading conditions. Furthermore, the dome--geometry has been defined taking the main fracture pattern into account; this means to include proper disconnections at midspan of four horizontal edges, between an upper and lower fracture-level.

The actual double-layer vault geometry has been replaced by a simple layer node mesh, both for the Dome and the vertical structures. Furthermore, a forced--response by modal superposition technique has been used to solve the so-obtained problem. This required the previous computation of p M-orthonormalized eigenvectors. This one proves to be a very efficient technique especially when several acceleration input diagrams must be considered, or when a fairly long time interval is defined for dynamic loads, particularly when little time steps are used for final integration. The latter, besides, ought to account also for the higher frequencies wich contribute to the dynamic response. Furthermore, we should remark that a particular structural assembly and geometry suggested to properly extend the modal analysis, i.e. the dimensions of the subspace defined by eigenvectors. The aim was to get sufficient accuracy of results, within the limits of the accuracy degree of the structural model, both for geometrical and mechanical aspects.

A first step of the computational procedure was the proper definition of the integration time step to be used for the integration of dynamic equilibrium equations. The inconditionally stable theta-Wilson integration scheme has been used, and in this first step direct-response and modal superposition techniques have been both extensively used and compared to each other, in order to achieve the desired accuracy in the p-dimensional principal subspace and with a computed time integration step.

The used computational procedure is the well-known SAP IV program, implemented by K.J.Bathe, E.L.Wilson and F.E.Peterson for the static and dynamic analysis of linear structures. The finite element model more extensively used was the shell & plate one, including membrane and bending stresses, while few elements were isoparametric plane-stress. We need to remark that the required accuracy in the problem formulation allows a linear approach, since the stresses produced by slight traffic-accelerations would justify - at least partially - the use of a linearly-elastic material model.

The same reasons suggested not to take into account damping effects in masonry, because sufficient experimental data and results are not completely yet in this field of investigation. Even within such and other approximations, due for istance to the geometrical and mechanical uncertainties of the assumed model, a linear dynamic analysis of a many-degrees of freedom elastic structure seems to be a useful tool for effective comparison with experimental results. Furthermore, this could be a starting point toward the implementation - in a close future - of an optimal control procedure i.e. of a real-time static and dynamic analysis of such monumental structures, already affected by wide-spreading fracture patterns. Besides, the usefulness of a numerical model ought to be viewed also in the possibility of easily changing the dynamic input, using the same procedure.

From this point of view we wanted to draw the attention of Public Municipal Authorities - contractors of some investigations - on the problem of seismic reliability of such structures. Taking into account some recent aspects of italian seismic codes about masonry structures, once again a linear approach to this problem seemed to be quite sufficient; so, an El-Centro, 1940, NS earthquake accelerogram has been given as input in the same procedure of forced response by modal superposition. Obviously, in this case the assumed elasticity behaviour does not comply with the resulting relevant stresses, due to the most unfavorable static plus dynamic loading condition. On the other hand, we shall recall that the italian seismic code for masonry structures allows linear-elastic computations, providing at the same time rather high values of the safety factors, much greater than those assumed for r.c. or steel structures.

4. Hypothesis of structural non-consolidation

The History of the dome and the results of the theoretical, experimental and numerical researches bring about the following considerations:
i) The fracture pattern of the dome has remote origins in respect to the evolutions that we have been able to control in the last forty years.
ii) The history of this monument shows that the fracture pattern is in progress towards a "statically determinate" configuration. This has been the object of particular attention so as to evaluate the ultimate safety factor.
iii) The accuracy of the ultimate safety factor can be furtherly increased through a nonlinear analysis wich accounts for other phenomena , for example: the fluage, the creep, the fatigue due to mechanical and thermal loads.
iv) The presence in site of a thick mesh of thermo-dilatometers measuring devices will permit the control in real time of the actual state of the structure. This will allow the evaluation of the ratio between the istantaneous safety factor and the ultimate one cumputed in ii) and iii).
Taking into consideration the cyclical mechanisms of fractures due mainly to thermal loads, we may assume a first stage restoration of the fracture-scheme, viewed as an elimination of the constraints that prevent the natural behaviour of the structure. Thus, the mesh of measuring devices will be able to furnish information about the progress of the fractures, without all possible perturbations due to the falling of fragments within the fractures. A future project for the strengthening of materials or the stiffening of the structure can be brought forward on basis of the former considerations.

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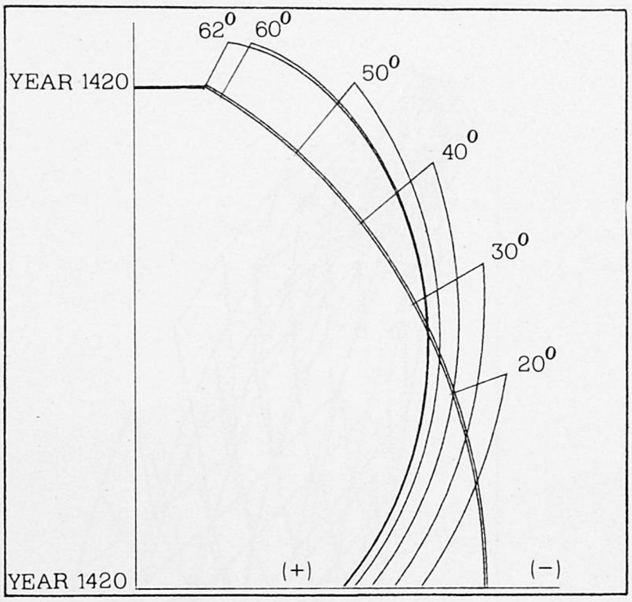
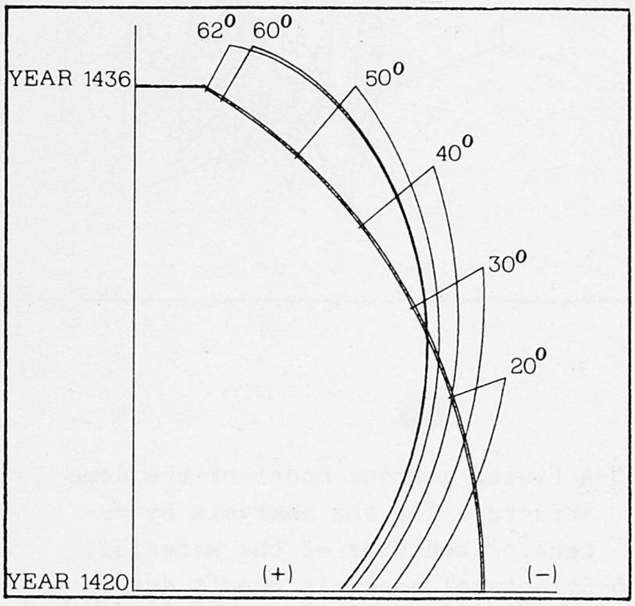
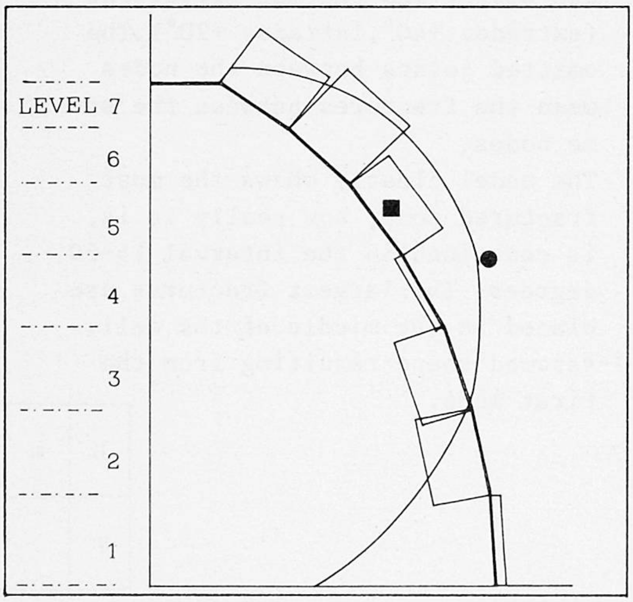
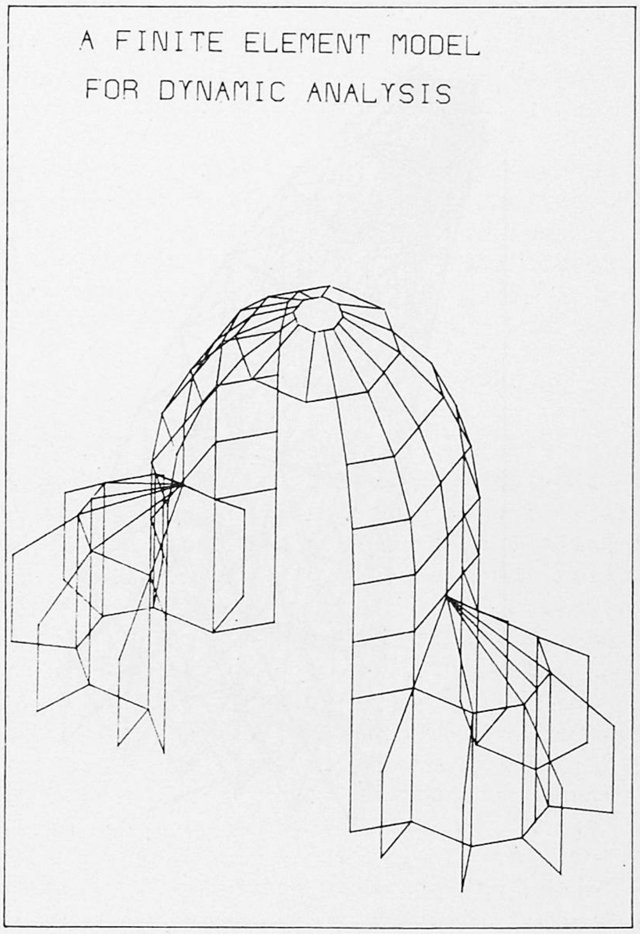






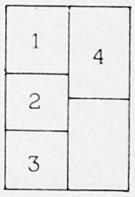


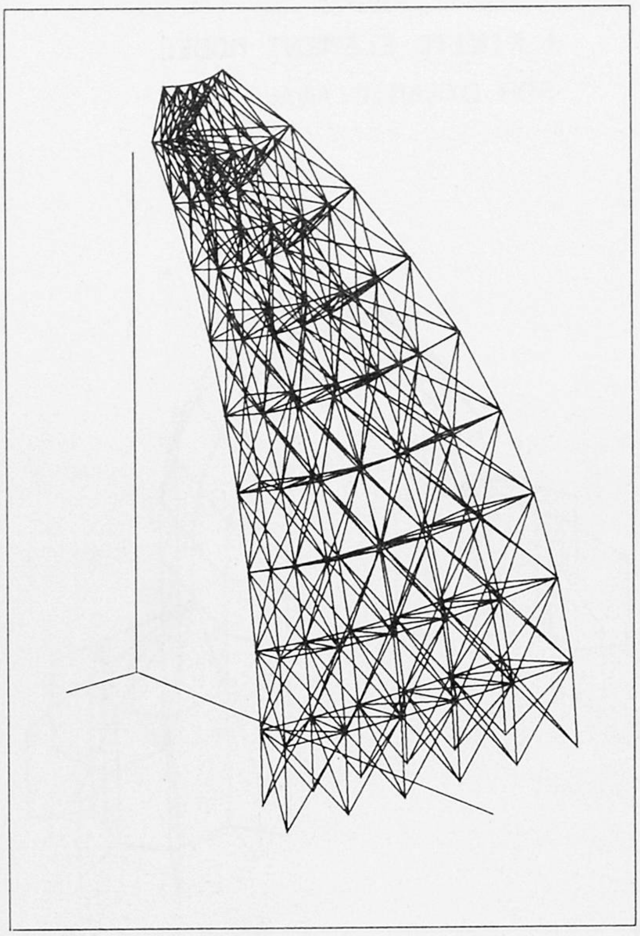
1-Membrane behavior analysis for a medium rotational dome. The building stages from 1420 to 1436 are simulated for the section in the middle of the cell: the zero point of the stresses along the parallels goes up.

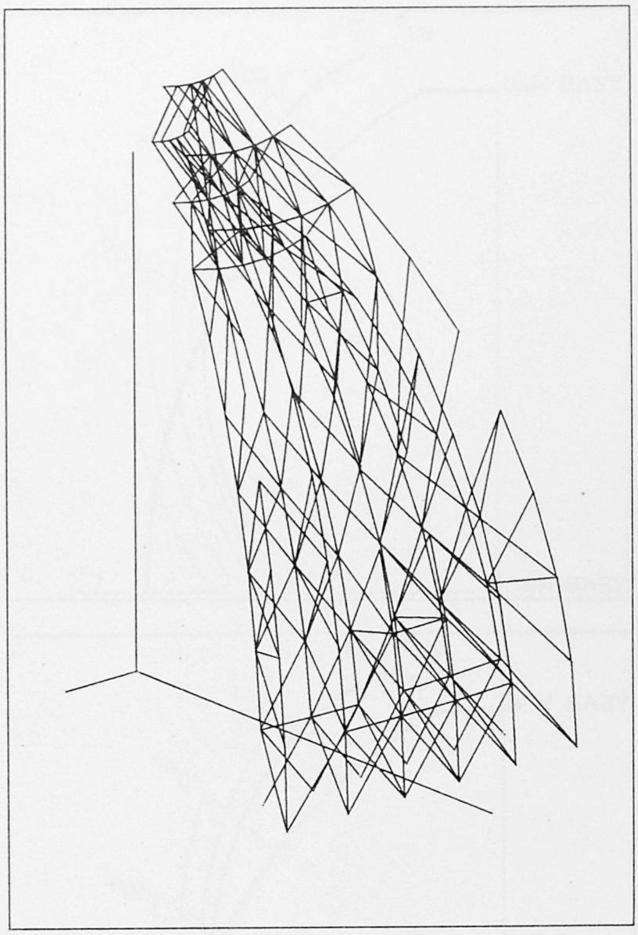
2-Membrane behavior analysis for a poligonal dome (Dischinger theory). The building stages from 1420 to 1436 are simulated for the section in the middle of the cell: the zero point of the stresses along the parallels goes up.

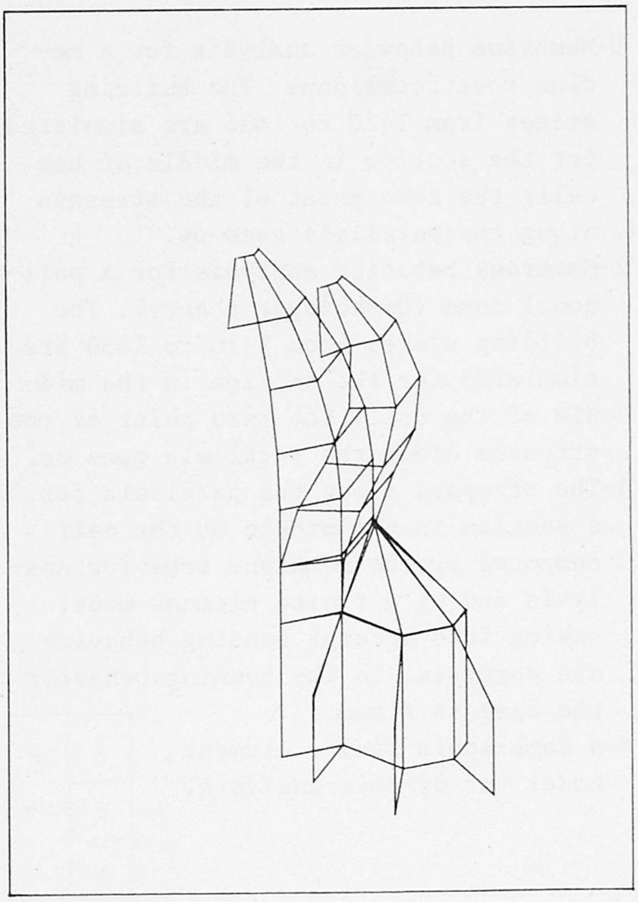
3-The stresses along the parallels for a section in the middle of the cell computed by the membrane behavior analysis and by a finite element model taking into account bending behavior are compared. In the bending behavior the base is fixed.

4-A dome-apsis finite element model for dynamic analysis.









5-A finite element model of the dome structure for the analysis by no-tension behavior of the material.
6-Structural analysis result due to own weight and thermal variations (extrados +40°,intrados +20°).The omitted joints between the nodes mean the fractures between the same nodes.
The model clearly shows the most fractured zone, how really it is, is conteined in the interval 15-50 degrees. The largest fractures are placed in the middle of the cell.
7-Assumed shape resulting from the first mode.

5	6
7	