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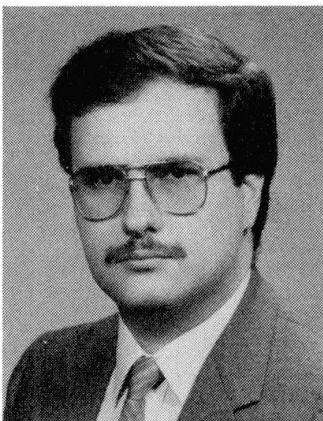
## Behaviour of Waffle Flat Slabs under Horizontal Load

Comportement des dalles nervurées soumises à des charges horizontales

Verhalten der Rippen-Flachdecken unter Horizontal-Belastung

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Valter Lucio has been a research assistant at UTL Lisbon since 1980 and has been preparing his Ph.D. thesis on waffle slabs for the last 4 years, 3 of them at the PCL.

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Paul Regan has been at PCL since 1972 and has been active in research on flat slabs and particularly shear for many years.

### SUMMARY

This paper presents a model of the behaviour of column-supported slabs under lateral actions. A method is proposed for quantifying the rotational stiffness of slab-column connections for both solid and waffle slabs. The method takes account of the cracking of the slab. The results obtained from the model are compared with those from tests of reinforced concrete slabs.

### RÉSUMÉ

Cet article présente un modèle destiné à étudier le comportement de dalles sur colonnes soumises à des actions latérales. Une méthode est proposée afin de quantifier la rigidité de rotation des joints dalle-colonne pour des dalles pleines et nervurées tout en tenant compte de la fissuration des dalles. Les résultats du modèle sont comparés avec ceux provenant d'essais effectués sur des dalles en béton armé.

### ZUSAMMENFASSUNG

Der Artikel stellt ein Modell für das Tragverhalten von punktgestützten Platten unter Horizontal-lasten vor. Es wird ein Berechnungsverfahren für die Steifigkeiten des Stützen-Platten-Knotens bei Kassettendecken und massiven Decken vorgeschlagen. Dabei wird auf den Einfluss der Rissbildung unter Eigenlasten und Horizontallasten berücksichtigt. Die Berechnungsergebnisse wurden mit Versuchswerten verglichen.



## 1. INTRODUCTION

Equivalent frame methods offer an understandable and reasonable modelling of flat slab structures. In their usual forms they do however have problems in representing the behaviour of slab-column connections. These problems arise from geometry and from the effects of cracking.

## 2. EXPERIMENTAL STUDIES

Experimental data from realistic models subjected to lateral loading is available for tests by Lucio [1,2] on a waffle slab and by Long and Kirk [3] on solid slabs.

### 2.1 Waffle slab model

A column-supported waffle slab with 9 panels (Fig. 1) was tested at the Polytechnic of Central London. The slab, which represented typical prototypes to a scale of about 1 in 3.5, was supported on 16 columns and had solid infill panels with plan dimensions equal to  $3/8$  times the spans.

The slab was subjected to various levels of vertical and horizontal loading to study its behaviour under lateral load with different amounts of cracking caused primarily by uniform vertical loading. The rotations at the column heads and the reactions at their bases were measured with inclinometers and triaxial load cells [4]. The base measurements allowed the moments at the slab-column joints to be determined.

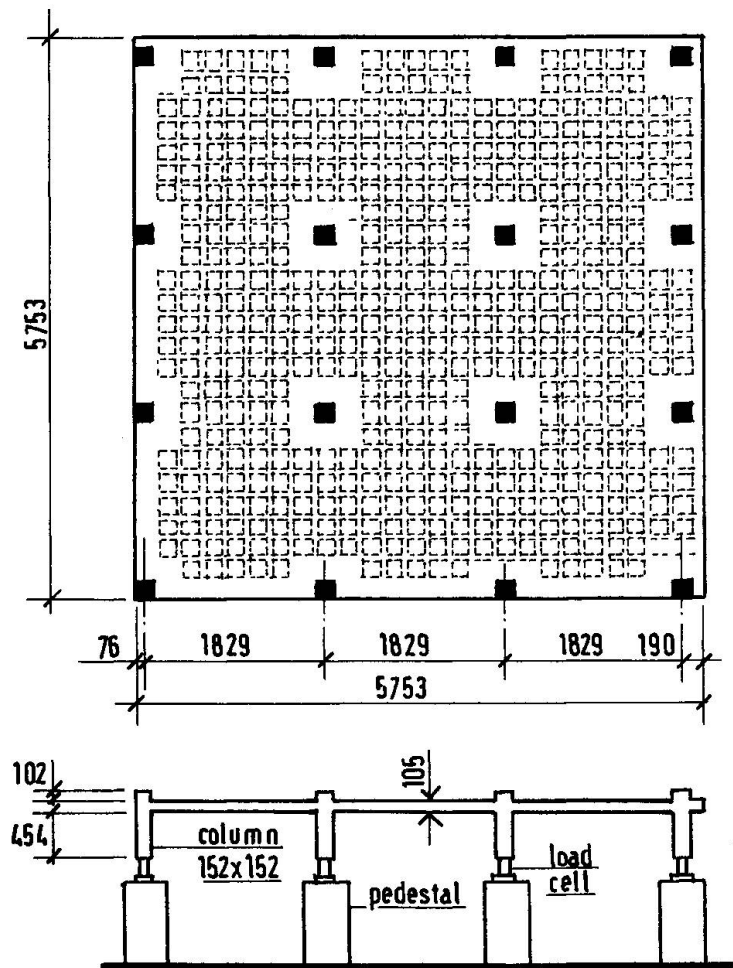


Fig. 1 Waffle slab tested by Lucio

Fig. 2 shows the rotations of an interior column head as functions of lateral load for various levels of prior vertical loading. The vertical loads during the lateral load tests were about 0.8 times the previous maxima.

Table 1 gives the stiffnesses of the structure at maximum horizontal loads in terms of  $H/a$ , where  $H$  is the total horizontal load and  $a$  is the lateral displacement at the level of the slab.

Prior Vert. Load (kN/m <sup>2</sup> )	H (kN)	H/a (kN/mm)
9.4	40	176
12.0	50	147
17.0	80	97

Table 1. Stiffness from lateral load tests

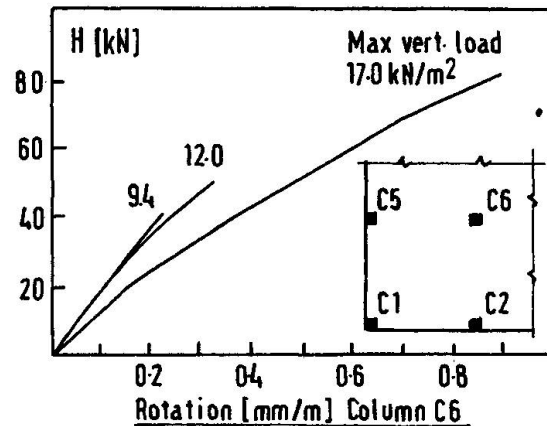


Fig. 2 Results of lateral load tests

## 2.2 Solid slab models

Long and Kirk tested three models representing parts of flat slab structures (Fig. 3). The supports were two columns and one line support. The long edges of the models represented mid-panel lines and rotations at and normal to these edges were prevented by the application of moments.

Model 2 included an edge beam and is not considered here. The other two differed in terms of column dimensions. For slab 1, the edge column was 100 x 150mm and the internal column was 150mm square. In model 3 the corresponding dimensions were 150 x 225mm and 225mm square.

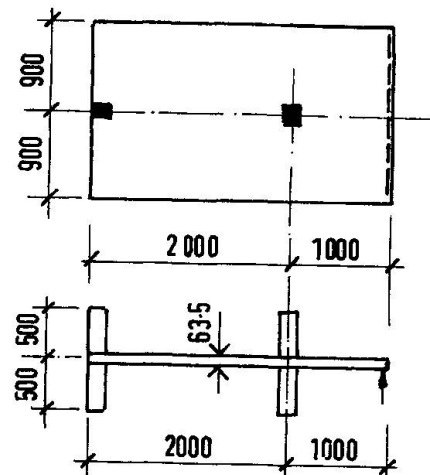


Fig. 3 Models of Long and Kirk

Displacements of 3mm were applied at the tops of the columns and horizontal reactions were measured at their bases. During horizontal loading, the vertical load represented the dead load of a prototype 3 times the model size, but the slabs had been preloaded to the prototype service load.

## 3. FRAME ANALYSIS

### 3.1 General

Equivalent frame methods for the analysis of flat slabs include those of ACI 318-89 [5], BS 8110 [6] and the flexible joint method (FJM) proposed by Regan [7]. In the code methods, vertical and horizontal loadings are treated by different models. For vertical loading the properties of the members may be directly those of the structure, ie.  $I = bh^3/12$  [6], in which case no allowance is made for the flexibility of the connections. Alternatively the column stiffnesses may be reduced to take account of this flexibility [5]. For horizontal loading the slab stiffness is reduced by considering an effective



breadth less than the full width between panel centre lines.

The FJM uses a torsional spring (Fig. 4c) to simulate the flexibility of a slab-column joint. This allows the same model to be used for any loading. All three methods are based on uncracked elastic theory.

3.2 Elastic behaviour

If the column of Fig. 4a has the same stiffness as the wall of Fig. 4b and the slabs are similar, the application of equal moments will produce a greater rotation in the former case. For elastic conditions the stiffnesses ( $M/\theta$ ) for both cases can be calculated by the Finite Element Method. The results depend somewhat on details but published data [8-11] for solid slabs is quite consistent.

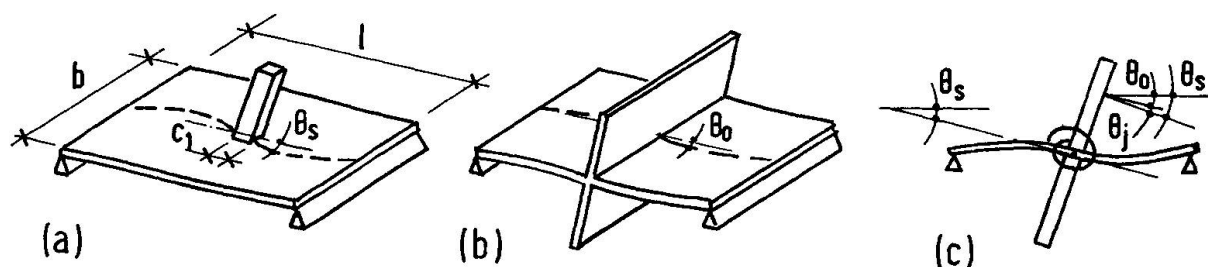


Fig. 4 Models for horizontal loading

The data can be presented in a form in which the slab rotation  $\theta_s$  of Fig. 4a is the sum of  $\theta_o$  (Fig. 4b) and a joint rotation  $\theta_j$ . Each rotation is equal to the applied moment divided by the relevant stiffness ( $K_s, K_o, K_j$ ).

$$\theta_s = \theta_o + \theta_j \quad \dots\dots(1)$$

$$\frac{1}{K_s} = \frac{1}{K_o} + \frac{1}{K_j} \quad \dots\dots(2)$$

where  $K_o/D = 12(b/l) (1-c_1/l)^{-3}$  and  $D$  is the flexural stiffness per unit width of slab.

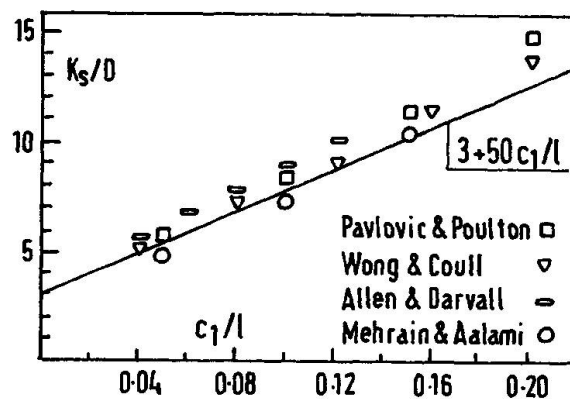


Fig. 5 Comparison of eqn. (3) with FEM results

Fig. 5 shows that FEM results for square columns ( $c_1 = c_2$ ) and square panels ( $b=l$ ) can be approximated by  $K_s/D = 3 + 50(c_1/l)$   $\dots\dots(3)$

Moderate variations of  $c_1/c_2$  have little effect on stiffness. Expression (3) can be used directly for square panels. For rectangular ones  $K_j$  can be calculated from equation (2) and combined with the appropriate  $K_o$ .

Fig. 6 shows the model used to analyse waffle slabs by the Finite Element Method. For the coffers available on the market, the

ratio between the flexural stiffnesses of the solid and waffle zones varies only in the range from 3 to 4. Equation (4) is proposed [12] to approximate elastic analysis of square waffle slabs and Fig. 7 compares it with FEM results.

$$K_S/D = (6.25 + 50c_1/l) (l_s/l + b_{SS}/l)^{0.75} (b_s/l)^{0.25} \dots\dots(4)$$

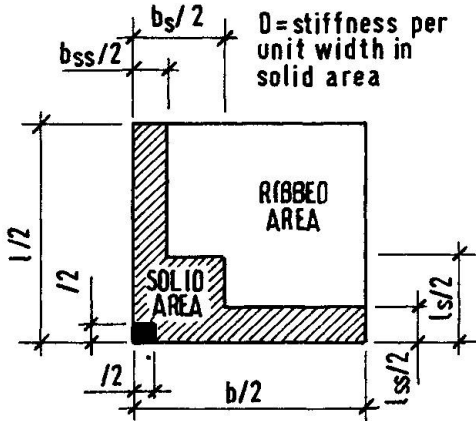


Fig. 6 Waffle slab model

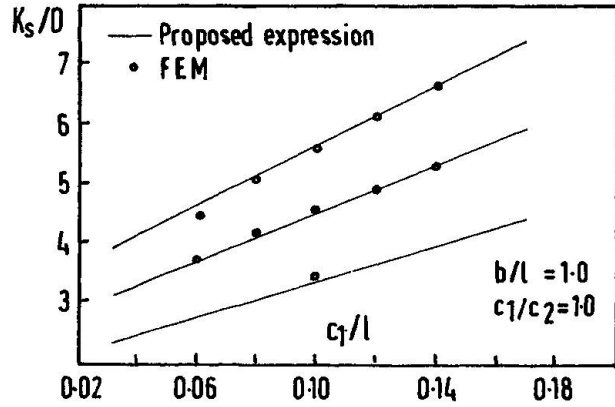


Fig. 7 Eqn. (4) compared with FEM data

### 3.3 Influence of cracking

The effect of vertical loading can be accounted for by a reduction of  $K_j$  to  $K_{jv}$ .

$$1/K_{jv} = \beta_1\beta_2 (m_{cr}/m)^2(1/K_j) + [1 - \beta_1\beta_2(m_{cr}/m)^2] [D/K_j D_{Cr}] \dots(5)$$

where  $\beta_1 = 1.0$  for high bond bars and 0.5 for plain bars and  $\beta_2 = 1.0$  for first loading and 0.5 for long term or repeated loading.  $m_{cr}$  is the cracking moment and  $m$  the applied moment at the column line and both of them may be averaged for the width of the column strip. For practical design the value of  $m$  should be that for the full service load. It is here taken as that for the maximum prior load.  $D$  is the uncracked and  $D_{Cr}$  the fully cracked (bare steel) stiffness again averaged for the width of the column strip.  $K_0$  can be taken to retain its uncracked value.

As shown in Fig. 2 the influence of horizontal loading itself is a further reduction of stiffness which can be accounted for by reducing  $K_{sv}$  to  $K_{sh}$  where

$$K_{sh} = K_{sv}/(1 + M/M_R) \dots\dots(6)$$

where  $M$  is moment acting between the column and the slab and  $M_R$  is the calculated resistance moment corresponding to flexural failure.

### 4. COMPARISON WITH TEST RESULTS

Table 2 presents the results of a comparison between the experimental stiffnesses of the three slabs discussed with values calculated by the method proposed. The agreement between the two sets of values can be seen to be very good.

Slab	H/a (kN/mm)	
	Test	Calc
Waffle	97	89
Long 1	1.66	1.66
Long 3	2.12	2.11

Table 2 Experimental and calculated stiffnesses



## 5. CONCLUSION

If the lateral load responses of flat slab structures are to be assessed realistically the models used in analysis must take account of the limited connections between the slabs and columns and also of the effects of cracking in the slabs.

There is no unique moment/rotation response for a slab-column connection as the local stiffness is greatly influenced by cracking produced by vertical loading, even if this involves no transfer of moment between the slab and column.

The model proposed provides a method of estimating joint stiffnesses and thence slab and column rotations, by accounting for geometric factors within an elastic approach and then modifying the results to allow for cracking.

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