Control of crack width under imposed deformations

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Contrôle de l'ouverture des fissures sous déformation imposée

Risskontrolle bei aufgezwungenen Verformungen

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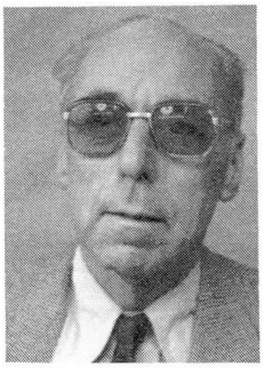

A.S.G. Bruggeling, born 1923, obtained his degree in Civil Engineering from the Delft University of Technology in 1947. Among other functions, he was Director of a firm producing precast prestressed concrete and headed a firm of consulting engineers. From 1969 to 1986 he was Professor of Concrete Construction at Delft. He received two honorary doctorates, from the Technological University of Stuttgart and from the University of Leuven (Louvain) respectively.

SUMMARY
An engineering model for the calculation of crack width under imposed deformations is presented in this paper. The influence that several of the parameters used in the calculation have upon crack width is discussed, and it is explained how the model can be used to control crack width under given conditions. Its application is illustrated with the aid of an example.

RÉSUMÉ
Un modèle pour le calcul de l'ouverture des fissures est proposé quand la construction en béton est soumise aux déformations imposées. L'influence de quelques paramètres est discutée. Le contrôle de l'ouverture des fissures est donné. La contribution est illustrée par un exemple de calcul.

ZUSAMMENFASSUNG
Es wird ein Modell für die Berechnung der Rissbreite in Betonbauteilen, welche durch eine vorgegebene Verformung beansprucht sind, vorgestellt. Der Einfluss verschiedener Parameter auf die Rissbreite wird diskutiert. Es wird dargestellt, wie das Modell unter gegebenen Bedingungen zur Beschränkung der Rissbreite verwendet werden kann. Die Anwendung wird anhand eines Beispieles illustriert.

1. INTRODUCTION

Sufficient durability of concrete structures can be obtained by means of various measures such as:
- sound design of the structure, in such a way that the exposure of the surface of the structure to an agressive environment is limited or avoided;
- careful execution of the concrete with respect to the design of the mix, the quality of the formwork, the cover to the reinforcement and the curing applied;
- good detailing of the reinforcement in order to control crack width.

This paper deals with the problem of how to control crack width efficiently, especially, in the case of imposed deformations. The approach to this problem is, however, based on compliance with the conditions of sound design and a careful execution.

2. IMPOSED DEFORMATIONS

2.1 The effect of imposed deformations on concrete structures

In many cases concrete structures, or parts of them, are subjected to the effect of imposed deformations which cannot take place because the structure is not free to deform. Due to this restraint, compressive or tensile stresses will be generated in the structure.

2.1.1 Compressive stresses

If compressive stresses are generated in the concrete structure, time-dependent effects (creep) will also take place. In many cases imposed deformations are variable with time. This means that during a certain time interval they will increase, resulting in larger compressive stresses, which are subsequently reduced by creep effects. After some time the imposed deformation will decrease. If a concrete structure is regularly subjected to this type of imposed deformations, there will be a tendency for the compressive stresses to decrease in magnitude and, after they have faded away, the above-mentioned effects will result in tensile stresses.

2.1.2 Tensile stresses

Prevention of shortening of structural elements subjected to imposed deformations will result in tensile stresses in the concrete. Especially in the case where a concrete structure is subjected to cyclic imposed deformations the tensile strength of the concrete will be reduced. Therefore cracks are very likely to be initiated. Control of the phenomenon of cracking in concrete structures is important in view of their durability. The aim of this paper is to introduce a simple engineering model for the control of crack width by the designer.

2.2 The nature of imposed deformations

Imposed deformations are caused by several factors. The most important causes are:
- changes of temperature, daily or seasonal factors;
- shrinkage of concrete;
- settlements of foundations.

In this respect the temperature effects, combined with shrinkage of concrete, are of primary importance because they may change rapidly, sometimes several times a day. In considering the behaviour of concrete structures under cyclic temperature effects, it has to be realised that also the magnitude of the temperature influx into the structure may vary considerably and that this magnitude is not very well known. It depends of the duration of this heat influx, the angle under which sun's rays strike the surface of the concrete, the effect of the wind, the effect of clouds and also the effect of the moisture content in the concrete "skin". As opposed to the magnitude of the loads acting on our structures, which can be estimated very well, the action associated with these effects cannot be estimated accurately enough for assessing its influence on the structure. This means that our concrete structures have to be detailed in such a way that they are not very sensitive to the effects of a major difference in the magnitude of imposed deformations.

3. THE BEHAVIOUR OF A FIXED REINFORCED CONCRETE TENSION MEMBER

3.1 The relationship between an imposed shortening and a generated force

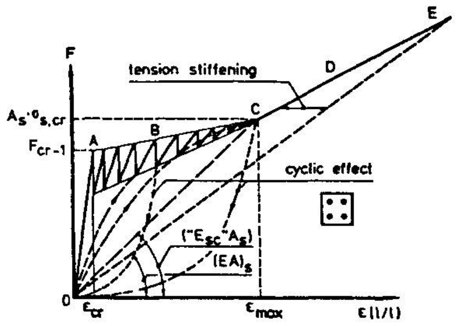

Fig. 1 The relationship between tensile force and imposed shortening of a reinforced concrete tension member

The behaviour of a reinforced concrete tension member is shown in Fig. 1. The following parts of the relationship between deformation and force can be considered:

OA The tension member is uncracked. This is only of importance for the first loading.

ABC In this part an increase in the imposed shortening results in the initiation of new cracks and a slightly increased tensile force. It means that under these conditions the structure is less sensitive to the effect of considerable differences in the magnitude of the imposed deformations. The first crack will be initiated in the weakest part (in terms of tensile strength) of the tension member, the second crack in the next weakest part, etc. With increasing imposed shortening the crack pattern will become "denser" with a certain overlap of influence zones of several cracks.

C If the imposed deformation has the magnitude ε_{max}, then the crack pattern is fully developed.

CDE In this part an increase of the imposed shortening results in the widening of already existing cracks and not in the initiation of new cracks. Initiation of new cracks is in this part only exceptional. The tensile force F increases considerably in magnitude with increasing imposed deformations. It means that in this part the crack width is sensitive to differences in the magnitude of the imposed deformations.

OE This line represents the force-elongation relationship of the reinforcement, uninfluenced by the surrounding concrete.

3.2 Crack width

3.2.1 Gradually increasing imposed deformation

If the imposed shortening is increasing gradually it can be shown that the mean value of the crack width can be determined analytically from the engineering model which will be explained here [1]. The following factors are of importance in the model:

- As a first assumption is assumed that no transverse reinforcement is present in the concrete cover.
- The concrete cover is at least twice the bar diameter.
- The bond-slip relationship of the high-bond reinforcement is written in the form $\tau_{cs} = C.\delta^N$. In this formula C is related to the concrete strength, the shape of the ribs, N is related to the bond behaviour and therefore to the development of stresses along the transmission length.
- The ultimate tensile stress in the concrete σ_{cr} at the initiation of the first crack. Due to equilibrium of forces over the length of the tension member there exists a simple relationship between the tensile stress $\sigma_{s,cr}$ in the

reinforcement in a crack and the tensile stress in the concrete just before the initiation of the next crack:

$$\sigma_{s,cr} = \sigma_{cr}(n + \frac{1}{\rho}) \qquad (1)$$

- The "Goto effect" is taken into account with a correction factor in the calculation of the maximum crack width.

The engineering model cannot be explained within the scope of this paper. Therefore only the result of the analytical solution will be given here.

Crack width: $$w_{mean} = 2\left\{\frac{1}{1+N} \cdot \frac{\phi_k}{4} \cdot \frac{1}{C.E_s} \cdot \frac{\sigma^2_{s,cr}}{1 + n.\rho}\right\}^{\frac{1}{1+N}} \qquad (2)$$

Length of the transmission zone on one side of a crack:

$$l_{st} = \frac{w_{mean}.E_s}{(1-N).\sigma_{s,cr}} \qquad (3)$$

Investigations have shown that, due to dispersion, the maximum crack width can be calculated by multiplying the mean value by a factor 1.5. The "Goto effect" is also taken into account in this value. Therefore we can write:

$$w_{cr-0.95} = 1.5\ w_{mean} \qquad (4)$$

The same investigations have shown that the mean value of the distance between two cracks can be calculated from:

$$\Delta l = 1.5\ l_{st} \qquad (5)$$

3.2.2 Cyclic imposed shortening

Experiments have shown that cyclic effects can increase the crack width considerably. In the case of a not fully developed crack pattern (Fig. 1: AB < C) the magnification factor is 1.1, 1.2 and 1.5 for cyclic stress levels of 215, 260 and 325 N/mm² respectively. This shows that for stress levels higher than 200 N/mm² this factor becomes important. In certain cases one crack can widen considerably in relation to adjoining cracks. In the case of a fully developed dense crack pattern (Fig. 1 CDE) the influence of cyclic effects is limited. In this case the most unfavourable assumption in the calculation of the maximum crack width is to neglect the tension stiffening of the concrete and to assume unrestrained shrinkage between two cracks. In this case the maximum crack width can be calculated by assuming a maximum crack spacing of 2 l_{st}. Therefore we can calculate the maximum crack width in this case from:

$$w_{cr-0.95} = 2\ l_{st}(\frac{\sigma_s}{E_s} + \varepsilon_{cs,\infty}) \qquad (6)$$

3.2.3 Sustained shortening

Sustained shortening will result in a decrease of the tensile strength of the concrete and, if the crack pattern is not fully developed, in the initiation of new cracks and/or widening of already existing cracks. It is therefore recommended to assume in this case the same magnification of the crack width as under cyclic effects.

3.3 Maximum elongation

With the assumption of a mean value of the crack spacing of 1.5 l_{st} the magnitude of the maximum elongation can be calculated from the formula:

$$\varepsilon_{max} = \frac{\sigma_{s,cr}}{"E_{sc}"} \quad \text{with } "E_{sc}" = \frac{E_s(1 + n.\rho)}{0.67(1-N) + n.\rho} \qquad (7)$$

A_c."E_{sc}" can be conceived as the deformation modulus of the tension member associated with the inclination of the line OC in Fig. 1. In the general case of high-bond bars, with N = 0.18, the magnitude of "E_{sc}" can be assumed to be:

$$1.7\ E_s \text{ (line OC)} \qquad (7a)$$

3.4 The magnitude of the tensile force F

The magnitude of the tensile force F depends on the tensile strength of the concrete. In the model used by the author [1] the following values of the tensile stress σ_{cr-1}, at the initiation of the first crack, are used:

- slowly imposed deformations : $\sigma_{cr-1} = 0.62\ f_{ctm}$ (settlement, shrinkage)
- sustained load : $\sigma_{cr-1} = 0.5\ f_{ctm}$
- rapidly imposed deformations: $\sigma_{cr-1} = 0.75\ f_{ctm}$ (solar radiation).

In this case f_{ctm} is the mean value of the concrete tensile strength:

$$f_{ctm} = 1.25(1 + 0.05\ f_{cc}) \qquad (8)$$

With these values the force F at initiation of the first crack can be calculated from the formula:

$$F_{cr-1} = A_c.\sigma_{cr-1} + A_s.\varepsilon_{cu}.E_s \quad \text{(see Fig. 1)} \qquad (9)$$

ε_{cu} is the ultimate strain of the concrete ($\sim 120 \times 10^{-6}$).
In general the magnitude of $E_s.\varepsilon_{cu}$ can taken as 25 N/mm².

3.5 The relationship between the force F and the elongation ε

The values of the force and the elongation can now be quantified, and the diagram can be drawn.

	F	ε
O	0	0
A	$A_c.\sigma_{cr-1} + 25\ A_s$	120×10^{-6}
C	$1.2(A_c.\sigma_{cr-1} + 25\ A_s)$	$\sigma_{s,cr}/1.7\ E_s$
E	$A_s.f_{sy}$	f_{sy}/E_s

Note: In this model it is assumed that there is no tension stiffening effect when yielding of the reinforcement occurs.

3.6 Detailing of the reinforcement

- Bar spacing:
  To control crack width efficiently, it is necessary for the bar spacing to be related to the crack spacing in a fully developed crack pattern. If the bar spacing is too large, the influence of bars crossing a crack on the limitation of the crack width is slight. Therefore a bar spacing equal to the mean crack spacing, but with a maximum of 200 to 250 mm, can be adopted.
- Transverse reinforcement:
  If transverse reinforcement is present in the concrete cover to the crack-controlling main bars, it will mostly initiate cracks at its own bars. It means that these cracks follow this reinforcement, which therefore is exposed over a certain length to the environment and its possibly detrimential effects. Therefore it is recommended that the crack-controlling reinforcement should be placed a little deeper within the concrete, so as to be just clear of the actual depth of cover, if possible.

4. EXAMPLE OF THE DETAILING OF THE REINFORCEMENT IN A CANTILEVERING BALCONY SLAB

Balcony slab $\Delta T = 25$ °C ($\varepsilon = 300 \times 10^{-6}$).
Concrete grade B 20; $f_{ccm} = 24$ N/mm²; $E_c = 26000$ N/mm²; $f_{ctm} = 2.5$ N/mm²;
$\sigma_{cr-1} = 0.62 \times 2.5 = 1.6$ N/mm²; $n = 7.9$.
Reinforcement (longitudinally) 32 ϕ_k 10 FeB 400; $\rho = 0.01$; $E_s = 205000$ N/mm²
bond behaviour: $N = 0.18$; $C = 0.38\ f_{ccm} = 9.1$ N/mm².
Control of maximum crack width.
Formula 1 : After first crack $\sigma_{s,cr} = 1.6(7.9 + 100) = 173$ N/mm².
Formula 2 : Fully developed crack pattern $\sigma_{s,cr} = 1.2 \times 173 = 207$ N/mm²

$$w_{mean} = 2\left\{\frac{1}{118} \times \frac{10}{4} \times \frac{1}{9.1 \times 205000} \times \frac{207^2}{1.08}\right\}^{\frac{1}{1.18}} = 0.14 \text{ mm.}$$

Formula 3 : $l_{st} = \frac{0.14 \times 205000}{0.82 \times 207} = 169$ mm.

Formula 5 : $\Delta l = 1.5 \times 169$ mm $= 254$ mm (mean crack distance).

Formula 4 : $w_{cr-0.95} = 1.5 \times 0.14 = 0.21$ (static loading).

Formula 7/7a: $\varepsilon_{max} = \frac{207}{1.7 \times 205000} = 594 \times 10^{-6}$.

In this case is assumed an imposed deformation of 300×10^{-6}, therefore the crack pattern will not be fully developed.

Cyclic effect of solar radiation, not fully developed crack pattern and cyclic stress level 207 N/mm².

Therefore magnification factor is 1.1. $w_{cr-0.95} = 1.1 \times 0.21 =$ 0.23 mm.

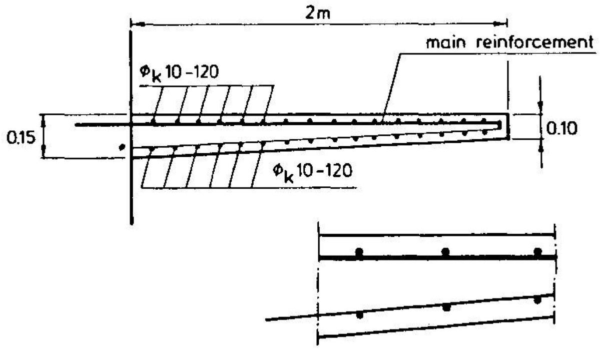

Fig. 2 Reinforcement of the balcony slab

5. CONCLUSION

This paper presents an engineering model for the control of crack width in reinforced concrete tension members. For beams a comparable model has been developed, but not presented here [1]. The model offers the designer the possibility to choose in a given case the correct solution to guarantee sufficient durability.

ACKNOWLEDGEMENT

Ir. C. van der Veen of the Stevin Laboratory of the Delft University of Technology gave me considerable assistance in developing the reinforced concrete tension member and improving this model. He will publish more details of the model in the near future.

NOTATION

A_s, A_c	cross-sectional area of reinforcement and concrete respectively
B	indication of concrete strength - f_{cc}
C, N	factors in bond-slip relationship
E_s, E_c	moduli of elasticity; "E_{sc}" deformation modulus (ε_{max})
F	tensile force
ΔT	change in temperature
f_{cc}	characteristic concrete strength; f_{ccm} - mean value
f_{sy}	characteristic bond stress of reinforcement
Δl	mean value of crack spacing (fully developed crack pattern)
n	E_s/E_c
w	crack width; w_{mean}; $w_{cr-0.95}$
$\varepsilon_{cs,\infty}$	effective shrinkage of concrete
ε_{cu}	ultimate tensile strain of concrete
σ_{cr-1}	tensile strength of concrete - initiation first crack
$\sigma_{s,cr}$	steel stress in a crack
ϕ_k	bar diameter
ρ	A_s/A_c
τ_{cs}	bond stress

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