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Serviceability Design with Prestressing

Calcul des structures précontraintes à l'état de service

Bemessung vorgespannter Bauteile für die Gebrauchslasten

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SUMMARY
Control of deformations and cracking under service conditions requires knowledge of the stresses and strains after occurrence of the long-term effects of creep, shrinkage and relaxation and after cracking due to transient live loads. The approach used in current practice for stress and strain predictions is critically reviewed and a more accurate and more general procedure is suggested. It applies to reinforced and prestressed concrete and to composite members, with or without cracking.

RÉSUMÉ
Le contrôle des déformations et de la fissuration à l'état de service requiert la connaissance des contraintes et des allongements spécifiques en tenant compte du fluage, du retrait et de la relaxation après une fissuration due aux charges mobiles. La méthode utilisée en pratique pour calculer les contraintes et les allongements spécifiques est revue et critiquée, et une procédure plus précise et plus générale est proposée. Cette procédure s'applique, avant et après fissuration, aux éléments en béton armé et précontraint ou aux structures mixtes.

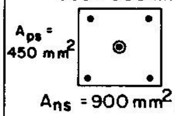
ZUSAMMENFASSUNG
Für die Beschränkung der Deformationen und der Rissöffnungen im Gebrauchszustand ist die Kenntnis des Spannungs- und Dehnungszustandes nach dem Abdingen der Langzeiteffekte Kriechen, Schwinden und Relaxation und nach der Rissbildung infolge der Verkehrslasten notwendig. Die in der Praxis angewendeten Methoden für die Bestimmung der Spannungen und Dehnungen wird kritisch überprüft und ein sorgfältigeres und allgemeineres Vorgehen wird vorgeschlagen. Es kann bei schlaff bewehrten, vorgespannten und gemischten Betonbauteilen für den gerissenen und ungerissenen Zustand angewendet werden.

1. CRITICAL REVIEW OF CURRENT PRACTICE

In current practice the effect of prestressing is represented by a compressive force applied on a plain concrete section. Creep, shrinkage and relaxation result in a reduction in the tension in the prestressed tendon; equal reduction of the compressive force on the concrete is assumed and the time-dependent changes in stresses and strains are calculated accordingly.

Prestressed cross sections are usually provided with non-prestressed steel; 0.5 to 1.2 percent reinforcement ratio is not uncommon. The cross section shown in Fig. 1 has 1 percent non-prestressed steel. When an initial force 590 kN is applied, 7 percent of the compressive force is taken by the non-prestressed steel and its share of the force gradually increases with time to 26 percent. This means that ignoring the non-prestressed steel, as often done in practice, substantially overestimates the long-term compressive stress on the concrete. Some results of two analyses are given in Fig. 1: one ignoring the non-prestressed steel and the other with the non-prestressed steel considered. The basis of the analysis is compatibility of strain in the concrete and the steel and equilibrium of forces in the components. The method of analysis will be discussed in a separate section.

$E_c(t_0) = 30$ GPa ; $E_{ps} = E_{ns} = 200$ GPa;
$\alpha = 2.5$; $\chi = 0.8$;
300 x 300mm2 $\epsilon_{cs} = -300 \times 10^{-6}$; $\Delta\bar{\sigma}_{pr} = -20$ MPa;



INITIAL PRESTRESS 590 kN		NONPRESTRESSED STEEL IGNORED	NONPRESTRESSED STEEL CONSIDERED
CONCRETE STRESS MPa	INITIAL AT t_0	−6.5	−6.0
	CHANGE BETWEEN t_0 AND t	+0.9	+2.0
	FINAL AT t	−5.6	−4.0
FORCE IN NONPRESTRESSED STEEL kN	INITIAL AT t_0	—	−40
	CHANGE BETWEEN t_0 AND t	—	−116
	FINAL AT t	—	−156

Fig. 1 Long-term stress in a tie

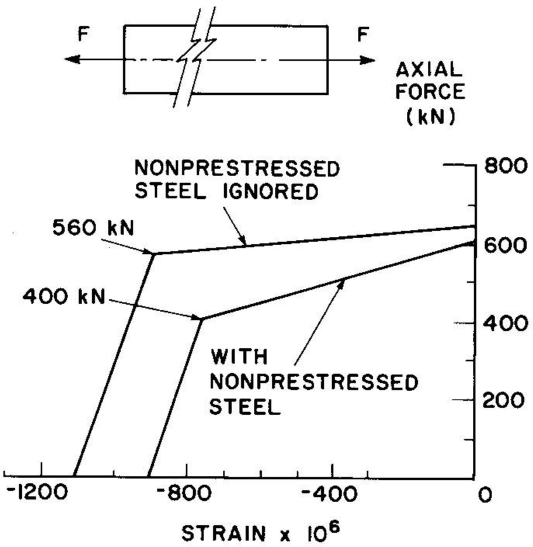
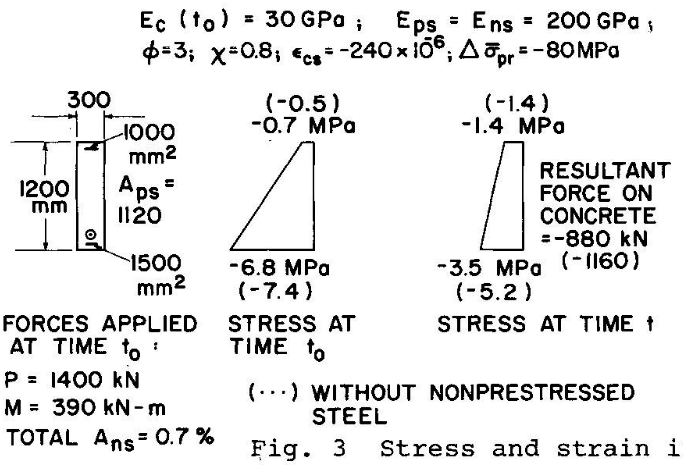


Fig. 2 Strain due to axial force applied at time t in the tie of Fig. 1

Without the non-prestressed steel the initial compressive stress (at time t_o) is -6.5 MPa and drops with time (at time t >> t_o) to -5.6 MPa. With non-prestressed steel, the initial compression is -6.0 MPa and the final is -4.0 MPa. The value -5.6 obtained by ignoring the non-prestressed steel is 40 percent higher in absolute value than the more accurate value -4.0 MPa. This is so because when the -590 kN prestressing force is applied, the non-prestressed steel instantaneously picked up -40 kN and gradually picked up additional -116 kN because of creep and shrinkage.

The main purpose of prestressing is to produce compression in a zone which will be subjected to tension when the live load is applied. Figure 2 represents the strain versus an axial tensile force F, representing the effect of live load applied at time t on the cross section in Fig. 1. After creep, shrinkgae and relaxation the strain is -900×10^{-6} (compared with -1100×10^{-6} when the nonprestressed steel is ignored). The stress in concrete becomes zero when F = 400 kN; this may be called the decompression force. For simplicity in presentation, the strength of concrete in tensin is ignored; thus cracking occurs at the decompression level and the stiffness (the slope of graph) is reduced to that of the reinforcements. Without the nonprestressed steel cracking occurs at a higher force (560 kN). The graph clearly shows the large difference in strain values calculated with and without the nonprestressed



	WITH NONPRESTRESSED STEEL	WITHOUT NONPRESTRESSED STEEL
AXIAL STRAIN AT t_0	-130	-130
AXIAL STRAIN AT t	-600	-690
CURVATURE AT t_0	$-170\ m^{-1}$	$-190\ m^{-1}$
CURVATURE AT t	$-300\ m^{-1}$	$-480\ m^{-1}$
MULTIPLIER	10^{-6}	

Fig. 3 Stress and strain in a member in flexure

steel, particularly when F is between 400 and 560 kN.
The above example shows that large errors in the calculated stresses and strains result from ignoring the non-prestressed steel in a section with concentric prestressing. The same conclusion can be reached by considering the cross section of a member in flexure with eccentric prestressing and 0.7 percent total non-prestressed steel (Fig. 3). After creep, shrinkage and relaxation the resultant force on the concrete is -880 kN compared with -1160 kN when the non-prestressed steel is ignored and the corresponding curvatures are respectively $-300\text{x}10^{-6}$ and $-480\text{x}10^{-6}\text{m}^{-1}$. If the curvature values are used to predict deflections, the error in camber would be approximately 60 percent.

In any of the above examples, presence of the non-prestressed steel has small influence on the final tension in the prestressed steel. Thus, use of this force to calculate stresses and deformations at service conditions is erroneous and this practice should be abandoned. In the following sections, a simple procedure will be presented to give the final stresses and strains in the concrete and the reinforcements, without the need to predict or estimate by empirical equations the loss in tension in the prestressing tendon. For the presentation, it is necessary to define few parameters.

2. RELAXATION OF STEEL AND CREEP AND SHRINKAGE OF CONCRETE

The stress in a tendon stretched between two fixed points drops gradually with time. The amount of drop, referred to as the intrinsic relaxation, $\Delta\sigma_{pr}$ is heavily dependent upon the initial steel stress σ_{po}; the magnitude of relaxation drops drastically when the initial stress is reduced. In a concrete member the length shortens due to creep and shrinkage and the stress in the tendon drops faster. The reduction in tension caused by creep and shrinkage has the same effect on the relaxation as if the initial tension were smaller. Thus, for prediction of stresses and deformation of composite members, use should be made of a reduced relaxation value:

$$\Delta\bar{\sigma}_{pr} = \chi_r\ \Delta\sigma_{pr} \tag{1}$$

where χ_r is a relaxation reduction factor given by [1]:

$$\chi_r = e^{(-6.7 + 5.3\lambda)\Omega} \tag{2}$$

with $\lambda = (\sigma_{po}/f_{ptk})$ and $\Omega = -(\Delta\sigma_{ps} - \Delta\sigma_{pr})/\sigma_{po}$. σ_{po} is the initial tension; f_{ptk} is the characteristic tensile strength; $\Delta\sigma_{ps}$ is the change in stress in the prestressed steel due to the combined effect of creep, shrinkage and relaxation and $\Delta\sigma_{pr}$ is the intrinsic relaxation.

The continuous curve in Fig. 4 represents the variation of strain in concrete with time due to a stress increment, $\Delta\sigma_c(t_o)$ introduced at time t_o and sustained, without change in magnitude to t. The total strain, instantaneous plus creep, at a later time t is

$$\Delta\varepsilon(t) = \frac{\Delta\sigma_c(t_o)}{E_c(t_o)}(1+\phi) \qquad (3)$$

where $\phi = \phi(t,t_o)$ is the creep coefficient. When a stress increment $\Delta\sigma_c(t,t_o)$ is gradually introduced between t_o and t, the total strain during the same period is:

$$\Delta\varepsilon(t,t_o) = \frac{\Delta\sigma_c(t,t_o)}{\bar{E}_c(t,t_o)} \qquad (4)$$

where $\bar{E}_c(t,t_o)$ is the age-adjusted modulus of elasticity on concrete

$$\bar{E}_c(t,t_o) = \frac{E_c(t_o)}{1+\chi\phi} \qquad (5)$$

χ is the aging coefficient. An approximate value: 0.8 may be used for each of the two coefficients χ_r and χ. The creep coefficient ϕ and the free shrinkage value ε_{cs} depend upon the size of the member, the relative humidity of the air, the ages of the concrete t_o and t at the start and end of the period considered. Suggested values for ϕ, ε_{cs} and χ are given in Refs. 2 and 3.

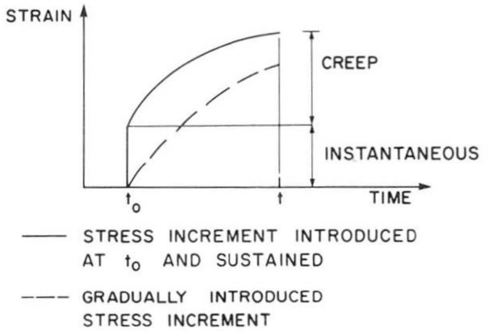


Fig. 4 Time variation of strain due to a stress increment

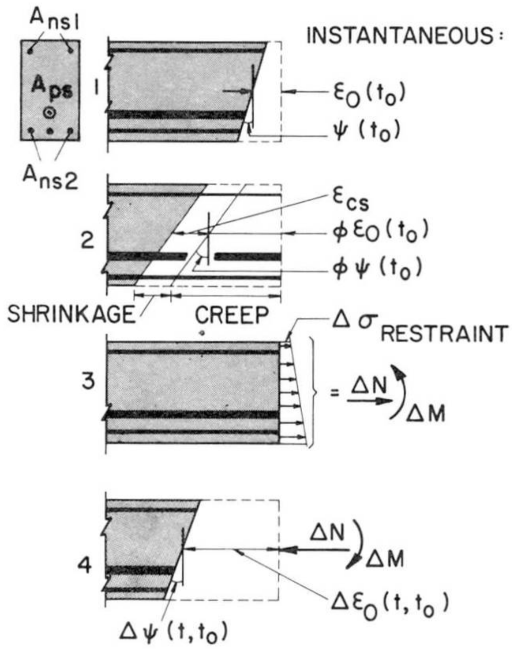


Fig. 5 Steps of analysis of instantaneous and time dependent stresses and strains

3. ANALYSIS OF STRESS AND STRAIN

The instantaneous and time dependent stress and strain in a prestressed concrete section may be calculated in four steps (Fig. 5):

Step 1: Apply the initial prestressing force and the dead load bending moment, which becomes effective at the time of prestressing, on a transformed section composed of the concrete area A_c plus α $(A_{ns} + A_{ps})$ where $\alpha = E_{sn}$ or $E_{ps}/E_c(t_o)$, with E_{ns} or E_{ps} being the modulus of elasticity of the non-prestressed or the prestressed steel. A_{ns} is the area of the non-prestressed steel; A_{ps} is the area of the prestressed steel. When post-tensioning is used, A_{ps} includes only the tendons prestressed in earlier stages. The diagram of the instantaneous strain at time t_o is defined by the value $\varepsilon_o(t_o)$ of the strain at an arbitrary reference point 0 and the slope (the curvature) $\psi(t_o)$. Equations A.4 of Appendix A may be used to calculate the two quantities.

Step 2: Determine the hypothetical strain which would occur due to creep and shrinkage of concrete if it were free to deform.

Step 3: Calculate the stress required which would artificially prevent the strains determined in step 2. This stress is simply equal to $(-\bar{E}_c)$ multiplied by the hypothetical strain determined in step 2. At any fibre at distance y below 0 the restraining stress

$$\Delta\sigma_{restraint} = -\bar{E}_c\{\phi[\varepsilon_o(t_o) + \psi(t_o)y] + \varepsilon_{cs}\} \qquad (6)$$

The variation of this stress may be defined by two quantities: $\{\Delta\sigma_o, \Delta\gamma\}_{restraint}$ where $\Delta\sigma_o$ represents the stress at 0 and γ the slope $(d\sigma/dy)$.

Step 4: Determine a force at 0 and a moment which are the resultants of the stress calculated in step 3 (Eqs. A.3). The strain in concrete due to the relaxation can be artificially prevented by the application, at the level of the prestressed steel, of a force equals $(\Delta\bar{\sigma}_{pr} A_{ps})$. This force is substituted by an equivalent force of the same magnitude at 0 plus a couple. Summing up gives ΔN and ΔM, the restraining force and couple required to prevent artificially the deformations due to creep, shrinkage and relaxation. Eliminate the artificial restraint by the application of ΔN and ΔM in reversed directions on a transformed section composed of A_c plus $\bar{\alpha}_{ns} A_{ns}$ plus $\bar{\alpha}_{ps} A_{ps}$ and determine the corresponding changes in strains and stresses (Eqs. A.2 and A.4). $\bar{\alpha}_{ns}$ or $\bar{\alpha}_{ps}$ is equal to E_{ns} or E_{ps} divided by $\bar{E}_c$.

The procedure used above is kin to the displacement method of structural analysis in which the displacements are artificially prevented by restraining forces and these are subsequently eliminated by application of the same forces in a reversed direction. The strain or stress changes in any of the four steps in Fig. 5 invoke simple calculations which structural designers are familiar with. Superposition of strains determined in steps 1 and 4 and the stresses in steps 1,3 and 4 gives the final values, instantaneous plus time dependent. It is to be noted that the above analysis gives directly the strains and stresses, without the necessity of preceding the analysis with an estimate of the loss in tension in the tendon. No empirical equations are involved. The compatibility of strains in concrete and steel is maintained at all reinforced layers. The procedure can be used for any reinforced concrete section with or without prestressing composed of more than one type of concrete or of concrete and structural steel [2].

When the stress and strain analysis is required at several sections and the strain and curvature are to be integrated to determine the changes in length or deflections, it is expedient to use programmable calculators or micro-computers to perform the analysis [4].

In deriving the equations of Appendix A, the stress or strain diagram is defined by a value at an arbitrary reference point 0 and the slope. When 0 is the centroid, the more general equations A.3 to A.4 take the well-known forms A.5 and A.6. The computations in Steps 1, 3 and 4 (Fig. 5) refer to three different sections having different centroids. The superposition will be simpler if the calculations are done using a fixed reference point.

4. TIME-DEPENDENT CHANGE OF COMPRESSION IN CONCRETE

In the special case when the prestressed and non-prestressed steels, are situated in one layer or when the centroid of A_c, A_{ns} and A_{ps} coincide and the section subjected to concentric force (without moment), the procedure presented in the preceding section leads to the following equation for the change in the resultant force in the concrete during the period t_o to t [2]:

$$\Delta P_c = -\beta[\phi\sigma_{cst}(t_o)\frac{E_{st}}{E_c(t_o)} + \varepsilon_{cs} E_{st} A_{st} + \Delta\bar{\sigma}_{pr} A_{ps}] \qquad (7)$$

where
$$\beta = [1 + \frac{A_{st}}{A_c}\frac{E_{st}}{\bar{E}_c}(1 + \frac{y_{st}^2}{r_c^2})]^{-1} \qquad (8)$$

where $A_{st} = A_{ns} + A_{ps}$ = total steel area; E_s = modulus of elasticity of steel assumed the same for the two types of steel; y_{st} is the y coordinate of the centroid of the total reinforcement, measured downwards from the centroid of A_c; $\sigma_{cst}(t_o)$ is the stress of concrete at y_{st} at time t_o; $r_c^2 = I_c/A_c$, with I_c being the moment of inertia of the concrete area A_c about an axis through its centroid.

The force ΔP_c (usually tensile) represents the prestress loss in concrete; it is equal to the loss in tension in the prestressed steel only in the absence of A_{ns}. The time-dependent changes in strain or stress may now be determined as the sum of the free (unrestrained) shrinkage and creep (see step 2 in Fig. 5) plus the effect of ΔP_c applied at y_{st} on a plain concrete section with modulus of elasticity $\bar{E}_c$.

Equation 7 may be used to calculate ΔP_c also in the general case when the section is subjected to an axial force and moment and when several layers of prestressed and non-prestressed steel are provided. In this case ΔP_c should be considered to act at the centroid of the total steel area. The analysis in this way will involve approximation; the compatibility of strain in concrete and steel is not exactly ensured at all reinforcement layers.

For structural designers who are used to calculating the loss of prestress, Equation 7 may be employed (in leiu of following the 4 steps in Sec. 3) noting that it gives the loss in compression in the concrete, which is in fact of more concern than the loss of tension in the prestressed steel.

5. CRACKED SECTIONS

Partially prestressed sections are often designed such that cracking is allowed only due to transient live load. The procedure discussed in the preceding sections gives the stress distribution at time t before application of the live load. This stress distribution may be defined by the stress value $\sigma_0(t)$ at arbitrary reference point 0 and the slope, $\gamma(t)$. Consider the stress and strain changes after cracking caused by additional bending M and axial force N at 0. For the purpose of analysis, partition each of M and N as follows:

$$N = N_1 + N_2 \quad ; \quad M = M_1 + M_2 \qquad (9)$$

The pair N_1 and M_1 represents the decompression forces which will bring the stresses in the concrete to zero. The values M_1 and N_1 may be calculated by Eq. A.3 as the forces necessary to produce a stress distribution defined by $-\sigma_o(t)$ and $-\gamma(t)$. The corresponding strain distribution is determined by division of the two values by $E_c(t)$. The strain and stress due to the decompression forces are added to the strain and stress due to N_2 and M_2 to obtain the total effects of the live load. Cracking is produced by N_2 and M_2; the stress and strain due to this pair should be derived in the same way as for a reinforced concrete section without prestressing. For this purpose, the concrete in tension is ignored giving the stress and strain at a cracked section.

Away from a crack, the concrete in the tension zone is capable of resisting some tensile stress and thus contribute to the rigidity of concrete members. The strains calculated ignoring the concrete in tension may be adjusted [2,3] to account for the stiffening effect of the concrete in the tension zone, giving reduced strains to be used in calculating the displacements for cracked members.

The procedure described here is combined with the analyses discussed in earlier sections in the computer program CRACK [4] which is suitable for reinforced concrete and composite cross sections with or without prestressing.

6. STATICALLY INDETERMINATE STRUCTURES

In general, creep, shrinkage, relaxation and cracking produce changes in the reactions of statically indeterminate structures. The analyses described above can be supplemented by numerical integration of strains to determine displacements from which the changes in the statically indeterminate internal forces may be determined. The equations necessary for this analysis and a computer program CPF [5,6] is used in the following example.

Figure 6a shows the elevation and cross section of a composite bridge. The dimensions and method of construction are similar to a recently constructed bridge in Idaho, USA. The steel U-shaped section is first erected without shoring, then the precast deck is placed and prestressed without connection to the steel. The connection is delayed until after prestressing, when pockets are concreted over grouped anchor studs. The stress distributions at two selected cross sections are shown in Fig. 6b and c at the end of construction and after occurrence of the time-dependent effects. The drastic changes in compression in concrete, caused by the restraining effect of the heavy structural steel section to the shortening due to creep and shrinkage of concrete, cannot be predicted using conventional equations for the loss in tension in the prestressed steel. For this structure, large change in bending moment gradually develops with time (Fig. 6d).

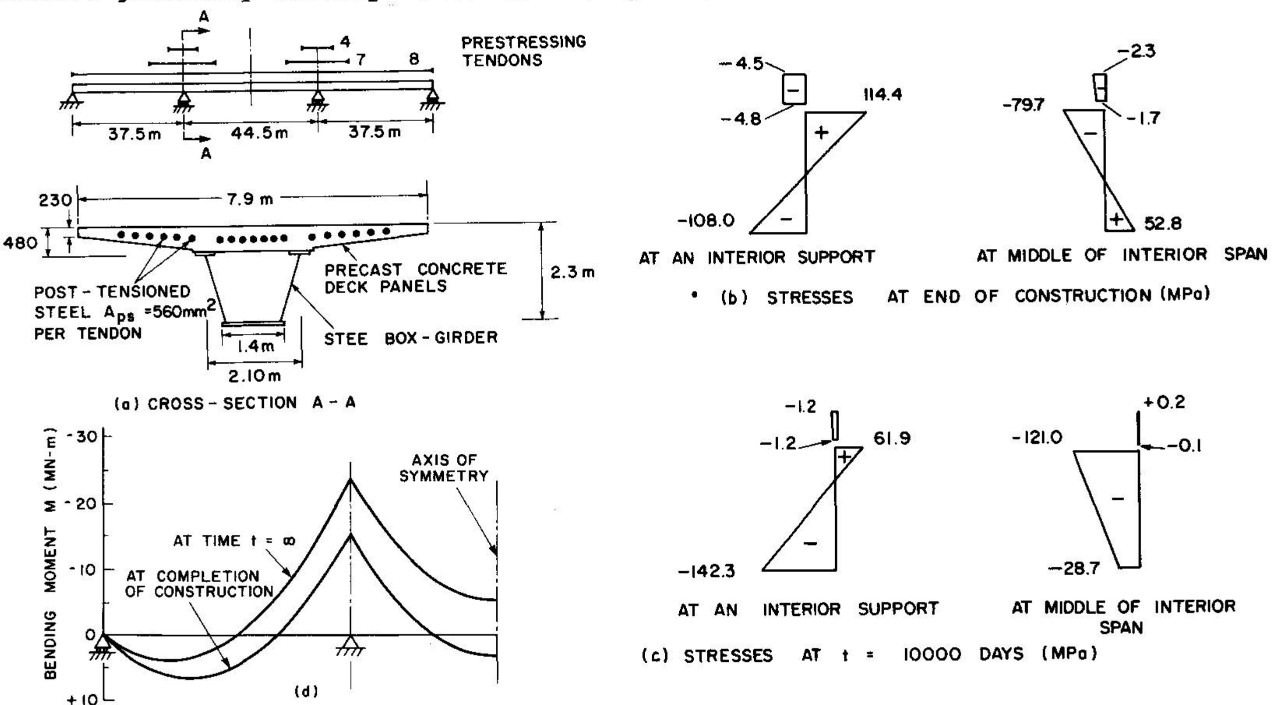


Fig. 6 Time-dependent changes in bending moment and stresses in a composite bridge

7. CONCLUSIONS

The tension in the prestressed steel is equal to the compressive force on a concrete section only in the absence of nonprestressed steel. The practice of calculating the effect of prestressing as that of a compressive force on a plain concrete section, ignoring the nonprestressed steel, may result in important errors in the stresses and the strains existing after creep, shrinkage and relaxation. A proposed procedure, composed of four simple steps give directly the time-dependent stresses and strains without the need to estimate the loss in tension in the prestressed steel. The stresses after losses are required to determine the effects of transient live load producing cracking. The strains can be used to determine the changes in the displacements and the statically indeterminate forces, if any.

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9. NOTATION

- A area
- B first moment of area
- E modulus of elasticity
- $\bar{E}$ age-adjusted modulus of elasticity
- I second moment of area
- M bending moment
- N normal force
- t time or age of concrete
- y coordinate (see Fig. A.1)
- σ normal stress
- α ratio of modulus of elasticity of steel to that of concrete
- $\bar{\alpha}$ ratio of modulus of elasticity of steel to age-adjusted modulus for concrete
- χ aging coefficient for concrete
- χ_r relaxation reduction coefficient
- Δ increment
- ε normal strain
- ϕ creep coefficient
- ψ curvature
- γ $(d\sigma/dy)$ = slop of stress diagram

subscripts

- c concrete
- cs shrinkage
- ns nonprestressed steel
- O reference point
- pr prestressed steel relaxation
- ps prestressed steel
- o instant of time

10. APPENDIX A: RELATIONSHIP BETWEEN STRESS, STRAIN AND FORCES ON A SECTION

The cross section shown in Fig. A.1 is subjected to a normal force N at an arbitrary reference point 0 and a bending moment M. The usual assumptions that plane cross sections remain plane and the stress is proportional to the strain are expressed as:

$$\varepsilon = \varepsilon_o + \psi y \quad ; \quad \sigma = E(\varepsilon_o + \psi y) \tag{A.1}$$

The equilibrium requires:

$$N = \int \sigma dA \quad ; \quad M = \int \sigma y dA \tag{A.2}$$

Substitution of Eq. A.1 into A.2 gives:

$$N = A\sigma_o + B\gamma \; ; \; M = B\sigma_o + I\gamma \tag{A.3}$$

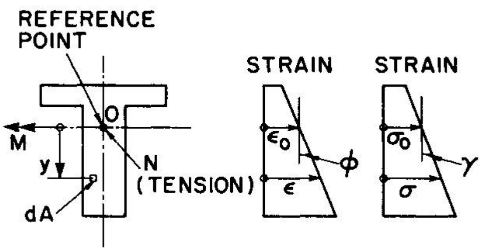

Fig. A.1 Sign convention

where A, B and I are the area, its first and its second moment about an axis through 0; σ_o is the stress at 0 and γ is the slope of the stress diagram. Equations A.3 can be used to determine the stress resultants N and M for a specified stress distribution. When N and M are given, the strain at 0 and the curvature may be determined by:

$$\varepsilon_o = \frac{IN - BM}{E(AI-B^2)} \quad ; \quad \psi = \frac{-BN + AM}{E(AI-B^2)} \tag{A.4}$$

When 0 is chosen at the centroid, B=0 and Eqs. A.3 and A.4 become:

$$N = EA\sigma_o \quad ; \quad M = EI\gamma \tag{A.5}$$

$$\varepsilon_o = \frac{N}{EA} \quad ; \quad \psi = \frac{M}{EI} \tag{A.6}$$