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Time-Dependent Shear Transfer of Cracked Reinforced Concrete

Béton fissuré soumis à des charges de cisaillement de longue durée

Das zeitabhängige Schubverhalten von gerissenem Stahlbeton

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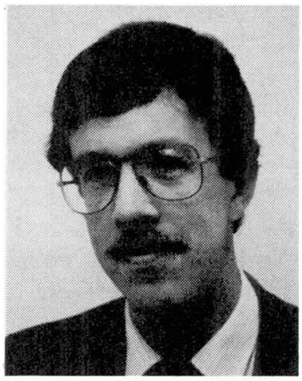

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SUMMARY
Recent experimental results are presented which show the displacement behaviour of a single crack in reinforced concrete. The push-off specimens used were subjected to sustained shear loading. The time-dependent shear transfer has been described quasi-statically with regard to the effects of aggregate interlock according to Walraven's model as well as dowel action of the reinforcing bars. The implementation of the observed phenomena in non-linear finite element programs is demonstrated qualitatively.

RÉSUMÉ
Des résultats expérimentaux récents montrent l'évolution d'une fissure unique dans du béton armé. Les éprouvettes ont été soumises à des charges de cisaillement de longue durée. La résistance au cisaillement dans le temps est considérée comme étant quasi-statique. L'explication se trouve dans l'effet d'interpénétration des granulats suivant le modèle de Walraven ainsi que l'effet de goujon de l'armature. L'interprétation du phénomène observé par un programme non-linéaire d'éléments finis est illustrée.

ZUSAMMENFASSUNG
Neue Versuchsergebnisse zeigen das Verformungsverhalten eines einzigen Risses in bewehrtem Beton. Die Proben wurden durch eine Dauerschubbelastung beansprucht. Die zeitabhängige Schubtragfähigkeit wird quasi-statisch beschrieben. Für eine theoretische Erklärung werden sowohl die Kornverzahnung als auch die Dübelwirkung der Bewehrungsstäbe berücksichtigt. Die Übertragung dieser Phänomene in nicht-lineare Finit-Element-Programme wird qualitativ erläutert.

1. INTRODUCTION

As a result of the increase in scale and complexity of new structures, advanced numerical methods are being used for design purposes. These methods take account of the non-linear behaviour of cracked reinforced concrete. Whereas in the case of bending the behaviour of reinforced concrete has been extensively investigated, there is still a lack of knowledge and modelling in the case of shear forces, especially when the concrete is cracked.
As an example of a heavily loaded complex structure an offshore production platform may be taken which rests on the sea bed. The substructure consists of a caisson subdivided into several compartments by means of concrete shear walls. Usually a dense and high-strength concrete is used with mean cylinder strengths ranging from 40 to 60 N/mm². Due to differential settlements and temperature gradients (for example storage of hot oil in the compartments) cracks may form. Also considerable redistribution may take place causing shear displacements along existing cracks. There is a need to analyse the behaviour of these cracks under various types of loading in order to design safely and cost-effectively.
This paper will deal with experimental investigations carried out on reinforced concrete specimens with a single crack, which were subjected to sustained shear loading. A physical explanation for the observed time-dependent crack displacements is based on Walraven's rough crack model [1] for plain concrete and on existing dowel action formulas for the reinforcing bars crossing the crack plane. These models should be adapted to sustained shear loading conditions. Moreover, attention will be paid to the numerical implementation of this research into finite element programs. This study is supported by the Netherlands Centre for Civil Engineering Research, Codes and Specifications (CUR).

2. STATIC SHEAR STRENGTH

For the actual research program [2] shear loading tests were carried out on push-off specimens similar to those used by Walraven [1]. See Figure 1. The dimensions of the crack plane were 120mm x 300mm. In order to prevent premature failure and to improve the gradual introduction of the external force into the cracked shear plane, the bottom and top of the specimen have been post-tensioned transversely.

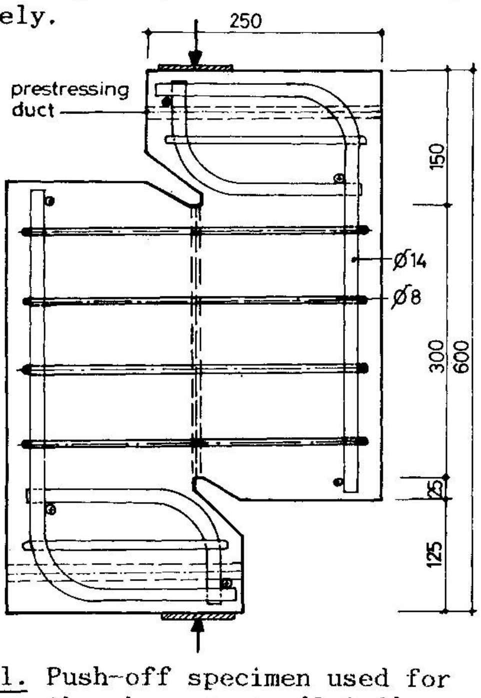

Fig. 1. Push-off specimen used for the shear tests [1,2,3].

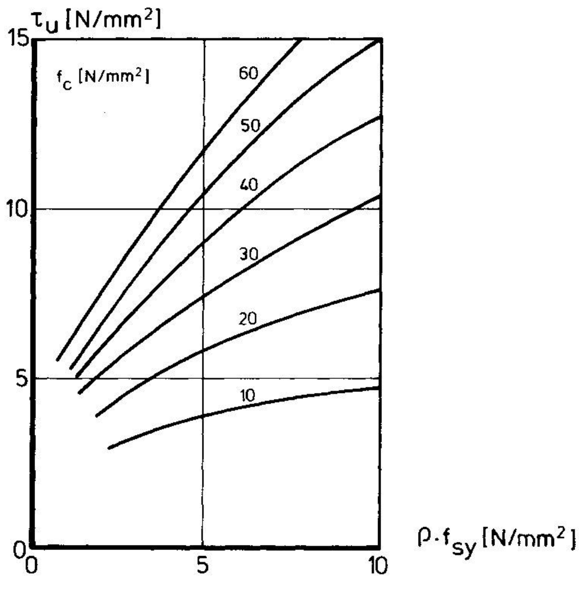

Fig. 2. Static shear strength according to formula (1).

For the series of sustained tests the static shear strength τ_u was taken as a calibration value. A formula for τ_u has been derived based on four sources [1-4]. The initial crack width of the push-off specimens was between 0.01 and 0.10mm. Concrete cylinder strengths varied from 17 to 60 N/mm². River gravel aggregates were used having a maximum particle size D_{max}= 16-32mm. The bar diameter of the stirrups ranged between 8 and 16mm and the value of ρf_{sy} lay between 0.35 and 12.32 N/mm². After analysing a total number of 88 static tests [5] an empirical expression was found:

$$\tau_u = \alpha(\rho f_{sy})^{\beta} \text{ [N/mm}^2\text{]} \qquad (1)$$

in which: $\alpha = 0.878 f_c^{0.406}$
$\beta = 0.167 f_c^{0.303}$ with f_c in [N/mm²]

Formula (1) is shown graphically in Figure 2. The average ratio of measured and calculated τ_u-values is 1.001 with a coefficient of variation of less than 11%.

3. SHEAR TRANSFER IN CRACKED CONCRETE UNDER SUSTAINED LOADING

3.1 Experimental variables

Recently, experiments were carried out on 32 push-off specimens for sustained shear loading conditions. Only a few details of the testing program [2] will be given. The main variables were:

- *cube compressive strength of the concrete*
  f_{cc}=51 or 70 N/mm², which corresponds to cylinder strengths f_c of 43 and 60 N/mm² respectively for an assumed strength ratio f_c/f_{cc}= 0.85. The two mixes contained Portland cement B and glacial river aggregates having a 16mm maximum diameter and a grading curve according to Fuller. Both a medium-strength and a high-strength concrete were investigated. Usually cracks will be initiated in the bond zone between the aggregate particles and the cement matrix. For high-strength concrete cracks were expected to extend predominantly through the aggregates. This should result in different mechanisms of shear transfer. However, no significantly different crack patterns were observed with regard to the percentage of fractured particles in the crack plane [2];
- *normal restraint stiffness*
  Embedded 8mm diameter reinforcing bars were used. The reinforcement ratios were ρ=0.0112, 0.0168 or 0.0224, realised by 4, 6 or 8 stirrups respectively, all perpendicularly crossing the crack plane. The yield strength of the deformed bars was f_{sy}= 460 and 550 N/mm² respectively;
- *initial crack width w_o*
  The initial crack width ranged between w_o= 0.01-0.05 mm;
- *sustained shear stress level τ/τ_u*
  τ=5.7-11.5 N/mm², i.e. 45%-89% of the static shear strength τ_u.

3.2 Testing procedure

The specimens were cured in a fog chamber (19°C: 95% RH) for 22 days. Next, they were stored in the laboratory (20°C: 50% RH). Tests started when the concrete had reached an age of 28 days. Prior to the shear test each specimen was pre-cracked in a vertical position. The average remaining crack width was the initial crack width. After application of the shear loading at a loading rate of about 0.02 N/mm² per second, the displacements parallel and perpendicular to the shear plane were recorded periodically. The load was sustained for at least 91 days. After unloading the displacements were measured during the following 25 days, after which a static failure test was carried out.

3.3 Statistical analysis of test results

Mean values of the measured crack widths w and the parallel displacements s can be presented as functions of the duration of load application t. The overall displacements consist of instantaneous values w_{el} and s_{el} (at t=0 hrs) plus the time-dependent increments $w_c(t)$ and $s_c(t)$. According to Fig. 3, it follows that:

$$t \geqslant 0 \text{ hrs}: \quad w(t) = w_{el} + w_c(t) \quad [\text{mm}]$$
$$s(t) = s_{el} + s_c(t) \quad [\text{mm}] \qquad (2)$$

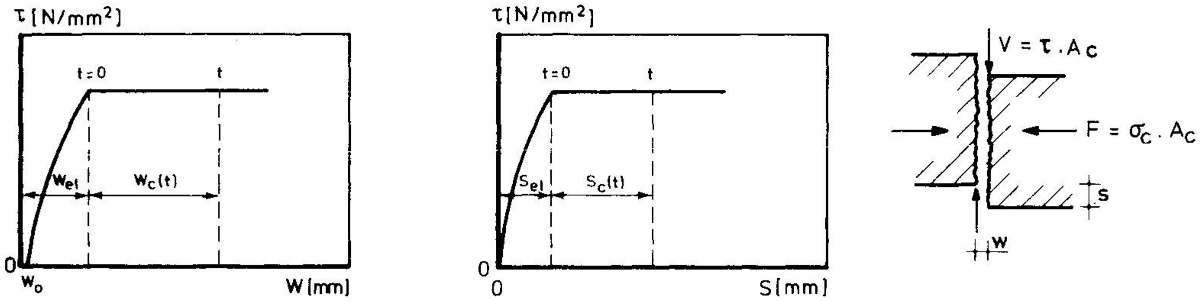

Fig. 3. Definitions of instant and creep displacements as functions of τ and t.

Due to the small displacement increments and their observed scatter, the instantaneous and the time-dependent displacements were statistically described as functions of the experimental variables by multiple regression analysis [2,6]. Also, the effect of scatter of the experimental variables (e.g. variations in concrete strength) on the displacement response could be quantified. Extrapolation to non-tested circumstances is possible to a certain extent. An empirical formula has been derived for the crack width [6]:

$$w(t) = \alpha_1 (\tau/\tau_u)^{\alpha_2} + \alpha_3 (\tau/\tau_u)^{\alpha_4} . \rho f_{sy} \quad [\text{mm}] \qquad (3)$$

with values: $30 \leqslant f_{cc} \leqslant 75$ N/mm²
$4 \leqslant \rho f_{sy} \leqslant 12$ N/mm²
$0.60 \leqslant \tau/\tau_u \leqslant 0.90$ provided that $\tau > 3$ N/mm²

in which $\alpha_1 - \alpha_4$ are non-linear functions of compressive strength and time of load application. A similar formula has been derived for s(t).

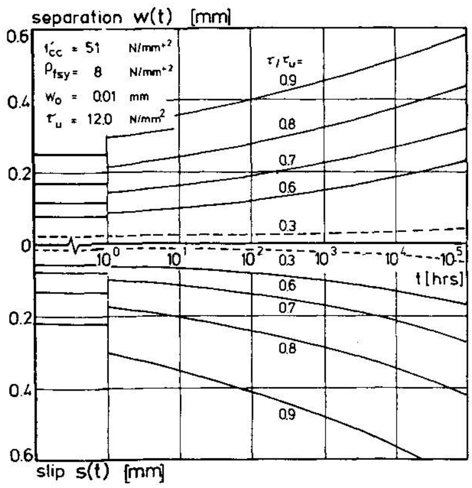

Fig. 4. w(t) and s(t) curves for ρf_{sy}=8 N/mm²; f_{cc}=51 N/mm².

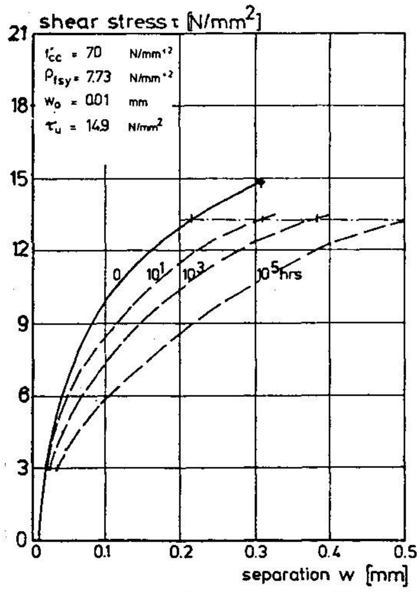

Fig. 5. τ-w(t) curves for ρf_{sy}=7.73 N/mm²; f_{cc}=70 N/mm².

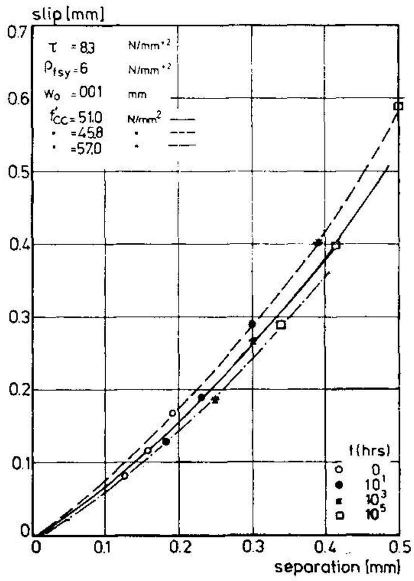

Fig. 6. s(w)-curves for τ=8.3 N/mm²; variation of f_{cc}.

Equation (3) is worked out in Figs. 4 and 5. It can be seen that there is a significant effect of the shear stress level τ/τ_u on the instantaneous and the time-dependent displacements. At low stresses, for instance $\tau/\tau_u = 0.30$, the time-dependent displacements are small within the first 10^4 hours. With the aid of equation (3), creep coefficients $\varphi_w = w_c(t)/w_{el}$ and $\varphi_s = s_c/s_{el}$ can be calculated [6]. Examples of the crack opening curves are given in Fig. 6; a variation of the concrete strength considerably influences the displacement behaviour [2]. For a safe physical basis a model should be provided which accounts for the most likely shear transfer mechanisms, i.e. aggregate interlock and dowel action.

4. PHYSICAL MODELLING OF SHEAR TRANSFER

4.1 Static shear loading

The external shear force V causes a shear stress $\tau = V/A_c$, which is transferred in the crack plane by both dowel action τ_d of the bars and by aggregate interlock τ_a of the opposing crack faces of the plain concrete.

Aggregate interlock
For static shear loading the aggregate interlock mechanism has been described by the rough crack model of Walraven [1]. Concrete is modelled as a two-phase material consisting of rigid spherical aggregate particles embedded in a cement matrix. As far as normal-strength concrete is concerned the preformed crack will run into the matrix along the surface of the aggregate particles.
If a shear stress τ_a is applied to the cracked concrete specimen, the crack opening increases and the rigid spheres of one crack face are pushed into the matrix material of the opposing crack face; see Figure 7. The required normal stress σ_a is obtained by embedded reinforcement or by external restraint rods.

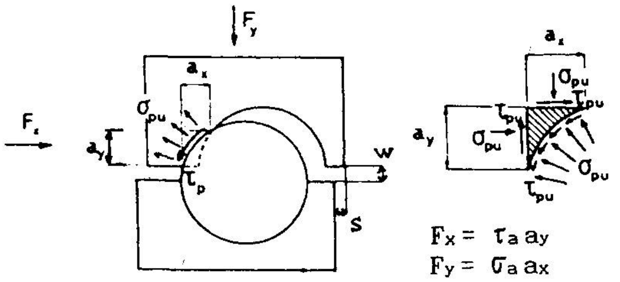

Fig. 7. Contact area between matrix and matrix material [1].

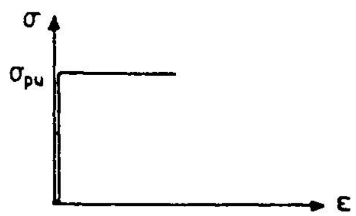

Fig. 8. Rigid-plastic behaviour of matrix material [1].

The total projected contact areas are $A_x = \Sigma a_x$ and $A_y = \Sigma a_y$. For a unit surface area of the crack, the equilibrium condition can be formulated:

$$\tau_a = \sigma_{pu}(A_y + \mu A_x) \quad [N/mm^2]$$
$$\sigma_a = \sigma_{pu}(A_x - \mu A_y) \quad [N/mm^2] \qquad (4)$$

The rough crack model implies that there is a unique relation between the displacements w and s of the crack and the corresponding stresses τ_a and σ_a. The shear stress as well as the normal stress are both functions of A_x and A_y (which can be analytically calculated: for a given mix proportion these areas depend on the shear displacements w and s), of the matrix strength σ_{pu} and of the coefficient of friction μ of the matrix material. It follows that $\tau_{pu} = \mu.\sigma_{pu}$. Empirical values of μ and σ_{pu} were derived from static shear tests on plain concrete push-off specimens [1]:

$$\sigma_{pu} = 6.39 f_{cc}^{0.56} \quad [N/mm^2] \quad \text{and} \quad \mu = 0.40 \quad [-] \qquad (5)$$

Dowel action

The ultimate dowel force F_{du} of a reinforcing bar, which will be reached for a sufficiently large parallel displacement $s > w_o$ of the crack faces, is given by [7]:

$$F_{du} = 1.3 d_{bar}^2 \sqrt{f_{cc} f_{sy}} \quad [N] \tag{6}$$

The maximum shear stress due to n bars is then: $\tau_{du} = n.F_{du}/A_c$. The total shear stress is the sum of the two components τ_a (aggregate interlock) and τ_{du} [3]:

$$\tau = \tau_a + \gamma_d \tau_{du} \quad [N/mm^2] \tag{7}$$

in which $\gamma_d = \tau_d/\tau_{du} = \sqrt{1 - (\sigma_s/f_{sy})^2}$ takes account of the yield criterion [8]. Now, a stepwise overview of the calculation procedure can be given if the crack opening curve is assumed to be known:

- for given crack displacements w and s the corresponding stresses σ_a and τ_a in the plain concrete are computed by means of the equations (4)-(5);
- equations (6)-(7) provide the contribution of the embedded reinforcing bars;
- if $\sigma_s = \sigma_a/\rho > f_{sy}$ then in eq. (7) τ_a should be multiplied by $\gamma_a = f_{sy}/\sigma_s$. This means that an equal reduction of σ_a and τ_a is proposed [3].

4.2 Sustained shear loading

In a first attempt to analyse theoretically the sustained shear tests it is assumed that τ_a and τ_d are both affected by the same damage parameter $\lambda(t) < 1$. This parameter takes account of the influence of the sustained loading period upon the concrete compressive strength.

$$\tau = \lambda(t)(\tau_a + \gamma_d \tau_{du}) \qquad [N/mm^2] \tag{8}$$

Combining the equations (4)-(8) with $\rho = n\pi d_{bar}^2/(4A_c)$ leads to:

$$\tau = \lambda(t) f_{cc}^{0.67}[4.42(A_y + \mu A_x] + 0.84\rho\gamma_d\sqrt{f_{sy}}] \quad [N/mm^2] \tag{9}$$

The values of λ were calculated for the complete test series using the time-dependent crack opening curves - as presented in Fig. 6 - as the input data. For t= 0 the sustained shear stress was just applied; it was found that $\bar{\lambda} = 1.077$ with a coefficient of variation of 8.9%. In fact λ should be 1.0. This calculation discrepancy could be due to the load-controlled application method used [2], wheras Walraven's specimens were loaded displacement-controlled [1].

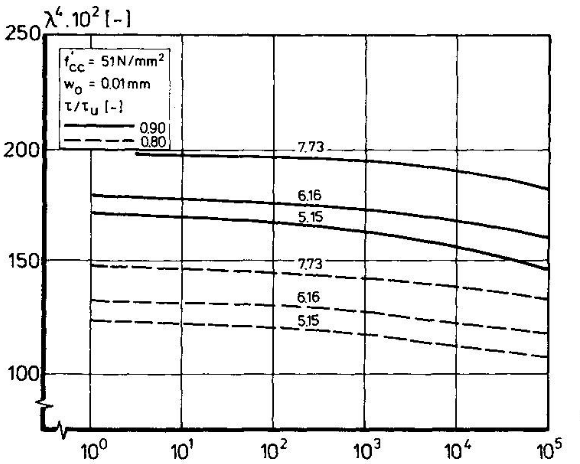

Fig. 9. Development of λ^4 as a function of t.

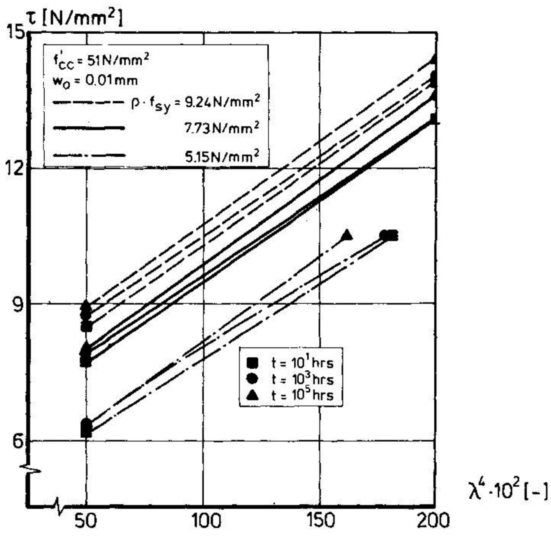

Fig. 10. Development of λ^4 as a function of the sustained shear stress τ.

For f_{cc}= 51 N/mm² and t= 10^0–10^5 an empirical relation was found statistically for the damage parameter with $D_j = C_{1j} + C_{2j}.\ln(t) + C_{3j}[\ln(t)]^2$ for j= 1,2,3 and 4:

$$\lambda(t) = \sqrt[4]{0.01[D_1 + D_2\rho f_{sy} + (D_3 + D_4\rho f_{sy})(\tau/\tau_u)]} \quad [-] \qquad (10)$$

The average ratio of calculated and empirically found λ^4-values is 1,017 with a coefficient of variation of 12.4%. A few examples of the development of λ^4 are presented in Figs. 9 and 10. As a result of the development of $\lambda(t)$ the dowel force will gradually decrease. Note that f_{cc} and μ are kept constant in this analysis. An increase of f_{cc} (due to hydratation) or μ would reduce the λ-values.

5. NUMERICAL IMPLEMENTATION

5.1 Static shear loading

Pruijssers [9] used the measured static crack behaviour for the smeared crack concept. For this purpose one incremental stress-strain relation is needed describing the behaviour of both the uncracked and the cracked concrete sections. Cracks develop in a direction perpendicular to the principal tensile stress σ_{nn}. Once the static tensile strength f_{ct} has been reached, at first a band of microcracks forms. In Fig. 11, ζ is defined as the tension softening factor or the normal retention factor. For the partially cracked area $\zeta = E_t.(\varepsilon_{nn}-\varepsilon_u)/(E.\varepsilon_{nn})$.

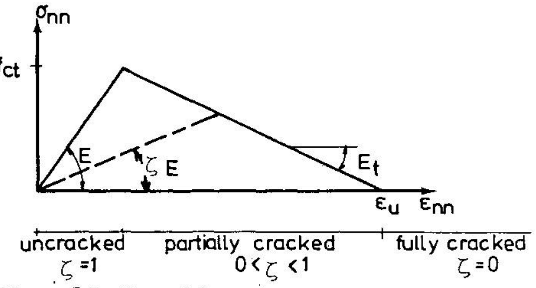

Fig. 11. Tensile stress-strain relation [9].

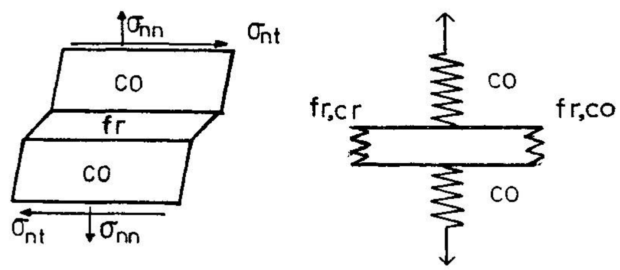

Fig. 12. Rheological model of the fracture zone [9].

The deformation of an element can be modelled by the contribution of the uncracked and cracked concrete sections. See Fig. 12. The fracture zone (fr) is divided into two parts: a fully cracked part (fr,cr) with ζ=0 and an uncracked part (fr,co). Using the model of Figure 11 the incremental stress-strain relation for each section of the element can be described. For the uncracked section (co) and the uncracked part of the fracture zone (fr,co) the incremental stress-strain relations are governed by Young's modulus E, Poisson's ratio ϑ, and ζ. For the cracked part of the fracture zone (fr,cr) the two-dimensional stiffness matrix has four coefficients which can be deduced from the rough crack model of Walraven [1,3,9]. Finally, a stiffness matrix for the complete element can be found.

5.2 Sustained shear loading

Now, the crack displacements will increase depending on the level and the time-duration of the shear loading. In Fig. 12 the gradual weakening of the concrete can be expressed by adding dashpots placed in series with the springs which represent the uncracked and the cracked part of the fracture zone. For the uncracked part of this zone, the material degradation can be formulated by reducing Young's modulus. Concerning the cracked part damage parameters λ_μ and λ_σ, as reported in Section 4.2, can take account of changes in concrete strength and friction coefficient. This converts equation (4) into:

$$\begin{aligned} \tau_a &= \lambda_\sigma.\sigma_{pu}(A_y + \lambda_\mu.\mu A_x) \quad [\text{N/mm}^2] \\ \sigma_a &= \lambda_\sigma.\sigma_{pu}(A_x - \lambda_\mu.\mu A_y) \quad [\text{N/mm}^2] \end{aligned} \qquad (11)$$

The development of the theoretical model will be a subject of further study.

6. CONCLUSIONS

Some conclusions can be drawn:

- Cracked concrete exhibits time-dependent crack sliding and crack opening under sustained shear loading. The magnitude depends strongly on concrete quality, reinforcement ratio and stress level. In the range which has been investigated all these effects are non-linear.
- The variation of concrete strength has a considerable effect on the time-dependent crack displacements.
- Time-dependent shear transfer can be modelled by the use of a damage parameter λ affecting both the aggregate interlock and the dowel action mechanism. The time dependency of the concrete compressive strength and the coefficient of friction should be further quantified.
- Time dependent shear displacements under service loads seem to be small.

7. NOTATION

Unless otherwise stated, the dimensions are N, mm or N/mm².

d_{bar}	-diameter of reinforcing bar	A_c	-area of shear plane [mm²]
f_c	-cylinder compressive strength	C_{ij}, D_j	- coefficients [-]
f_{cc}	-cube compressive strength	V	-shear force
f_{sy}	-yield strength of steel	λ	-damage parameter [-]
s	-parallel displacement (or:slip)	ζ	-normal retention factor [-]
s_c	-slip increment	μ	-friction coefficient [-]
s_{el}	-instantaneous slip	ρ	-reinforcement ratio [-]
t	-load application period [hours]	σ_{pu}	-yield strength of matrix material
w	-crack width (or:separation)	τ	-shear stress
w_c	-separation increment	τ_a	-shear stress due to aggregate interlock
w_{el}	-instantaneous separation		
w_o	-initial crack width	τ_d	-shear stress due to dowel action
A_x, A_y	-projected contact areas [mm²]	τ_u	-static shear strength

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