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Elastically Braced Light Gauge Beams with Open Sections

Poutres à parois minces et section ouverte tenues latéralement de façon élastique

Berechnung von dünnwandigen Pfetten mit offenen Querschnitten

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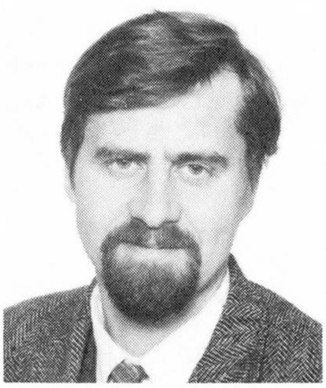

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SUMMARY

Light gauge steel purlins have been studied both theoretically and experimentally. The classical equations for bending-torsion problems have been solved numerically with the finite-difference method regarding the bracing structure as elastic springs acting against torsion and sidesway and taking the nonlinear effects into account. Simple design formulae have been derived taking account of local and global buckling of the purlin. Structures that can be analyzed are both purlins under gravity load or wind suction and purlins in compression with one or both flanges braced. Both elastic and inelastic design are considered.

RÉSUMÉ

Des pannes à parois minces en acier ont été étudiées aussi bien en théorie qu'expérimentalement. Les équations classiques propres aux problèmes de déversement ont été résolues numériquement par la méthode des différences finies. La structure destinée à stabiliser la poutre contre la torsion et le déversement a été représentée par des ressorts élastiques. Le comportement non-linéaire a été pris en compte. Des formules simples de dimensionnement tenant compte du voilement local et de la stabilité globale de la poutre sont proposées. Les structures que l'on peut analyser sont aussi bien les pannes soumises aux charges de gravité, à l'aspiration du vent que les pannes soumises à la compression, avec une aile ou les deux tenues latéralement. L'analyse peut être élastique ou non-élastique.

ZUSAMMENFASSUNG

Dünnwandige Pfetten sind theoretisch und experimentell untersucht worden. Die klassischen Beziehungen für Biegung und Torsion wurden numerisch mit der Differenzenmethode gelöst. Der Effekt der stützenden Konstruktion wurde als elastische Federkonstanten angenommen. Nichtlineare Effekte sind beachtet. Einfache Berechnungsregeln, die örtlichem Beulen und der Gesamtstabilität Rechnung tragen, werden vorgestellt. Die untersuchten Konstruktionen sind sowohl Pfetten unter Eigengewicht oder Windsog als auch Pfetten unter Druck mit einem oder beiden Flanschen gestützt. Elastische sowie nicht elastische Bemessung werden betrachtet.

1. INTRODUCTION

A thin walled beam with an open cross section shown in fig. 1 has the shear centre often out of the loading plane. Thus it is subjected to torsion and in the case of nonsymmetry which is affected by local buckling also to bi-axial bending. Owing to the weak torsional stiffness the warping stresses due to torsion can be of the same order as those due to bending if the beam is free to rotate. The Z- and C-purlins in the roof or wall structures are often fixed to the sheeting with mechanical fasteners which give an elastic restraint to the purlins against rotation and sidesway. This causes a complex problem for exact solutions. The solution is in this work obtained in accordance with classical theory of elasticity [1], [2].

The analytical solutions have formed the base for the derivation of design formulae. Despite of the fact that similar studies have earlier been made in the 70'ies, there has been a lack of models suitable for engineering calculations taking into account the partial restraint of sidesway and rotation. In the beginning of 1980's some models [3, 4, 5] have been presented taking into account the torsional effect of the bracing structure mostly applied to structures subjected to wind uplift. In this work a model is presented where also the partial restraint of the sidesway of the flange in compression in the case of soft material between the purlin and the sheeting structures subjected to gravity load, wind uplift and structures in compression are included. The calculation model is also extended to give a tool to take into account the inelastic reserve of the bending capacity in continuous purlin systems.

2. ANALYTICAL SOLUTION

For an unsupported beam-element we have the formulas

$$EI_y \cdot u^{IV} = q_x \tag{1a}$$

$$EI_x \cdot v^{IV} = q_y \tag{1b}$$

$$EI_\omega \cdot \theta^{IV} - GI_d \cdot \theta'' = m \tag{1c}$$

where I_x, I_y are the moments of inertia for the effective area (mm^4)
I_ω is the warping constant for the effective area (mm^4)
I_d is the torsion constant for the gross area (mm^4)
q_x, q_y are the components of the applied load q (N/mm)
$m = q \cdot e$ is the torque (Nmm/mm)
E is the modulus of elasticity. (N/mm^2)

The elastic support is according to Vlasov [1] regarded as external forces acting in the shear centre:

$$\begin{aligned} \bar{q}_x &= -k_x \cdot u_H \\ \bar{q}_y &= -k_y \cdot v_H \\ \bar{m} &= -k_\theta \cdot \theta_H + (h_x - a_x) \cdot \bar{q}_y - (h_y - a_y)\, \bar{q}_x \end{aligned} \tag{2}$$

where k_x, k_y are the stiffnesses of the foundation against sidesway (N/mm/mm)
k_θ is the stiffeness of the foundation against rotation (Nmm/mm/rad)
u_H, v_H, θ_H are the deformations in the supporting point (mm).

In the analysis the forces causing instability have been expressed in the terms derived by Roik et. al. [2]:

$$
\begin{aligned}
\tilde{q}_x &= [N(u' + a_y \cdot \theta)']' - (M_x\,\theta)'' \\
\tilde{q}_y &= [N(v' - a_x \cdot \theta')]' - (M_y\theta)'' \qquad (3) \\
\tilde{m} &= a_y\,(N \cdot u')' - a_x\,(N \cdot v')' - M_x \cdot u'' - M_y \cdot v'' \\
&\quad + [(r^2\,N + 2\,\beta_y\,M_x - 2\,\beta_x\,M_y)\,\theta']' \\
&\quad - [q_x\,(e_x - a_x) + q_y\,(e_y - a_y)]\theta
\end{aligned}
$$

where $r^2 = \dfrac{I_x + I_y}{A} + a_x^2 + a_y{}^2$

$$\beta_x = \frac{U_y}{2\,I_y} - a_x \quad ; \quad U_y = \int_A x^3\,dA + \int_A y^2\,x\,dA$$

$$\beta_y = \frac{U_x}{2I_x} - a_y \quad ; \quad U_x = \int_A y^3\,dA + \int_A x^2\,y\,dA$$

N is the normal force (N).

By substituting the formulas (2) and (3) to the right side of the equation (1) we get three differential equations that are coupled and suitable for nonlinear analysis with step by step solution. They are solved with the difference-method. Calculations have been made for a number of one-bay and two-bay structures with a computer program specially made for these studies.

3. A PROPOSAL FOR DESIGN FORMULAE

3.1 General

The design formulae are derived based on computer calculations of the combined expressions (1), (2) and (3) for Z- and C-profiles and are based on the assumptions that the transversal load is acting in the direction of the web and that the lateral support is acting perpendicular to the web.

The derivation of the formulae is done outgoing from the idea also used in other similar calculation models [3, 4, 5] to solve the problem by calculating the critical buckling force for a column formed by the compressed flange and a part of the web (fig. 2). The column is on an elastic foundation. The stiffness coefficients of the foundation k_x and k_θ are determined experimentally.

3.2 Purlin under bending

3.2.1 Purlin with both flanges free

$$\sigma_{el1} = \frac{\pi^2 E}{\lambda^2} \cdot k \quad (N/mm^2) \tag{4}$$

where $k = \dfrac{e_x}{\| e_x + \frac{m}{3} \|} < 1$

$$\lambda = l/\sqrt{\frac{I_1}{A_1}}$$

A_1 is the area of the column (mm^2)
I_1 is the moment of inertia about the axis y - y for the section of the column (mm^4)
l is the length of the span (mm)
e_x is the distance between the web and the loading point (mm)
m is the distance between the web and the shear centre (mm)

3.2.2 Purlin with one flange braced

For laterally supported beams the critical buckling stress is determined as:

$$\sigma_{el2} = \frac{2}{A_1} \sqrt{\frac{E\, I_1}{\delta}} \cdot k > \sigma_{el1} \tag{5}$$

where δ is the lateral deformation due to the unit force in the supporting point H.

The lateral deformation δ depends on the two coefficients of the stiffness k_θ and k_x as follows:

pressure load:

$$\delta = \frac{30 \cdot e_y^2}{10 \cdot e_y^2 \cdot k_x + 3 \cdot k_\theta} \quad (mm/N/mm) \tag{6}$$

uplift load:

$$\delta = \frac{e_y^2}{4 \cdot k_\theta} \quad \text{for } k_x > 0{,}01 \ (N/mm^2) \tag{7}$$

Expression (6) is derived on the assumption that two springs work parallel-coupled together and in the case of uplift load that no sidesway occurs in the tension flange (7). The numbers 4, 10 etc are determined as results from analysis.

3.2.3 Both flanges braced

In the case of both flanges being braced the rotation stiffness of the bracing structure to the tension flange is added to that of the flange in compression reduced with the factor $(e_y^c/e_y^t)^2$.

Having calculated the critical stress σ_{el} (σ_{el1} or σ_{el2}), the design strength f_{cd} is obtained from the ECCS curve c for flexural buckling with $\lambda = \sqrt{f_y/\sigma_{el}}$. The reduction of the bending capacity due to lateral buckling is then $r = f_{cd}/f_y$.

The bending capacity M_D is calculated from:

$$M_D = \eta \cdot f_{cd} \cdot W_e \tag{8}$$

where η = 0,9 for structures braced along one flange and 1.0 with both flanges braced taking into account the effect of shear deformation of the profile [3]. W_e is determined taking into account the local buckling in accordance with fig. 3.

3.3 The inelastic reserve M_{pl}

The inelastic reserve of the bending capacity of the profile is calculated for an edge strain that is three times the yielding strain, that is putting $f_y' = 3 \cdot f_y$ in the formulae in the fig. 3 to calculate the effective area A_e in the inelastic stage. This method is based on the notation (Rockey et. al.) that the formulae for the effective area gives a good agreement using the edge strain in the post-yielding stage for plates in compression. Though this gives an overestimating of the inelastic reserve for both trapezoidal sheets and sections (see fig. 4), it has been assumed in accordance with [6] that the strain in the first plastic hinge at the support in a 2-bay beam is $\varepsilon_m = 3 \cdot \varepsilon_y$ when the bending capacity is reached in the field. For purlins the inelastic reserve is reduced in the same manner as in the elastic stage with the coefficient $r = f_{cd}/f_y$. For a 2-span beam the ultimate load is thus:

$$q_u = \frac{M_D}{l_2} \left[(4 + 2 \cdot j \cdot m) + 4\sqrt{1 + j \cdot m}\right] \tag{9}$$

where M_D is the moment capacity in the field reduced with the factor r
- j is the inelastic reserve at the support calculated for $\varepsilon_m = 3 \cdot \varepsilon_y$; $j = M_{pl}/M_D$
- m is the relation between the moment capacity at the support and in the field
- l is the span.

3.4 Purlin in compression

For the case of structure in compression the critical stress is in a similar way calculated for the flange with the weaker restraint by putting (10) into (5)

$$\delta = \frac{1}{k_x} + \frac{e_y^2}{k_\theta} \cdot 10 \tag{10}$$

The capacity for the compression force is

$$N_d = f_{cd} \cdot A_e \tag{11}$$

with f_{cd} obtained from the ECCS curve c and A_e is the effective area of the whole section.

4. TEST RESULTS FOR PURLINS SUBJECTED TO BENDING, COMPARISON WITH CALCULATED VALUES

In table 1 results are given for tests on one-bay purlins (Z and C profiles) under pressure load and uplift load.

Table 1. Results from a test series at the Technical University of Helsinki in 1982 - 83.

Test nr	Profile h/t	Yield strength f_y N/mm^2	Steel thickness t_{Fe} mm	Span l (mm)	Load direction	Fasteners (screws)	Bracing structure	Material between purlin and bracing	k_θ N	k_x N/mm^2	Bending capacity (kNm) Plane bending without lateral buckling	Difference method	Calculation model	Test results
1	Z 200/2.5	341	2.37	7200	pressure	ø 6.3 cc 750 mm	Trap.sheet 45/0.7	min.wool 50 mm	400	0,01	17.8	8.2	11.1	10.6 10)
2	Z 300/3	333	2.98	7200	pressure	ø 6.3 cc 300	Trap.sheet 45/0.7	min.wool 50 mm	400	0.025	52.0	20.7	30.6	31.2
3	Z 200/2	416	1.86	7200	uplift	ø 5.5 cc 300	Trap.sheet 45/0.7	no	2290	0.71	-16.4	-14.0	-10.1	-11.0
4	Z 300/2.5	353	2.39	7200	uplift	ø 6.3 cc 150	Trap.sheet 45/0.7	min.wool 50 mm	400	0.05	-39.1	-19.8	-14.8	-19.6
5	C 100/1.2	387	1.08	7200	pressure	2 ø 5.5 cc 1200	20/0.8	no	290	0.77	2.44	2.49	1.66	2.81
6	C 200/2.5	353	2.39	7200	pressure	2 ø 5.5 cc 1200	20/0.8	no	290	0.77	18.8	12.6	11.7	15.0
7	C 100/1.2	387	1.08	7200	uplift	2 ø 5.5 cc 1200	20/0.8	no	900	0.77	- 2.44	- 2.44	- 1.54	- 2.39
8	C 200/2	433	1.91	7200	uplift	2 ø 5.5 cc 1200	20/0.8	no	900	0.77	-17.6	-11.8	- 7.67	- 7.41

In table 2 results are given for tests on two-bay purlins. The reduction factor is determined for the section at the middle of the span.

Table 2. Results from tests on 2-bay beams. Span is 4200 + 4200 mm. Purlin Z 200/1.5. In tests nr 9 to 15 the profile was overlapped 1400 mm at the midspan. In the test nr 16 the purlin was continous without overlapping.

test	yield strength (N/mm^2)	plate thickness (mm) 2)	load direction	stiffeness coefficients k_x (N/mm^2)	k_θ (Nmm/mm)	ultimate load q kN/m test result	diff.method	calculation model
9	412	1.46	pressure	1.25	1050	5.78	6.46	4.87
10	416	1.63	pressure	1.25	1050	6.89	7.36	5.72
11	420	1.62	uplift	1.25	1050 430 1)	5.91	5.36 1)	5.49 1)
12	412	1.64	pressure	0.17	1220	5.95	6.42	5.99
13	400	1.63	uplift	0.17	1220 470 1)	5.32	5.92 1)	5.38 1)
14	410	1.64	pressure	0.22	1450	5.37 2) 6.43	6.43	5.71
15	422	1.62	uplift	0.22	1450 540 1)	4.48 2)	5.83 1)	5.55
16 3)	278	1.43	pressure	0.44	1530	4.28 (yield at 3.43)	- (yield at 2.97)	2.93 (yield at 2.50)

1) the deflection of the web is included in the rotation θ
2) slipping occured in the seams of the bracing structure leading to failure
3) continous beam without overlapping

5. CONCLUSIONAL REMARKS

A calculating model for a wide range of use has been presented. Tests that have been performed show a satisfactory agreement between tests results and calculated values. The results show also the need of taking into account the side slip of the flange in compression even when it is braced especially if there is material between the purlin and the bracing stucture.

6. REFERENCES

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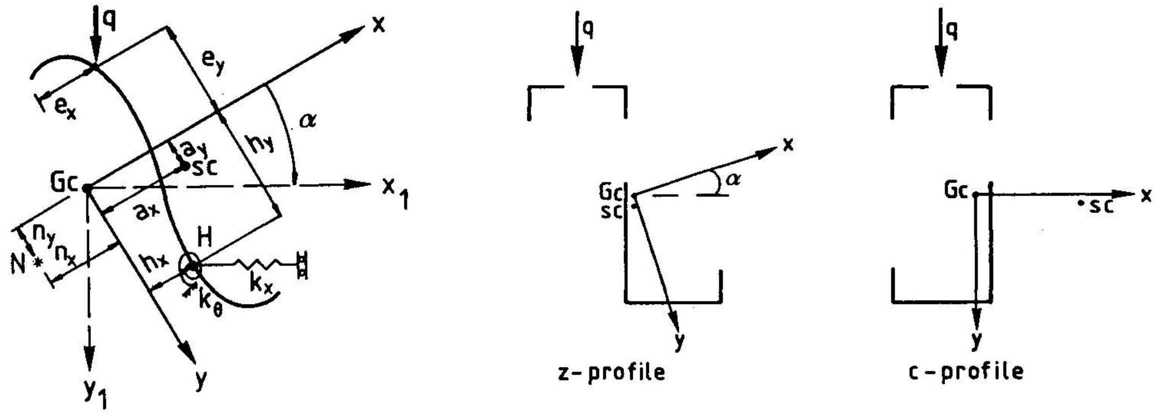


Fig. 1 Thin-walled open cross sections. Gc = gravity centre. Sc = shear centre. x, y are main axes. y_1 is load direction.

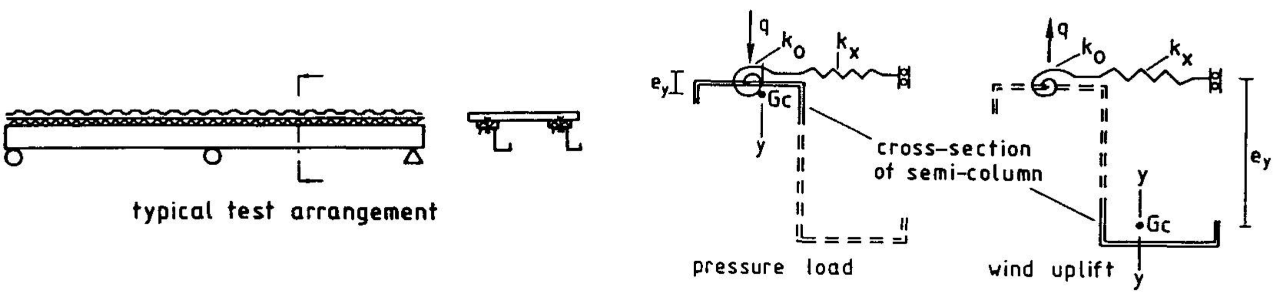


Fig. 2 Real structure and column for which critical stress is determined by means of calculations.

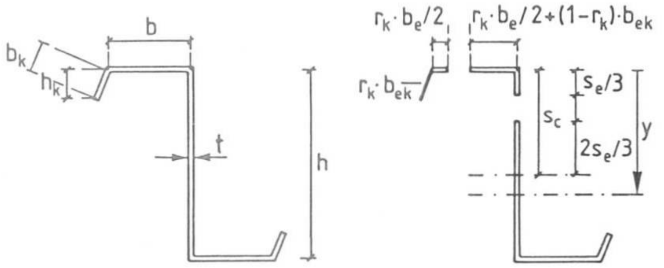


Fig. 3 Calculation of the effective cross-section for a Z-and C-profile.

$$b_e = \frac{1}{\lambda_f} \left(1 - \frac{0.22}{\lambda_f} \right) \cdot b$$

$$\lambda_f = \frac{1.05}{\sqrt{4}} \cdot \frac{b}{t} \cdot \sqrt{f_y/E}$$

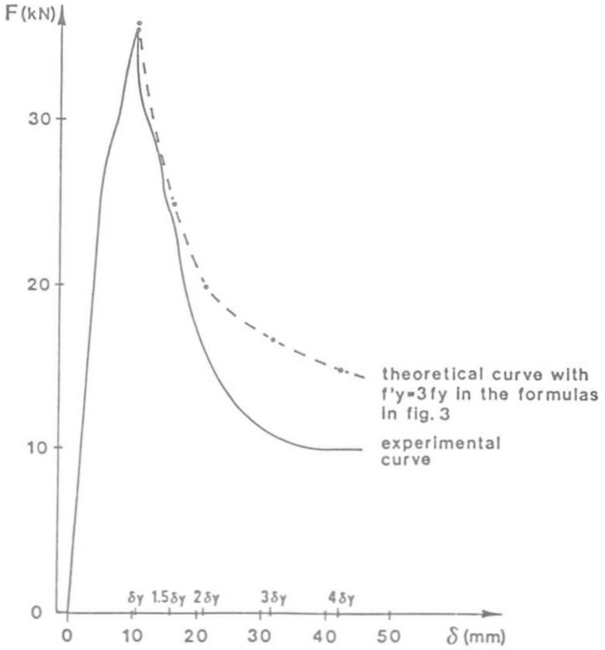
$$b_{ek} = 0.61 \cdot t \cdot \sqrt{E/f_y}$$

$$s_e = \frac{1}{\lambda_w} \left(1 - \frac{0.22}{\lambda_w}\right) \cdot s_c$$

$$\lambda_w = \frac{1.05}{\sqrt{7.81}} \cdot \frac{s_c}{t} \sqrt{f_y/E}$$

$$r_k = 1.49 - 0.6 \cdot \alpha$$

$$\alpha = 1.86 \cdot \sqrt{\frac{f_y \cdot b}{E \cdot t} \sqrt{\frac{b + 2h}{h_k - 2t}}}$$



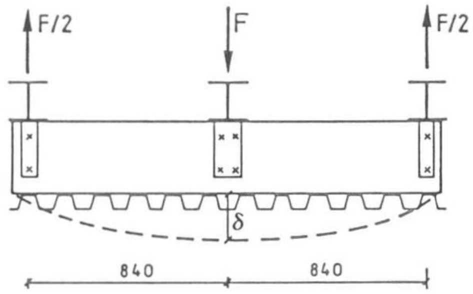


Fig. 4 Inelastic reserve for a Z-section 200/1.5 determined experimentally and theoretically.

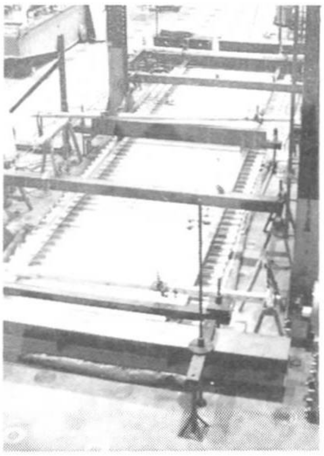


Fig. 5 Test no. 16 with air-bag system.

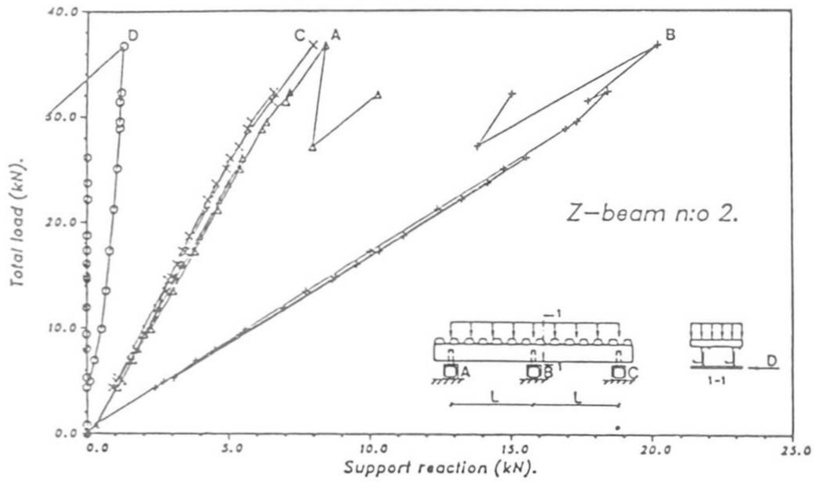


Fig. 6 Results for test no. 16.