Distortion and Warping of Profiled Metal Sheeting in Shear

Distorsion et gauchissement des plaques profilées en métal soumises au cisaillement

Verwindung und Verzerrung von Stahlprofilblechen unter Schubbeanspruchung

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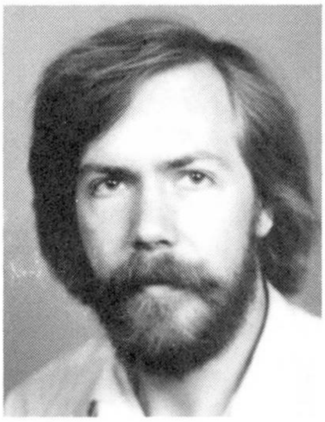

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SUMMARY

In this paper a mathematical basis is presented for the calculation of shear deformation of trapezoidally corrugated sheeting. Expressions are given for the calculation of shear strain and distortion both combined with warping of corrugations. Various boundary conditions along profiled edges can be treated. Explicit formulae are given for the two wavelengths, belonging to a profiled sheet with all corrugations equally attached. For shear diaphragms longer than wavelengths distortion is independent of the length: this can be simulated physically by discrete springs.

RÉSUMÉ

Dans cet article on présente une base mathématique pour le calcul de la déformation due au cisaillement de panneaux en tôle ondulée de forme trapézoïdale. On donne des expressions pour le calcul du glissement et de la distorsion, en combinaison avec le gauchissement. Des conditions aux limites différentes sur les bords des profilés peuvent être traîtées. On donne des formules explicites pour les deux longueurs d'onde attribuées à un panneau de tôle profilée fixé de façon égale dans chaque onde. Pour les panneaux plus longs que les longueurs d'onde, la distorsion est indépendante de la longueur; ceci peut être représenté physiquement par des ressorts ponctuels.

ZUSAMMENFASSUNG

In diesem Beitrag wird ein mathematisches Modell für die Berechnung der Schubverformung von Trapezblechscheiben präsentiert. Ausdrücke für die Berechnung von Schub- und Querschnittverformung werden gegeben, beide kombiniert mit Verwölbung. Verschiedene Bedingungen an den Querrändern können behandelt werden. Für Bleche, die gleichmässig in allen Rippentälern befestigt sind, werden Formeln für die Schubverformung mit zwei Beulwellen gegeben. Für Scheiben, welche länger als die Wellenlänge sind, ist die Querschnittverformung von der Blechlänge unabhängig: das kann physikalisch mit Einzelfedern simuliert werden.

1. SCOPE

Trapezoidally profiled metal sheeting in stressed-skin structures is capable of transferring considerable in-plane loads. For that purpose it must be connected to proper intermediate or edge members. Then normal forces or in-plane bending moments are basically taken by these members, where shear forces are transferred by the sheeting. In ECCS recommendations |1| this is called diaphragm action.

The application of shear diaphragms depends to a major extent on the resistance against in-plane deformation. Deformation of an individual diaphragm in its plane is generally caused by extension and deflection of the edge members in plane of the panel, by local deformation of fastenings and finally by shear deformation of profiled sheeting itself. In this paper attention will be paid to the latter only: shear deformation of profiled sheets.

Object of considerations will be sheeting with trapezoidally profiled cross section, that can be described with 5 relevant parameters (figure 1a). Few other interesting parameters then can be derived (figure 1b).

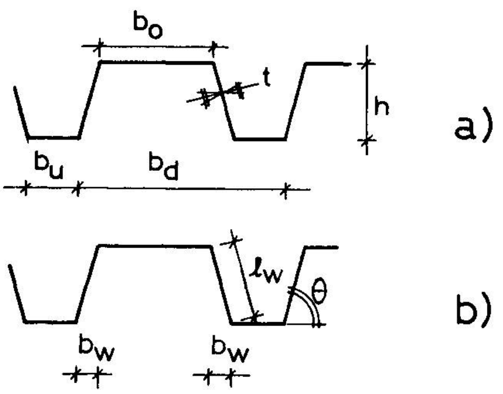

Figure 1: a) relevant parameters
b) derived parameters

For short notation following dimensionless shape factors are denoted:

$$\frac{b_o}{b_d} = \alpha_o \quad (1)$$

$$\frac{2\,\ell_w}{b_d} = \alpha_w \quad (2)$$

$$\frac{b_u}{b_d} = \alpha_u \quad (3)$$

$$\frac{b_o + 2\,\ell_w + b_u}{b_d} = \alpha \quad (4)$$

2. SHEAR DEFORMATION

2.1 Shear strain

For a global deformation only two independent displacements of a shear diaphragm are relevant: a displacement u in x-direction (perpendicular to foldlines) and a displacement v in y-direction (parallel to foldlines). In-plane rotation of a diaphragm as a rigid body is expressed by:

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \quad (5)$$

If a profiled sheet is in stress, due to a constant shear force s per unit length, every strip will be strained to a magnitude of $\gamma = s/Gt$. Two parallel lines in y-direction with pitch b_d will translate with respect to each other, giving a parallel shift Δv:

$$\Delta v = \gamma(b_o + 2\ell_w + b_u) = \frac{s}{Gt}(\alpha b_d) \quad (6)$$

Thus the parallel shift per unit width becomes:

$$\frac{\Delta v}{b_d} = \frac{\alpha s}{Gt} \quad \text{where} \quad \frac{\Delta}{b_d} = \frac{\partial}{\partial x} \quad (7)$$

This is shear deformation merely as a result of shear strain. A top view of a strained element (figure 2) shows, that an originally plane cross section does not remain plane. This phenomenon is known as warping of a cross section.

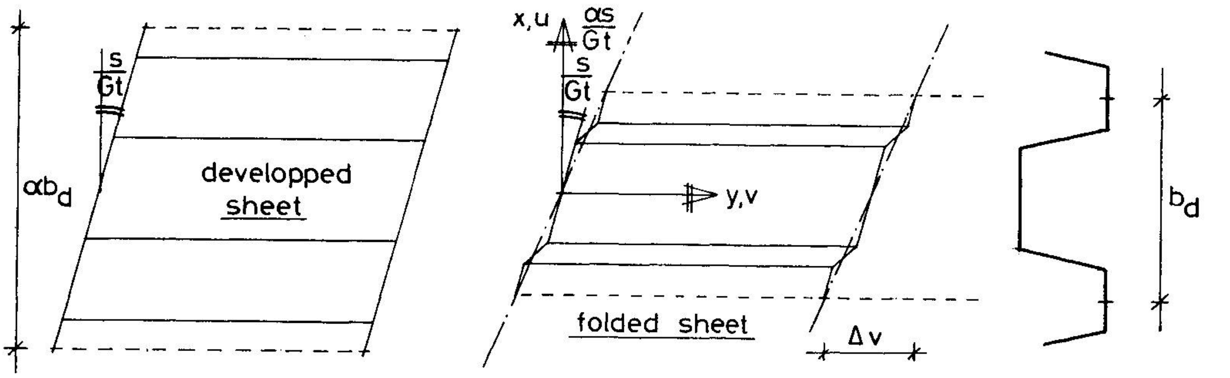

Figure 2: shift and warping due to shear strain

Combination of rigid body rotation and shear strain yields the kinematic condition:

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\alpha s}{Gt} \tag{8}$$

2.2 Distortion

A constant shear force per unit length can only exist if along the profiled sheet edges in every flange and web the shear force is transmitted to an edge member. This will hardly ever be possible in practice. Usually profiled sheeting is attached at one side to a supporting structure. In that situation the uniform distribution of shear stresses is disturbed, because the entire shear force over a cross section in the vicinity of fasteners is concentrated at one side of the sheet (figure 3). This causes distortion of the cross section, mainly due to inextensional bending.

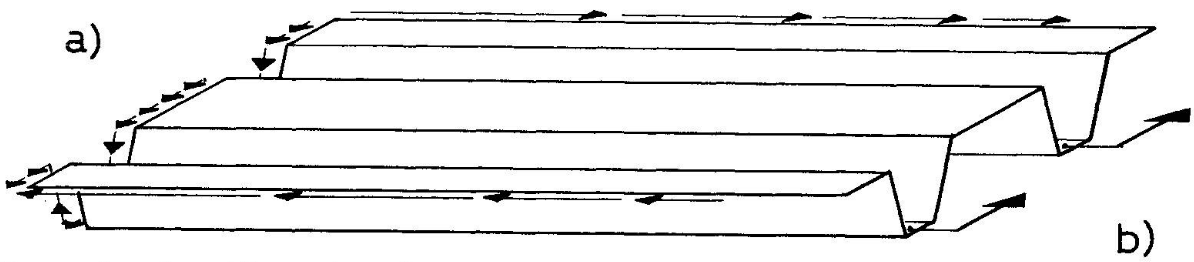

Figure 3: a) uniform shearforce
b) shearforce concentrated in bottom flanges

Further considerations will be confined to sheeting that is attached to supporting members identically in every corrugation, where distortion of all corrugations is equal. Distortion of a cross section now can be composed of two basic deformation modes (figure 4). Displacements in x-direction are denoted u_o for top flanges and u_u for bottom flanges, which means:

$$u_o + u_u = u_1 = 0 \quad \text{for basic mode 1 and}$$

$$u_o - u_u = u_2 = 0 \quad \text{for basic mode 2}$$

A top view of the deformed corrugation (figure 4) shows, that individual plate elements curve in their own plane. Initially plane cross sections do not remain plane and distortion always occurs simultaneously with warping.

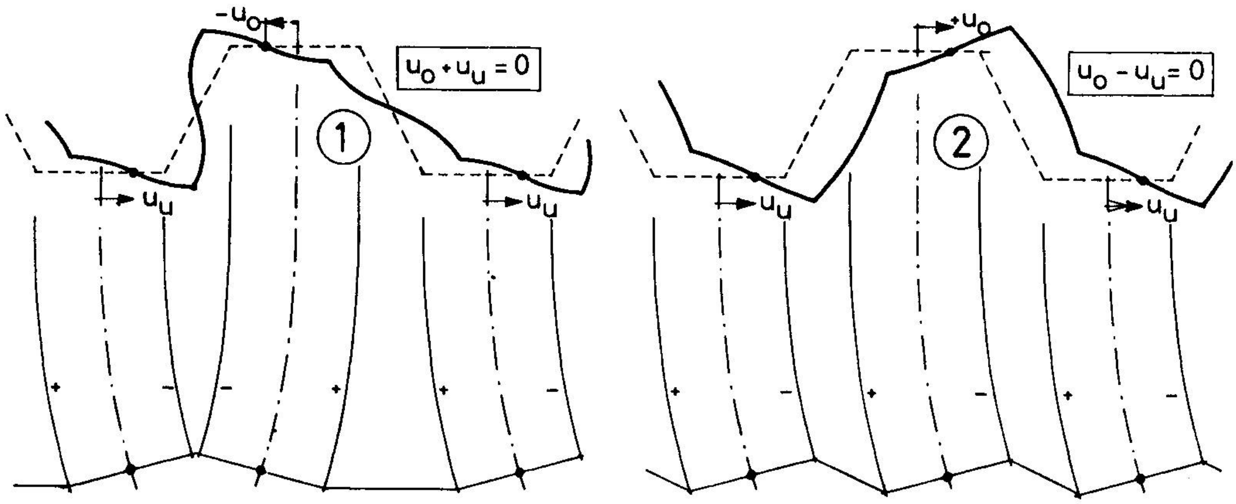

Figure 4: basic modes of distortion and warping

Distortion of corrugations can be exposed by a vector function $\underline{u}$, that contains two displacement functions of y:

$$\underline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} u_o \\ u_u \end{bmatrix} \tag{9}$$

The relation between inextensional bending and warping of a cross section |2| leads to the following differential equations:

$$(K^{-1}J)(K^{-1}J)\, \underline{u}^{IV} + 4\, \beta^4\, \underline{u} = \underline{0} \tag{10}$$

where:

$$\underline{u}^{IV} = \frac{d^4}{dy^4}\, \underline{u} \quad , \quad \beta = \frac{1}{b_d} \sqrt{\frac{t}{h} \sqrt{\frac{3}{1-\upsilon^2}}} \tag{11}$$

$$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 2(\alpha_o - \alpha_u) \end{bmatrix} \tag{12}$$

$$J = \begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22} \end{bmatrix} = \begin{bmatrix} \frac{\eta_o + \eta_u}{2} & \frac{\eta_o - \eta_u}{2} \\ \frac{\theta_o - \theta_u}{2} & \frac{\theta_o + \theta_u}{2} \end{bmatrix} \tag{13}$$

Matrix J contains a number of non-linear combinations of dimensionless shape parameters:

$$\eta_o = \alpha_o^3 + \alpha_w \alpha_o^2 - \tfrac{1}{2}\, \alpha_o \alpha_w \alpha_u \qquad \eta_u = \alpha_u^3 + \alpha_w \alpha_u^2 - \tfrac{1}{2}\, \alpha_o \alpha_w \alpha_u \tag{14}$$

$$\theta_o = \alpha_o^3 + \alpha_w \alpha_o^2 + \tfrac{1}{2}\, \alpha_o \alpha_w \alpha_u \qquad \theta_u = \alpha_u^3 + \alpha_w \alpha_u^2 + \tfrac{1}{2}\, \alpha_o \alpha_w \alpha_u \tag{15}$$

Notice, that $(\eta_o - \eta_u) = (\theta_o - \theta_u)$ so that J is a symmetrical matrix. For corrugations with all flanges of equal width, where $(\alpha_o - \alpha_u) = 0$, also K is a symmetrical matrix.

The general solution for $\underline{u}$ of the differential problem can be written by means of two eigen vectors and two hyperbolic periodic functions:

$$\underline{u} = \begin{bmatrix} 1 \\ e_1 \end{bmatrix} f_1 + \begin{bmatrix} e_2 \\ 1 \end{bmatrix} f_2 = \begin{bmatrix} 1 & e_2 \\ e_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{16}$$

where:

$$f_1 = \cosh\frac{\pi y}{\ell_1}\left(A_1\cos\frac{\pi y}{\ell_1} + A_2\sin\frac{\pi y}{\ell_1}\right) + \sinh\frac{\pi y}{\ell_1}\left(A_3\cos\frac{\pi y}{\ell_1} + A_4\sin\frac{\pi y}{\ell_1}\right) \tag{17}$$

$$f_2 = \cosh\frac{\pi y}{\ell_2}\left(B_1\cos\frac{\pi y}{\ell_2} + B_2\sin\frac{\pi y}{\ell_2}\right) + \sinh\frac{\pi y}{\ell_2}\left(B_3\cos\frac{\pi y}{\ell_2} + B_4\sin\frac{\pi y}{\ell_2}\right) \tag{18}$$

$$\ell_1 = \frac{7}{3} b_d \sqrt{\frac{h}{t}} \sqrt[4]{\frac{\eta_o\theta_u + \eta_u\theta_o}{2}} \left(\zeta - \sqrt{\zeta^2 - 1}\right) \tag{19}$$

$$\ell_2 = \frac{7}{3} b_d \sqrt{\frac{h}{t}} \sqrt[4]{\frac{\eta_o\theta_u + \eta_u\theta_o}{2}} \left(\zeta + \sqrt{\zeta^2 - 1}\right) \tag{20}$$

$$\zeta = 1 + \frac{\{(\eta_o - \eta_u) - (\alpha_o - \alpha_u)(\eta_o + \eta_u)\}^2}{\eta_o\theta_u + \eta_u\theta_o} \tag{21}$$

$$e_1 = -e_2 \left\{ \frac{(\eta_o + \eta_u)}{(\theta_o + \theta_u) - 2(\alpha_o - \alpha_u)(\theta_o - \theta_u)} \right\} \tag{22}$$

$$e_2 = -(\alpha_o - \alpha_u)\left\{ 1 - \sqrt{1 + \frac{(\theta_o + \theta_u) - 2(\alpha_o - \alpha_u)(\theta_o - \theta_u)}{(\alpha_o - \alpha_u)^2(\eta_o + \eta_u)}} \right\} \tag{23}$$

For corrugations with all flanges of equal width it is obvious that $\zeta = 1$ and $\ell_1 = \ell_2$; for all other kinds of profiled sheeting $\ell_1 < \ell_2$.
Hyperbolic functions f_1 and f_2 represent the fact, that distortion is caused locally and vanishes rapidly going along the foldlines. Characteristic lengths ℓ_1 and ℓ_2 indicate the area near fastening arrangements, where distortion and warping have impact. Beyond that range sheeting is not notably affected and a uniform distribution of shear stresses is achieved. The magnitude of distortion depends on the constants of integration, which can only be solved by considering profiled-edge conditions.

2.3 Boundary conditions at profiled edges

As a result of warping, according to figure 4, membrane stresses arise: longitudinal stresses linearly over each strip and transverse stresses parabolically over each strip (figure 5).

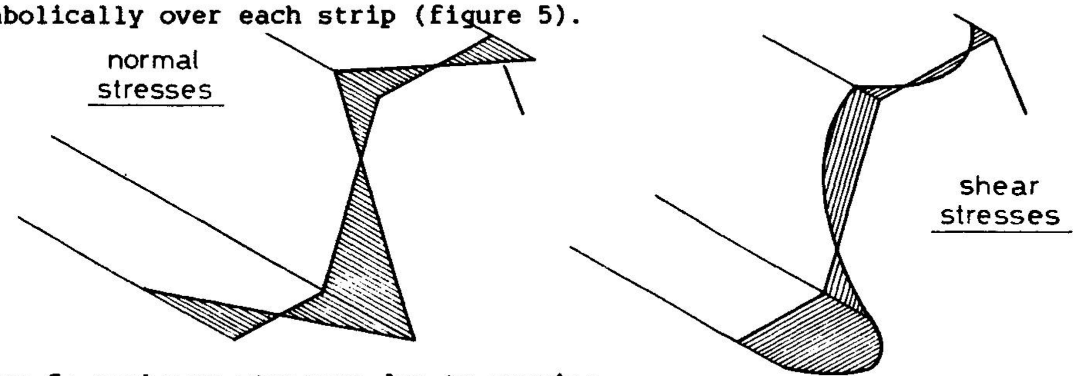

Figure 5: membrane stresses due to warping

Longitudinal stresses can be represented by longitudinal forces, concentrated in the foldlines |3|. Thus one pair of forces N_{ow} is found for every top flange, forming a bimoment $N_{ow}b_o$, and one pair of forces N_{uw} for every bottom flange, forming a bimoment $N_{uw}b_u$ (figure 6).

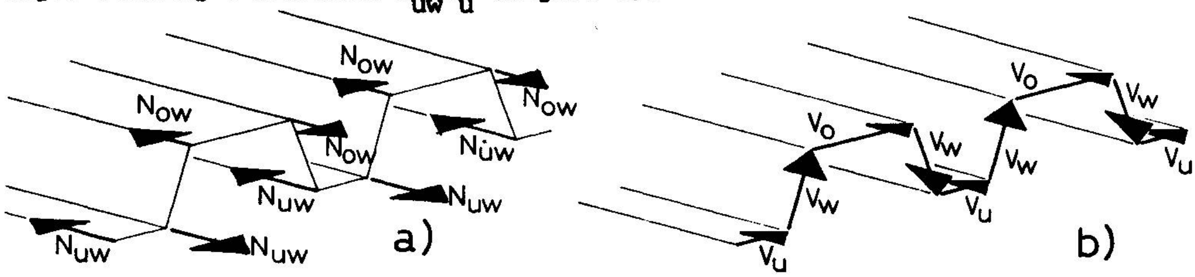

Figure 6: a) internal longitudinal forces concentrated in the foldlines
b) internal transverse forces concentrated in the strips

In a similar way transverse stresses can be represented by transverse forces V_o, V_w, V_u concentrated in topflanges, webs and bottom flanges respectively. The sum of transverse forces over one corrugation projected in-plane is constant and equal to the uniform shear force per pitch, hence:

$$V_o + 2V_w \cos\Theta + V_u = s.b_d \tag{24}$$

The relation between concentrated forces and distortion is given by:

$$\underline{m} = -Et\,\frac{b_d^3}{12}\,J\,\underline{u}^{II} \tag{25}$$

$$\underline{q} = -Et\,\frac{b_d^3}{12}\,J\,\underline{u}^{III} = \underline{m}^{I} \tag{26}$$

where :

$$\underline{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} N_{ow}\,.\,b_o \\ N_{uw}\,.\,b_u \end{bmatrix} \tag{27}$$

$$\underline{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} V_o - b_o.V_w/\ell_w \\ V_u - b_u.V_w/\ell_w \end{bmatrix} \tag{28}$$

These expressions must be used in order to satisfy equilibrium at profiled edges. For example, at the very end of a diaphragm with overhang no longitudinal forces nor transverse forces are transmitted, which means mathematically: $\underline{m} = \underline{0}$, $\underline{q} = \underline{0}$ or rather $\underline{u}^{II} = \underline{0}$, $\underline{u}^{III} = \underline{0}$ at the end. Thus four constants of integration are solved per edge.
Along edges, where deformation is prevented, compatibility conditions must be satisfied. If distortion is prevented at the edge, the condition is mathematically: $\underline{u} = \underline{0}$.
Warping of a cross section is related to distortion on one hand and shear strain on the other hand; if the edge is forced to remain plane (edge welded to a beam), both modes of warping must compensate each other, mathematically formulated:

$$\underline{u}^{I} + \left(\frac{\alpha s}{Gt} - \frac{s}{Gt}\right).\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \underline{0} \text{ at the edge} \tag{29}$$

Compatibility conditions must also be used, when two diaphragms A and B are continuously connected to each other; over the connection line conditions are

for distortion: $$\underline{u}_A = \underline{u}_B \tag{30}$$

and warping: $$\underline{u}_A^I + (\alpha-1)\,\frac{s}{Gt}\begin{bmatrix}2\\0\end{bmatrix}_A = \underline{u}_B^I + (\alpha-1)\,\frac{s}{Gt}\begin{bmatrix}2\\0\end{bmatrix}_B \tag{31}$$

Warping related to distortion ($\underline{u}^I$) generally overwhelms warping related to shear strain. For cross sections, where distortion is not entirely prevented, the latter may therefore be simplified to:

for warping: $$\underline{u}_A^I = \underline{u}_B^I \tag{32}$$

2.3 Deformation of a shear diaphragm

As a result of distortion, supporting members translate in x-direction with respect to the sheeting. This translation, denoted by û, is equal to the displacement u_u, if sheeting is fastened through bottom flanges, hence:

$$\hat{u} = u_u = [\,\tfrac{1}{2}\ \ -\tfrac{1}{2}\,]\ \underline{u} \tag{33}$$

Two supporting members at the profiled edges with intermediate span b (figure 7) translate in opposite direction, showing a parallel shift $\Delta u = 2\ \hat{u}$. The parallel shift per unit length is:

$$\frac{\Delta u}{b} = \frac{2\ \hat{u}}{b} \quad \text{where} \quad \frac{\Delta}{b} = \frac{\partial}{\partial y} \tag{34}$$

This is shear deformation of a diaphragm due to distortion. The latter in combination with the previous result of 2.1 yields the complete kinematic condition for profiled sheeting in shear:

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{\alpha s}{Gt} + \frac{2\ \hat{u}}{b} \tag{35}$$

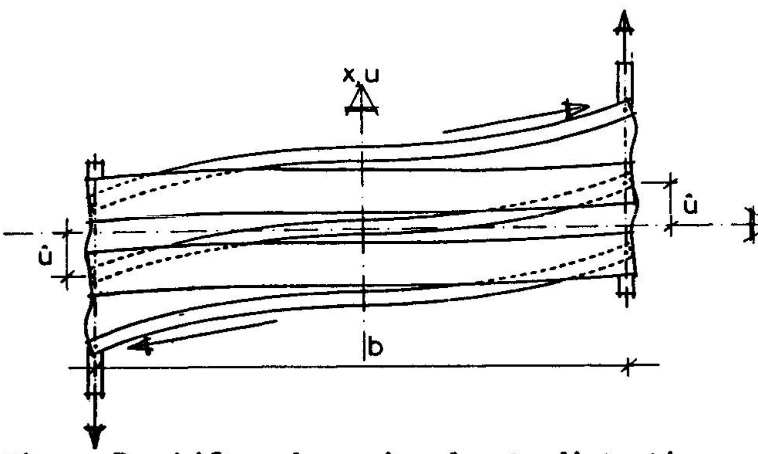

Figure 7: shift and warping due to distortion

3. INFLUENCE OF WARPING RESTRAINT

Shear deformation $\Delta v/a$ of the diaphragm of figure 8 is found to be:

$$\frac{\Delta v}{a} = \frac{\alpha s}{Gt} + \frac{2\ \hat{u}}{b} \qquad \text{where : } s = \frac{Q}{b} \tag{36}$$

and $$\hat{u} = 6\,\frac{s b_d}{Et} \sum_{i=1,2} \left(\frac{\ell_i}{\pi b_d}\right)^3 F_i D_i W_i \tag{37}$$

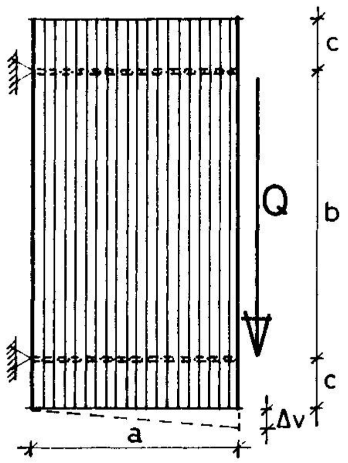

Figure 8: diaphragm with overhang

Sheeting is fastened to purlins through bottom flanges, therefore:

$$F_1 = \frac{1 - e_1}{1 - e_1 e_2} \; \frac{\theta_o + \eta_b e_2}{\eta_u \theta_o + \eta_o \theta_u} \qquad (38)$$

$$F_2 = \frac{1 - e_2}{1 - e_1 e_2} \; \frac{\eta_o + \theta_o e_1}{\eta_o \theta_u + \eta_u \theta_o} \qquad (39)$$

Distortional restraint is represented by the term D_i; For a type of fastening, where distortion is free $D_i = 1$, otherwise $D_i < 1$.

The influence of span b and overhang c (providing warping restraint) is expressed by the term W_i, in general form :

$$W_i = \frac{1}{4} \frac{(\cosh\frac{\pi b}{\ell_i} - \cos\frac{\pi b}{\ell_i})(2 + \cosh\frac{2\pi c}{\ell_i} + \cos\frac{2\pi c}{\ell_i}) + (\sinh\frac{\pi b}{\ell_i} - \sin\frac{\pi b}{\ell_i})(\sinh\frac{2\pi c}{\ell_i} - \sin\frac{2\pi c}{\ell_i})}{(\cosh\frac{\pi b}{\ell_i}\sinh\frac{2\pi c}{\ell_i} + \cosh\frac{2\pi c}{\ell_i}\sinh\frac{\pi b}{\ell_i}) - (\cos\frac{\pi b}{\ell_i}\sin\frac{2\pi c}{\ell_i} + \cos\frac{2\pi c}{\ell_i}\sin\frac{\pi b}{\ell_i})} \qquad (40)$$

If b and 2c $\ll \frac{\ell_i}{\pi}$: $W_i \approx 3 \left(\frac{\ell_i}{\pi b_d}\right) \left(\frac{b_d}{b}\right) \left(\frac{b}{b + 2c}\right)^3$ (41)

If c is taken zero, Bryan's solution |4| for short diaphragms is found and (37) changes into:

$$\hat{u} = 18 \frac{s b_d}{Et} \frac{b_d}{b} \sum_{i=1,2} \left(\frac{\ell_i}{\pi b_d}\right)^4 F_i D_i \qquad (42)$$

In normal practice however, span b will be larger than both ℓ_1 and ℓ_2, and (40) can be reduced to :

$$W_i = \frac{1}{4} \left(1 + \frac{1 + \cos\frac{2\pi c}{\ell_i}}{e^{2\pi c/\ell_i}} + \frac{1 - \sin\frac{2\pi c}{\ell_i}}{e^{2\pi c/\ell_i}} \right) \qquad (43)$$

$W_i = 1$ for $c = 0$ (edge purlin) and $W_i = 0.25$ for $c \gg \ell_i$ (intermediate purlins). Obviously $\hat{u}$ is now independent of span b; the relation between displacement $\hat{u}$ and force per fastener $s b_d$ is constant for a given corrugation and a given overhang, which can be simulated by springs. Then rigidity of springs is proportional to $t^{2.5}$.

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