Crack width and deflection of partially prestressed and reinforced concrete members

Autor(en): **Zhao, Guofan / Wang, Qingxiang**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **55 (1987)**

PDF erstellt am: **02.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-42727

Crack Width and Deflection of Partially Prestressed and Reinforced Concrete Members

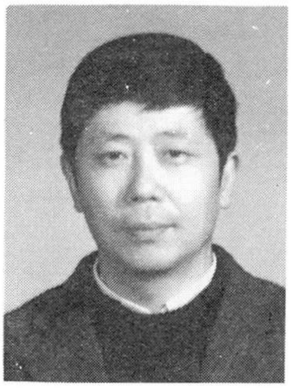
Ouverture des fissures et déformation de structures en béton armé ou partiellement précontraint

Rissbreite und Biegeverformung von teilweise vorgespannten und schlaff bewehrten Betonbauteilen

Guofan ZHAO
Professor
Dalian Inst. of Technol.
Dalian, China

Guofan Zhao, born 1925, obtained his Civil Engineering degree from the Jiao-Tong University, Shanghai, China, in 1949. Since 1950 he has been actively engaged in teaching and research in R.C. and P.C. and structural safety. Currently, he is a member of the IABSE, RILEM.

Qingxiang WANG
Lecturer
Dalian Inst. of Technol.
Dalian, China



Qingxian Wang, born 1945, obtained his Civil Engineering degree from Dalian Institute of Technology, Dalian, China, in 1969. Since 1969 he has been actively engaged in teaching and research in reinforced concrete.

SUMMARY
Based on test results, a trigonometric function is assumed to express the tensile strain distribution of steel bar and concrete between the adjacent cracked sections, and a series of simple formulas are then derived to predict the crack width of partially prestressed concrete beams and R.C. members subjected to bending, pure tension, and bending combined with tension or compression, as well as the rigidity of R.C. members subjected to bending.

RÉSUMÉ
D'après des résultats d'essai, une fonction trigonométrique est déduite pour exprimer la distribution des tensions du béton et des tensions de l'armature entre les fissures. On peut en tirer directement une série de formules de calcul pour prédire la largeur des fissures d'une poutre à précontrainte partielle ou en béton armé sollicitée en flexion, tension pure et flexion composée. On peut aussi obtenir une formule pour le calcul de la rigidité modifiée.

ZUSAMMENFASSUNG
Aufgrund von Versuchsergebnissen wird die Verteilung der Zugdehnungen im Stahl und im Beton zwischen zwei Rissen mit einer trigonometrischen Funktion beschrieben. Eine Anzahl einfacher Formeln für die Bestimmung der Rissbreite in teilweise vorgespannten und schlaff bewehrten Stahlbetonbalken unter Biegung, reinem Zug und Biegung mit Normalkraft wurden hergeleitet ; wie auch für die Berechnung der Biegesteifigkeit von gerissenen Stahlbetonbauteilen.

1. CALCULATION OF CRACK WIDTH

The calculation of crack width is a complex problem in the field of the theory of the reinforced and partially prestressed concrete members. The rather strict relationship for calculating the crack width may be expressed as follows:

$$W_{cr}=\int_0^{l_{cr}}\varepsilon_{sx}\,dx-\int_0^{l_{cr}}\varepsilon_{cx}\,dx=\int_0^{l_{cr}}(\varepsilon_{sx}-\varepsilon_{cx})\,dx \tag{1}$$

Where ε_{sx}, ε_{cx} ---tensile strain of steel and concrete at distance x away from crack section respectively.

The concrete strain is not important since it is much smaller in comparison with ε_{sx}, if the tensile strain of concrete is neglected, the formula (1) can be simplified to the following formula :

$$W_{cr}=\int_0^{l_{cr}}\varepsilon_{sx}\,dx \tag{2}$$

The key problem to calculate the crack width from formulas (1) or (2) is to find out a function to express ε_{sx} in good agreement with the experimental results on the strain of tensile steel, and this function should be easy to integrate.

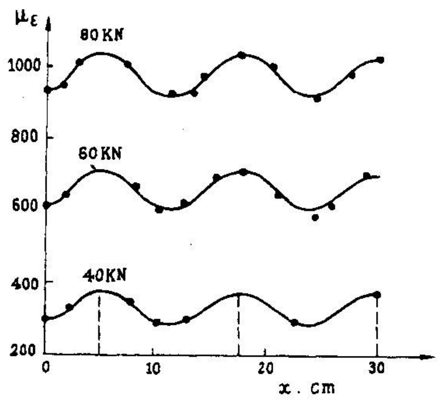


Fig. 1 The tensile strain diagram of steel of specimen LA-1

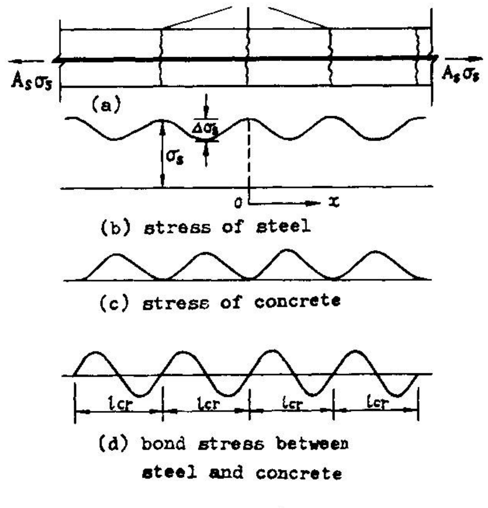


Fig. 2 Distribution diagram of steel stress, concrete stress and bond stress between two adjacent cracked sections

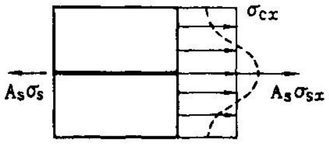
From the analysis of the curve of the typical experimental steel strain shown in Fig. 1 and Ref. [3], it can be seen that the strain diagram of steel is a gradually varied curve and the reasonable stress diagram can be simply expressed by the trigonometric function as shown in Fig. 2. The distribution of the steel stress σ_{sx} can be derived as follows:

$$\sigma_{sx}=\sigma_s-\Delta\sigma_{sx}=\sigma_s-B\left(1-\cos\frac{2\pi x}{l_{cr}}\right) \tag{3}$$

Where $\sigma_{sx}=\varepsilon_{sx}E_s$, $\sigma_s=\varepsilon_s E_s$

ε_s --- strain of the tensile steel in the cracked section
$\Delta\sigma_s$ --- the decrease of the steel stress at distance x away from the cracked section caused by influence of the uncracked concrete
B --- parameter determined by formula (5)

When the distribution of σ_{sx} is known, the stress distribution of concrete σ_{cx} can be obtained as follows (Fig. 3).



$$A_s\sigma_s = A_s\sigma_{sx} + A_c\sigma_{cx}$$

$$\sigma_{cx} = \frac{A_s}{A_c}(\sigma_s - \sigma_{sx}) = \mu_e B\left(1-\cos\frac{2\pi x}{l_{cx}}\right) \quad (4)$$

Fig. 3 Distribution of concrete stress in the cross section

where $\mu_e = \frac{A_s}{A_c}$

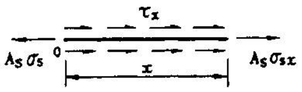
A_s --- area of tensile steel bar
A_c --- area of cross section of an axial tension member

At $x = \frac{l_{cr}}{2}$, the peak value of σ_{cx} is equal to $2\mu_e B \leq f_t$, $\therefore B \leq 0.5\frac{f_t}{\mu_e}$

$$\text{or} \qquad B = \frac{\alpha f_t}{\mu_e} \qquad (\alpha \leq 0.5) \qquad (5)$$

where f_t --- tensile strength of concrete

The bond stress τ_x can be obtained as follows:



$$\sigma_s A_s = A_s\sigma_{sx} + \int_0^x \tau_x u\,dx$$

Fig.4 Force equilibrium diagram of steel bar

$$\tau_x = \frac{2\pi A_s B}{l_{cr} u}\sin\frac{2\pi x}{l_{cr}} = \tau_{max}\sin\frac{2\pi x}{l_{cr}} \quad (6)$$

$$\tau_{max} = \frac{2\pi A_s B}{l_{cr} u} = \frac{2\pi d B}{4 l_{cr}} = \frac{\pi d \alpha f_t}{2 l_{cr}\mu_e} \quad (7)$$

Where u --- the perimeter of the steel bar

The theoretical formula of the crack spacing l_{cr} can be obtained from formulas (7) and modified by test data as follows :

$$l_{cr} = \left(2c + 0.08\frac{d}{\mu_e}\right)K_2 \quad (8)$$

Where c --- thickness of concrete cover in cm
d --- diameter of steel bar in cm
K_2 --- coefficient denoting the bond effect of the bars
K_2 =1.0 for deformed bars
K_2 =1.3 for plain bars

The formulas (3), (4) and (6) are the expressions of the steel stress, concrete stress and bond stress of the axial tension member with the crack spacing l_{cr} , and have been verified in good agreement with the experimental results given in Fig. 1 and Ref.[3].

When the tensile stress diagrams of steel and concrete are known as shown in Fig. 2, the formula for evaluating the crack width of a reinforced concrete member can be directly obtained from equation (1) as follows :

$$W_{cr} = \int_0^{l_{cr}} (\varepsilon_{sx} - \varepsilon_{cx}) dx$$

$$= \frac{\sigma_s}{E_s}\left[1 - \frac{B(1+n\mu_e)}{\sigma_s}\right] l_{cr} = \frac{\sigma_s}{E_s}\left[1 - \frac{\alpha f_t(1+n\mu_e)}{\mu_e \sigma_s}\right] l_{cr} \tag{9}$$

where $n = \frac{E_s}{E_c}$

E_c--- modulus of elasticity of concrete

$\mu_e = \frac{A_s}{A_{ce}}$

A_{ce}--- effective area of tensile concrete

$A_{ce}=0.4bh+(b_f - b)h_f$ for bending, eccentric compression and tension members

$A_{ce}=bh$ for pure tension members

b --- web width of section

h --- overall depth of section

b_f --- width of tensile flange of inverted T or I section

h_f --- thickness of tensile flange of inverted T or I section

If the strain of tensile concrete may be neglected, W_{cr} can be obtained as follows :

$$W_{cr} = \frac{\sigma_s}{E_s}\left(1 - \frac{\alpha f_t}{\mu_e \sigma_s}\right) l_{cr} \tag{10}$$

The coefficient α can be approximately chosen as 0.5 with the experimental results. The value of μ_e is determined by the ratio of the area of tensile steel to the effective area of tensile concrete. Finally, the effect of variation of crack widths and the effect of long-term loads on the average crack width should be taken into account , thus, the unified formula for evaluating the maximum crack width of reinforced concrete members subjected to pure tension, bending, eccentric tension or compression may be expressed as follows :

$$W_{max} = 1.5\, K_1 K_2 K_3 \frac{\sigma_s}{E_s}\left(1 - \frac{0.5 f_t}{\mu_e \sigma_s}\right)\left(2C + 0.08 \frac{d}{\mu_e}\right)$$

$$= K_1 K_2 K_3 \frac{\sigma_s}{E_s}\left(1 - \frac{0.5 f_t}{\mu_e \sigma_s}\right)\left(3C + 0.12 \frac{d}{\mu_e}\right) \tag{11}$$

where K_1 --- coefficient denoting the load effect

K_1 = 1.0 for flexural members

K_1 = 0.9 for eccentric compression members

K_1 = 1.1 for eccentric tension members

K_1 = 1.2 for axial tension members

K_3 --- coefficient denoting the effect of the duration of the loading

K_3 = 1.0 for the short-term load

K_3 = 1.5 for the long-term load

For design practice , the calculated value of W_{max} should meet the requirement :

$$W_{max} \leqslant [W_{max}] \tag{12}$$

where $[W_{max}]$ --- allowable value of maximum crack width, which has been recommended in China Design Code based on the investigated data of existing structures.

For evaluating the crack width of a partially prestressed concrete members , it is necessary to give the change in net stress $\Delta\sigma_p$ in the prestressing steel and the magnitude of the tensile stress σ_s in the non-prestressed steel at any crack width load level in which the decompression load is taken as the reference point, which have been recommended in Ref.[4]. Meantime, the values of μ_e and d introduced in Eq. (11) for ordinary reinforced concrete members should be replaced by $\mu_e=(A_p+A_s)/[0.4bh+(b_f-b)h_f]$ and $d_e=(n_1d_1+n_2d_2)/(n_1+n_2)$ respectively, where A_p =area of prestressing steel, n_1 and n_2 =number of prestressing tendons or bars and non-prestressed bars or wires, respectively, d_1 and d_2 = diameter of prestressing tendons or bars and non-prestressed bars or wires, respectively.

2. THE RIGIDITY OF REINFORCED CONCRETE MEMBER WITH FLEXURE

For evaluating the deflection of a cracked reinforced concrete beam, the rigidity EI for beam made of elastic materials should be replaced with B_l . According to the steel strain distribution shown in Fig. 2 and referring to equation (3), the rigidity B_s under short-term loads was derived as follows (see Ref[5]) :

$$B_s=\frac{A_sE_sh_0^2}{1.6+6n\mu-\dfrac{0.4f_tbh_0^2}{M}} \tag{13}$$

The formula for evaluating the rigidity of a flexural member with rectangular section has been verified by the test data obtained in Dalian Institute of Technology and elsewhere. As for T ,inverted T and I section, the procedures for deriving the rigidity formulas are the same as for the rectangular beam. But for the purpose of design practice, the rigidity B_s for these sections may be calculated by multiplying the rigidity B_s for rectangular section with a factor β_1[5][6]:

$$\beta_1=1+0.7r'+0.2r_1 \tag{14}$$

where $r'=\dfrac{(b_f'-b)h_f'}{bh_0}$ $(r'\leqslant 1.5)$, b_f' ,h_f' are the width and thickness of the compressive flange respectively

$r_1=\dfrac{(b_f-b)h_f}{bh}$ $(r_1\leqslant 1.5)$, b_f ,h_f are the width and thickness of the tensile flange respectively

Under the action of long-term loads, the rigidity B_l may be evaluated by the following empirical formula :

$$B_l=\frac{M}{M_l(Q-1)+M}B_s \tag{15}$$

where M_l --- bending moment under long-term loads

$Q = 2.0-\dfrac{\mu'}{\mu}$

$\mu=\dfrac{A_s}{bh_0}$, $\mu'=\dfrac{A_s'}{bh_0}$

A_s' --- area of compressive steel

M --- bending moment under total characteristic loads.

For design practice, the maximum deflection of a beam calculated by the conventional formula with rigidity B_l should meet the requirement :

$$f_{max}\leqslant[f_{max}] \tag{16}$$

where [f_{max}] --- allowable maximum value of deflection recommended in China Design Code.

REFERENCES

1. Zhao Guofan, Liao Wanqing, Wang Qingxiang, Experiments and Calculating Method for the Cracking Strength and Maximum Crack Width in Reinforced Concrete Members, Journal of Building Structures, Vol. 1, No. 4, 1980

2 Design Code of Port Construction (JTJ220-82), Publishing House of Communication, 1982.

3. Zhao Guofan, Wang Qingxiang, Stress Distribution and Calculation Formula for Crack Width Analysis of Reinforced Concrete Members, Journal of Dalian Institute of Technology, Vol. 23, No. 4, Dec. 1984

4 Zhao Guofan , Liu Yaping, Approximate method for Calculating the Steel Stress of a Partially Prestressed Concrete Beam, Journal of Shandong Architectural and Civil Engineering Institute, No. 1, 1986

5 Zhao Guofan, The Flexural Rigidity of Reinforced Concrete Members, Journal of Dalian Institute of Technology, Vol. 24, No. 4, Sept.1985.

6 Chen Yongchun. et al, Simplified Expression for Stiffness of R. C. Flexural Members, China Civil Engineering Journal, Vol. 16, No. 2, 1983.